Appendix C

Some Fourier transform pairs

Note:

$$G(k) = \int_{-\infty}^{\infty} g(x) e^{-jkx}\, dx, \quad g(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(k) e^{jkx}\, dk, \quad g(x) \leftrightarrow G(k).$$

$$\text{rect}(x) \leftrightarrow 2 \operatorname{sinc} k \tag{C.1}$$

$$\Lambda(x) \leftrightarrow \operatorname{sinc}^2 \frac{k}{2} \tag{C.2}$$

$$\text{sgn}(x) \leftrightarrow \frac{2}{jk} \tag{C.3}$$

$$e^{jk_0 x} \leftrightarrow 2\pi \delta(k - k_0) \tag{C.4}$$

$$\delta(x) \leftrightarrow 1 \tag{C.5}$$

$$1 \leftrightarrow 2\pi \delta(k) \tag{C.6}$$

$$\frac{d^n \delta(x)}{dx^n} \leftrightarrow (jk)^n \tag{C.7}$$

$$x^n \leftrightarrow 2\pi j^n \frac{d^n \delta(k)}{dk^n} \tag{C.8}$$

$$U(x) \leftrightarrow \pi \delta(k) + \frac{1}{jk} \tag{C.9}$$

$$\sum_{n=-\infty}^{\infty} \delta\left(t - n\frac{2\pi}{k_0}\right) \leftrightarrow k_0 \sum_{n=-\infty}^{\infty} \delta(k - nk_0) \tag{C.10}$$

$$e^{-ax^2} \leftrightarrow \sqrt{\frac{\pi}{a}} e^{-\frac{k^2}{4a}} \tag{C.11}$$

$$e^{-ax} U(x) \leftrightarrow \frac{1}{a + jk} \tag{C.12}$$

$$e^{-a|x|} \leftrightarrow \frac{2a}{a^2 + k^2} \tag{C.13}$$

$$e^{-ax} \cos bx\, U(x) \leftrightarrow \frac{a + jk}{(a + jk)^2 + b^2} \tag{C.14}$$

$$e^{-ax} \sin bx\, U(x) \leftrightarrow \frac{b}{(a + jk)^2 + b^2} \tag{C.15}$$

$$\cos k_0 x \leftrightarrow \pi[\delta(k + k_0) + \delta(k - k_0)] \tag{C.16}$$

$$\sin k_0 x \leftrightarrow j\pi[\delta(k + k_0) - \delta(k - k_0)] \tag{C.17}$$

$$\frac{1}{2} b e^{-\frac{1}{2}bx} \left[I_0\left(\frac{1}{2}bx\right) + I_1\left(\frac{1}{2}bx\right) \right] U(x) \leftrightarrow \sqrt{\frac{jk + b}{jk}} - 1 \tag{C.18}$$

$$g(x) - a e^{-ax} \int_{-\infty}^{x} e^{au} g(u)\, du \leftrightarrow \frac{jk}{jk + a} G(k) \tag{C.19}$$