CHAPTER 10.2
FILTER DESIGN

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PRACTICAL FILTER DESIGN

Table 10.1.10 gives element value for normalized networks. Three general steps lead to practical filters.

1. Statement of the filter specifications
2. Translation of the specifications to equivalent statements for the normalized prototype and their use to determine the parameters necessary to enter Table 10.1.10
3. Application of frequency transformations and impedance-level scaling to the normalized prototype to determine the filter network

This process is described in the following paragraphs for the various types of filters, accompanied by examples.

THE LOW-PASS FILTER

The low-pass filter is related to the normalized prototype by frequency and impedance-level scaling. The frequency in the prototype is related to the frequency in the low-pass filter by

$$\omega = \omega'/\omega'_c \tag{1}$$

where the primed quantities refer to the practical low-pass filter, with ω'_c being its cutoff frequency. The application of the frequency scaling implied by Eq. (1) and the impedance-level scaling required to change the load resistance from 1.0 to R_L results in the elements of the low-pass filter becoming those indicated in Fig. 10.2.1.

Example. A low-pass filter is to be designed to have (*a*) a maximally flat amplitude-versus-frequency characteristic at $f = 0$, (*b*) a cutoff (half-power) frequency of 2 kHz, (*c*) a load resistance of 200 Ω, (*d*) a voltage source input with zero resistance, and (*e*) an attenuation of not less than 20 dB at frequencies greater than 3.2 kHz. A Butterworth approximation is indicated. The frequency in the prototype equivalent to 3.2 kHz is

$$\omega = \frac{(3.2 \times 10^3)(2\pi)}{(2 \times 10^3)(2\pi)} = 1.6 \tag{2}$$

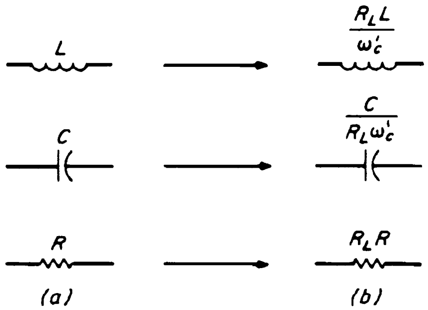

FIGURE 10.2.1 Relations between elements of (*a*) a normalized prototype and those of (*b*) a practical low-pass filter.

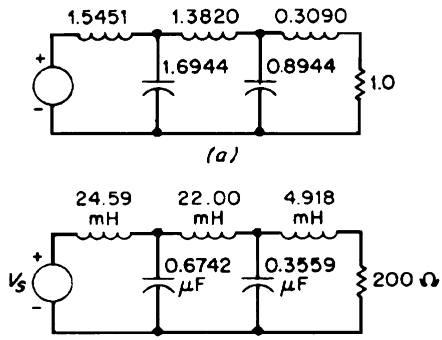

FIGURE 10.2.2 (*a*) A normalized prototype and (*b*) resulting low-pass filter.

and the equivalent design specifications for the prototype require that the attenuation be no less than 20 dB for $\omega \geq 1.6$. This, using Eq. (16), results in

$$n \geq \frac{20}{20 \log 1.6} = 4.899 \tag{3}$$

and the next larger integer value of 5 is used. The appropriate prototype appears in Fig. 10.2.2*a* with element values taken from Table 10.1.10 for the Butterworth approximation with $n = 5$ and $R_s = 0$. The application of frequency and magnitude scaling by means of the relations in Fig. 10.2.1 results in the low-pass filter of Fig. 10.2.2*b*.

A TIME-DELAY NETWORK

The frequencies in the time-delay network and its normalized prototype are related by

$$\omega = \tau'_a \omega' \tag{4}$$

where the primed quantities again refer to the practical network, τ'_d being the group delay evaluated at $\omega' = 0$. The element values of the practical network are related to those of the normalized prototype by frequency and impedance-level scaling. These relations are shown in Fig. 10.2.3.

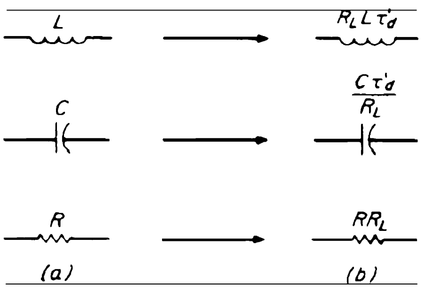

FIGURE 10.2.3 Relations between elements for (*a*) a normalized prototype and those of (*b*) a time-delay network.

Example. A time-delay network is to be designed to have (*a*) a time delay of 1 ms, the error being no greater than 5 percent at 500 Hz; (*b*) a voltage input with a resistance of 500 Ω; and (*c*) a load resistance of 500 Ω. The prototype frequency equivalent to 500 Hz is

$$\omega = 10^{-3}(2\pi \times 500) = 3.142 \tag{5}$$

and the equivalent specifications for the prototype require a time delay of 1 s, the error being no greater than 5 percent at $\omega = 3.142$. Figure 10.1.12 is used to determine that $n = 5$ is required to satisfy this requirement. The prototype can be of the form of either Fig. 10.1.16*d* or Fig. 10.1.17*b*. The latter is chosen with the element values taken from Table 10.2.1 for

TABLE 10.2.1 Element Choices for GIC Biquadratic Filters

Filter type	Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_A	Y_B
Low-pass	0	$G_1 + sC_1$	G_2	sC_3	G_4	G_5	0	G_B
High-pass	G_0	G_1	sC_2	G_3	G_4	G_5	0	sC_B
Bandpass	sC_0	G_1	sC_2	G_3	G_4	G_5	0	G_B
Band-stop, version 1	0	sC_1	G_2	G_3	G_4	G_5	sC_A	G_B
Version 2	G_0	G_1	G_2	G_3	sC_4	0	G_A	sC_B
All-pass	G_0	G_1	sC_2	G_3	G_4	0	$G_A = G_4$	sC_B

the Bessel approximation with $n = 5$ and $G_s = 1$. The prototype is shown in Fig. 10.2.4*a* and the application of the relation is Fig. 10.2.3 results in the time-delay network of Fig. 10.2.4*b*.

THE HIGH-PASS FILTER

The frequency in the high-pass filter is related to that in the normalized prototype by

$$\omega = \omega_c' / \omega' \tag{6}$$

where the primed quantities refer to the high-pass filter, ω_c' being the cutoff frequency. The relations between the elements of the normalized prototype and those of the high-pass filter are given in Fig. 10.2.5.

Example. A high-pass filter is to be designed to have (*a*) a cutoff frequency of 7.5 kHz, (*b*) an equiripple characteristic in the passband with a 0.5-dB ripple ($\epsilon = 0.3493$), (*c*) an attenuation of at least 35 dB at frequencies below 3 kHz, (*d*) equal load and source resistance of 2000 Ω, and (*e*) a voltage source input. The prototype frequency equivalent to 3 kHz is

$$\omega = \frac{2\pi(7.5\times10^3)}{2\pi(3\times10^3)} = 2.5 \tag{7}$$

Thus the prototype is based on a Chebyshev approximation with a 0.5-dB ripple, equal load and source terminations, voltage source input, and an attenuation of at least 35 dB for frequencies greater than $\omega = 2.5$. The last requirement, used with Eq. (29), results in $C_n(2.5) \approx 161.0$. This result, used in Eq. (30), yields $n = 3.69$, and $n = 4$ is used. Since the regular Chebyshev approximation is not available, the modified Chebyshev approximation is used. The networks of Figs. 10.1.16*c* and 10.1.17*a* are possible. The former is chosen, and the

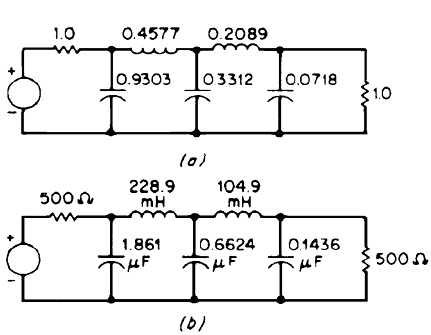

FIGURE 10.2.4 (*a*) A normalized prototype and (*b*) resulting time-delay network.

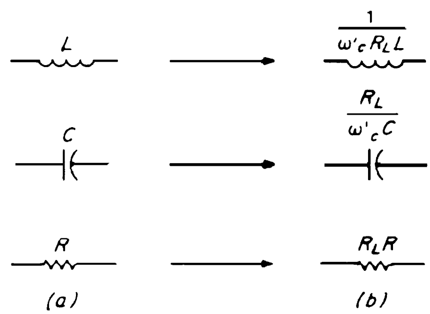

FIGURE 10.2.5 Relations between elements of (*a*) a normalized prototype and those of (*b*) a high-pass filter.

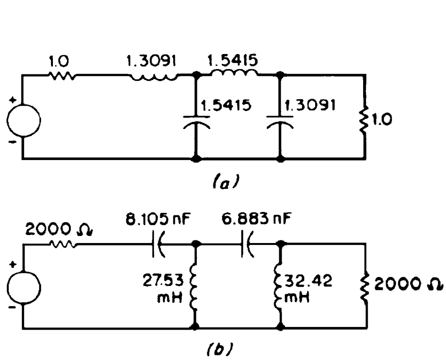

FIGURE 10.2.6 (*a*) A normalized prototype and (*b*) resulting high-pass filter.

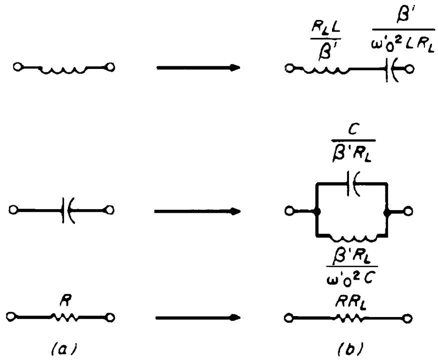

FIGURE 10.2.7 Relations between elements of (*a*) a normalized prototype and those of (*b*) a bandpass filter.

resulting prototype is shown in Fig. 10.2.6*a*. The relations of Fig. 10.2.5 are then used to determine the element values of the high-pass filter of Fig. 10.2.6*b*.

THE BANDPASS FILTER

The frequency in the bandpass filter is related to that in the normalized prototype by

$$\omega = \frac{\omega_0'}{\beta'}\left(\frac{\omega'}{\omega_0'} - \frac{\omega_0'}{\omega'}\right) \tag{8}$$

where the primed quantities refer to the bandpass filter, ω_0' being the center frequency and β' the bandwidth, $\omega_{c2}' - \omega_{c1}'$, as defined in Fig. 10.1.2*c*. The elements of the normalized prototype and those of the bandpass filter are related as shown in Fig. 10.2.7.

Example. A bandpass filter is to be designed to have (*a*) a center frequency of 4.0 kHz, (*b*) a bandwidth of 900 Hz, (*c*) an equiripple characteristic in the passband with a 1-dB ripple ($\epsilon = 0.5088$), (*d*) an attenuation of at least 18 dB at 4.9 kHz, (*e*) equal load and source resistances of 200 Ω, and (*f*) a current source input. The prototype frequency equivalent to 4.9 kHz is, from Eq. (8), 1.816 and the prototype is based on a Chebyshev approximation with a 1-dB ripple, equal load and source terminations, current source input, and an attenuation of at least 18 dB at $\omega = 1.816$. The last requirement, used with Eq. (29), results in $C_n(1.8163) \approx 19.65$. This result, used in Eq. (30), yields $n = 2.86$, and $n = 3$ is used. Two networks, those of Figs. 10.1.16*a* and 10.1.17*d*, are possible. The former is chosen, and the resulting prototype is shown in Fig. 10.2.8*a*. The relations of Fig. 10.2.7 are then used to determine the element values of the bandpass filter of Fig. 10.2.8*b*.

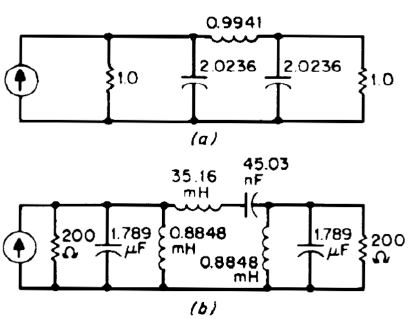

FIGURE 10.2.8 (*a*) A normalized prototype and (*b*) resulting bandpass filter.

The results of the preceding example illustrate a problem in the transformation from prototype to bandpass filter. For even the moderate ratio of center frequency to bandwidth of the example, the ratio of series-arm to shunt-arm inductances in the bandpass filter is large. This creates some practical problems arising from the stray capacitances associated with large inductances. The reader is referred to Humpherys[8] for an excellent

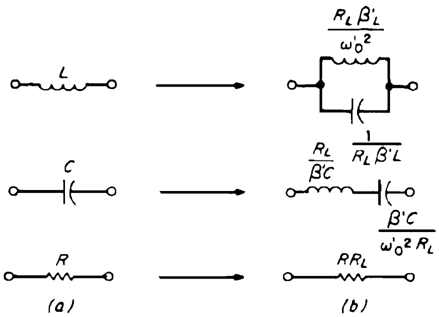

FIGURE 10.2.9 Relations between elements of (*a*) a normalized prototype and those of (*b*) a band-elimination filter.

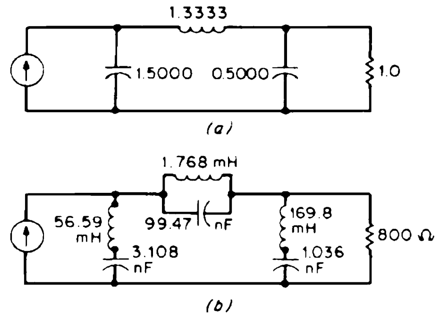

FIGURE 10.2.10 (*a*) A normalized prototype and (*b*) resulting band-elimination filter.

discussion of this problem and possible ways of overcoming the difficulty. A similarly large ratio of shunt-arm to series-arm capacitances is also present, although the practical problems are not so severe.

THE BAND-ELIMINATION FILTER

The frequency in the band-elimination filter is related to that in the normalized prototype by

$$\omega = \frac{1}{(\omega_0'/\beta')[(\omega_0'/\omega') - \omega'/\omega_0']} \tag{9}$$

where the primed quantities refer to the band-elimination filter, ω_0' being the center frequency and β' the bandwidth, $\omega_{c2}' - \omega_{c1}'$, as defined in Fig. 10.1.2*d*. The elements of the normalized prototype and those of the band-elimination filter are related as shown in Fig. 10.2.9.

Example. A band-elimination filter is to be designed to have (*a*) a center frequency of 12 kHz, (*b*) a bandwidth (half-power) of 1.5 kHz, (*c*) a maximally flat characteristic at $\omega' = 0$, (*d*) an attenuation of at least 16 dB at 11.6 kHz, (*e*) a current source input, and (*f*) $R_I = 800\ \Omega$ and $G_S = 0$. Since the prototype frequency equivalent to 11.6 kHz is, from Eq. (9), 1.8432, the prototype is based on a Butterworth approximation with an attenuation of at least 16 dB at this frequency. This requirement, used with Eq. (16), results in $n = 2.99$, and $n = 3$ is used. Since $G_S = 0$, the only network available is that of Fig. 10.2.17*b*, and the resulting prototype is shown in Fig. 10.2.10*a*. The relations of Fig. 10.2.9 are then used to determine the element values of the band-elimination filter of Fig. 10.2.10*b*.

This example illustrates the large ratios of shunt-arm to series-arm inductances and of series-arm to shunt-arm capacitances that result with even a moderate ratio of center frequency to bandwidth. The reader is again referred to Humpherys[8] for a discussion of this difficulty and steps that can be taken to overcome it.