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Step Motor Drives

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14.1 Introduction

Step motors are used in many low-lost positioning applications due to their inherent ability to stop at discrete positions and follow position vs. time profiles while being controlled open loop. A step motor is a synchronous machine, but historically has been used almost exclusively in positioning and position tracking applications. Recently, however, some types of step motors have been applied in variable speed drives applications.

Step motors can be driven without feedback to stop at discrete angular positions, known as detent positions. The number of detent positions can be as low as 12 steps or detent positions per revolution to 400 or 500 or more steps per revolution. The location accuracy of the detent positions varies typically within 5% of the step size. The repeatability of the motor is also high, in that the rotor can start on one position, move away to other positions, and then return to within typically 3% of step size of the original position.

The motors with relatively few steps are typically used in higher-speed applications, where the motors with many steps per revolution are often used in high-torque, low-speed, direct drive applications or applications where many repeatable discrete positions are required. Step motors have been successfully applied in many applications such as computer peripherals (e.g., disk drives, pen plotters, and printers), office machines (e.g., copiers, scanners), automotive (e.g., seat positioning, speed control), aerospace (e.g., flap control, starter-generators), and industrial (e.g., robots, scanners, machine tools), to name a few.

Many step motor drives are driven with digital pulses, thus it is easy to interface and control step motors from computers or microcontrollers without digital to analog circuitry. For control purposes, the step motor and drive can be thought of as a digital to angular position converter.

It is perhaps easier to understand how a step motor works than any other rotating machine. However, the mathematical models of step motors are nonlinear, since the inductance and torque vary sinusoidally with position. The non-linear nature of the models requires that the engineer carefully design the controllers for the motors. In the following section the three types of step motors are discussed along with the operation and drive circuits of each. The mathematical models of each type are discussed in the second section. In the third section, the control of step motors is presented.

14.2 Types and Operation of Step Motors

The three common types of step motors are the variable reluctance, the permanent magnet, and the hybrid step motors. The variable reluctance and the hybrid step motors are double salient structures, i.e., teeth on both the stator and the rotor, with multiple windings or phases on the stator and no windings on the rotor (thus brushless machines). The variable reluctance step motors can have three, four, or even five phases, while the permanent magnet and the hybrid step motors usually have two phases.

Principle of Operation: A first understanding of the principle of operation can be easily seen from the variable reluctance step motor, but is common for all types. When a single winding or phase is energized, the motor generates a torque in the direction to align the rotor teeth with the teeth of the energized phase. The torque generated by current in a single phase, for example, phase A, is

$$T_e = -k_T i_A \sin(N_r \theta) \tag{14.1}$$

where T_e is the generated torque, k_T is the torque constant, i_A is the current in phase A, N_r is number of electrical cycles per mechanical revolution, and θ is the mechanical rotor position. The cross-sectional view of a variable reluctance step motor is illustrated in Fig. 14.1, showing only the phase A winding. The rotor is shown in the aligned position, which is the detent position. The generated torque by the current in phase A vs. rotor position for this motor is shown in Fig. 14.2. If a load on the rotor were to displace the rotor in the position θ or clockwise direction, the generated torque would act in the direction to realign the rotor. From Fig. 14.2, we see that a displacement in the positive θ direction generates negative torque, which will push the rotor in the negative direction, thus trying to restore the detent position. If a load on the rotor were to displace the rotor in the negative θ or counterclockwise direction, the generated torque would also act in the direction to realign the rotor. From Fig. 14.2, we see that a displacement in the negative θ direction generates positive torque, which in turn is in the direction to restore the rotor to the detent position.

Now, suppose the current in phase A is deenergized and phase B is energized. It is apparent from Fig. 14.1 that the rotor will rotate 15° in the positive direction so that rotor teeth will align with the phase B stator teeth. Next, suppose that phase B is deenergized and phase C is energized. The rotor will rotate 15° additionally in the positive direction, aligning phase C stator teeth with rotor teeth. One more phase switching, deenergizing phase C and energizing phase A causes the rotor to rotate yet an additional

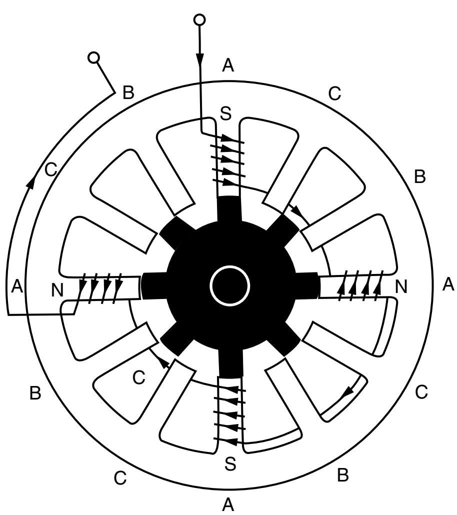

FIGURE 14.1 Cross-sectional view.

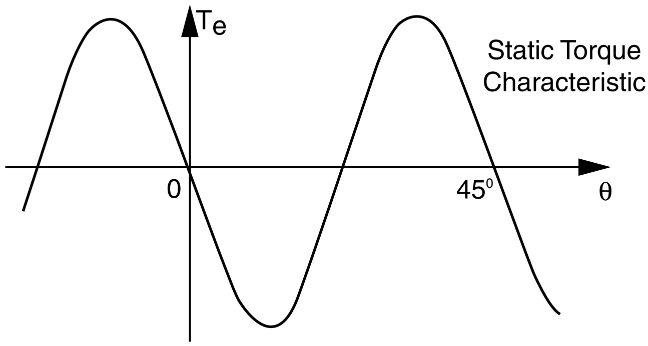

FIGURE 14.2 Static torque characteristic.

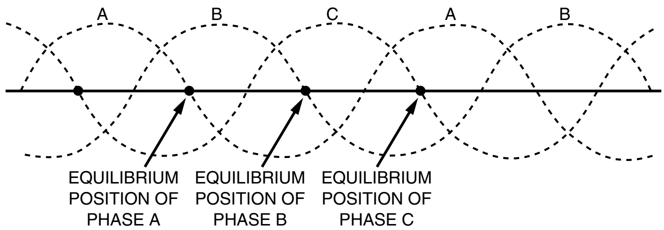

FIGURE 14.3 Static torque characteristics for all three phases.

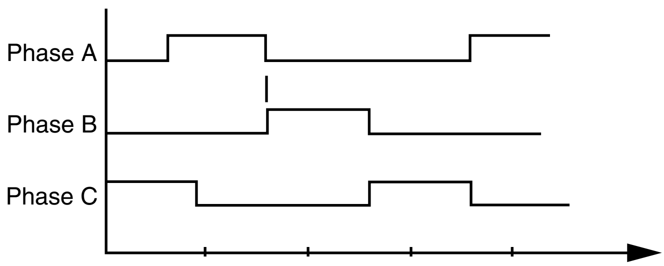

FIGURE 14.4 By exciting the motor windings during the positive portions of their torque curve, the motor can be made to produce nonzero average torque.

15° in the positive direction. Now the rotor has moved a total of 45° in the positive direction. This detent position is at 45° in Fig. 14.2. Detent positions occur where the torque vs. position curve crosses zero torque with negative slope.

The torque vs. position for all three phases of the motor (energized one at a time) are shown in Fig. 14.3. By exciting the motor phases in order during intervals of positive torque, as shown in Fig. 14.4, the motor can be made to run in the positive direction. Conversely, by exciting the motor phases in reverse order during intervals of negative torque, the motor can be made to run in the opposite direction.

Variable Reluctance Step Motor

As mentioned above, the variable reluctance step motors can have three, four, or five phases. The mode of operation discussed above is common to all variable reluctance step motors, and will not be repeated here. The motor discussed above is known as a 12/8 variable reluctance step motor, in that the stator has 12 teeth and the rotor has 8 teeth. This motor takes 15° steps and has 24 steps per revolution. The number of detent positions per revolution can be as low as 12 for a 6/4 motor and as high as 200 or 400 steps

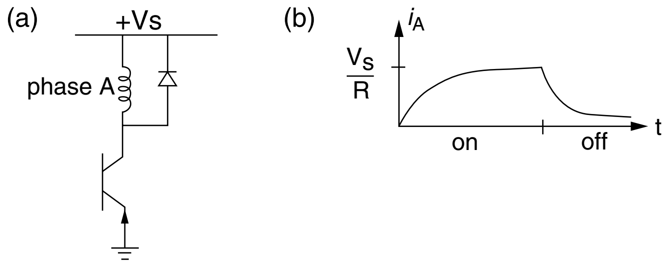

FIGURE 14.5 Drive circuit and phase current for one phase.

per revolution. The size of the motor can range from small fractional hp, with detent torque in the few ounce-inch range to 10 hp to more. The larger variable reluctance step motors are more commonly called switch reluctance motors and are usually used in variable speed applications.

Drive Circuits for Variable Reluctance Step Motors

The drive circuits are quite simple for the variable reluctance step motor. Figure 14.5a shows the simplest drive circuit for one phase (each phase requires its own identical drive circuit). The transistor acts as a switch, either off or on. When the switch is on, the current flows from the supply, through the phase winding, through the switch, to ground. When the switch is off, the current in the winding cannot drop to zero instantaneously due to the winding inductance. A path for the decay current is provided through the diode. The voltage across the winding when the switch is on is the supply voltage, V_s (neglecting the voltage drop across the switch). The current in the phase takes time to reach the value of V_s/R, where R is the phase resistance due to the phase inductance, L. Neglecting the back EMF, the phase current is approximately:

$$i_A \approx \frac{V_s}{R}(1 - e^{-(R/L)t}) \tag{14.2}$$

as shown in Fig. 14.5b. When the switch is off, the diode forms a short-circuit and the current flows through the diode, thus the voltage across the winding is zero (neglecting the voltage drop across the diode). Thus the current decays to zero with the L/R time constant. Once the current decays to zero, neglecting leakages, the winding is open circuited and no current flows.

The drive circuit in Fig. 14.6a can be used for higher-performance operation of the variable reluctance step motor. In this circuit, both transistors act as switches. When both switches are on, the current flows from the supply, through the top switch, through the phase winding, through the bottom switch, to ground. When both switches are off, the current in the winding cannot drop to zero instantaneously due to the winding inductance, so the current flows from ground through the lower diode, through the winding, through the top diode, and back into the supply. The voltage across the winding when both switches are on is the supply voltage (neglecting the voltage drop across the switches). When the switches are off, the voltage across the winding is the negative of the supply voltage (neglecting the voltage drop across the diodes). Thus the current decays toward $-V_s/R$ with the L/R time constant. Once the current decay reaches zero the diodes block and, neglecting leakages, the winding is open circuited and no current flows. The current decay time is much less with this circuit than with the circuit in Fig. 14.5. The current waveform of this operation of this circuit is shown in Fig. 14.6b.

Even greater performance of the variable reluctance step motor can be achieved with the circuit in Fig. 14.6a. V_s is set five to ten times larger than the motor rated voltage and the current is controlled by "chopping" one of the switches on and off. When the phase is energized, the current rises with the L/R time constant towards V_s/R as before, but now V_s/R is five to ten times larger than rated current, so the current reaches rated current much sooner. At this time, one of the two switches is then turned off,

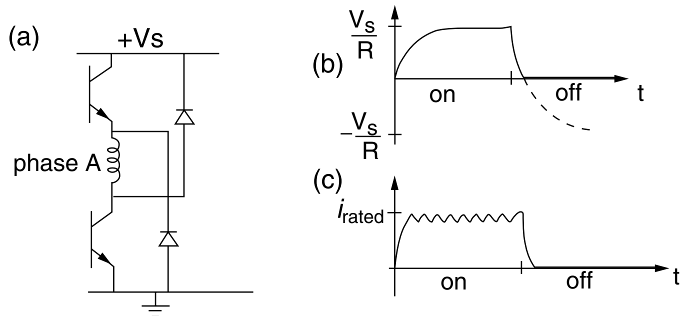

FIGURE 14.6 Drive circuit and current waveforms for higher-performance operation.

allowing the current to decay toward zero. A short time later, the switch is turned on again until current reaches rated current again. This process is repeated until the phase is to be deenergized. The current waveforms for this operation of the circuit is shown in Fig. 14.6c.

Permanent Magnet (Can-Stack) Step Motor

The permanent magnet step motor has a smooth, permanent magnet rotor. The rotor is constructed to have many pairs of magnetic poles. The windings are not wrapped around poles as in the variable reluctance motor, but around the circumference of the air gap. The stator poles are wrapped around the windings to form north and south magnetic poles to attract and repel the magnetic poles on the rotor. Two sets of windings and stator poles are required, with each set of stator poles offset by half a tooth pitch. This motor usually has a low number of steps or detent positions per revolution, and detent positions are less accurate than the other types of motors. The motor does have an unenergized detent torque.

The permanent magnet step motor has two phases, but can be wound in two different ways. If the motor is wound unifilar, that is, one winding per phase, bidirectional currents are required for proper operation. With bifilar windings, that is, two windings or a center tapped winding per phase, unidirectional currents can be used to run the motor.

Hybrid Step Motor

The hybrid step motor can be described as two two-phase (unidirectional) variable reluctance step motors put together with an axially mounted permanent magnet between the rotors. The magnetic flux paths are three-dimensional, aligned axially between the rotor halves and radially in the air gaps. The possible winding configurations are similar to the permanent magnet type motor, either unifilar, requiring bidirectional currents, or bifilar, requiring only unidirectional currents. As with variable reluctance motors, the number of steps per revolution typically range from 24 to 400. As with permanent magnet motors, there is some unenergized detent torque.

Drive Circuitry for Permanent Magnet and Hybrid Step Motors

The drive circuits for the permanent magnet and hybrid step motors are different from the drive circuits for the variable reluctance step motor. The drive circuit also depends on if the motor is wound unifilar or bifilar. Bifilar wound motors require fewer drive circuit components than for the unifilar wound motors, but at most only half the phase winding is energized at one time.

Figures 14.7 and 14.8 show partial drive circuits for unifilar wound step motors. The circuits shown are for only half or one winding of the motor. A second identical circuit is needed for the other motor winding. The drive circuit in Fig. 14.7, known as a half-H-bridge, requires both positive and negative voltage supplies. The transistors act as switches, connecting one end of the phase winding to either the

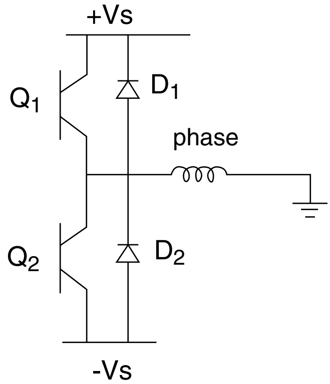

FIGURE 14.7 Half-H-bridge drive circuit.

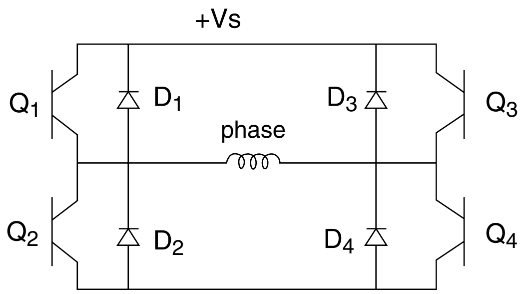

FIGURE 14.8 H-bridge drive circuit.

positive supply voltage, $+V_s$, or the negative supply voltage, $-V_s$. When switch Q_1 is on, the current flows from $+V_s$ through the switch, through the phase winding to ground. When switch Q_1 is off, the current cannot instantaneously drop to zero due to the winding inductance, thus the decay current flows through diode D_2 from $-V_s$ to the winding to ground. Once the current decays to zero, the diode blocks the current and, neglecting leakage, the winding is open circuited. When Q_1 is on, the phase voltage is $+V_s$ (neglecting the voltage drop across the switch). When Q_1 is off, the phase voltage is $-V_s$ (neglecting the voltage drop across the diode) until the current decays to zero, then the phase is open circuited. Similarly, when switch Q_2 is on, the current flows from ground through the phase winding in the opposite direction as before, through the switch to $-V_s$. When switch Q_2 is off the decay current flows through diode D_1 to $+V_s$ from the winding from ground.

The drive circuit in Fig. 14.8, known as an H-bridge, requires only a positive supply voltage. The four transistors act as switches, connecting each end of the phase winding to either the positive supply voltage, $+V_s$, or ground. When switches Q_1 and Q_4 are on, the current flows from $+V_s$ through Q_1, through the phase winding through Q_4 to ground, applying $+V_s$ across the winding. When switches Q_1 and Q_4 are off, the current in the winding cannot instantaneously drop to zero due to the winding inductance, thus the decay current flows through diodes D_2 and D_3, applying $-V_s$ across the winding until the current decays to zero, when the diode blocks the current and, neglecting leakage, the winding is open circuited. Similarly, when switches Q_2 and Q_3 are on, the current flows from $+V_s$ through Q_3, through the phase winding in the opposite direction as before, through Q_2 to ground, applying $-V_s$ across the winding. When switches Q_2 and Q_3 are off, the decay current flows through diodes D_1 and D_4, applying $+V_s$ across the winding until the current decays to zero, when the diode blocks the current and, neglecting leakage, the winding is open circuited.

Higher motor performance can be achieved from the circuit in Fig. 14.8 if the supply voltage is set at five to ten times the motor rated voltage and the phase currents are regulated by chopping either switch

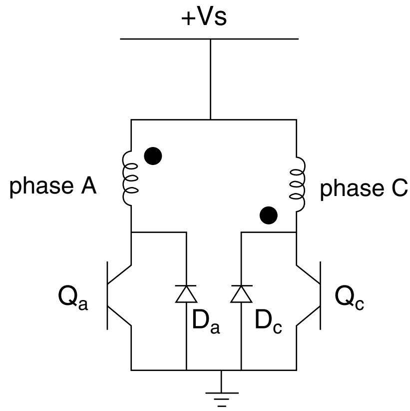

FIGURE 14.9 Inverse diode clamped drive circuit.

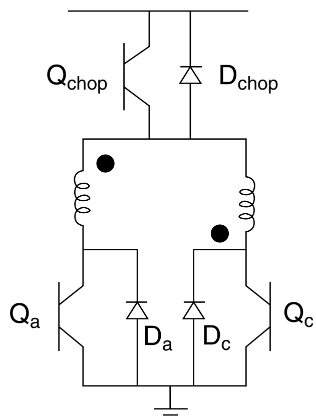

FIGURE 14.10 Drive circuit of Fig. 14.9 as a chopper drive.

Q_1 or Q_3. For example, when Q_1 and Q_4 are on, the phase current rises toward V_s/R with the L/R time constant, where R is the winding resistance and L is the winding inductance. When the phase current reaches rated current, Q_1 is off and the circuit path is through D_2, the phase winding, and Q_4, thus the applied voltage is zero (neglecting the diode and transistor voltage drops), and the current now starts to decay toward zero. A short time later, Q_1 is turned on again and the current builds towards rated current. Once the current reaches rated current, the cycle is repeated until the phase is to be deenergized.

Figures 14.9 and 14.10 show drive circuits for bifilar wound step motors. Both of these circuits are known as inverse diode clamped drive circuits. The circuits shown assume the center tapped winding configuration and are for only one bifilar winding of the motor. A second identical circuit is needed for the other motor winding. It is best to think of a bifilar wound step motor as having four phases, A, B, C, and D, but unlike with the variable reluctance step motor, phases A and C are inversely mutually coupled and phases B and D are inversely mutually coupled.

The drive circuit in Fig. 14.9 assumes that the supply voltage is set to motor rated voltage. The transistors in the circuit act as switches. When Q_a is on, the current flows from the supply through the phase A, through Q_a to ground. When Q_a is off, and since the current in the bifilar winding cannot decay to zero instantaneously, the mutual coupling in the bifilar winding couples the current to phase C, where the current flows from ground up through the diode, D_c, through phase C, into the supply. This applies $-V_s$ across the phase C, causing the current to decay quickly.

The drive circuit in Fig. 14.10 works the same way as the drive circuit in Fig. 14.9 when Q_{chop} is on. The addition of Q_{chop} and the additional diode allows the inverse diode clamp drive circuit to be a chopper drive. The supply voltage is set to five to ten times the motor rated voltage and when either leg of the circuit is on, the current is regulated using Q_{chop}. When Q_{chop} is off, the phase current drops to half of its original value, half of the conducting current couples to the opposite phase, and the current flows up through the clamp diode in the opposite phase, backward through the opposite phase, through the on phase, and through the on phase transistor.

14.3 Step Motor Models

When a constant current is passed through one phase of a step motor, the motor generates a torque. This torque is typically a sinusoidal function of rotor displacement from the detent position that causes the rotor to minimize this displacement. When the phases of the motor are excited so that the motor "runs," the generated torque is still a function of position and current, but the current becomes a varying quantity, dependent on time, position, velocity, and of course, the drive circuit and drive scheme. Selection of a motor, drive circuit, and drive scheme depends on predicting the performance and the dynamic torque–speed characteristics of a particular motor with a drive circuit and drive scheme. These performances of step motors can be predicted to within reasonable accuracy using mathematical models for both the motor and drive circuit. Ways to model the motor and drive circuit are presented in this section. As with modeling most physical systems, more accurate models produce more accurate results. Tradeoffs between accuracy and simplicity are also discussed with each model.

The variable reluctance step motor model needs to be modeled separately from the permanent magnet and hybrid step motor models, and separate models are needed for unifilar and bifilar windings.

Variable Reluctance Step Motor Model

Precise mathematical modeling of variable reluctance step motors requires knowledge of both the geometry of the machine and of the ferromagnetic material characteristics. These requirements are often relaxed and assumptions are made to simplify the model to a set of nonlinear differential equations.

Assumption 1. The ferromagnetic material does not saturate. This is a poor assumption for variable reluctance step motors in that the motors are usually operated with a high degree of saturation. This assumption is replaced after the "non-saturated" model is presented.

Assumption 2. The inductance for each phase varies sinusoidally around the circumference of the air gap, for example, the phase A inductance is $L_A(\theta) = L_0 + L_1 \cos(N_r\theta)$. This assumption required Assumption 1, otherwise L_A is a function of both θ and i_A.

The terminal voltage for phase A can be found using Faraday's law as:

$$V_A = R_A i_A + \frac{d\lambda_A}{dt} \tag{14.3}$$

where V_A is the terminal voltage, R_A is the winding resistance, i_A is the winding current, and λ_A is the phase flux linkages. Since $\lambda_A = L_A i_A$:

$$\frac{d\lambda_A}{dt} = L_A\frac{di_A}{dt} + i_A\frac{dL_A}{dt} = L_A\frac{di_A}{dt} - L_1 i_A \omega N_r \sin(N_r\theta) \tag{14.4}$$

where the first term is the magnetizing voltage and the second term is the speed voltage. Equation (14.6) can be rewritten as

$$\frac{di_A}{dt} = \frac{1}{L_A}V_A - \frac{R_A i_A}{L_A} + \frac{L_1 N_r}{L_A} i_A \omega \sin(N_r\theta) \tag{14.5}$$

The differential equations for the remaining phases are the same as the above equations, replacing the subscripts with the appropriate phase letter. The inductances for the other phases, however, need to be shifted in position. For a three-phase motor, the inductances are

$$
\begin{aligned}
L_B(\theta) &= L_0 + L_1 \cos\left(N_r\theta - \frac{2\pi}{3}\right) \\
L_C(\theta) &= L_0 + L_1 \cos\left(N_r\theta - \frac{4\pi}{3}\right)
\end{aligned}
\tag{14.6}
$$

For a four-phase motor, the inductances are

$$
\begin{aligned}
L_B(\theta) &= L_0 + L_1 \cos\left(N_r\theta - \frac{\pi}{2}\right) \\
L_C(\theta) &= L_0 + L_1 \cos(N_r\theta - \pi) \\
L_D(\theta) &= L_0 + L_1 \cos\left(N_r\theta - \frac{3\pi}{2}\right)
\end{aligned}
\tag{14.7}
$$

The mechanical equations can be found from Newton's law and conservation of energy. Newton's law states

$$
J\frac{d\omega}{dt} = T_e - T_L - B\omega \tag{14.8}
$$

where J is the rotor and load moment of inertia, ω is the rotor velocity in mechanical radians per second, T_e is the torque generated by the motor, T_L is the load torque, and B is the rotor and load viscous friction coefficient. Using conservation of energy, the torque generated by i_A for the variable reluctance step motor assuming no magnetic saturation is

$$
T_A = -\frac{L_1 N_r}{2} \sin(N_r\theta)\, i_A^2 \tag{14.9}
$$

where T_A is the torque generated by the current in phase A.

Summarizing, the differential equations for the three-phase variable reluctance step motor are

$$
\begin{aligned}
\frac{di_A}{dt} &= \frac{1}{L_A}V_A - \frac{R_A}{L_A}i_A + \frac{L_1 N_r}{L_A} i_A\omega \sin(N_r\theta) \\
\frac{di_B}{dt} &= \frac{1}{L_B}V_B - \frac{R_B}{L_B}i_B + \frac{L_1 N_r}{L_B} i_B\omega \sin\left(N_r\theta - \frac{2\pi}{3}\right) \\
\frac{di_C}{dt} &= \frac{1}{L_C}V_C - \frac{R_C}{L_C}i_C + \frac{L_1 N_r}{L_C} i_C\omega \sin\left(N_r\theta - \frac{4\pi}{3}\right) \\
\frac{d\omega}{dt} &= \frac{T_e}{J} - \frac{T_L}{J} - \frac{B}{J}\omega \\
\frac{d\theta}{dt} &= \omega
\end{aligned}
\tag{14.10}
$$

where

$$T_e = -\frac{L_1 N_r}{2}\left[\sin(N_r\theta)i_A^2 + \sin\left(N_r\theta - \frac{2\pi}{3}\right)i_B^2 + \sin\left(N_r\theta - \frac{4\pi}{3}\right)i_C^2\right] \tag{14.11}$$

The differential equations for the four-phase variable reluctance step motor are

$$\begin{aligned}
\frac{di_A}{dt} &= \frac{1}{L_A}V_A - \frac{R_A}{L_A}i_A + \frac{L_1 N_r}{L_A}i_A\omega\sin(N_r\theta)\\
\frac{di_B}{dt} &= \frac{1}{L_B}V_B - \frac{R_B}{L_B}i_B + \frac{L_1 N_r}{L_B}i_B\omega\sin\left(N_r\theta - \frac{\pi}{2}\right)\\
\frac{di_C}{dt} &= \frac{1}{L_C}V_C - \frac{R_C}{L_C}i_C + \frac{L_1 N_r}{L_C}i_C\omega\sin(N_r\theta - \pi)\\
\frac{di_D}{dt} &= \frac{1}{L_D}V_D - \frac{R_D}{L_D}i_D + \frac{L_1 N_r}{L_D}i_D\omega\sin\left(N_r\theta - \frac{3\pi}{4}\right)\\
\frac{d\omega}{dt} &= \frac{T_e}{J} - \frac{T_L}{J} - \frac{B}{J}\omega\\
\frac{d\theta}{dt} &= \omega
\end{aligned} \tag{14.12}$$

where

$$T_e = -\frac{L_1 N_r}{2}\left[\sin(N_r\theta)i_A^2 + \sin\left(N_r\theta - \frac{\pi}{2}\right)i_B^2 + \sin(N_r\theta - \pi)i_C^2 + \sin\left(N_r\theta - \frac{3\pi}{2}\right)i_D^2\right] \tag{14.13}$$

The above model does not account for magnetic saturation of the ferromagnetic material used to construct the variable reluctance step motor. A common and effective way to account for the magnetic saturation is to replace the torque expressions with an expression that is linear, instead of quadratic, in phase current. For example, the torque due to the current in phase A is modeled as:

$$T_A = -k_T\sin(N_r\theta)\,i_A \tag{14.14}$$

Equation (14.11) is replaced by

$$T_e = -k_T\left[\sin(N_r\theta)\,i_A + \sin\left(N_r\theta - \frac{2\pi}{3}\right)i_B + \sin\left(N_r\theta - \frac{4\pi}{3}\right)i_C\right] \tag{14.15}$$

and Eq. (14.13) is replaced by

$$T_e = -k_T\left[\sin(N_r\theta)i_A + \sin\left(N_r\theta - \frac{\pi}{2}\right)i_B + \sin(N_r\theta - \pi)i_C + \sin\left(N_r\theta - \frac{3\pi}{2}\right)i_D\right] \tag{14.16}$$

where k_T is the torque constant, equal to the zero speed one phase on holding torque.

Bifilar-Wound Hybrid Step Motor Model

A two-phase bifilar-wound hybrid step motor is wound with two windings or one center-tapped winding per pole. A positive current in one center-tapped winding will cause the magnetic flux to align in one direction, while a positive current in the other half of the winding causes the flux to align in the reverse direction. In both cases, the current can be supplied through the center tap. Thus, this motor can be driven from a single, or unipolar, supply. For convenience, the bifilar-wound hybrid step motor is considered to have four phases, with each center-tapped winding consisting of two opposite phases. One center-tapped winding consists of phases A and C, the other consists of phases B and D. Nearly perfect flux coupling exists between phases A and C as well as between phases B and D, whereas practically no flux coupling exists between the two separate winding pairs. As a result, the flux linkage in the kth phase is due to current in the kth phase winding, the current in the other half of the winding pair, and the flux due to the permanent magnet. These relationships are

$$\begin{aligned} \lambda_A &= \lambda_{AA} + \lambda_{AC} + \lambda_{AF} \\ \lambda_B &= \lambda_{BB} + \lambda_{BD} + \lambda_{BF} \\ \lambda_C &= -\lambda_A \\ \lambda_D &= -\lambda_B \end{aligned} \tag{14.17}$$

where λ_k is the total flux in winding k: λ_{kj} is the flux in winding k due to the j current winding and λ_{kf} is the flux in winding k due to the permanent magnet.

If the per phase inductances, L_k, are assumed to be equal, i.e., in the kth phase $L_k = L$ for all k, with the assumption of no saturation, and with eddy currents neglected, the flux linkage due to self-inductance is $\lambda_{kk} = L_{ik}$ and flux linkage in the opposite (jth) phase due to mutual inductance is $\lambda_{jk} = -\lambda_{kk}$ where k and j are phases on the same pole. With these relationships between flux linkage and current, Eq. (14.17) reduces to

$$\begin{aligned} \lambda_A &= L(i_A - i_C) + \lambda_{AF} \\ \lambda_B &= L(i_B - i_D) + \lambda_{BF} \end{aligned} \tag{14.18}$$

where λ_{AF}, λ_{BF} are the flux linkages due to the permanent magnet given by $\lambda_{AF} = k_o \cos(\theta)$ and $\lambda_{BF} = k_o \sin(\theta)$; where k_o is the flux constant due to the permanent magnet; θ is the rotor angular position in electrical radians; and $\theta = N_r\theta_m$, where θ_m is the mechanical position or displacement of the rotor with respect to the detent position of phase A which is at $\theta = 0$ rad. N_r is the number of rotor teeth (one mechanical period = N_r electrical period).

The phase voltages, being the voltages measured on the motor terminals, are modeled as

$$V_k = R_k i_k + \dot{\lambda}_k \tag{14.19}$$

where $k = A, B, C, D$. R_A, R_B, R_C, and R_D represent the resistance of phases A, B, C, and D, respectively, which are not assumed equal, and the super-dot denotes derivative with respect to time.

From Eqs. (14.18) and (14.19), four differential equations corresponding to the four phases can be derived with the flux linkages as the state variables; however, two of these four state variables are dependent since the flux linkage in phase C is equal to the opposite of that in phase A and flux linkage in phase D is equal to the opposite of that of phase B, as stated in Eq. (14.17). Only two differential equations are necessary to model the flux linkages in the motor, one for each half of the motor.

Let the flux linkages in phase A and phase B be the two state variables. Flux linkages are used as state variables instead of currents because step discontinuities can occur in the current when a phase in switched, while the flux linkages are continuous in time. For phase A, replacing for i_C and i_A (for $k = A, C$) in Eq. (14.18), replacing for λ_C from Eq. (14.17), and rearranging terms yields

$$\begin{aligned} \dot{\lambda}_A &= \frac{1}{2}V_A\left[1 - \frac{R_A - R_C}{R_A + R_C}\right] - \frac{1}{2}V_C\left[1 + \frac{R_A - R_C}{R_A + R_C}\right] - \frac{R_A R_C(\lambda_A - \lambda_{AF})}{L(R_A + R_C)} \\ \dot{\lambda}_B &= \frac{1}{2}V_B\left[1 - \frac{R_B - R_D}{R_B + R_D}\right] - \frac{1}{2}V_D\left[1 + \frac{R_B - R_D}{R_B + R_D}\right] - \frac{R_B R_D(\lambda_B - \lambda_{BF})}{L(R_B + R_D)} \end{aligned} \tag{14.20}$$

The torque generated by the motor can be modeled by

$$T_E = \frac{k_T}{L}(-\lambda_A \sin(\theta) + \lambda_B \cos(\theta)) - k_m \sin(4\theta) \tag{14.21}$$

where

k_T = torque constant
k_m = maximum torque due to the permanent magnet

Drive Circuit Modeling

The easiest and most common way to model a motor and drive circuit is to model the model as presented above with all the resistances equal to the phase resistance, and the phase voltages as V_s (or $-V_s$ when appropriate) when a phase is on and either 0 or $-V_s$, whichever is appropriate when a phase is off. However, this models the unenergized phases as short circuited, which, after the current has decayed, would be better modeled as an open circuit. In general, this has the effect of underpredicting the performance of the motor, especially at higher speeds.

When more accurate motor and drive system models are indicated, the phase voltages should be set to V_s, and the transistors and diodes modeled as variable resistors. Conducting transistor and diode resistances can be set to zero and nonconducting transistor and diode resistances can be set to a large value.

14.4 Control of Step Motors

Successful application of a step motor to a positioning or position tracking application requires careful attention to the control of the step motor. In this section, techniques are discussed that show how to increase the torque, double the number of detent positions, and open-loop control characteristics. The drive circuits for the various types of step motors and winding configurations was discussed in Section 14.2.

Excitation of Step Motors

Although energizing one phase at a time is the simplest way to control the step motor, greater performance from the motor is possible by exciting two phases at a time, or by switching between one phase on and two phases on at a time. This latter switching scheme is known as half-stepping.

One Phase-On Excitation: By exciting the phases one at a time, the motor will move from detent position to detent position, for example, A-B-C-D-A-B-C … for a four-phase motor.

Two Phase-On Excitation: When two phases are energized at a time, the torque curves for the individual phases add. The stable detent position is halfway between the detent positions of the motor when the phases are energized one at a time. By exciting the phases two at a time, the motor will move from the new detent position to new detent position, for example, AB-BC-CD-DA-AB-BC-CD … for a four-phase motor, where AB is the position halfway between detent position A and detent position B. Exciting the four-phase motor with two phases on at a time produces $\sqrt{2}$ more torque but consumes twice the power of exciting the motor one phase on at a time. Exciting a three-phase motor with two phases on at a time produces no additional torque but also consumes twice the power of exciting the motor one phase on at a time.

Half-Stepping Excitation: Switching the excitation alternately between one phase on and two phases on is called half-stepping excitation. This mode of operation doubles the number of detent positions of the motor, in that all of the one phase on detent positions and all of the two phase on detent positions are available. For example, by exciting a four-phase motor using half-stepping, the rotor can be made to step from detent position to detent position such as A, AB-B-BC-C-CD-D-DA-A ….

Open-Loop Control

The most common way to control a step motor is open loop, that is, without position and/or velocity feedback. Once the motor and drive scheme have been chosen, the step command sequence must be chosen. The following examples illustrate some of the characteristics and hazards of open-loop controlled step motors.

The One-Step Move: Fig. 14.11a illustrates the position vs. time and Fig. 14.11b illustrates the velocity vs. time for a one-step move achieved by deenergizing phase A and energizing phase B. The characteristic position overshoot and ringing can be seen from these plots. Figure 14.11c shows the velocity vs. position or phase–plane plot of this one-step move. Observe that the peak overshoot is almost half a step. The peak velocity reaches over 500 steps per second.

A Six-Step Move at a Rate of 50 Steps per Second: Fig. 14.12a shows the position vs. time and Fig. 14.12b shows the velocity vs. time for a six-step move at a rate of 50 steps per second. The stair step in Fig. 14.12a is the commanded position, the straight line in Fig. 14.12b is commanded velocity. We can see the motor is moving in discrete steps, in that the position and velocity profiles are almost six individual single-step responses with overshoots close to half a step and peak velocities over 500 steps per second and close to –400 steps per second.

A Six-Step Move at a Rate of 500 Steps per Second: Fig. 14.13a shows the position vs. time and Fig. 14.13b shows the velocity vs. position for a six-step move at a rate of 500 steps per second. The stair step in Fig. 14.13a is the commanded position and the straight line in Fig. 14.13b is the commanded velocity. In these figures, we no longer see the individual step responses in that the next phase is energized before the rotor has reached the peak overshoot for the previously energized phase. The velocity profile shows that the motor is still not running at a constant speed, but the wild velocity oscillations are gone. The characteristic ringing is seen at the end of the move.

A Six-Step Move at a Rate of 1000 Steps per Second: Fig. 14.14a shows the position vs. time and Fig. 14.14b shows the velocity vs. position for a six-step move at a rate of 1000 steps per second. The stair step in Fig. 14.14a is the commanded position and the straight line in Fig. 14.14b is the commanded velocity. In these figures, we see that the rotor is unable to keep up with the commanded position. A step motor is a synchronous machine, and can generate nonzero average torque only at synchronous speed.

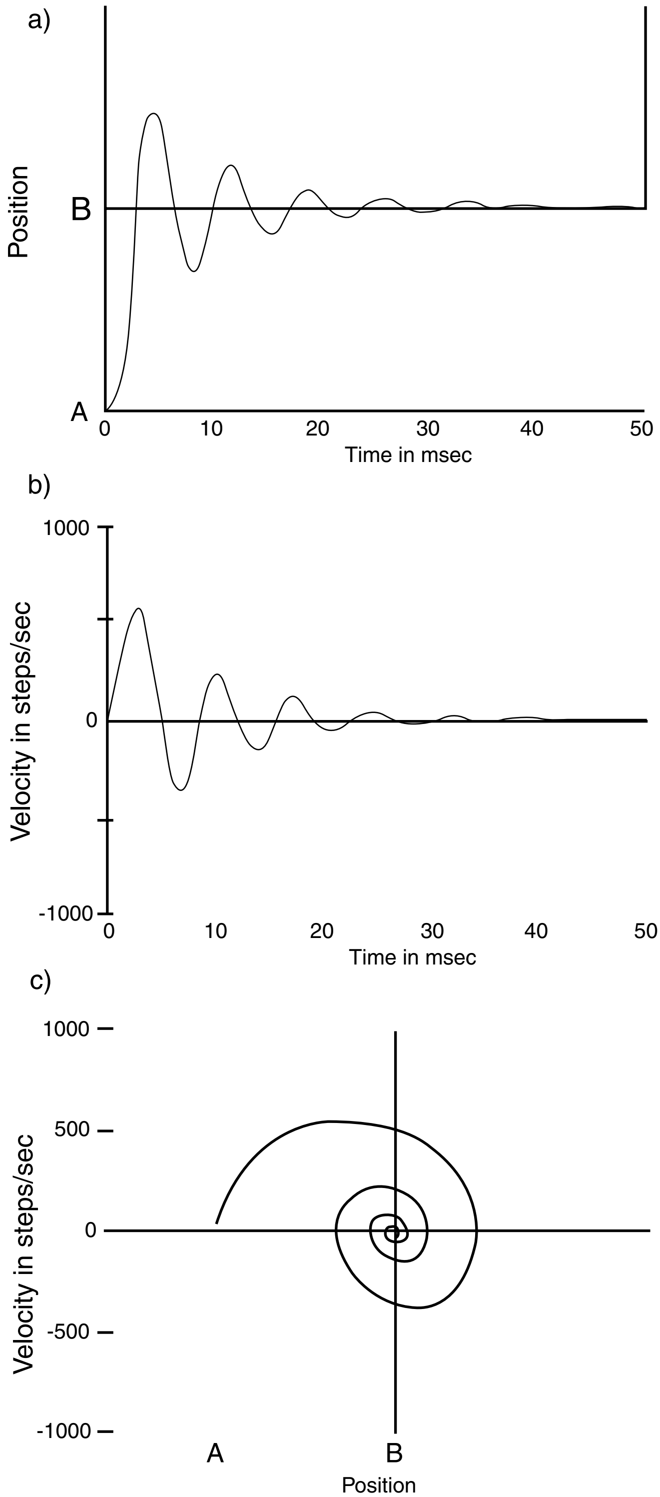

FIGURE 14.11 The one-step move: (a) Position vs. time; (b) velocity vs. time.

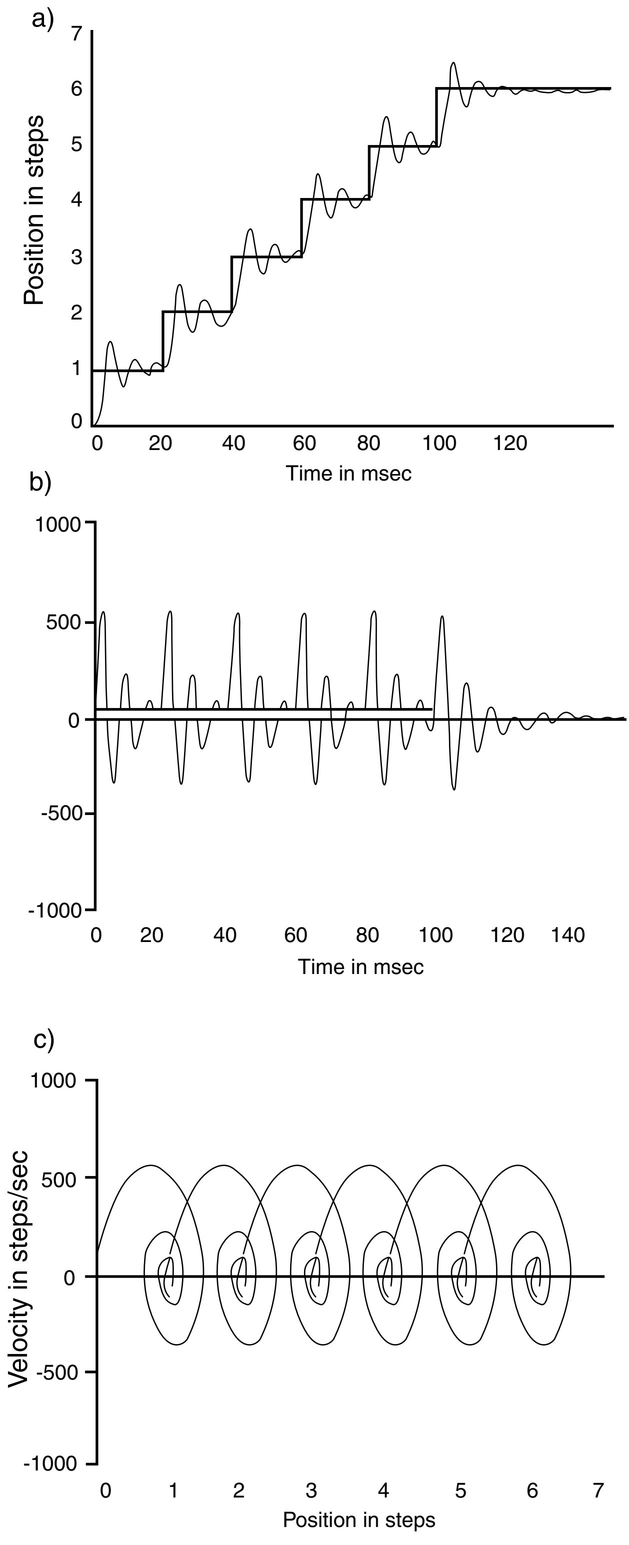

FIGURE 14.12 A six-step move at 50 steps/sec.

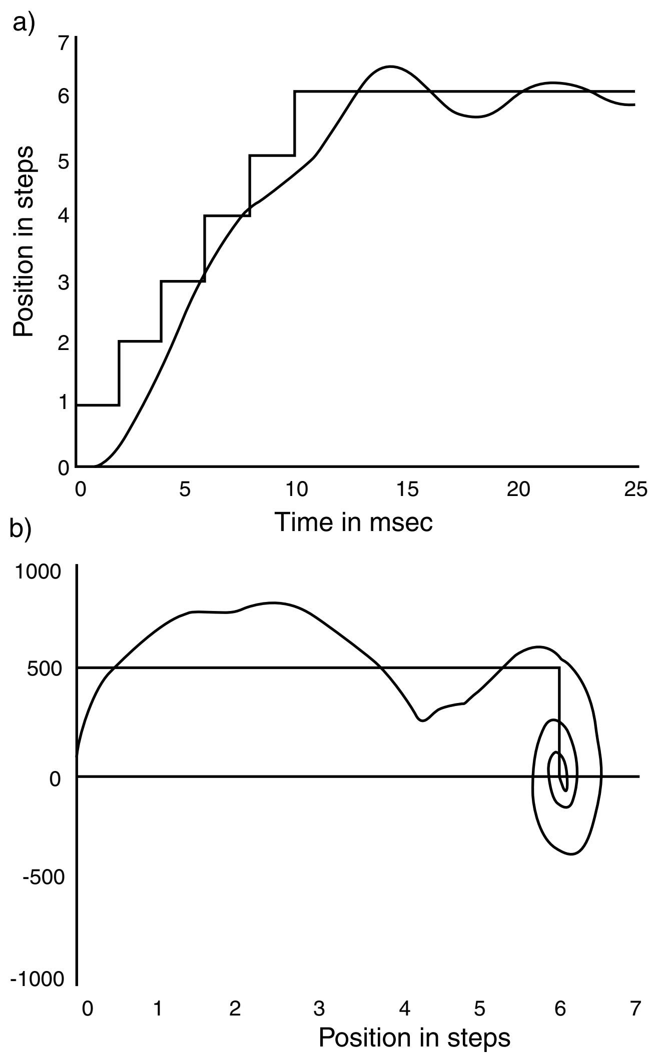

FIGURE 14.13 A six-step move at 500 steps/sec.

Typically, the motor will just vibrate when it loses synchronism. Here we see that the motor is attracted to the wrong detent position after the move is completed, which is four steps away from the desired position.

Error-Free Start-Stop Rate: From the above examples we see that the motor can run at speeds up to 500 steps per second, but cannot start at a speed of 1000 steps per second. The error-free start-stop rate for this motor is between 500 and 1000 steps per second. Speed greater than the error-free start-stop rate can be achieved by starting the motor at or below the error-free start-stop rate and accelerating the motor up to a higher speed. Near the end of the move, the motor must decelerate to a speed at or below the error-free start-stop rate or it may not be able to stop at the desired position, but somewhere beyond.

Low-Frequency Resonance: When a step motor is run at or near the natural frequency of its one-step response, synchronism with the commanded position can be lost due to low-frequency resonance.

Midfrequency Resonance: Fig. 14.15 illustrates another problem known as midfrequency resonance. In this position vs. time plot, the motor is started out at 400 steps per second, well below its error-free start-stop rate, and accelerated up to a speed of 1200 steps per second. At this speed, large oscillations

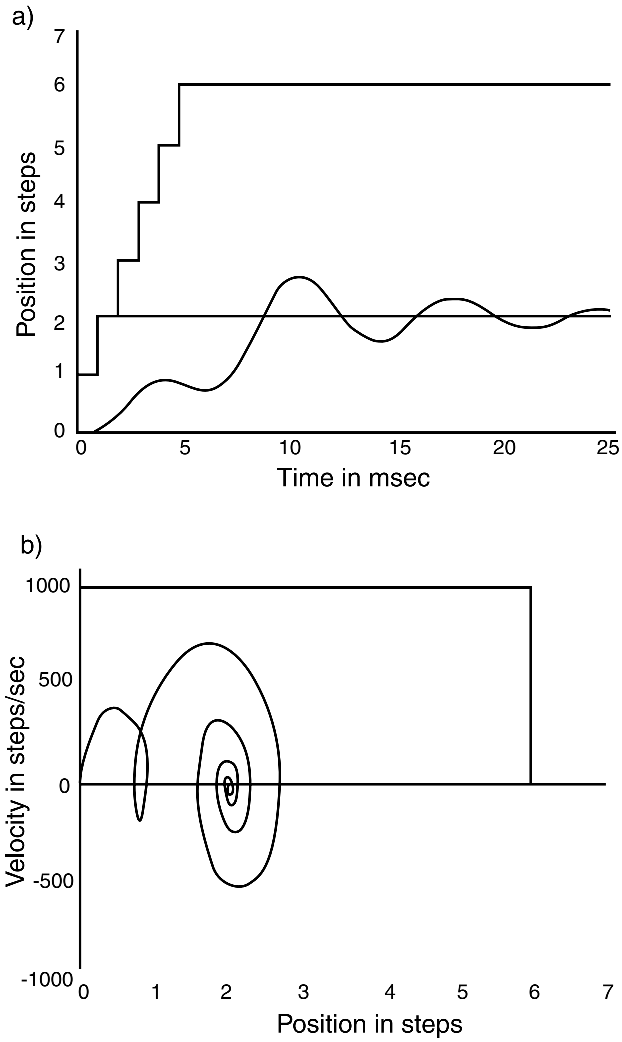

FIGURE 14.14 A six-step move at 1000 steps/sec.

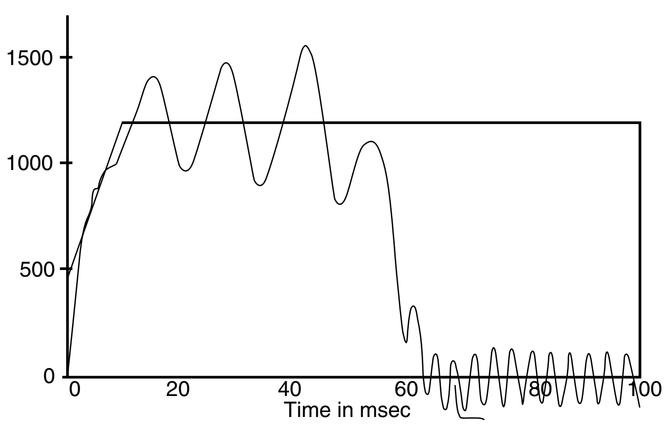

FIGURE 14.15 Midfrequency resonance.

occur in the velocity until, finally, the rotor loses synchronism with the commanded signal. This phenomenon can be avoided by accelerating through this speed range.

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