II

Power Electronic Circuits and Controls

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2

DC-DC Converters

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2.1 Overview

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The purpose of a DC-DC converter is to supply a regulated DC output voltage to a variable-load resistance from a fluctuating DC input voltage. In many cases the DC input voltage is obtained by rectifying a line voltage that is changing in magnitude. DC-DC converters are commonly used in applications requiring regulated DC power, such as computers, medical instrumentation, communication devices, television receivers, and battery chargers [1, 2]. DC-DC converters are also used to provide a regulated variable DC voltage for DC motor speed control applications.

The output voltage in DC-DC converters is generally controlled using a switching concept, as illustrated by the basic DC-DC converter shown in Fig. 2.1. Early DC-DC converters were known as choppers with silicon-controlled rectifiers (SCRs) used as the switching mechanisms. Modern DC-DC converters classified as switch mode power supplies (SMPS) employ insulated gate bipolar transistors (IGBTs) and metal oxide silicon field effect transistors (MOSFETs).

The switch mode power supply has several functions [3]:

1. Step down an unregulated DC input voltage to produce a regulated DC output voltage using a buck or step-down converter.
2. Step up an unregulated DC input voltage to produce a regulated DC output voltage using a boost or step-up converter.

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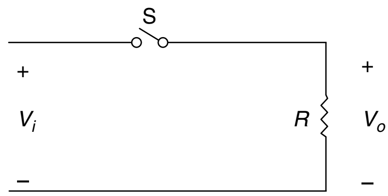

FIGURE 2.1 Basic DC-DC converter.

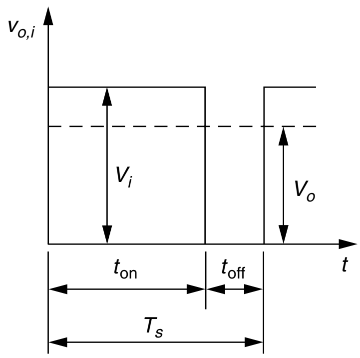

FIGURE 2.2 DC-DC converter voltage waveforms. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

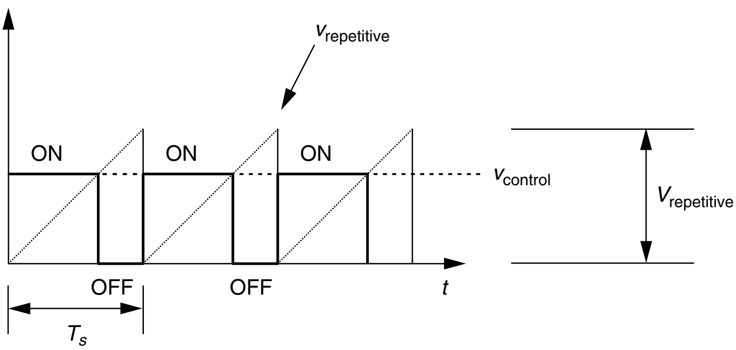

FIGURE 2.3 Pulsewidth modulation concept. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

3. Step down and then step up an unregulated DC input voltage to produce a regulated DC output voltage using a buck–boost converter.
4. Invert the DC input voltage using a Cúk converter.
5. Produce multiple DC outputs using a combination of SMPS topologies.

The regulation of the average output voltage in a DC-DC converter is a function of the on-time t_{on} of the switch, the pulse width, and the switching frequency f_s as illustrated in Fig. 2.2. Pulse width modulation (PWM) is the most widely used method of controlling the output voltage. The PWM concept is illustrated in Fig. 2.3. The output voltage control depends on the duty ratio D. The duty ratio is defined as

$$D = \frac{t_{on}}{T_s} = \frac{v_{control}}{V_{repetitive}} \tag{2.1}$$

based on the on-time t_{on} of the switch and the switching period T_s. PWM switching involves comparing the level of a control voltage $v_{control}$ to the level of a repetitive waveform as illustrated in Fig. 2.3 [2]. The on-time of the switch is defined as the portion of the switching period where the value of the repetitive

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waveform is less than the control voltage. The switching period (switching frequency) remains constant while the control voltage level is adjusted to change the on-time and therefore the duty ratio of the switch. The switching frequency is usually chosen above 20 kHz so the noise is outside the audio range [2, 3].

DC-DC converters operate in one of two modes depending on the characteristics of the output current [1, 2]:

1. Continuous conduction
2. Discontinuous conduction

The continuous-conduction mode is defined by continuous output current (greater than zero) over the entire switching period, whereas the discontinuous conduction mode is defined by discontinuous output current (equal to zero) during any portion of the switching period. Each mode is discussed in relationship to the buck and boost converters in subsequent sections.

References

1. Agrawal, J. P., *Power Electronics Systems: Theory and Design,* Prentice-Hall, Upper Saddle River, NJ, 2001, chap. 6.
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3. Venkat, R., *Switch Mode Power Supply,* University of Technology, Sydney, Australia, March 1, 2001, available at http://www.ee.uts.edu.au/~venkat/pe_html/pe07_nc8.htm.

2.2 Choppers

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Choppers are DC-DC converters that are used for transferring electrical energy from a DC source into another DC source, which may be a passive load. These converters are widely used in regulated switching power supplies and DC motor drive applications.

DC-DC converters that are discussed in this section are one-quadrant, two-quadrant, and four-quadrant choppers. Step-down (buck) converter and step-up (boost) converters are basic one-quadrant converter topologies. The two-quadrant chopper, which, in fact, is a current reversible converter, is the combination of the two basic topologies. The full-bridge converter is derived from the step-down converter.

One-Quadrant Choppers

In one-quadrant choppers, the average DC output voltage is usually kept at a desired level, as there are fluctuations in input voltage and output load. These choppers operate only in first quadrant of v–i plane. In fact, output and input voltages and currents are always positive. Therefore, these converters are called one-quadrant choppers.

One method of controlling the output voltage employs switching at a constant frequency, i.e., a constant switching time period ($T = t_{on} + t_{off}$), and adjusting the on-duration of the switch to control the average output voltage. In this method, which is called pulse-width modulation (PWM), the switch duty ratio d is defined as the ratio of the on-duration to the switching time period.

$$d = \frac{t_{on}}{T} \tag{2.2}$$

In the other control method, both the switching frequency and the on-duration of the switch are varied. This method is mainly used in converters with force-commutated thyristors.

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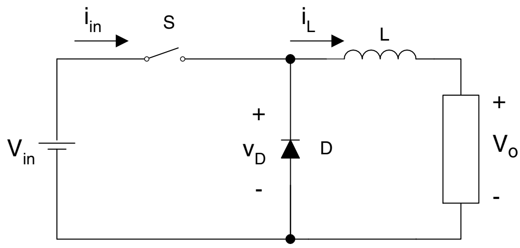

FIGURE 2.4 Step-down buck converter.

Choppers can have two distinct modes of operation, which have significantly different characteristics: continuous-conduction and discontinuous-conduction modes. In practice, a converter may operate in both modes. Therefore, converter control should be designed for both modes of operation.

Step-Down (Buck) Converter

A step-down converter produces an average output voltage, which is lower than the DC input voltage V_{in}. The basic circuit of a step-down converter is shown in Fig. 2.4.

In continuous-conduction mode of operation, assuming an ideal switch, when the switch is on for the time duration t_{on}, the inductor current passes through the switch, and the diode becomes reverse-biased. This results in a positive voltage $(V_{in} - V_o)$ across the inductor, which, in turn, causes a linear increase in the inductor current i_L. When the switch is turned off, because of the inductive energy storage, i_L continues to flow. This current flows through the diode and decreases. Average output voltage can be calculated in terms of the switch duty ratio as:

$$v_{o,\,\text{ave.}} = \frac{1}{T}\int_0^T v_o(t)\,dt = \frac{1}{T}\left(\int_0^{t_{on}} V_{in}\,dt + \int_{t_{on}}^{T} 0.0\right) = \frac{t_{on}}{T}V_{in} = dV_{in} \tag{2.3}$$

$v_{o,\,\text{ave.}}$ can be controlled by varying the duty ratio $(d = t_{on}/T)$ of the switch. Another important observation is that the average output voltage varies linearly with the control voltage. However, in the discontinuous-conduction mode of operation, the linear relation between input and output voltages is not valid. Figure 2.5 shows $(v_{o,\,\text{ave.}}/v_{in,\,\text{ave.}}) - i_{o,\,\text{ave.}}$ characteristic of a step-down converter in continuous and discontinuous conduction modes of operation.

Step-Up (Boost) Converter

Schematic diagram of a step-up boost converter is shown in Fig. 2.6. In this converter, the output voltage is always greater than the input voltage. When the switch is on, the diode is reversed-biased, thus isolating the output stage. The input voltage source supplies energy to the inductor. When the switch is off, the output stage receives energy from the inductor as well as the input source.

In the continuous-conduction mode of operation, considering d as the duty ratio, the input–output relation is as follows:

$$v_{o,\,\text{ave.}} = \frac{1}{1-d}V_{in} \tag{2.4}$$

If input voltage is not constant, V_{in} is the average of the input voltage. In this case, relation (2.3) is an approximation. In the discontinuous-conduction mode of operation, relation (2.3) is not valid. Figure 2.7 shows $(v_{in,\,\text{ave.}}/v_{o,\,\text{ave.}}) - i_{L,\,\text{ave.}}$ characteristic of a step-up converter in the continuous- and discontinuous-conduction modes of operation.

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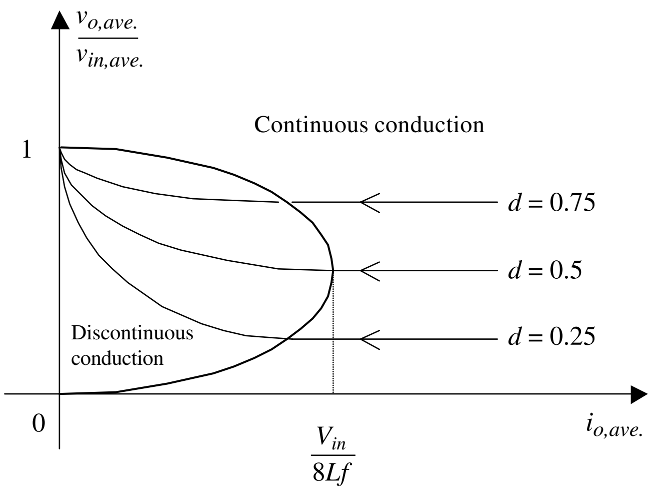

FIGURE 2.5 $(v_{o,\text{ave.}}/v_{\text{in, ave.}}) - i_{o,\text{ave.}}$ characteristic of a step-down converter.

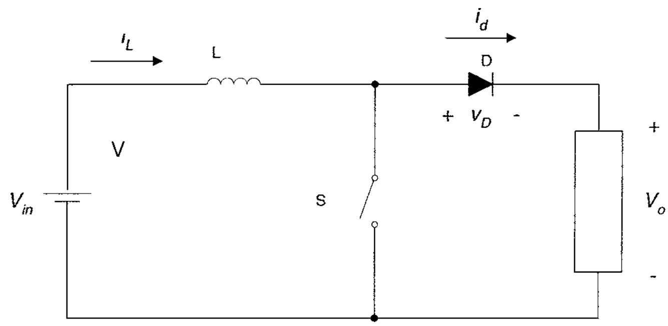

FIGURE 2.6 Step-up boost converter.

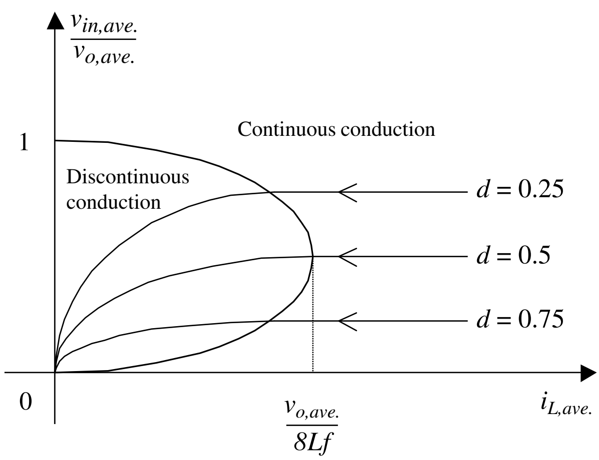

FIGURE 2.7 $(v_{\text{in, ave.}}/v_{o,\text{ ave.}}) - i_{L,\text{ ave.}}$ characteristic of a step-down converter.

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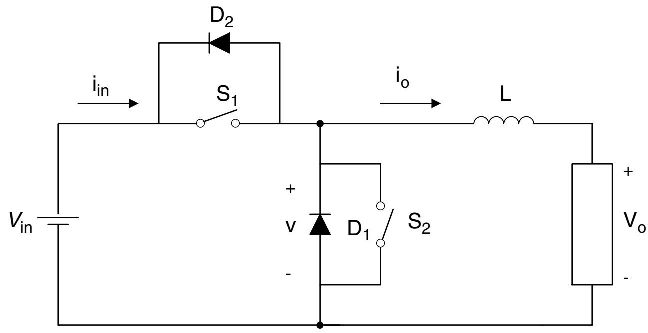

FIGURE 2.8 A current reversible chopper.

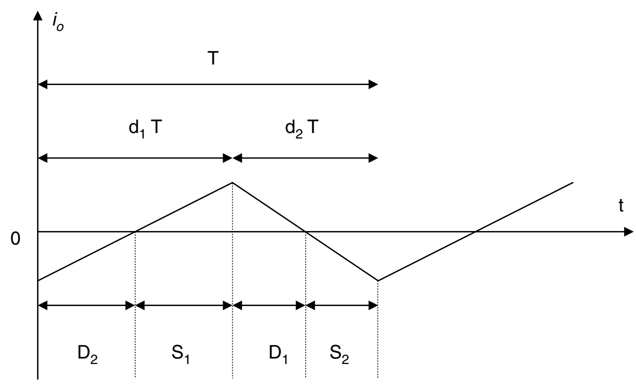

FIGURE 2.9 Output current of a two-quadrant chopper.

Two-Quadrant Choppers

A two-quadrant chopper has the ability to operate in two quadrants of the (v–i) plane. Therefore, input and output voltages are positive; however, input and output currents can be positive or negative. Thus, these converters are also named current reversible choppers. They are composed of two basic chopper circuits. In fact, a two-quadrant DC-DC converter is achieved by a combination of two basic chopper circuits, a step-down chopper and a step-up chopper, as is shown in Fig. 2.8.

The step-down chopper is composed of S_1 and D_1, and electric energy is supplied to the load. The step-up chopper is composed of S_2 and D_2; electric energy is fed back to the source. Reversible current choppers can transfer from operating in the power mode to operating in the regenerative mode very smoothly and quickly by changing only the control signals for S_1 and S_2, without using any mechanical contacts.

Figure 2.9 depicts the output current of a two-quadrant chopper. d_1 and $d_2 = 1 - d_1$ are the duty ratios of step-down and step-up converters, respectively. By changing d_1 and d_2, not only the amplitude of the average of the output current changes, but it can also be positive and negative, leading to two-quadrant operation.

For each of step-down and step-up operating mode, relations (2.3) and (2.4) are applicable for continuous currents. However, in discontinuous-conduction modes of operation, relations (2.3) and (2.4) are not valid. Figure 2.10 shows the $(v_{o,\text{ ave.}}/v_{\text{in, ave.}}) - i_{o,\text{ ave.}}$ characteristic of a two-quadrant converter in continuous- and discontinuous-conduction modes of operation. As is shown in Fig. 2.10, for changing the operating mode both from step-down to step-up operation and in the opposite direction,

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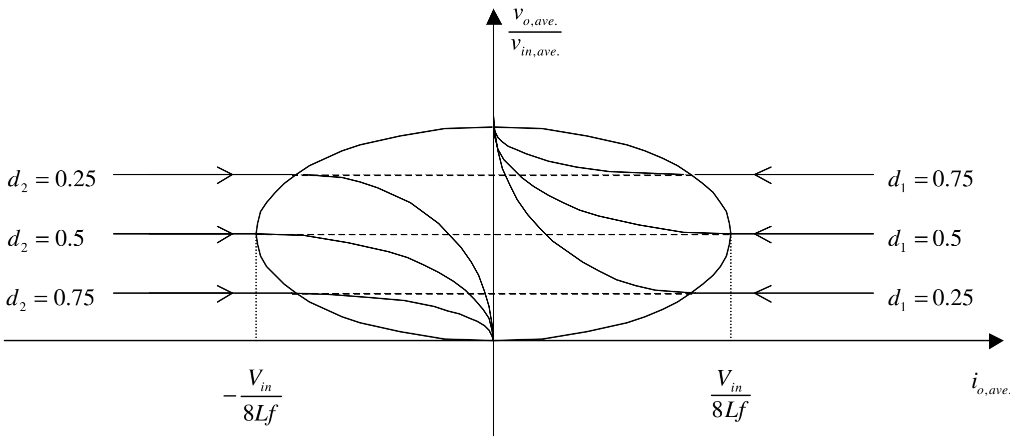

FIGURE 2.10 $(v_{o,\text{ave.}}/v_{\text{in, ave.}}) - i_{o,\text{ ave.}}$ characteristic of a two-quadrant converter.

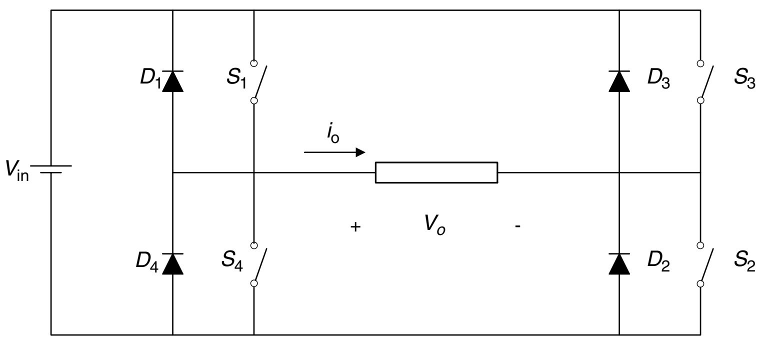

FIGURE 2.11 A full-bridge four-quadrant chopper.

the operating mode must move from the discontinuous-current region. However, by applying $d_2 = 1 - d_1$, the operating point will never move into the discontinuous-conduction region of the two basic converters. In Fig. 2.10, the broken lines indicate passage from step-down operation to step-up operation, and vice versa. In fact, because of this specific command—the relation between the two duty ratios—the converter operating point always stays in the continuous-conduction mode.

Four-Quadrant Choppers

In four-quadrant choppers, not only can the output current be positive and negative, but the output voltage also can be positive and negative. These choppers are full-bridge DC-DC converters, as is shown in Fig. 2.11. The main advantage of these converters is that the average of the output voltage can be controlled in magnitude as well as in polarity. A four-quadrant chopper is a combination of two two-quadrant choppers in order to achieve negative average output voltage and/or negative average output current.

The four-quadrant operation of the full-bridge DC-DC converter, as shown in Fig. 2.12, for the first two quadrants of the (v–i) plane is achieved by switching S_1 and S_2 and considering D_1 and D_2 like a two-quadrant chopper. For the other two quadrants of the (v–i) plane, the operation is achieved by switching S_3 and S_4 and considering D_3 and D_4 as another two-quadrant chopper, which is connected to the load in the opposite direction of the first two-quadrant chopper.

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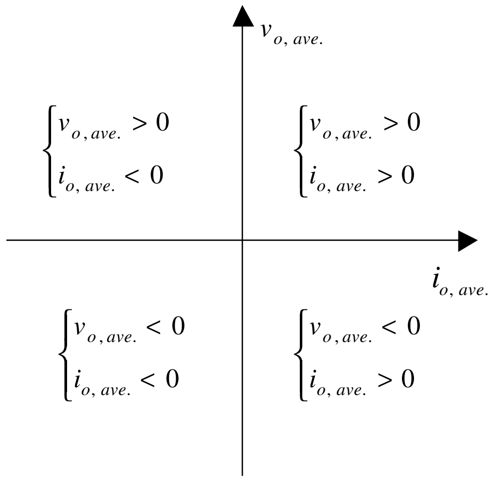

FIGURE 2.12 Four-quadrant operation of a full-bridge chopper.

2.3 Buck Converters

Richard Wies, Bipin Satavalekar, and Ashish Agrawal

The buck or step-down converter regulates the average DC output voltage at a level lower than the input or source voltage. This is accomplished through controlled switching where the DC input voltage is turned on and off periodically, resulting in a lower average output voltage [1]. The buck converter is commonly used in regulated DC power supplies like those in computers and instrumentation [1, 2]. The buck converter is also used to provide a variable DC voltage to the armature of a DC motor for variable speed drive applications [2].

Ideal Buck Circuit

The circuit that models the basic operation of the buck converter with an ideal switch and a purely resistive load is shown in Fig. 2.13. The output voltage equals the input voltage when the switch is in position 1 and the output voltage is zero when the switch is in position 2. The resulting output voltage is a rectangular voltage waveform with an average value as shown in Fig. 2.2 (in Section 2.1). The average output voltage level is varied by adjusting the time the switch is in position 1 and 2 or the duty ratio. The resulting average output voltage V_o is given in terms of the duty ratio and the input voltage V_i by Eq. (2.5) [2].

$$V_o = DV_i \tag{2.5}$$

The square wave output voltage for the ideal circuit of the buck converter contains an undesirable amount of voltage ripple. The circuit is modified by adding an inductor L in series and a capacitor C in parallel with the load resistor as shown in Fig. 2.14. The inductor reduces the ripple in the current through

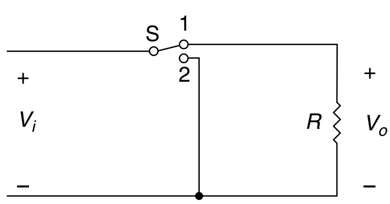

FIGURE 2.13 Ideal buck converter.

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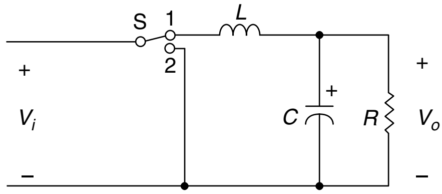

FIGURE 2.14 Modified buck converter with *LC* filter. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design*, 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

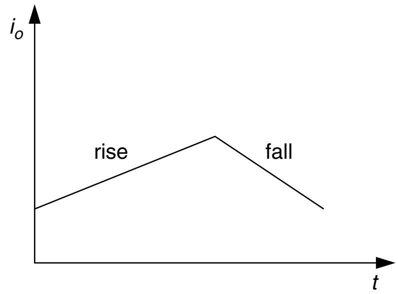

FIGURE 2.15 Rise and fall of load current in buck converter.

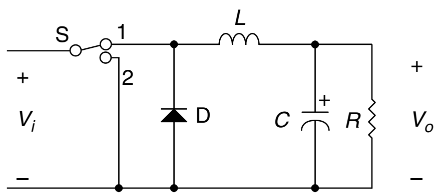

FIGURE 2.16 Buck converter with practical switch.

the load resistor, while the capacitor directly reduces the ripple in the output voltage. Since the current through the load resistor is the same as that of the inductor, the voltage across the load resistor (output voltage) contains less ripple.

The current through the inductor increases with the switch in position 1. As the current through the inductor increases, the energy stored in the inductor increases. When the switch changes to position 2, the current through the load resistor decreases as the energy stored in the inductor decreases. The rise and fall of current through the load resistor is linear if the time constant due to the *LR* combination is relatively large compared with the on- and off-time of the switch as shown in Fig. 2.15 [3]. A capacitor is added in parallel with the load resistor to reduce further the ripple content in the output voltage. The combination of the inductor and capacitor reduces the output voltage ripple to very low levels.

The circuit in Fig. 2.14 is designed assuming that the switch is ideal. A practical model of the switch is designed using a diode and power semiconductor switch as shown in Fig. 2.16. A freewheeling diode is used with the switch in position 2 since the inductor current freewheels through the switch. The switch is controlled by a scheme such as pulse width or frequency modulation.

Continuous-Conduction Mode

The continuous-conduction mode of operation occurs when the current through the inductor in the circuit of Fig. 2.14 is continuous. This means that the inductor current is always greater than zero. The average output voltage in the continuous-conduction mode is the same as that derived in Eq. (2.5) for the ideal circuit. As the conduction of current through the inductor occurs during the entire switching period, the average output voltage is the product of the duty ratio and the DC input voltage. The operation

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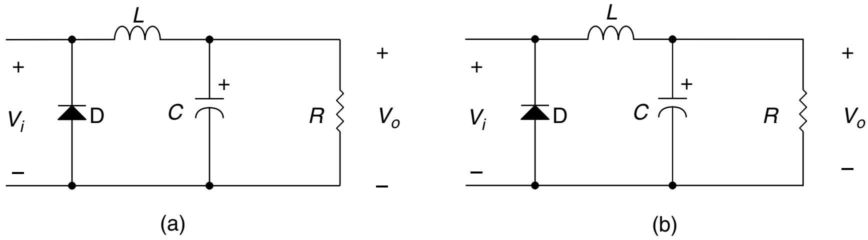

FIGURE 2.17 Buck converter switch states: (a) switch in position 1; (b) switch in position 2. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

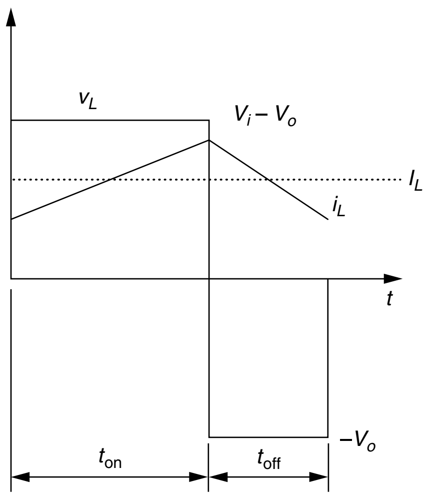

FIGURE 2.18 Inductor voltage and current for continuous mode of buck converter. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

of this circuit resembles a DC transformer according to Eq. (2.6) based on the time-integral of the inductor voltage equal to zero over one switching period [2].

$$D = \frac{V_o}{V_i} = \frac{I_i}{I_o} \tag{2.6}$$

The operation of the circuit in steady state consists of two states as illustrated in Fig. 2.17 [2, 4]. The first state with the switch in position 1 has the diode reverse-biased and current flows through the inductor from the voltage source to the load. The switch changes to position 2 at the end of the on-time and the inductor current then freewheels through the diode. The process starts again at the end of the switching period with the switch returning to position 1. A representative set of inductor voltage and current waveforms for the continuous-conduction mode is shown in Fig. 2.18.

Discontinuous-Conduction Mode

The discontinuous mode of operation occurs when the value of the load current is less than or equal to zero at the end of a given switching period. Assuming a linear rise and fall of current through the inductor, the boundary point between continuous- and discontinuous-current conduction occurs when the average inductor current over one switching period is half of the peak value, as illustrated in Fig. 2.19. The average inductor current at the boundary point is calculated using Eq. (2.7) [2].

$$I_{LB} = \frac{1}{2} i_{L(\text{peak})} = \frac{DT_s}{2L}(V_i - V_o) \tag{2.7}$$

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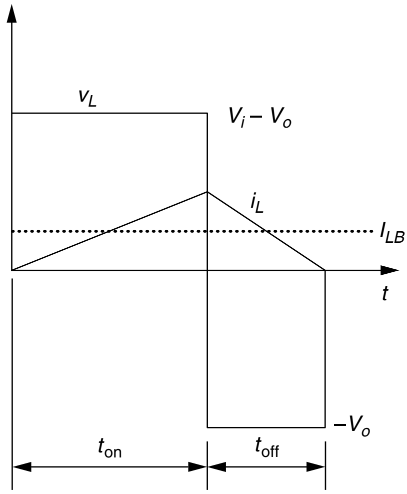

FIGURE 2.19 Inductor current at boundary point for discontinuous mode of buck converter. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

The input voltage or output voltage is kept constant depending on the application. If the input voltage remains constant, then the average inductor current at the boundary is calculated by replacing the output voltage in Eq. (2.7) with Eq. (2.5), which yields the expression in Eq. (2.8) [2].

$$I_{LB} = \frac{DT_s}{2L}(V_i)(1 - D) \tag{2.8}$$

The voltage ratio is now defined according to Eq. (2.9) [2]:

$$\frac{V_o}{V_i} = \frac{D^2}{D^2 + \frac{1}{4}\left(\frac{I_o}{I_{LB(\max)}}\right)} \tag{2.9}$$

If the output voltage remains constant, then the average inductor current at the boundary is calculated by replacing the input voltage in Eq. (2.7) with Eq. (2.5), which yields the expression in Eq. (2.10) [2]:

$$I_{LB} = \frac{T_s}{2L}(V_o)(1 - D) \tag{2.10}$$

The duty ratio is defined according to Eq. (2.11) by manipulating Eq. (2.9) [2]:

$$D = \frac{V_o}{V_i}\left(\frac{I_o/I_{LB(\max)}}{1 - \left(\frac{V_o}{V_i}\right)}\right)^{\frac{1}{2}} \tag{2.11}$$

Output Voltage Ripple

In DC-DC converters the output voltage ripple is a measure of the deviation in the output voltage from the average value. The peak-to-peak voltage ripple for the buck converter in Figure 2.16 for the continuous conduction mode can be calculated for a specified value of output capacitance by calculating the additional charge ΔQ provided by the ripple current in the inductor. This analysis assumes that all of the ripple current flows through the capacitor, while the average value of the inductor current flows through the load resistor. The peak-to-peak voltage ripple is calculated by taking the area under the inductor current i_L (the additional charge ΔQ) and dividing by the capacitance resulting in Equation 2.12 [2]:

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$$\Delta V_o = \frac{\Delta Q}{C} = \frac{1}{C}\frac{1}{2}\frac{\Delta I_L}{2}\frac{T_s}{2} = \frac{T_s}{8C}\frac{V_o}{L}(1-D)T_s \tag{2.12}$$

It is customary to refer to ripple in terms of the percentage ripple as illustrated in Equation 2.13 [2]:

$$\frac{\Delta V_o}{V_o} = \frac{1}{8}\frac{T_s^2(1-D)}{LC} = \frac{\pi^2}{2}(1-D)\left(\frac{f_c}{f_s}\right)^2 \tag{2.13}$$

where f_s is the switching frequency and f_c is the corner frequency of the low-pass LC filter on the output. The voltage ripple is minimized by selecting a corner frequency for the lowpass filter which is much less than the switching frequency.

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2.4 Boost Converters

Richard Wies, Bipin Satavalekar, and Ashish Agrawal

A boost converter regulates the average output voltage at a level higher than the input or source voltage. For this reason the boost converter is often referred to as a step-up converter or regulator. The DC input voltage is in series with a large inductor acting as a current source. A switch in parallel with the current source and the output is turned off periodically, providing energy from the inductor and the source to increase the average output voltage. The boost converter is commonly used in regulated DC power supplies and regenerative braking of DC motors [1, 2].

Ideal Boost Circuit

The circuit that models the basic operation of the boost converter is shown in Fig. 2.20 [2, 3]. The ideal boost converter uses the same components as the buck converter with different placement. The input voltage in series with the inductor acts as a current source. The energy stored in the inductor builds up when the switch is closed. When the switch is opened, current continues to flow through the inductor to the load. Since the source and the discharging inductor are both providing energy with the switch open, the effect is to boost the voltage across the load. The load consists of a resistor in parallel with a filter capacitor. The capacitor voltage is larger than the input voltage. The capacitor is large to keep a constant output voltage and acts to reduce the ripple in the output voltage.

Continuous-Conduction Mode

The continuous-conduction mode of operation occurs when the current through the inductor in the circuit of Fig. 2.20 is continuous with the inductor current always greater than zero. The operation of the circuit in steady state consists of two states, as illustrated in Fig. 2.21 [2, 3]. The first state with the switch closed has current charging the inductor from the voltage source. The switch opens at the end of the on-time and the inductor discharges current to the load with the input voltage source still connected. This results in an output voltage across the capacitor larger than the input voltage. The output

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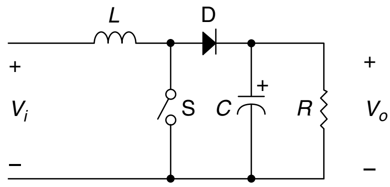

FIGURE 2.20 Basic boost converter. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

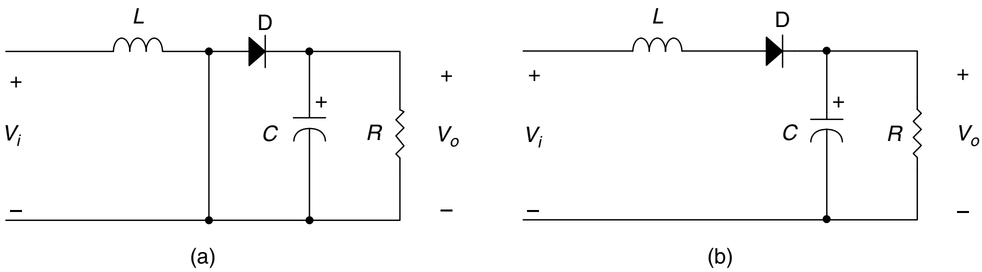

FIGURE 2.21 Basic boost converter switch states: (a) switch closed; (b) switch open. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

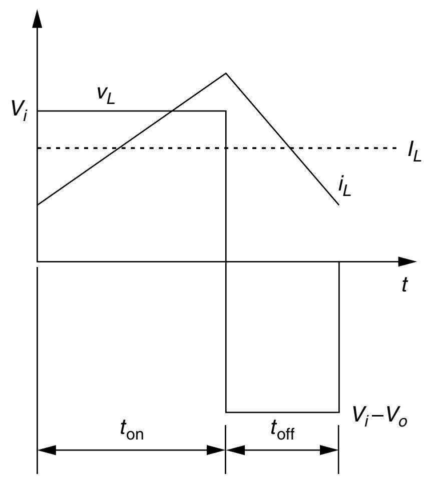

FIGURE 2.22 Inductor voltage and current waveforms for continuous mode of boost converter. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

voltage remains constant if the RC time constant is significantly larger than the on-time of the switch. A representative set of inductor voltage and current waveforms for the continuous conduction mode is shown in Fig. 2.22 [2].

The voltage ratio for a boost converter is derived based on the time-integral of the inductor voltage equal to zero over one switching period. The voltage ratio is equivalent to the ratio of the switching period to the off-time of the switch as illustrated by Eq. (2.14) [2].

$$\frac{V_o}{V_i} = \frac{I_i}{I_o} = \frac{T_s}{t_{\text{off}}} = \frac{T_s}{T_s - t_{\text{off}}} = \frac{T}{1 - D} \tag{2.14}$$

The current ratio is derived from the voltage ratio assuming that the input power is equal to the output power, as with ideal transformer analysis.

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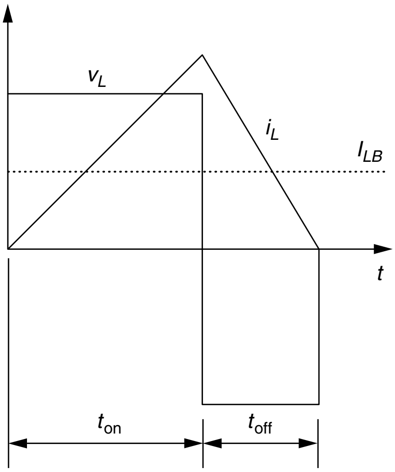

FIGURE 2.23 Inductor current at boundary point for discontinuous mode of boost converter. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

Discontinuous-Conduction Mode

The discontinuous mode of operation occurs when the value of the load current is less than or equal to zero at the end of a given switching period. Assuming a linear rise and fall of current through the inductor, the boundary point between continuous- and discontinuous-current conduction occurs when the average inductor current over one switching period is half the peak value, as illustrated in Fig. 2.23 [2]. The average inductor current at the boundary point is calculated using Eq. (2.15) [2].

$$I_{LB} = \frac{1}{2} i_{L(\text{peak})} = \frac{V_o T_s}{2L} D(1-D) \tag{2.15}$$

The output current at the boundary condition is derived by using the current ratio of Eq. (2.14) in Eq. (2.15) with the inductor current equal to the input current. This results in Eq. (2.16) [2]:

$$I_{OB} = \frac{V_o T_s}{2L} D(1-D)^2 \tag{2.16}$$

For the boost converter in discontinuous mode, the output voltage V_o is generally kept constant while the duty ratio D varies in response to changes in the input voltage V_i.

The duty ratio is defined as a function of the output current for various values of the voltage ratio according to Eq. (2.17) [2]:

$$D = \left[\frac{4}{27}\frac{V_o}{V_i}\left(\frac{V_o}{V_i} - 1\right)\frac{I_o}{I_{oB(\max)}}\right]^{\frac{1}{2}} \tag{2.17}$$

Output Voltage Ripple

The peak-to-peak voltage ripple for the boost converter in Figure 2.20 for the continuous conduction mode can be calculated for a specified value of output capacitance by calculating the additional charge ΔQ provided by the ripple current in the inductor. This analysis is similar to that discussed for the buck converter. The peak-to-peak voltage ripple is calculated by taking the area under the inductor current i_L (the additional charge ΔQ) and dividing by the capacitance resulting in Equation 2.18 [2]:

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$$\Delta V_o = \frac{\Delta Q}{C} = \frac{I_o D T_s}{C} = \frac{V_o D T_s}{R \; C} \tag{2.18}$$

The percentage output voltage ripple is calculated as in Equation 2.19 [2]:

$$\frac{\Delta V_o}{V_o} = \frac{D T_s}{RC} = D\frac{T_s}{\tau} \tag{2.19}$$

where τ is the RC time constant of the output filter. The voltage ripple is minimized by increasing the time constant of the output filter.

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1. Agrawal, J. P., *Power Electronics Systems: Theory and Design,* Prentice-Hall, Upper Saddle River, NJ, 2001, chap. 6.
2. Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995, chap. 7.
3. Venkat, R., *Switch Mode Power Supply,* University of Technology, Sydney, Australia, 01 March 2001, available at http://www.ee.uts.edu.au/~venkat/pe_html/pe07_nc8.htm.

2.5 Cúk Converter

Richard Wies, Bipin Satavalekar, and Ashish Agrawal

The Cúk converter is a switched-mode power supply named after the inventor Dr. Slobodan Cúk. The basic nonisolated Cúk converter shown in Fig. 2.24 is designed based on the principle of using two buck–boost converters to provide an inverted DC output voltage [1]. The advantage of the basic nonisolated Cúk converter over the standard buck–boost converter is to provide regulated DC output voltage at higher efficiency with identical components due to an integrated magnetic structure, reduced ripple currents, and reduced switching losses [2, 3]. The integrated magnetic structure of the isolated Cúk converter consists of the isolation transformer and the two inductors in a single core. As a result, the ripple currents in the inductors are driven into the primary and secondary windings of the isolation transformer. Also, the single core results in reduced flux paths, which improves the overall efficiency of the converter.

Nonisolated Operation

The basic nonisolated Cúk converter is a switching power supply with two inductors, two capacitors, a diode, and a transistor switch as illustrated in Fig. 2.24 [1, 2]. The transfer capacitor C_t stores and transfers energy from the input to the output. The average value of the inductor voltages for steady-state operation is zero. As a result, the voltage across the transfer capacitor is assumed to be the average value V_{C_t} in steady state and is the sum of the input and output voltages. The inductor currents are assumed to be continuous for steady-state operation.

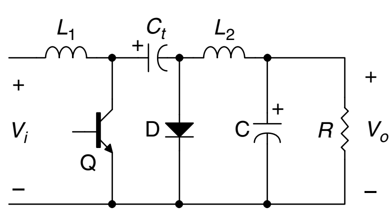

FIGURE 2.24 Nonisolated Cúk converter. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

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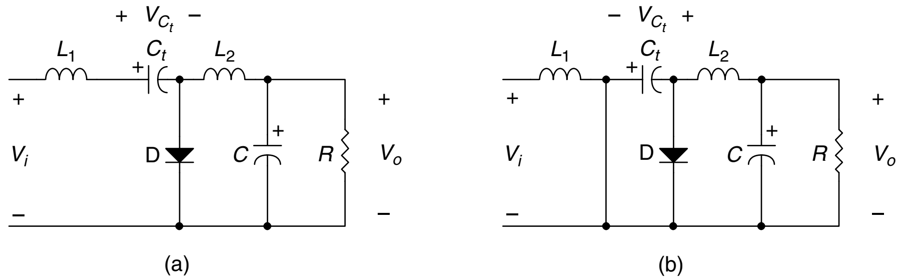

FIGURE 2.25 Cúk converter switch states: (a) switch open; (b) switch closed. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

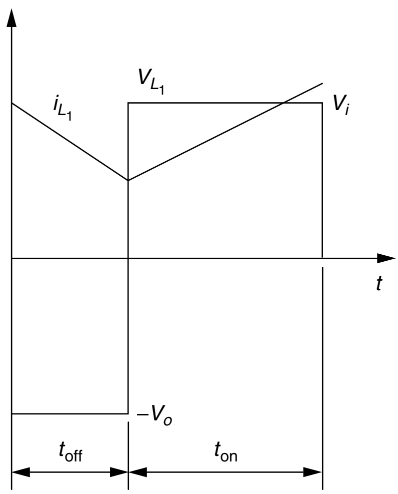

FIGURE 2.26 Inductor 1, voltage and current waveforms for Cúk converter. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

The operation of the basic nonisolated Cúk converter in steady state consists of two transistor states, as illustrated in Fig. 2.25 [1, 2]. In the first state when the transistor is off, the inductor currents flow through the diode and energy is stored in the transfer capacitor from the input and the inductor L_1. The energy stored in the inductor L_2 is transferred to the output. As a result, both of the inductor currents are linearly decreasing in the off-state. In the second state when the transistor is on, the inductor currents flow through the transistor and the transfer capacitor discharges while energy is stored in the inductor L_1. As the transfer capacitor discharges through the transistor, energy is stored in the inductor L_2. Consequently, both of the inductor currents are linearly increasing in the on-state. A representative set of inductor voltage and current waveforms for the nonisolated Cúk converter are shown in Figs. 2.26 and 2.27 [1].

The voltage and current ratio for the nonisolated Cúk converter can be derived by assuming the inductor currents, which correspond to the input current and output current, are ripple-free [1]. This results in an equal charging and discharging of the transfer capacitor during the off-state and the on-state. The charging and discharging are defined in Eq. (2.20) in terms of the product of current and time [1].

$$I_{L_1} t_{\text{off}} = I_{L_2} t_{\text{on}} \tag{2.20}$$

The resulting current ratio is expressed in Eq. (2.21) by substituting $I_{L_1} = I_i$, $I_{L_2} = I_o$, $t_{\text{off}} = (1 - D)T_s$, and $t_{\text{on}} = DT_s$ into Eq. (2.20) [1].

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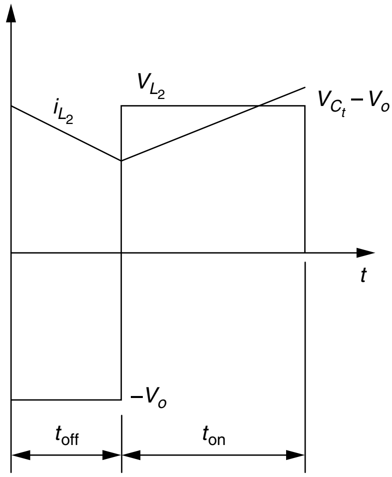

FIGURE 2.27 Inductor 2, voltage and current waveforms for Cúk converter. (From Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995. With permission from John Wiley & Sons.)

$$\frac{I_o}{I_i} = \frac{1-D}{D} \tag{2.21}$$

If the input power is equal to the output power for the ideal case, the voltage ratio in Eq. (2.22) is determined as the inverse of the current ratio using the analysis of an ideal transformer [1].

$$\frac{V_o}{V_i} = \frac{D}{1-D} \tag{2.22}$$

Practical Cúk Converter

The advantages of the practical isolated Cúk converter discussed earlier are the integrated magnetic structure, reduced ripple currents, and reduced switching losses. With the use of a single transformer to provide isolation and the two inductors required in the circuit, the ripple in the inductor currents is essentially reduced to zero. This reduces the amount of external filtering required, but the transfer capacitor carries the ripple from both inductors. This requires a transfer capacitor with a large ripple current capacity. For futher information and a more-detailed analysis of the practical Cúk converter, see Refs. 2 and 3.

References

1. Mohan, N., Undeland, T. M., and Robbins, W. P., *Power Electronics: Converters, Applications, and Design,* 2nd ed., John Wiley & Sons, New York, 1995, chap. 7.
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2.6 Buck–Boost Converters

Daniel Jeffrey Shortt

A schematic of the buck–boost converter circuit (in one of its simplest forms) is shown below in Fig. 2.28. The main power switch is shown to be a bipolar transistor, but it could be a power MOSFET, or any

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other device that could be turned on (and off) in a controlled fashion. This converter processes the power from a DC-biased source (high-voltage ripple) to a DC output (containing low-voltage ripple). The DC output voltage value can be chosen to be higher or lower than the input DC voltage. *Note*: The output load is represented by a resistor, R_L, but in real life can be something much more complicated. In a general sense, this circuit processes power from input to output with "square wave" technology, that is, the circuit produces waveforms that have sharp edges (such as those shown in Fig. 2.29). (There are converters that develop sine waves and semi-sine waves in the power process. They will not be discussed here.) The waveforms in Fig. 2.29 have a square-wave (or semi-square-wave) appearance and are indicative of current waveforms in a typical DC-DC converter. In fact, the i_L waveform is in a similar shape as the inductor (L) current in the buck–boost converter of Fig. 2.28, i_D can represent the diode current, and i_C, the capacitor current.

The operation of this converter is nonlinear and discrete; however, it can be represented by a cyclic change of power stage topologies. The three topologies for this converter, the equations for those topologies, and the small-signal transfer functions are presented in this section. For specific details of the derivation of each of these items, see the technical articles and papers listed in the References.

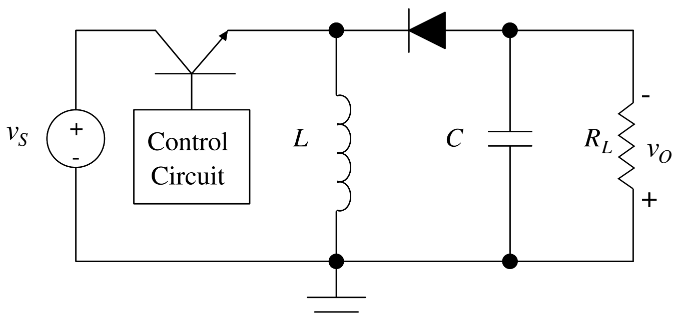

FIGURE 2.28 Buck–boost converter.

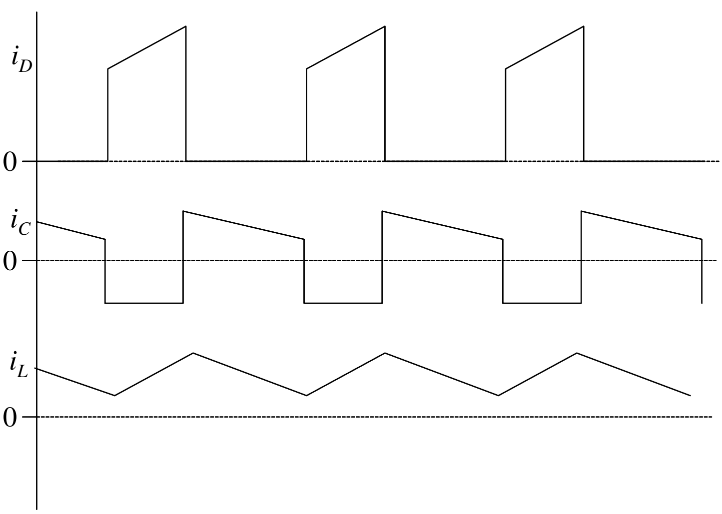

FIGURE 2.29 Typical current waveforms in a buck–boost converter.

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Circuit Analysis

The buck–boost converter has cyclic changes in topology due to the switching action of the semiconductor devices. During a cycle of operation, the main power switch is turned on and off; the diode responds to this by switching off and on.

Continuous-Current Mode

Figure 2.30 illustrates the topology where the main power switch is on and the diode is reverse-biased; thus, it is off. For the purpose of illustration the semiconductor devices are assumed to be ideal.

There are two independent state variables that contain the information describing the operation of this circuit: the inductor current, i_L, and the capacitor voltage, v_C. Two differential equations in terms of these variables, the output voltage, v_O, and the source voltage, v_S, for the designated Topology 1 are shown below.

$$\frac{di_L}{dt} = \frac{v_S}{L} \tag{2.23}$$

$$\frac{dv_C}{dt} = \frac{v_O}{R_L C} \tag{2.24}$$

Please note that the inductor is receiving energy from the source and being charged up, while the capacitor is being discharged into the output load, R_L, and the output voltage is falling.

Figure 2.31 shows the change in topology when the main power switch turns off. The inductor maintains current flow in the same direction so that the diode is forward-biased. The differential

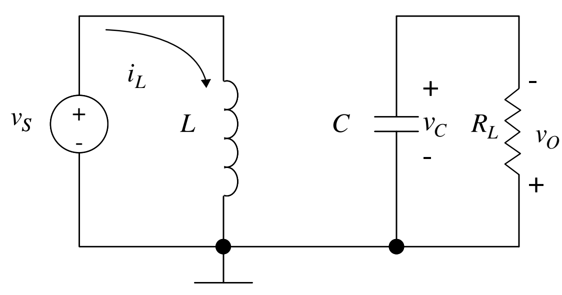

FIGURE 2.30 Topology 1 for the buck–boost converter.

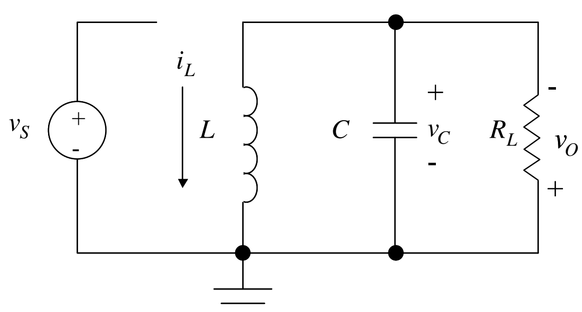

FIGURE 2.31 Topology 2 for the buck–boost converter.

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equations for the designated Topology 2 are shown below. Please note that the inductor is transferring the energy it has obtained from the source into the capacitor; the capacitor is being charged up as the inductor is being discharged, and the output voltage is rising.

$$\frac{di_L}{dt} = -\frac{v_C}{L} \tag{2.25}$$

$$\frac{dv_C}{dt} = \frac{i_L}{C} + \frac{v_O}{R_L C} \tag{2.26}$$

Another topology change will occur if the inductor has transferred all of its energy out into the capacitor. In that case the inductor current will fall to zero. This will be examined later in the section. The inductor current is assumed to be nonzero.

These four linear time-invariant differential equations describe the state of the buck–boost converter. The power stage analysis is linear for each interval; however, for the complete operational cycle, it becomes a piecewise linear problem. The on-time or off-time of the main power switch may vary from cycle to cycle, further complicating the analysis.

Various modeling schemes have been proposed using nonlinear techniques that would in essence "combine" these equations. Basically there are two approaches: numerical (universal) and analytical (mathematical) techniques [1, 2]. In analytical techniques, a closed-form expression representing the operation of the converter is obtained, enabling a qualitative analysis to be performed [1]. The numerical techniques use various algorithms to produce an accurate quantitative result. However, simple relations among the system parameters are not easily obtainable. Numerical techniques are not to be considered at this time, because the desire at this point is to obtain a closed-form solution from which a considerable amount of design insight can be obtained.

Analytical techniques can be divided into two different system descriptions, discrete and continuous. The discrete system description makes no simplifying assumption on the basis of converter application. This description could be used in any application where the linearization of a periodically changing structure is sought. This method is accurate, but very complicated. The derived expressions are complex and cumbersome, which impedes its practical usefulness, and physical insight into the system operation is not easily obtainable.

An important continuous analytical technique is the averaging technique by Wester and Middlebrook [3]. It is easy to implement and gives physical insight into the operation of a buck–boost converter. Through circuit manipulation, analytical expressions were derived to determine the appropriate expressions. Middlebrook and Cúk [4, 5] modified the technique to average the state space descriptions (variables) over a complete cycle. Shortt and Lee [6–8] used a discrete sample of the average state space representation to develop a modeling technique that would enable a judicious control selection to be made. Vorpérian et al. [9] developed an equivalent circuit model for a pulse width modulation (PWM) switch that can be used in the analysis of this converter.

For the averaging technique each interval in the cycle is described by its state space representation (differential equation). Figure 2.32 shows the waveform of the continuous, instantaneous inductor current (that is,

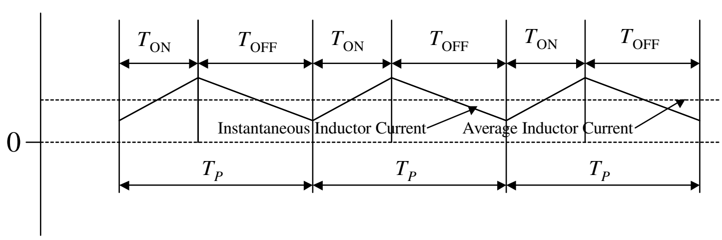

FIGURE 2.32 Continuous inductor current.

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i_L does not equal zero at any point in time) and the average inductor current for the buck–boost converter (Fig. 2.30). The instantaneous current is cyclic with a time period equal to T_P s; the main power switch is on for T_{ON} s and off for T_{OFF} s. The equations are averaged to give a single period representation, as shown below:

$$\dot{i}_L = -d'\frac{v_C}{L} + d\frac{v_S}{L} \tag{2.27}$$

$$\dot{v}_C = d'\frac{i_L}{C} + \frac{v_O}{R_L C} \tag{2.28}$$

where i_L and v_C are average state variables, $d = T_{ON}/T_P$ and $d' = T_{OFF}/T_P$. Please note: $d + d' = 1$.

To study the small signal behavior, the time-varying system described in Eqs. (2.27) and (2.28) can be linearized using small signal perturbation techniques. By using these techniques, the inputs are assumed to vary around a steady-state operating point. Taking a first-order Fourier series approximation, the inputs are represented by the sum of a DC or steady-state term and an AC variation or sinusoidal term. Introducing variations in the line voltage and duty cycle by the following substitutions

$$v_S = V_S + \hat{v}s, \qquad d = D + \hat{d}, \qquad d' = D' - \hat{d}$$

cause perturbations in the state and output, as shown below. In the above and following equations the variables in capital letters represent the DC or steady-state term; and the variables with the symbol "ˆ" above them represent the AC variation or sinusoidal term.

$$\dot{i}_L = \dot{I}_L + \dot{\hat{i}}_L, \qquad i_L = I_L + \hat{i}_L, \qquad \dot{v}_C = \dot{V}_C + \dot{\hat{v}}_C, \qquad v_C = V_C + \hat{v}_c, \qquad v_O = V_O + \hat{v}_O$$

Figure 2.33 shows the type of change that is being modeled for an inductor current perturbation of Fig. 2.32. Note the T_{ON} and T_{OFF} slowly change from cycle to cycle, which produces a slight change in the inductor current from cycle to cycle.

The derivative of a DC term is zero, so the above equations can be simplified to the following:

$$\dot{i}_L = \dot{\hat{i}}_L, \qquad i_L = I_L + \hat{i}_L, \qquad \dot{v}_C = \dot{\hat{v}}_C, \qquad v_C = V_C + \hat{v}_c, \qquad v_O = V_O + \hat{v}_O$$

Substituting these equations into (2.27) and (2.28), separating the DC (steady-state) terms and the AC (sinusoidal) terms results in the following:

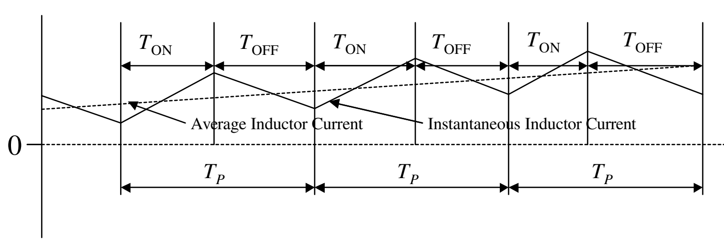

FIGURE 2.33 Inductor current perturbation.

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DC terms:

$$\frac{D'V_C}{L} + \frac{DV_S}{L} = 0 \tag{2.29}$$

$$\frac{D'I_L}{C} + \frac{V_O}{R_L C} = 0 \tag{2.30}$$

AC terms (neglecting the higher-order terms):

$$\dot{\hat{i}}_L = \frac{D'\hat{v}_C}{L} + \frac{D\hat{v}_S}{L} + \frac{V_S - V_C}{L}\hat{d} \tag{2.31}$$

$$\dot{\hat{v}}_C = \frac{D'}{C}\hat{i}_L + \frac{\hat{v}_O}{R_L C} + \frac{I_L}{C}\hat{d} \tag{2.32}$$

The equation

$$\frac{V_C}{V_S} = -\frac{D}{D'}$$

is derived from Eq. (2.29). Note that from Fig. 2.28, $v_C = -v_O$, giving $V_C = -V_O$ and $\hat{v}_C = -\hat{v}_O$; substituting this into the previous equation results in:

$$\frac{V_O}{V_S} = \frac{D}{D'} \tag{2.33}$$

Equation (2.33) states that the ratio of the DC output voltage to the DC input voltage is equal to the ratio of the power switch on-time to the power switch off-time. The expression for the DC inductor current term is

$$I_L = -\frac{V_O}{D'R_L} \tag{2.34}$$

Equations (2.31) and (2.32) constitute the small signal model of a buck–boost converter.

Another method that is utilized to extract the small signal model is to realize an equivalent circuit model from Eqs. (2.27) and (2.28). Figure 2.34 is the average circuit model of the buck–boost converter.

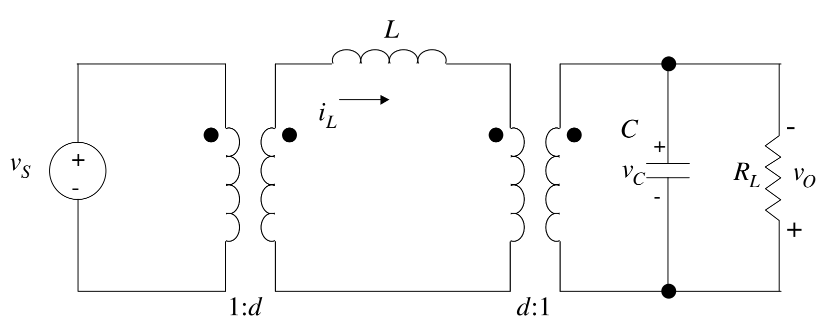

FIGURE 2.34 Average circuit model of the buck–boost converter.

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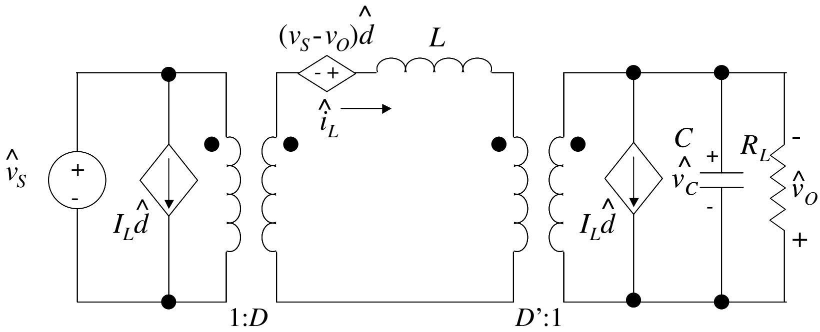

FIGURE 2.35 The small signal circuit model.

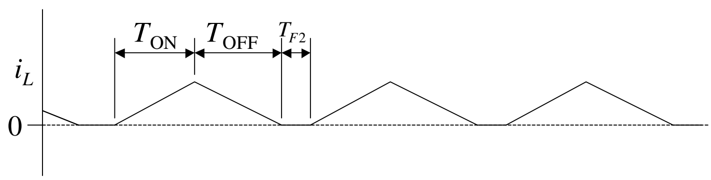

FIGURE 2.36 Discontinuous inductor current.

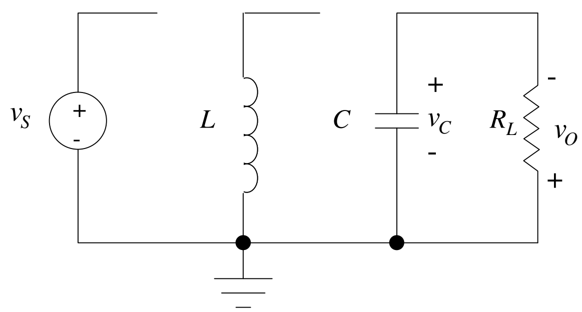

FIGURE 2.37 Topology 3 for the buck–boost converter (discontinuous inductor current).

(For a quantitative, numerical analysis, this circuit can be simulated with SPICE or an equivalent simulation package, as demonstrated in Ref. 10.)

Introducing perturbations into the state and output, removing the DC conditions, neglecting the small nonlinear terms, and simplifying the structure, results in Fig. 2.35.

Discontinuous-Current Mode

Figure 2.36 shows the waveform of the discontinuous inductor current for the buck–boost converter (Fig. 2.30). Note that the inductor current is equal to zero for T_{F2}s. This results in an additional (third) topology change, shown in Fig. 2.37.

Since the inductor current is zero for this portion of the switching cycle, there is only one state equation that can be determined.

$$\frac{dv_C}{dt} = \frac{v_O}{R_L C} \tag{2.35}$$

This equation indicates that the capacitor is now discharging its energy into the load resistor, R_L, and the output voltage is falling.

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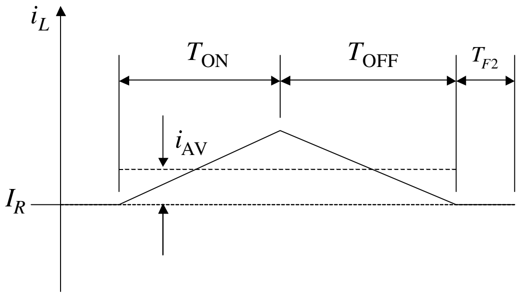

FIGURE 2.38 General form of discontinuous inductor current.

The modeling of this particular mode is presented in Refs. 5,11, and 12. A general discussion is provided here as it applies to the model of the buck–boost converter in the discontinuous inductor current mode.

For this case, the inductor current does not behave as a true state variable, since $di_L/dt = 0$, thereby reducing the system order by one. Figure 2.38 illustrates the general form of the inductor current. The equations for the T_{on} time interval are the same as Eqs. (2.23) and (2.24), except $i_L = I_R + i_L^*$, where I_R represents the DC level at which the inductor current begins and i_L^*, the value of the time-varying inductor current. The equations for the T_{OFF} interval are the same as Eqs. (2.25) and (2.26) except $i_L = I_R + i_L^{**}$, where i_L^{**} represents the value of the time-varying inductor current. By combining these sets of equations with Eq. (2.31) by the averaging technique, the equations listed below are obtained.

$$\frac{di_L}{dt} = \frac{1}{T_P}\int_0^{T_{ON}} \frac{v_S}{L}\,dt + \frac{1}{T_P}\int_{T_{ON}}^{T_{ON}+T_{OFF}} \left(\frac{-v_C}{L}\right) dt = 0 \tag{2.36}$$

$$\frac{dv_C}{dt} = \frac{1}{T_P}\int_0^{T_{ON}} \left(-\frac{v_C}{R_L C}\right) dt + \frac{1}{T_P}\int_{T_{ON}}^{T_{ON}+T_{OFF}} \left(\frac{I_R + i_L^*}{C} - \frac{v_C}{R_L C}\right) dt + \frac{1}{T_P}\int_{T_{ON}+T_{OFF}}^{T_{ON}+T_{OFF}+T_{F2}} \left(-\frac{v_C}{R_L C}\right) dt \tag{2.37}$$

For the buck–boost converter case $I_R = 0$; also, from Fig. 2.38, note that

$$\int_{T_{ON}}^{T_{ON}+T_{OFF}} i_L^*\,dt = \left(\frac{1}{2}\frac{v_S}{L}T_{ON}\right)T_{OFF} = i_{AV}T_{OFF} \tag{2.38}$$

The variable i_{AV} is the average value of the inductor during the $T_{ON} + T_{OFF}$ time, not for the whole cycle. Substituting into Eqs. (2.36) and (2.37) results in the following:

$$\frac{dv_C}{dt} = -\frac{T_{ON}}{T_P}\frac{v_C}{R_L C} + \frac{T_{OFF}}{T_P}\frac{i_{AV}}{C} - \frac{T_{OFF}}{T_P}\frac{v_C}{R_L C} - \frac{T_{F2}}{T_P}\frac{v_C}{R_L C} \tag{2.39}$$

Let

$$d_1 = \frac{T_{ON}}{T_P}, \qquad d_2 = \frac{T_{OFF}}{T_P}, \qquad d_3 = \frac{T_{F2}}{T_P}$$

and substitute into the above equation.

$$\dot{v}_C = d_2\frac{i_{AV}}{C} - \frac{v_C}{R_L C} \tag{2.40}$$

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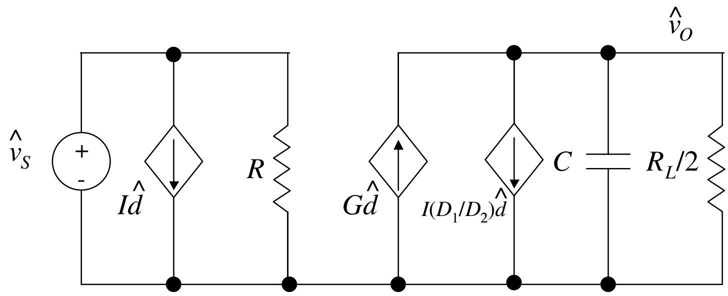

FIGURE 2.39 Buck–boost converter small signal model for the discontinuous mode.

Note that, $d_1 + d_2 + d_3 = 1$ and

$$i_{\text{AV}} = \frac{1}{2}\frac{v_s}{L}T_{\text{ON}} \tag{2.41}$$

At this point, the same perturbation techniques, as presented previously, are used to obtain the small signal model. Introducing variations in the line voltage and duty cycle

$$v_S = V_S + \hat{v}_S, \qquad d_1 = D_1 + \hat{d}_1, \qquad d_2 = D_2 + \hat{d}_2, \qquad d_3 = D_3 + \hat{d}_3$$

produce perturbations in the state and output; separating the DC and AC terms and simplifying results in

$$\dot{v}_C = -\frac{\hat{v}_C}{R_L C} + \frac{V_S T_{\text{OFF}}}{2LC}\hat{d}_1 + \frac{V_S T_{\text{ON}}}{2LC}\hat{d}_2 + \frac{T_{\text{ON}}T_{\text{OFF}}}{2LCT_P}\hat{v}_S \tag{2.42}$$

where

$$T_{\text{ON}} = \frac{V}{V_S}\sqrt{\frac{2LT_P}{R_L}} \tag{2.43}$$

$$T_{\text{OFF}} = \sqrt{\frac{2LT_P}{R_L}} \tag{2.44}$$

$$\frac{V_C}{V_S} = \frac{D_1}{D_2} \tag{2.45}$$

A circuit model (Fig. 2.39) can be realized from the above equations. The process is not shown here; however, please see Ref. 5 for the details of the circuit derivation and presentation. This concludes the circuit analysis portion of this section. In the next section the appropriate transfer functions to be used in the design and implementation of the buck–boost converter are presented. The above small signal model is used to derive them. For more detail, please see Refs. 3, 5, and 9.

Small Signal Transfer Functions

The analysis done in the previous section enables the development of transfer functions that describe the buck–boost converter stability performance and input to output signal attenuation. The transfer functions are illustrated in Fig. 2.40. This figure assumes there is only one feedback (the output voltage) loop; for more complicated feedback schemes, please see Refs. 7 and 8.

The continuous-current mode transfer functions are derived by using the Laplace transform to solve for the output voltage and duty cycle variations in Eqs. (2.31) and (2.32). Equation (2.42) is used to derive the discontinuous-current mode transfer functions.

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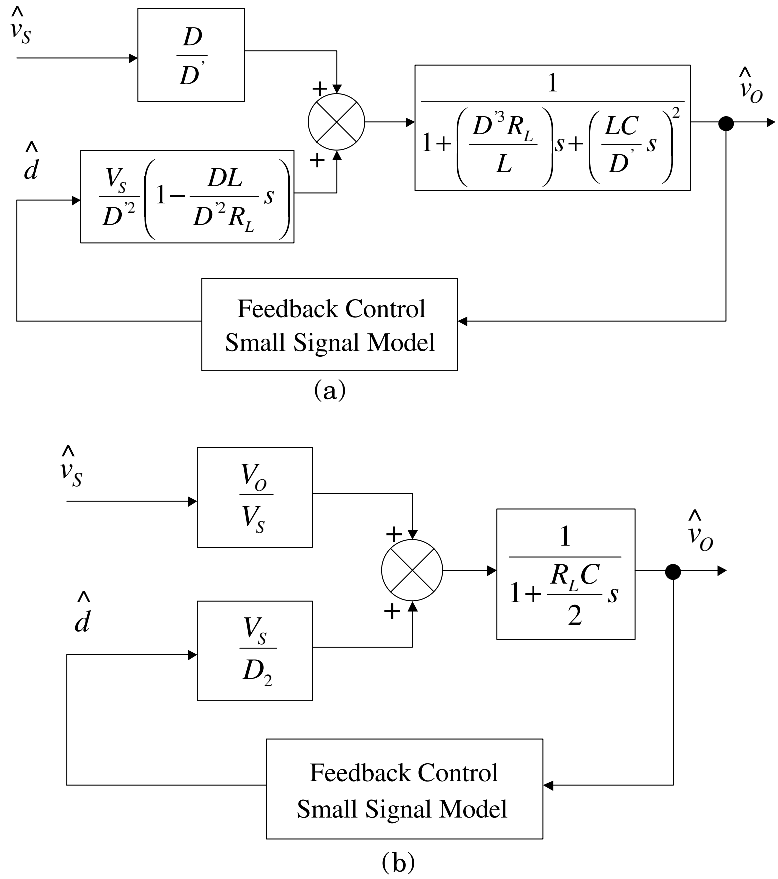

FIGURE 2.40 Block diagram of transfer functions for the buck–boost converter: (a) continuous-current mode; (b) discontinuous-current mode.

Component Selection

The component values can be chosen based on several constraints. The constraints that are to be discussed here are not exhaustive, but are only mentioned to provide an introduction into the selection process. Some component values can be based on an arbitrary selection. For example, the frequency of the converter is the designer's choice. As the frequency rises, the volume of the inductor (which is usually the biggest component in the converter) decreases and its temperature rises. The component value can be selected based on a given frequency value, which is assumed to be optimized based on the previously mentioned constraints. However, the frequency value can also be chosen based on experience. There is to be no discussion on the optimization of the switching frequency in this section; please see Ref. 13 for a detailed explanation of the process to optimize the converter switching frequency. Thus, an assumption made at this point is that the switching frequency has been selected.

Inductor Value

The inductor has to be large enough to handle the output power, according to the energy transfer equation shown below.

$$\frac{1}{2} L i_{\text{peak}}^{2} = P T_P \tag{2.46}$$

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where, i_{peak} is the maximum value of the inductor current, P is the output power, and T_P is the time period of the switching cycle. If the desire for the current is to be continuous, then the inequality shown below must be satisfied.

$$\frac{2L}{R_L} > \frac{T_P}{\left(\dfrac{V}{V_S} + 1\right)^2} \tag{2.47}$$

The inequality (2.47) is derived from Eqs. (2.43) and (2.44). The total of those equations has to be greater than the cycle time, T_P, for the converter to be in the continuous-current mode. If the designer desires the converter to be in the discontinuous mode, then the inequality sign in (2.47) is reversed so that $T_{\text{ON}} + T_{\text{OFF}}$ is less than T_P.

Satisfying the above two constraints, (2.46) and (2.47), should provide an inductor that is minimal, but probably not optimal. Using a circuit simulation package, such as PSpice, to simulate and check the converter action can help determine an optimal value.

Capacitor Value

The capacitor value is chosen based on the specified ripple voltage, V_{PP}, the switching frequency (actually the T_{OFF} for the buck–boost converter), and the allowable capacitor ripple current, $i_{\text{allowable}}$. The following inequality describes the relationship of the previously mentioned items:

$$C > \frac{i_{\text{allowable}}}{V_{pp}} T_{\text{OFF}} \tag{2.48}$$

As with the inductor value, this constraint provides a minimal capacitor value, but probably not an optimal one. Any value that is chosen should be used in a simulation to test the value for feasibility. The latent assumption made here is that the capacitor is an ideal one. In actuality, a practical capacitor can be modeled as a linear combination of resistors, inductors, and capacitors. This complicates the previously discussed models greatly. The equivalent series resistance (ESR) and the equivalent series inductance (ESL) (Fig. 2.41), probably have the biggest influence on the effective capacitance, because of their effect on the capacitor ripple voltage. Both, in general, tend to raise ripple voltage. This may require an iteration involving a simulation using the catalog or given values for the ESR and ESL in a more realistic model of the capacitor.

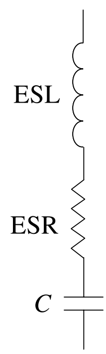

FIGURE 2.41 A practical capacitor model.

Main Power Switch and Output Power Diode

The main switching transistor and diode should be chosen based on the inductor current peak value. As with all of these components, the final component value selection should have appropriate design margins. These margins, however, do vary with the scope of the mission of the individual converter.

The main power switch function is to provide a path for the inductor to receive energy from the source; that is, the switch connects the source to the inductor at the appropriate time in the switching cycle. The switch can dissipate a significant amount of power if not chosen properly or not connected to an appropriately designed heat sink. So, in addition to ensuring that the switch can handle the peak current and voltage values, the power dissipation must be checked. For a bipolar transistor, assuming the efficiency

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of the converter is very high, the on-state power dissipation can be expressed as the following:

$$P_{\text{DISS}} = V_{\text{CEsat}}\left(\frac{D}{D'}I_O\right) \tag{2.49}$$

where $I_O = P/V_O$. If a MOSFET device is chosen, the on-state dissipation is the following:

$$P_{\text{DISS}} = \left(\frac{D}{D'}I_O\right)^2 R_{DS(\text{on})} \tag{2.50}$$

The output power diode provides the path for the inductor to discharge its energy to the output; it connects the inductor to the output when the main power switch is off. Its voltage drop is primarily responsible for power dissipation. If V_d is the on-state voltage drop of the diode, then its power dissipation is expressed as

$$P_{\text{DISS}} = V_D I_O \tag{2.51}$$

A judicious selection for the diode can be made using the above calculated power value, the peak output current, and output voltage.

Flyback Power Stage

A popular version of the buck–boost converter, shown in Fig. 2.28, is the variation shown in Fig. 2.42, the flyback converter. The flyback converter provides isolation from input to output: note the output voltage is not inverted as in the simpler buck–boost converter version of Fig. 2.28. These things are accomplished because of the two-winding or coupled inductor. The inductor now serves a dual purpose: it transfers energy from the source to the output and provides input to output voltage. This is a popular power stage used in off-line (110 V_{AC} or 220 V_{AC}) applications, particularly with multiple output voltages. Power diodes, capacitors, and windings on the two-winding inductor (power transformer) are added in the appropriate fashion to provide additional outputs.

The process discussed previously can be used to determine the small signal model and DC operating point of this converter. The state variables for this converter are the capacitor voltage, v_C, and the flux density, ϕ, of the two-winding inductor:

$$N_P\frac{d\phi}{dt} = L_P\frac{di}{dt} \quad \text{and} \quad N_S\frac{d\phi}{dt} = L_S\frac{di}{dt}$$

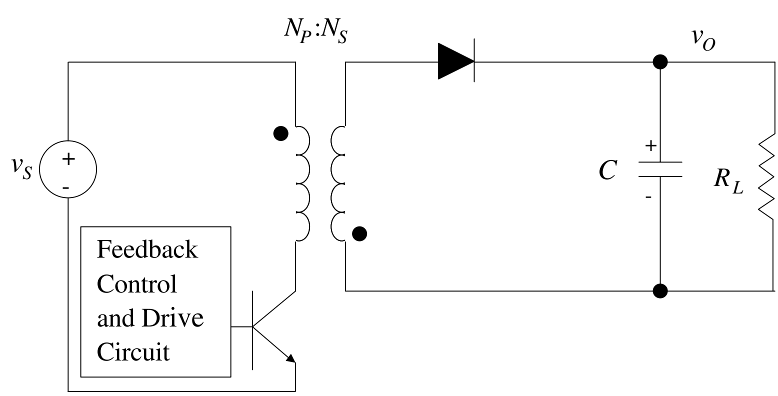

FIGURE 2.42 Flyback power converter.

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The input voltage can be expressed as $(N_S/N_P)v_S$, instead of just v_s. Making these substitutions allows the development of the small signal model that is shown below.

The continuous-current mode small signal model is

$$\dot{\hat{i}}_L = \frac{D'\hat{v}_C}{L} + \frac{D\frac{N_S}{N_P}\hat{v}_S}{L} + \frac{\frac{N_S}{N_P}V_S - V_C}{L}\hat{d} \tag{2.52}$$

$$\dot{\hat{v}}_C = \frac{D'}{C}\frac{N_S}{L_S}\hat{\phi} + \frac{\hat{v}_O}{R_L C} + \frac{\frac{N_S}{L_S}\Phi}{C}\hat{d} \tag{2.53}$$

with

$$\frac{V_O}{V_S} = \frac{N_S}{N_P}\frac{D}{D'} \tag{2.54}$$

and

$$\Phi = -\frac{V_O}{D'R_L}\frac{L_S}{N_S} \tag{2.55}$$

The discontinuous-current mode small signal model is

$$\dot{v}_C = -\frac{\hat{v}_C}{R_L C} + \frac{\frac{N_S}{N_P}V_S T_{\mathrm{OFF}}}{2L_S C}\hat{d}_1 + \frac{\frac{N_S}{N_P}V_S T_{\mathrm{ON}}}{2L_S C}\hat{d}_2 + \frac{T_{\mathrm{ON}}T_{\mathrm{OFF}}}{2L_S C T_P}\frac{N_S}{N_P}\hat{v}_S \tag{2.56}$$

with

$$T_{\mathrm{ON}} = \frac{V}{\frac{N_S}{N_P}V_S}\sqrt{\frac{2L_S T_P}{R_L}} \tag{2.57}$$

$$T_{\mathrm{OFF}} = \sqrt{\frac{2L_S T_P}{R_L}} \tag{2.58}$$

$$\frac{V_C}{V_S} = \frac{N_S}{N_P}\frac{D_1}{D_2} \tag{2.59}$$

Summary

This section has presented and analyzed a buck–boost converter (Fig. 2.28). The topological changes have been presented and a discussion of the state space has shown a modeling process (averaging), which can be used to design the converter. This process models a linear time-varying structure in a relatively simple way so that significant design insight can be obtained. The small signal model that was presented can be used to analyze the stability and the input-to-output signal attenuation of the buck–boost converter.

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