ELECTROMAGNETICS

Edward J. Rothwell Michael J. Cloud

Copyrighted Material

ELECTROMAGNETICS

Electrical Engineering Textbook Series

Richard C. Dorf, Series Editor University of California, Davis

Forthcoming Titles

Applied Vector Analysis Matiur Rahman and Issac Mulolani

> *Optimal Control Systems* Subbaram Naidu

Continuous Signals and Systems with MATLAB Taan ElAli and Mohammad A. Karim

Discrete Signals and Systems with MATLAB Taan ElAli

ELECTROMAGNETICS

Edward J. Rothwell

Michigan State University East Lansing, Michigan

Michael J. Cloud

Lawrence Technological University Southfield, Michigan

Boca Raton London New York Washington, D.C. CRC Press

Library of Congress Cataloging-in-Publication Data Rothwell, Edward J. Electromagnetics / Edward J. Rothwell, Michael J. Cloud. p. cm.—(Electrical engineering textbook series ; 2) Includes bibliographical references and index. ISBN 0-8493-1397-X (alk. paper) 1. Electromagnetic theory. I. Cloud, Michael J. II. Title. III. Series. QC670 .R693 2001 530.14′1—dc21 00-065158 CD

This book contains information obtained from authentic and highly regarded sources. Reprinted material is quoted with permission, and sources are indicated. A wide variety of references are listed. Reasonable efforts have been made to publish reliable data and information, but the author and the publisher cannot assume responsibility for the validity of all materials or for the consequences of their use.

Neither this book nor any part may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying, microfilming, and recording, or by any information storage or retrieval system, without prior permission in writing from the publisher.

The consent of CRC Press LLC does not extend to copying for general distribution, for promotion, for creating new works, or for resale. Specific permission must be obtained in writing from CRC Press LLC for such copying.

Direct all inquiries to CRC Press LLC, 2000 N.W. Corporate Blvd., Boca Raton, Florida 33431, or visit our Web site at<www.crcpress.com>

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation, without intent to infringe.

Visit our website at [www.crcpress.com.](www.crcpress.com)

© 2001 by CRC Press LLC

No claim to original U.S. Government works International Standard Book Number 0-8493-1397-X Library of Congress Card Number 00-065158 Printed in the United States of America 1 2 3 4 5 6 7 8 9 0 Printed on acid-free paper

Preface

This book is intended as a text for a first-year graduate sequence in engineering electromagnetics. Ideally such a sequence provides a transition period during which a student can solidify his or her understanding of fundamental concepts before proceeding to specialized areas of research.

The assumed background of the reader is limited to standard undergraduate topics in physics and mathematics. Worthy of explicit mention are complex arithmetic, vector analysis, ordinary differential equations, and certain topics normally covered in a "signals and systems" course (e.g., convolution and the Fourier transform). Further analytical tools, such as contour integration, dyadic analysis, and separation of variables, are covered in a self-contained mathematical appendix.

The organization of the book is in six chapters. In Chapter 1 we present essential background on the field concept, as well as information related specifically to the electromagnetic field and its sources. Chapter 2 is concerned with a presentation of Maxwell's theory of electromagnetism. Here attention is given to several useful forms of Maxwell's equations, the nature of the four field quantities and of the postulate in general, some fundamental theorems, and the wave nature of the time-varying field. The electrostatic and magnetostatic cases are treated in Chapter 3. In Chapter 4 we cover the representation of the field in the frequency domains: both temporal and spatial. Here the behavior of common engineering materials is also given some attention. The use of potential functions is discussed in Chapter 5, along with other field decompositions including the solenoidal–lamellar, transverse–longitudinal, and TE–TM types. Finally, in Chapter 6 we present the powerful integral solution to Maxwell's equations by the method of Stratton and Chu. A main mathematical appendix near the end of the book contains brief but sufficient treatments of Fourier analysis, vector transport theorems, complex-plane integration, dyadic analysis, and boundary value problems. Several subsidiary appendices provide useful tables of identities, transforms, and so on.

We would like to express our deep gratitude to those persons who contributed to the development of the book. The reciprocity-based derivation of the Stratton–Chu formula was provided by Prof. Dennis Nyquist, as was the material on wave reflection from multiple layers. The groundwork for our discussion of the Kronig–Kramers relations was provided by Michael Havrilla, and material on the time-domain reflection coefficient was developed by Jungwook Suk. We owe thanks to Prof. Leo Kempel, Dr. David Infante, and Dr. Ahmet Kizilay for carefully reading large portions of the manuscript during its preparation, and to Christopher Coleman for helping to prepare the figures. We are indebted to Dr. John E. Ross for kindly permitting us to employ one of his computer programs for scattering from a sphere and another for numerical Fourier transformation. Helpful comments and suggestions on the figures were provided by Beth Lannon–Cloud.

Thanks to Dr. C. L. Tondo of T & T Techworks, Inc., for assistance with the LaTeX macros that were responsible for the layout of the book. Finally, we would like to thank the staff members of CRC Press — Evelyn Meany, Sara Seltzer, Elena Meyers, Helena Redshaw, Jonathan Pennell, Joette Lynch, and Nora Konopka — for their guidance and support.

Contents

[Preface](#page-5-0)

1 [Introductory](#page-0-0) concepts

- 1.1 Notation, [conventions,](#page-0-0) and symbology
- 1.2 The field concept of [electromagnetics](#page-1-0)
	- 1.2.1 Historical [perspective](#page-1-0)
	- 1.2.2 [Formalization](#page-3-0) of field theory
- 1.3 The sources of the [electromagnetic](#page-4-0) field
	- 1.3.1 Macroscopic [electromagnetics](#page-5-0)
	- 1.3.2 [Impressed](#page-8-0) vs. secondary sources
	- 1.3.3 Surface and line source [densities](#page-9-0)
	- 1.3.4 Charge [conservation](#page-11-0)
	- 1.3.5 [Magnetic](#page-16-0) charge
- 1.4 [Problems](#page-17-0)

2 Maxwell's theory of [electromagnetism](#page-0-0)

- 2.1 The [postulate](#page-0-0)
	- 2.1.1 The [Maxwell–Minkowski](#page-1-0) equations
	- 2.1.2 [Connection](#page-4-0) to mechanics
- 2.2 The [well-posed](#page-5-0) nature of the postulate
	- 2.2.1 [Uniqueness](#page-6-0) of solutions to Maxwell's equations
	- 2.2.2 [Constitutive](#page-8-0) relations
- 2.3 [Maxwell's](#page-15-0) equations in moving frames
	- 2.3.1 Field conversions under Galilean [transformation](#page-16-0)
	- 2.3.2 Field conversions under Lorentz [transformation](#page-19-0)
- 2.4 The [Maxwell–Boffi](#page-25-0) equations
- 2.5 [Large-scale](#page-29-0) form of Maxwell's equations
	- 2.5.1 Surface moving with [constant](#page-30-0) velocity
	- 2.5.2 Moving, [deforming](#page-36-0) surfaces
	- 2.5.3 [Large-scale](#page-37-0) form of the Boffi equations
- 2.6 The nature of the four field [quantities](#page-39-0)
- 2.7 [Maxwell's](#page-40-0) equations with magnetic sources
- 2.8 Boundary (jump) [conditions](#page-42-0)
	- 2.8.1 Boundary conditions across a [stationary,](#page-42-0) thin source layer
	- 2.8.2 Boundary conditions across a stationary layer of field [discontinuity](#page-44-0)
	- 2.8.3 Boundary [conditions](#page-48-0) at the surface of a perfect conductor
- 2.8.4 Boundary conditions across a stationary layer of field [discontinuity](#page-49-0) using equivalent sources
- 2.8.5 Boundary conditions across a moving layer of field [discontinuity](#page-49-0)
- 2.9 [Fundamental](#page-50-0) theorems
	- 2.9.1 [Linearity](#page-50-0)
	- 2.9.2 [Duality](#page-51-0)
	- 2.9.3 [Reciprocity](#page-55-0)
	- 2.9.4 [Similitude](#page-56-0)
	- 2.9.5 [Conservation](#page-58-0) theorems
- 2.10 The wave nature of the [electromagnetic](#page-69-0) field
	- 2.10.1 [Electromagnetic](#page-70-0) waves
	- 2.10.2 Wave equation for [bianisotropic](#page-71-0) materials
	- 2.10.3 Wave equation in a [conducting](#page-73-0) medium
	- 2.10.4 Scalar wave equation for a [conducting](#page-74-0) medium
	- 2.10.5 Fields determined by Maxwell's equations vs. fields [determined](#page-74-0) by the wave equation
	- 2.10.6 Transient uniform plane waves in a [conducting](#page-74-0) medium
	- 2.10.7 [Propagation](#page-81-0) of cylindrical waves in a lossless medium
	- 2.10.8 [Propagation](#page-85-0) of spherical waves in a lossless medium
	- 2.10.9 [Nonradiating](#page-88-0) sources
- 2.11 [Problems](#page-89-0)

3 The static [electromagnetic](#page-0-0) field

- 3.1 Static fields and steady [currents](#page-0-0)
	- 3.1.1 [Decoupling](#page-1-0) of the electric and magnetic fields
	- 3.1.2 Static field [equilibrium](#page-2-0) and conductors
	- 3.1.3 Steady [current](#page-4-0)
- 3.2 [Electrostatics](#page-6-0)
	- 3.2.1 The [electrostatic](#page-6-0) potential and work
	- 3.2.2 Boundary conditions
	- 3.2.3 Uniqueness of the [electrostatic](#page-10-0) field
	- 3.2.4 Poisson's and Laplace's [equations](#page-11-0)
	- 3.2.5 Force and [energy](#page-25-0)
	- 3.2.6 Multipole [expansion](#page-29-0)
	- 3.2.7 Field produced by a [permanently](#page-35-0) polarized body
	- 3.2.8 [Potential](#page-36-0) of a dipole layer
	- 3.2.9 Behavior of electric charge density near a [conducting](#page-38-0) edge
	- 3.2.10 Solution to Laplace's equation for bodies immersed in an [impressed](#page-40-0) field

3.3 [Magnetostatics](#page-41-0)

- 3.3.1 The [magnetic](#page-44-0) vector potential
- 3.3.2 Multipole [expansion](#page-47-0)
- 3.3.3 Boundary conditions for the [magnetostatic](#page-49-0) field
- 3.3.4 Uniqueness of the [magnetostatic](#page-51-0) field
- 3.3.5 Integral solution for the vector [potential](#page-51-0)
- 3.3.6 Force and [energy](#page-54-0)
- 3.3.7 Magnetic field of a [permanently](#page-63-0) magnetized body
- 3.3.8 Bodies immersed in an impressed magnetic field: [magnetostatic](#page-65-0) shielding
- 3.4 Static field [theorems](#page-67-0)
- 3.4.1 Mean value theorem of [electrostatics](#page-67-0)
- 3.4.2 [Earnshaw's](#page-67-0) theorem
- 3.4.3 [Thomson's](#page-67-0) theorem
- 3.4.4 Green's [reciprocation](#page-69-0) theorem
- 3.5 [Problems](#page-70-0)

4 Temporal and spatial frequency domain [representation](#page-0-0)

- 4.1 [Interpretation](#page-0-0) of the temporal transform
- 4.2 The [frequency-domain](#page-1-0) Maxwell equations
- 4.3 Boundary conditions on the [frequency-domain](#page-2-0) fields
- 4.4 The constitutive and [Kronig–Kramers](#page-3-0) relations
	- 4.4.1 The complex [permittivity](#page-4-0)
	- 4.4.2 High and low frequency behavior of [constitutive](#page-5-0) parameters
	- 4.4.3 The [Kronig–Kramers](#page-5-0) relations
- 4.5 Dissipated and stored energy in a dispersive medium
	- 4.5.1 [Dissipation](#page-10-0) in a dispersive material
	- 4.5.2 Energy stored in a [dispersive](#page-13-0) material
	- 4.5.3 The energy [theorem](#page-17-0)
- 4.6 Some simple models for [constitutive](#page-18-0) parameters
	- 4.6.1 Complex permittivity of a [non-magnetized](#page-18-0) plasma
	- 4.6.2 Complex dyadic [permittivity](#page-23-0) of a magnetized plasma
	- 4.6.3 Simple models of [dielectrics](#page-25-0)
	- 4.6.4 Permittivity and [conductivity](#page-38-0) of a conductor
	- 4.6.5 [Permeability](#page-38-0) dyadic of a ferrite
- 4.7 [Monochromatic](#page-43-0) fields and the phasor domain
	- 4.7.1 The [time-harmonic](#page-44-0) EM fields and constitutive relations
	- 4.7.2 The phasor fields and [Maxwell's](#page-45-0) equations
	- 4.7.3 Boundary [conditions](#page-46-0) on the phasor fields
- 4.8 Poynting's theorem for [time-harmonic](#page-46-0) fields
	- 4.8.1 General form of [Poynting's](#page-47-0) theorem
	- 4.8.2 Poynting's theorem for [nondispersive](#page-48-0) materials
	- 4.8.3 [Lossless,](#page-50-0) lossy, and active media
- 4.9 The complex [Poynting](#page-52-0) theorem
	- 4.9.1 Boundary condition for the [time-average](#page-54-0) Poynting vector
- 4.10 Fundamental theorems for [time-harmonic](#page-54-0) fields
	- 4.10.1 [Uniqueness](#page-54-0)
	- 4.10.2 [Reciprocity](#page-57-0) revisited
	- 4.10.3 [Duality](#page-60-0)
- 4.11 The wave nature of the [time-harmonic](#page-63-0) EM field
	- 4.11.1 The [frequency-domain](#page-63-0) wave equation
	- 4.11.2 Field relationships and the wave equation for [two-dimensional](#page-64-0) fields
	- 4.11.3 Plane waves in a [homogeneous,](#page-67-0) isotropic, lossy material
	- 4.11.4 [Monochromatic](#page-78-0) plane waves in a lossy medium
	- 4.11.5 Plane waves in [layered](#page-88-0) media
	- 4.11.6 Plane-wave [propagation](#page-109-0) in an anisotropic ferrite medium
	- 4.11.7 [Propagation](#page-112-0) of cylindrical waves
	- 4.11.8 [Propagation](#page-129-0) of spherical waves in a conducting medium
	- 4.11.9 [Nonradiating](#page-133-0) sources
- 4.12 [Interpretation](#page-133-0) of the spatial transform
- 4.13 Spatial Fourier [decomposition](#page-135-0)
	- 4.13.1 Boundary value problems using the spatial Fourier [representation](#page-140-0)
- 4.14 Periodic fields and [Floquet's](#page-149-0) theorem 4.14.1 [Floquet's](#page-150-0) theorem
	- 4.14.2 [Examples](#page-151-0) of periodic systems
- 4.15 [Problems](#page-154-0)

5 Field [decompositions](#page-0-0) and the EM potentials

- 5.1 Spatial symmetry [decompositions](#page-0-0)
	- 5.1.1 Planar field [symmetry](#page-0-0)
- 5.2 [Solenoidal–lamellar](#page-5-0) decomposition
	- 5.2.1 Solution for potentials in an [unbounded](#page-15-0) medium: the retarded potentials
	- 5.2.2 Solution for potential [functions](#page-25-0) in a bounded medium
- 5.3 [Transverse–longitudinal](#page-27-0) decomposition
	- 5.3.1 [Transverse–longitudinal](#page-27-0) decomposition in terms of fields
- 5.4 TE–TM [decomposition](#page-30-0)
	- 5.4.1 TE–TM [decomposition](#page-30-0) in terms of fields
	- 5.4.2 TE–TM [decomposition](#page-31-0) in terms of Hertzian potentials
	- 5.4.3 [Application:](#page-33-0) hollow-pipe waveguides
	- 5.4.4 TE–TM [decomposition](#page-43-0) in spherical coordinates
- 5.5 [Problems](#page-52-0)

6 Integral solutions of [Maxwell's](#page-0-0) equations

- 6.1 Vector [Kirchoff](#page-0-0) solution
	- 6.1.1 The Stratton–Chu formula
	- 6.1.2 The [Sommerfeld](#page-4-0) radiation condition
	- 6.1.3 Fields in the excluded region: the [extinction](#page-5-0) theorem
- 6.2 Fields in an [unbounded](#page-6-0) medium
	- 6.2.1 The far-zone fields produced by sources in [unbounded](#page-7-0) space
- 6.3 Fields in a bounded, [source-free](#page-13-0) region
	- 6.3.1 The vector Huygens [principle](#page-13-0)
	- 6.3.2 The Franz [formula](#page-14-0)
	- 6.3.3 Love's [equivalence](#page-15-0) principle
	- 6.3.4 The [Schelkunoff](#page-17-0) equivalence principle
	- 6.3.5 Far-zone fields produced by [equivalent](#page-18-0) sources
- 6.4 [Problems](#page-21-0)

A [Mathematical](#page-0-0) appendix

- A.1 The Fourier [transform](#page-0-0)
- A.2 Vector [transport](#page-24-0) theorems
- A.3 Dyadic [analysis](#page-28-0)
- A.4 [Boundary](#page-34-0) value problems
- **B Useful [identities](#page-0-0)**

C Some Fourier [transform](#page-0-0) pairs

D [Coordinate](#page-0-0) systems

E [Properties](#page-0-0) of special functions

- E.1 Bessel [functions](#page-0-0)
- E.2 Legendre [functions](#page-6-0)
- E.3 Spherical [harmonics](#page-10-0)

[References](#page-0-0)

Electrical Engineering

ELECTROMAGNETICS **Edward J. Rothwell • Michael J. Cloud**

Electromagnetics provides a smooth transition from basic undergraduate EM courses to advanced and often specialized graduate studies. Emphasizing concepts over problem-solving techniques, it focuses on the topics most important to EM research and those most troublesome to beginning graduate students. The authors cover the required mathematical background and introduce the primary physical principles. building from a well-posed postulate to a coherent description of the EM field. Stressing both a physical understanding and a detailed mathematical description of each topic, this text provides an account of EM theory that is in-depth, lucid, and accessible.

Highly engaging prose, clear, concise explanations, and numerous examples relating concepts to modern engineering applications create a comfortable atmosphere that enhances the reader's grasp of the material. Electromagnetics thus builds a foundation that allows readers to proceed with confidence to advanced EM studies, research, and applications.

Features

- Provides the ideal transition between introductory EM courses and advanced graduate studies
- Gives readers a keen understanding of EM fields, building progressively ۰ from simple background material to the most significant concepts
- Covers each fundamental principle with clear, concise explanations ٠
- Includes many examples linking fundamental principles with engineering applications
- Solidifies fundamental principles with ٠ numerous end-of-chapter problems

