CHAPTER 1.5
NOISE AND INTERFERENCE

Geoffrey C. Orsak, H. Vincent Poor, John B. Thomas

GENERAL

In a general sense, *noise* and *interference* are used to describe any unwanted or undesirable signals in communication channels and systems. Since in many cases these signals are random or unpredictable, some study of random processes is a useful prelude to any consideration of noise and interference.

RANDOM PROCESSES

A random process $X(t)$ is often defined as an indexed family of random variables where the *index* or *parameter* t belongs to some set T; that is, $t \in T$.

The set T is called the *parameter set* or *index set* of the process. It may be finite or infinite, denumerable or nondenumerable; it may be an interval or set of intervals on the real line, or it may be the whole real line $-\infty < t < \infty$. In most applied problems, the index t will represent time, and the underlying intuitive notion will be a random variable developing in time. However, other parameters such as position and temperature may also enter in a natural manner.

There are at least two ways to view an arbitrary random process: (1) as a set of random variables: this viewpoint follows from the definition. For each value of t, the random process reduces to a random variable; and (2) as a set of functions of time. From this viewpoint, there is an underlying random variable each realization of which is a time function with domain T. Each such time function is called a *sample function* or *realization* of the process.

From a physical point of view, it is the sample function that is important, since this is the quantity that will almost always be observed in dealing experimentally with the random process. One of the important practical aspects of the study of random processes is the determination of properties of the random variable $X(t)$ for fixed t on the basis of measurements performed on a single sample function $x(t)$ of the process $X(t)$.

A random process is said to be *stationary* if its statistical properties are invariant to time translation. This invariance implies that the underlying physical mechanism producing the process is not changing with time. Stationary processes are of great importance for two reasons: (1) they are common in practice or approximated to a high degrees of accuracy (actually, from the practical point of view, it is not necessary that a process be stationary for all time but only for some observation interval that is long enough to be suitable for a given problem); (2) many of the important properties of common stationary processes are described by first and second moments. Consequently, it is relatively easy to develop a simple but useful theory (*spectral theory*) to describe these processes. Processes that are not stationary are called *nonstationary*, although they are also sometimes referred to as *evolutionary* processes.

The *mean* $m(t)$ of a random process $X(t)$ is defined by

$$m(t) = E\{X(t)\} \tag{1}$$

where $E\{\cdot\}$ is the mathematical expectation operator defined in Chap. 1.1. In many practical problems, this mean is independent of time. In any case, if it is known, it can be subtracted from $X(t)$ to form a new "centered" process $Y(t) = X(t) - m(t)$ with zero mean.

The *autocorrelation function* $R_x(t_1, t_2)$ of a random process $X(t)$ is defined by

$$R_x(t_1, t_2) = E\{X(t_1)X(t_2)\} \tag{2}$$

In many cases, this function depends only on the time difference $t_2 - t_1$ and not on the absolute times t_1 and t_2. In such cases, the process $X(t)$ is said to be *at least wide-sense stationary*, and by a linear change in variable Eq. (2) can be written

$$R_x(\tau) = E\{X(t)X(t + \tau)\} \tag{3}$$

If $R_x(\tau)$ possesses a Fourier transform $\varphi_x(\omega)$, this transform is called the *power spectral density* of the process and $R_x(\tau)$ and $\varphi_x(\omega)$ form the Fourier-transform pair

$$\varphi_x(\omega) = \int_{-\infty}^{\infty} R_x(\tau)e^{-j\omega r}\,d\tau \tag{4}$$

and

$$R_x(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi_x(\omega)e^{j\omega r}\,d\omega \tag{5}$$

For processes that are at least wide-sense stationary, these last two equations afford a direct approach to the analysis of random signals and noises on a power ratio or mean-squared-error basis. When $\tau = 0$, Eq. (5) becomes

$$R_x(0) = E\{X^2(t)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi_x(\omega)\,d\omega \tag{6}$$

an expression for the normalized power in the process $X(t)$.

As previously mentioned, in practical problems involving random process, what will generally be available to the observer is not the random process but one of its sample functions or realizations. In such cases, the quantities that are easily measured are various time averages, and an important question to answer is: Under what circumstances can these time averages be related to the statistical properties of the process?

We define the *time average* of a sample function $x(t)$ of the random process $X(t)$ by

$$A\{x(t)\} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\,dt \tag{7}$$

The *time autocorrelation function* $\mathfrak{R}_x(t)$ is defined by

$$\mathfrak{R}_x(\tau) = A\{x(t)x(t+\tau)\} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)x(t+\tau)\,dt \tag{8}$$

It is intuitively reasonable to suppose that, for stationary processes, time averages should be equal to expectations; e.g.,

$$E\{X(t)X(t + \tau)\} = A\{x(t)x(t + \tau)\} \tag{9}$$

for every sample function $x(t)$. A heuristic argument to support this claim would go as follows. Divide the parameter t of the random process $V(t)$ into long intervals of T length. If these intervals are long enough (compared with the time scale of the underlying physical mechanism), the statistical properties of the process in one interval T should be very similar to those in any other interval. Furthermore, a new random process $X(t)$ could be formed in the interval $(0, T)$ by using as sample functions the segments of length T from a single sample function of the original process. This new process should be statistically indistinguishable from the original process, and its ensemble averages would correspond to time averages of the sample function from the original process.

The foregoing is intended as a very crude justification of the condition of *ergodicity*. A random process is said to be *ergodic* if time averages of sample functions of the process can be used as approximations to the corresponding ensemble averages or expectations. A further discussion of ergodicity is beyond the scope of this treatment, but this condition can often be assumed to exist for stationary processes. In this case, time averages and expectations can be interchanged at will, and, in particular,

$$E\{X(t)\} = A\{x(t)\} = u = \text{a constant} \tag{10}$$

and

$$R_x(\tau) = \mathfrak{R}_x(\tau) \tag{11}$$

CLASSIFICATION OF RANDOM PROCESSES

A central problem in the study of random processes is their classification. From a mathematical point of view, a random process $X(t)$ is defined when all n-dimensional distribution functions of the random variables $X(t_1)$, $X(t_2), \ldots, X(t_n)$ are defined for arbitrary n and arbitrary times $t_1, t_2, \ldots, t_n$. Thus classes of random processes can be defined by imposing suitable restrictions on their n-dimensional distribution functions. In this way, we can define the following (and many others):

(*a*) *Stationary processes*, whose joint distribution functions are invariant to time translation.
(*b*) *Gaussian* (*or normal*) *processes*, whose joint distribution functions are multivariate normal.
(*c*) *Markov processes*, where given the value of $X(t_1)$, the value of $X(t_2)$, $t_2 > t_1$, does not depend on the value of $X(t_0)$, $t_0 < t_1$; in other words, the future behavior of the process, given its present state, is not changed by additional knowledge about its past.
(*d*) *White noise*, where the power spectral density given by Eq. (4) is assumed to be a constant N_0. Such a process is not realizable since its mean-squared value (normalized power) is not finite; i.e.,

$$R_x(0) = E\{X^2(t)\} = \frac{N_0}{2\pi}\int_{-\infty}^{\infty} d\omega = \infty \tag{12}$$

On the other hand, this concept is of considerable usefulness in many types of analysis and can often be postulated where the actual process has an approximately constant power spectral density over a frequency range much greater than the system bandwidth.

Another way to classify random processes is on the basis of a model of the particular process. This method has the advantage of providing insights into the physical mechanisms producing the process. The principal disadvantage is the complexity that frequently results. On this basis, we may identify the following (natural) random processes.

(*a*) *Thermal noise* is caused by the random motion of the electrons within a conductor of nonzero resistance. The mean-squared value of the thermal-noise voltage across a resistor of resistance R Ω is given by

$$\overline{v}^2 = 4kTR\,\Delta f \qquad (\text{volts}^2) \tag{13}$$

where k is Boltzmann's constant, T is the absolute temperature in kelvins, and Δf is the bandwidth of the measuring equipment.

(*b*) *Shot noise* is present in any electronic device (e.g., a transistor) where electrons move across a potential barrier in a random way. Shot noise is usually modeled as

$$X(t) = \sum_{i=-\infty}^{\infty} f(t - t_i) \tag{14}$$

where the t_i are random emission times and $f(t)$ is a basic pulse shape determined by the device geometry and potential distribution.

(*c*) *Defect noise* is a term used to describe a wide variety of related phenomena that manifest themselves as noise voltages across the terminals of various devices when dc currents are passed through them. Such noise is also called current noise, excess noise, flicker noise, contact noise, of $1/f$ noise. The power spectral density of this noise is given by

$$\varphi_x(\omega) = kI^{\omega}/\omega^{B} \tag{15}$$

where I is the direct current through the device, ω is radian frequency, and k, α, and β are constants. The constant α is usually close to 2, and β is usually close to 1. At a low enough frequency this noise may predominate because of the $1/\omega$ dependence.

ARTIFICIAL NOISE

The noises just discussed are more or less fundamental and are caused basically by the noncontinuous nature of the electronic charge. In contradistinction to these are a large class of noises and interferences that are more or less artificial in the sense of being generated by electrical or electronic equipment and hence, in principle, are under our control. The number and kinds here are too many to list and the physical mechanisms usually too complicated to describe; however, to some degree, they can be organized into three main classes.

1. *Interchannel Interference.* This includes the interference of one radio or television channel with another, which may be the result of inferior antenna or receiver design, variation in carrier frequency at the transmitter, or unexpectedly long-distance transmission via scatter of ionospheric reflection. It also includes crosstalk between channels in communication links and interference caused by multipath propagation or reflection. These types of noises can be removed, at least in principle, by better equipment design, e.g., by using a receiving antenna with a sufficiently narrow radiation pattern to eliminate reception from more than one transmitter.

2. *Co-channel Interference*. In wireless systems, this refers to interference from other communication systems operating in the same frequency band. It arises, for example, in cellular telephony systems, owing to interference from adjacent cells. It also arises in many types of consumer systems, such as wireless local area networks, that operate in lightly regulated bands.

3. *Multiple-access Interference*. This refers to intranetwork interference that results from the use of nonorthogonal modulation schemes in networks of multiple communicating pairs. Many cellular telephony systems, such as those using code-division multiple-access (CDMA) protocols, involve this kind of interference.

4. *Hum.* This is a periodic and undesirable signal arising from the power lines. Usually it is predictable and can be eliminated by proper filtering and shielding.

5. *Impulse Noise.* Like defect noise, this term describes a wide variety of phenomena. Not all of them are artificial, but the majority probably are. This noise can often be modeled as a low-density shot process or, equivalently, as the superposition of a small number of large impulses. These impulses may occur more or less periodically, as in ignition noise from automobiles or corona noise from high-voltage transmission lines. On the other hand, they may occur randomly, as in switching noise in telephone systems or the atmospheric noise from thunderstorms. The latter type of noise is not necessarily artificial, of course. This impulse noise tends to have an amplitude distribution that is decidedly non-gaussian, and it is frequently highly nonstationary. It is difficult to deal with in a systematic way because it is ill-defined. Signal processors that must handle this type of noise are often preceded by limiters of various kinds or by noise blankers which give zero transmission if a certain amplitude level is surpassed. The design philosophy behind the use of limiters and blankers is fairly clear. If the noise consists of large impulses of relatively low density, the best system performance is obtained if the system is limited or blanked during the noisy periods and behaves normally when the (impulse) noise is not present.