CHAPTER 10.3
ACTIVE FILTERS

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INTRODUCTION

Active filters are electronic filter circuits that incorporate one or more electronic devices in their realizations. They include operational amplifiers, operational transconductances amplifiers, and, in some cases, special transistors. Many are available in integrated circuit form. Active filters eliminate inductors and usually provide a voltage or current gain. Three basic techniques are synthetic inductance networks, infinite gain networks, and controlled source (including Sallen and Key) realizations.

The factors computed from Eqs. (17) (Butterworth), (33), and (44) (Chebyshev), or tabulated in Tables 10.1.2 (modified Chebyshev), 10.1.4 (Legendre-Papoulis), or 10.1.6 (Bessel) are all low-pass functions, and are all denominator polynomials. In the functions, the numerators are constants. To transform these functions to high pass, use the change of variable

$$s = 1/p \tag{1}$$

followed by frequency scaling.

To transform to bandpass functions, use the transformation

$$s = (p^2 + \omega_0^2)/Bp \tag{2}$$

on a factor-by-factor basis to avoid later factoring of high-ordered polynomials. Use the quadratic equation on Eq. (2) to transform bandpass variables to equivalent low-pass variables. An nth order, low-pass function is of order $2n$ when this transform is used, and the numerator includes a factor p^n and a constant. In this expression, B = bandwidth, $\omega_{c2} - \omega_{c1}$, and ω_0 = center frequency (Fig. 10.1.2, rad/s).

To transform to a band reject function, use the reciprocal of Eq. (2).

FIRST-ORDER STAGES

Most active network realizations require the cascading of second-order networks, each of which realizes one of the second-order denominator polynomials. To realize a first-order factor (real poles), use either of the networks of Fig. 10.3.1 as the initial or final stage in the cascade. In Fig. 10.3.1,

$$\frac{V_2}{V_1}(s) = -K_1\left(\frac{s+a}{s+b}\right) \tag{3}$$

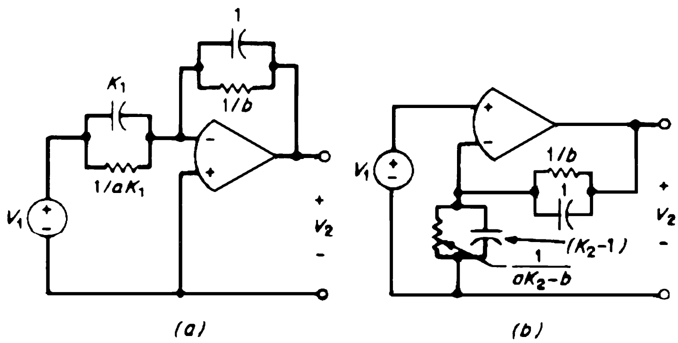


FIGURE 10.3.1 (*a*) Inverting and (*b*) noninverting amplifiers to realize real poles.

and in Fig. 10.3.1,

$$\frac{V_2}{V_1}(s) = K_2\left(\frac{s+a}{s+b}\right) \tag{4}$$

In Fig. 10.3.1, $K_2 \geq 1$, and $aK_2 \geq b$.

SYNTHETIC INDUCTANCE FILTERS—GYRATORS

It is possible to build filters with active elements that replace inductors. These realizations use the published tables for passive low-pass networks and the networks that are transformed from them. They also have the low-sensitivity properties of passive networks. One technique is based on the gyrator,[24] a two-port with a z-matrix representation

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & -a \\ a & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{5}$$

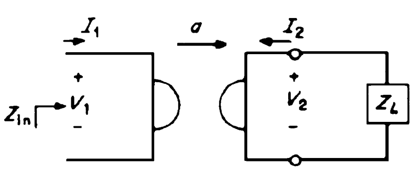


FIGURE 10.3.2 Gyrator terminated with Z_L.

where a is called the *gyration resistance*. A gyrator terminated in an impedance Z_L is shown in Fig. 10.3.2, and it is possible to show that

$$V_1/I_1 = Z_{IN} = a^2/Z_L \tag{6}$$

when $Z_L = 1/sC$,

$$Z_{IN} = a^2 sC \tag{7}$$

which is equivalent to an inductance of a^2C. The problem then becomes one of choosing a and C.

Gyrators are available in integrated-circuit form from several electronics manufacturers; the user simply adds the appropriate capacitor. Gyrators can also be built with operational amplifiers. One useful circuit is the Riordan gyrator, shown in Fig. 10.3.3 with Z_L as one of the five impedances it requires.

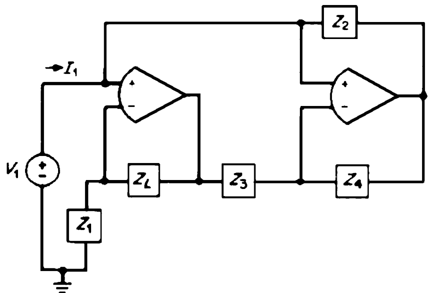


FIGURE 10.3.3 Riordan gyrator.

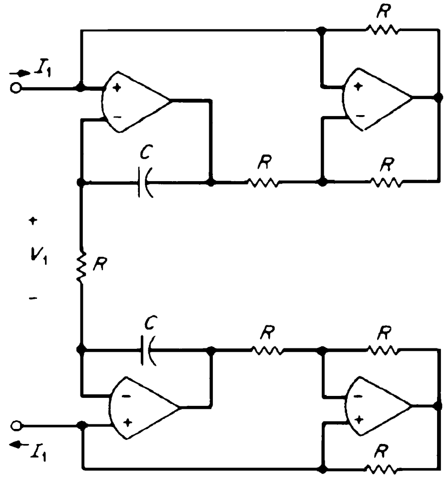


FIGURE 10.3.4 Riordan-type back-to-back gyrator for floating inductors.

It can be shown that

$$\frac{V_1}{I_1} = Z_{\text{IN}} = \frac{Z_1 Z_2 Z_3}{Z_4} \frac{1}{Z_L} \tag{8}$$

When all Z's are resistors R, and $Z_L = 1/sC$,

$$Z_{\text{IN}} = R^2 sC \tag{9}$$

This circuit must be used to replace a grounded inductor, e.g., in a high-pass network. If an ungrounded inductor is needed, the modification shown in Fig. 10.3.4 can be used, for which

$$Z_{\text{IN}} = R^2 sC \tag{10}$$

Other circuits, for which one side of the added capacitor C may be grounded, can also be used, though they may be more difficult to align.

SYNTHETIC INDUCTANCE FILTERS—FREQUENCY-DEPENDENT NEGATIVE RESISTORS

Bruton[25] introduced a new type of circuit, called the *frequency-dependent negative resistor* (FDNR); the symbol is shown in Fig. 10.3.5. To use this idea, consider a new type of impedance scaling, in which passive elements become scaled by A/s; this does not affect the transfer function:

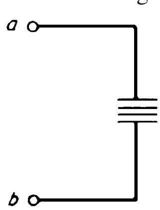
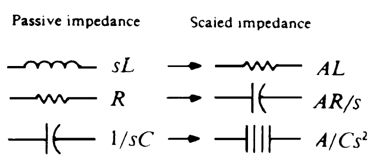


FIGURE 10.3.5 FDNR representation.



(11a)
(11b)
(11c)

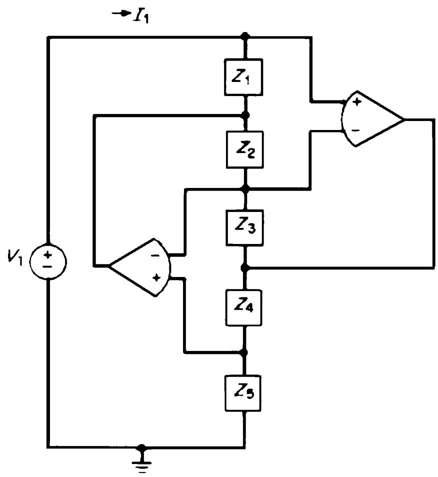


FIGURE 10.3.6 Generalized impedance converter.

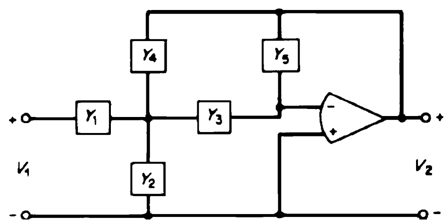


FIGURE 10.3.7 A multiple-feedback, infinite-gain active realization for low-pass, high-pass, and bandpass filters. Table 10.3.1 indicates the choice of the five active elements for each case.

When $s = j\omega$ or $s^2 = -\omega^2$, the last element becomes

$$Z(j\omega) = -A/D\omega^2 \tag{12}$$

a real, negative, frequency-dependent resistor. Thus, inductors are replaced by resistors, resistors by capacitors, and capacitors by FDNRs.

Bruton also gives a circuit for an FDNR, as a special case of the generalized impedance converter (GIC). It is similar to the gyrator and is shown in Fig. 10.3.6. For this circuit,

$$V_1/I_1 = Z_{IN} = Z_1Z_3Z_5/Z_2Z_4 \tag{13}$$

when

$$Z_1 = Z_3 = 1/sC \tag{14}$$

$$Z_2 = Z_4 = Z_5 = R \tag{15}$$

$$Z_{IN} = 1/s^2RC^2 \tag{16}$$

which is an FDNR. GICs are available from integrated-electronic-circuit manufacturers, and the user adds resistors and capacitors to make FDNRs. They can also be used to make gyrators. FDNRs are grounded when replacing capacitors in low-pass networks; floating FDNRs can be achieved by back-to-back GICs, as with gyrators.

INFINITE-GAIN, MULTIPLE-FEEDBACK REALIZATION

The circuit of Fig. 10.3.7 shows an operational amplifier with five passive elements, which are either resistors or capacitors. The general voltage transfer function is

$$\frac{V_2}{V_1} = \frac{-Y_1Y_3}{Y_5(Y_1+Y_2+Y_3+Y_4)+Y_3Y_4} \tag{17}$$

TABLE 10.3.1 Element Choices for Active Filter Circuit of Fig. 10.3.7.

Filter desired	Y_1	Y_2	Y_3	Y_4	Y_5
Low-pass	Resistor	Capacitor	Resistor	Resistor	Capacitor
High-pass	Capacitor	Resistor	Capacitor	Capacitor	Resistor
Bandpass	Resistor	Resistor	Capacitor	Capacitor	Resistor

Table 10.3.1 describes how the five passive elements can be chosen to implement a low-pass, high-pass, or bandpass network.

STATE-VARIABLE REALIZATION

This network is a special but important type of infinite-gain realization. It has the advantage that low-pass, high-pass, and bandpass configuration can be realized simultaneously, and it is also easy to adjust and to produce in quantity. The network is shown in Fig. 10.3.8, and the three possible transfer functions are

Low-pass:

$$\frac{V_{lp}}{V_1} = \frac{\dfrac{R_4(R_5+R_6)}{R_1R_2R_5C_1C_2(R_3+R_4)}}{s^2 + s\left[\dfrac{R_3(R_5+R_6)}{R_1C_1(R_3+R_4)}\right] + \dfrac{R_6}{R_1R_2R_5C_1C_2}} \tag{18}$$

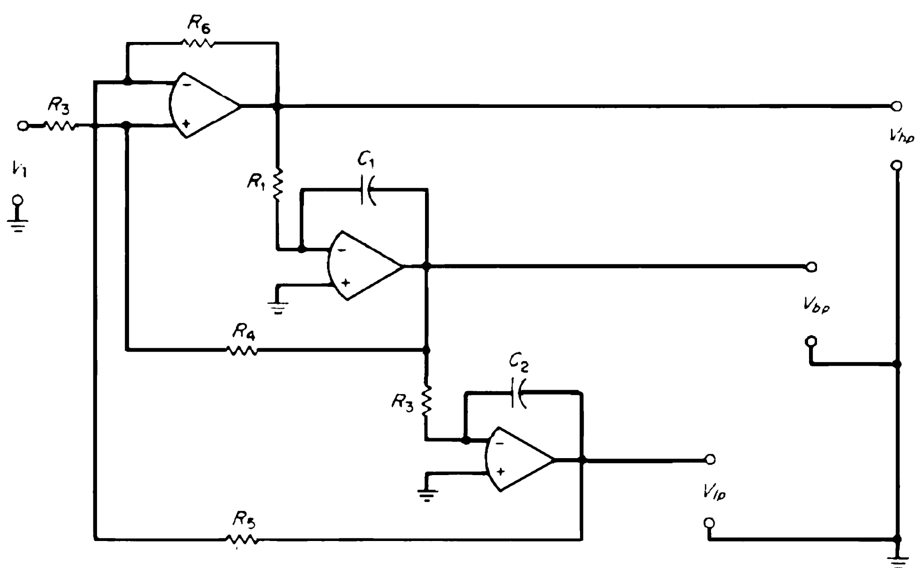


FIGURE 10.3.8 State-variable realization of second-order active filters.

High-pass:

$$\frac{V_{hp}}{V_1} = \frac{\dfrac{s^2 R_4(R_5+R_6)}{R_5(R_3+R_4)}}{s^2 + s\left[\dfrac{R_3(R_5+R_6)}{R_1C_1(R_3+R_4)}\right] + \dfrac{R_6}{R_1R_2R_5C_1C_2}} \tag{19}$$

Bandpass:

$$\frac{V_{bp}}{V_1} = \frac{\dfrac{-sR_4(R_5+R_6)}{R_1R_5C_1(R_3+R_4)}}{s^2 + s\left[\dfrac{R_3(R_5+R_6)}{R_1C_1(R_3+R_4)}\right] + \dfrac{R_6}{R_1R_2R_5C_1C_2}} \tag{20}$$

The network has a low output impedance at each terminal, so that RC sections can be added for odd-ordered networks. These sections can be cascaded so that networks of order 4 and higher can be built. They have the property that performance variations with parameter changes are comparable with those of strictly passive networks, but the disadvantage of requiring three operational amplifiers.

As with the other networks, design is a matter of choosing the appropriate quadratic factors, matching coefficients with other constraints that arise from technological and economic considerations, and finally, scaling impedance and frequency.

DELYIANNIS BANDPASS CIRCUIT

Daryanani[44] describes a useful bandpass circuit shown in Fig. 10.3.9. For this circuit

$$\frac{V_2}{V_1}(s) = \frac{-s/[R_1C_2(1-1/k)]}{s^2 + (\omega_p/Q_p)s + \omega_p^2} \tag{21}$$

where

$$k = 1 + \frac{R_B}{R_A} \tag{22}$$

$$\omega_p^2 = \frac{1}{R_1R_2C_1C_2} \tag{23}$$

$$\frac{\omega_p}{Q_p} = \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{1}{k-1}\frac{1}{R_1C_2} \tag{24}$$

The circuit can be used with $R_A = 0$; in this case $k \to \infty$, and Eq. (21) is readily simplified. Here it is necessary to limit Q_P to about 5.

When Q_P greater than 5 is desired, R_A is included and the difference term in the denominator permits a greater Q_P.

FRIEND BIQUADRATIC

Friend[26] has described a generalization of the Delyiannis circuit of the preceding section that can be used for high-pass, band-reject, and all-pass networks with proper choice of components. It is shown in Fig. 10.3.10. For this circuit

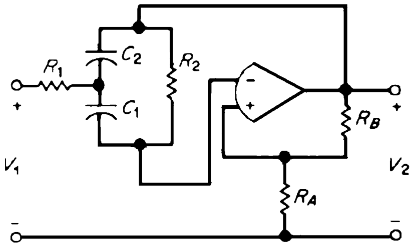


FIGURE 10.3.9 Delyiannis bandpass circuit.

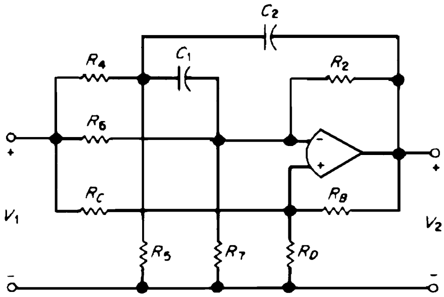


FIGURE 10.3.10 Friend biquadratic circuit.

$$\frac{V_2}{V_1}(s) = \frac{K_2 s^2 + as + b}{s^2 + (\omega_p / Q_p)s + \omega_p^2} \tag{25}$$

where

$$a = \frac{K_2}{C_2}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + \frac{K_2}{C_1}\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{K_1}{R_1 C_2}\left(1 + \frac{R_A}{R_B}\right)$$

$$- \frac{K_3}{R_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\left(1 + \frac{R_A}{R_B}\right)$$

$$b = \frac{1}{C_1 C_2}\left[\frac{K_2}{R_1}\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{K_3}{R_1 R_3}\left(1 + \frac{R_A}{R_B}\right)\right] \tag{26}$$

$$\frac{\omega_p}{Q_p} = \frac{C + C_2}{C_1 C_2}\left(\frac{1}{R_2} - \frac{R_A}{R_B R_3}\right) - \frac{R_A}{R_B R_1 C_2}$$

$$\omega_p^2 = \frac{1}{R_1 C_1 C_2}\left(\frac{1}{R_2} - \frac{R_A}{R_B R_3}\right)$$

and

$$K_1 = \frac{R_5}{R_1 + R_5} \qquad K_2 = \frac{R_D}{R_C + R_D} \qquad K_3 = \frac{R_7}{R_6 + R_7} \tag{27}$$

$$R_4 = \frac{R_C R_D}{R_C + R_D} \qquad R_1 = \frac{R_4 R_5}{R_4 + R_5} \qquad R_3 = \frac{R_6 R_7}{R_6 + R_7}$$

With this circuit, normal practice is to make $C_1 = C_2$. If a high-pass network is needed, $a = b = 0$; this leads to constraints on K_1, K_3, and the resistors. For band-reject or notch filters, $a = 0$, other constraints follow from Eqs. (26), but it is usually possible to adjust them so that all element values are positive.

BIQUADRATICS WITH GENERALIZED IMPEDANCE CONVERTERS

Temes describes a general method for designing any biquadratic transfer function using two operational amplifiers and eight impedances. The GIC, introduced earlier, can be used. A complete circuit is shown in Fig. 10.3.11. For this circuit,

$$\frac{V_2}{V_1}(s) = \frac{Y_A(Y_1 Y_3 - Y_0 Y_3) + Y_B Y_2 (Y_4 + Y_5)}{Y_1 Y_3 (Y_A + Y_5) + Y_2 Y_4 (Y_B + Y_0)} \tag{28}$$

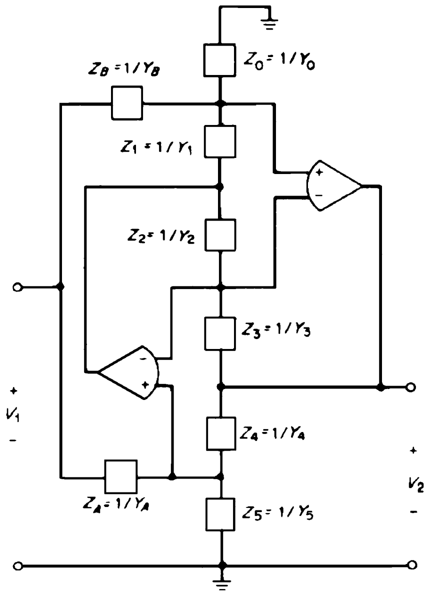


FIGURE 10.3.11 GIC biquadratic filter.

Table 10.2.1 shows to choose the elements for various types of transfer function. In version 1, the notch frequency is greater than the resonant frequency of the circuit, while in version 2, the notch frequency is less than the resonant frequency. In addition, version 2 requires $G_1G_3 > G_0G_2$. These circuits have the property of low sensitivity at the expense of two amplifiers.

SALLEN AND KEY NETWORKS

The circuits of Figs. 10.3.12, 10.3.13, and 10.3.14 are low-pass, high-pass, and bandpass circuits, respectively, having a positive gain K. Design of any of these circuits requires choice of suitable linear and quadratic denominator factors, transformation and frequency scaling, and coefficient matching. Since there are more elements

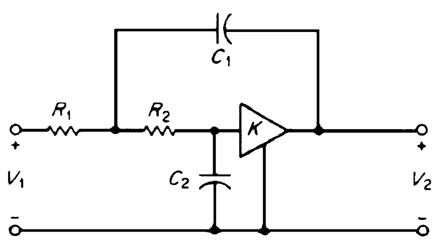


FIGURE 10.3.12 A low-pass active filter network with gain K greater than 0.

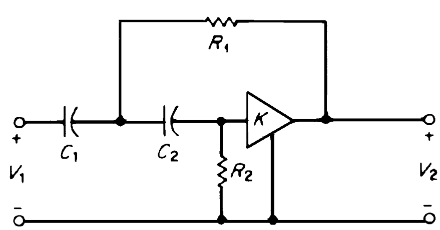


FIGURE 10.3.13 A high-pass active filter network with gain K greater than 0.

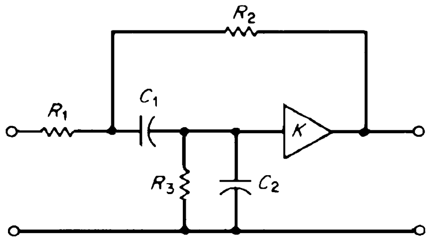


FIGURE 10.3.14 A bandpass active filter network with gain K greater than 0.

to be specified than there are constraints, two elements may be chosen arbitrarily. Often, $K = 1$ or $K = 2$ leads to a good network. For Fig. 10.3.12,

$$\frac{V_2}{V_1}(s) = \frac{K/(R_1R_2C_1C_2)}{s^2 + \left[(1-K)\frac{1}{R_2C_2} + \frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right]s + \frac{1}{R_1R_2C_1C_2}} \tag{29}$$

For Fig. 10.3.13,

$$\frac{V_2}{V_1}(s) = \frac{Ks^2}{s_2 + \left[(1-K)\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}\right]s + \frac{1}{R_1R_2C_1C_2}} \tag{30}$$

For Fig. 10.3.14,

$$\frac{V_2}{V_1}(s) = \frac{\dfrac{Ks}{R_1C_2}}{s^2 + \left[\frac{(1-K)}{R_2C_2} + \frac{1}{R_3C_2} + \frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_1C_2}\right]s + \frac{1}{R_3C_1C_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \tag{31}$$

CHAIN NETWORK

Figure 10.3.15 shows a chain network that realizes low-pass functions and is easily designed. For this circuit,

$$\frac{V_2}{V_1}(s) = \frac{\omega_1\omega_2\omega_3\cdots\omega_n}{s^n + \omega_1 s^{n-1} + \omega_1\omega_2 s^{n-2} + \cdots + \omega_1\omega_2\omega_3\cdots\omega_n} \tag{32}$$

where

$$\omega_i = 1/R_iC_i \tag{33}$$

As an example, consider a third-order Bessel filter, for which

$$\frac{V_2}{V_1}(s) = \frac{15}{s^3 + 6s^2 + 15s + 15} \tag{34}$$

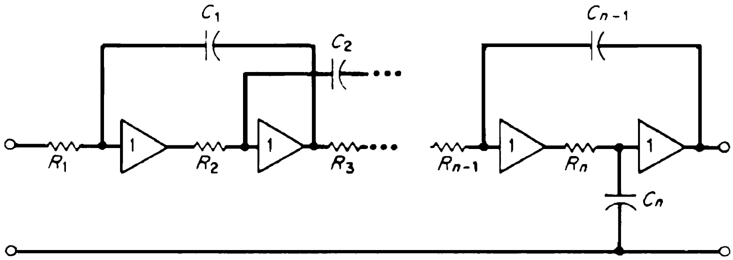


FIGURE 10.3.15 An *RC*-unity-gain amplifier realization of an active low-pass filter.

Choose $1/R_1C_1 = 6$, $6(1/R_2C_2) = 15$, and $15(1/R_3C_3) = 15$. If all C's are set to 1.0, then $R_1 = 1/6$, $R_2 = 2/5$, and $R_3 = 1$. Use frequency and impedance scaling as required.

LEAPFROG FILTERS

Also called active ladders or multiple feedback filters, these circuits use the tabulated element values from Table 10.1.10 to develop a set of active networks that have the sensitivity characteristics of passive ladders. The process may be extended from low-pass to bandpass networks using the transformation from prototype to bandpass filter disc under "Bandpass Filter" and techniques that will be discussed in this paragraph. The term "leapfrog" was suggested by Girling and Good,[28] the inventors, because of the topology.

Figure 10.3.16*a* shows a conventional fourth-order low-pass prototype network, and Fig. 10.3.16*b* shows a block diagram of a simulation with the same equations, which follow, using Laplace notation. In writing these equations and preparing the block diagram, current terms have been multiplied by an arbitrary constant R so that all variables appear to have the dimensions of voltage. This simplifies the block diagram and later examples.

$$I_1 = (V_1 - V_3)[R/(R_1 + sL_1)]$$

$$V_3 = (RI_1 - RI_3)(1/sC_2R)$$

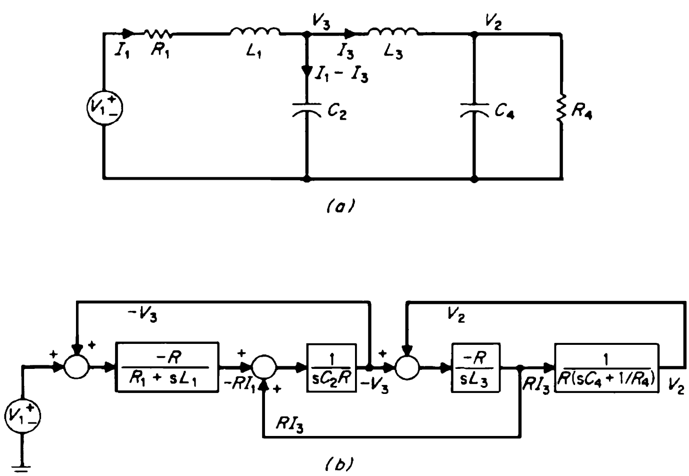


FIGURE 10.3.16 (*a*) Low-pass prototype ladder filter; (*b*) block-diagram simulation.

$$RI_3 = (V_3 - V_2)(R/sL_3) \tag{35}$$

$$V_2 = RI_3 \frac{1}{R(1/R_4 + sC_4)}$$

Though shown for a specific case, the technique is general and may be extended for any order of ladder network. In the simulation it should be noted that the algebraic signs of the blocks alternate. This variation is important in the realization.

In Fig. 10.3.16*b*, the currents are simulated by voltages, and this suggests the use of operational amplifiers as realization elements. The blocks in simulation require integrations, which the readily achieved with operational amplifiers, resistors, and capacitors. Figure 10.3.17 shows suitable combinations that will realize integrators, both inverting and noninverting, and also lossy integrators, which have a pole in the left half plane, rather than at the origin. Bruton[25] shows that, for integration, the circuit of Fig. 10.3.17*b* has superior performance compared with Fig. 10.3.17*a* when the imperfections of nonideal operational amplifiers are considered.

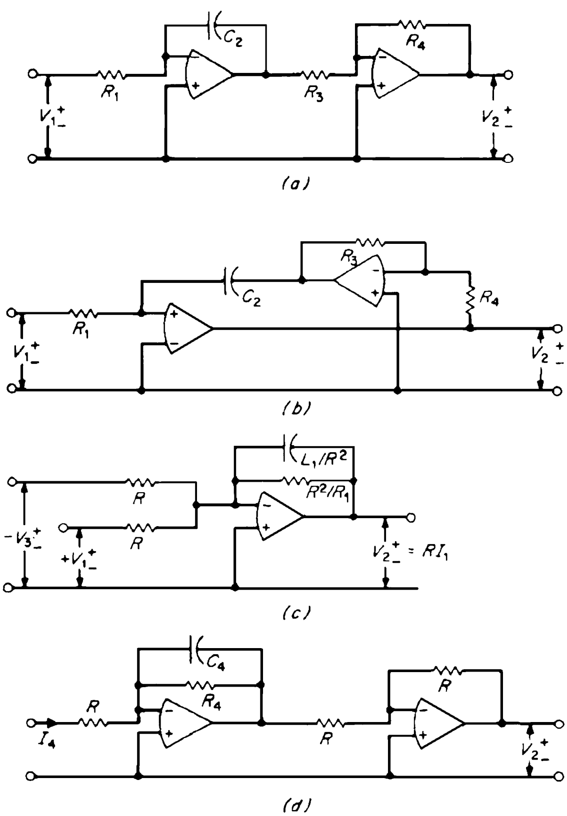


FIGURE 10.3.17 Building blocks for leapfrog filters: (*a*) noninverting integrator, for which $V_2/V_1 = R_4/sC_2R_1R_3$; (*b*) noninverting integrator, for which $V_2/V_1 = R_4/sC_2R_1R_3$; (*c*) lossy, summing integrator to realize $(V_1 - V_3)R/(R_1 + sL_1) = -RI_1$; (*d*) lossy, noninverting integrator to realize $V_2/RI_4 = 1/R(1/R_4 + sC_4)$.

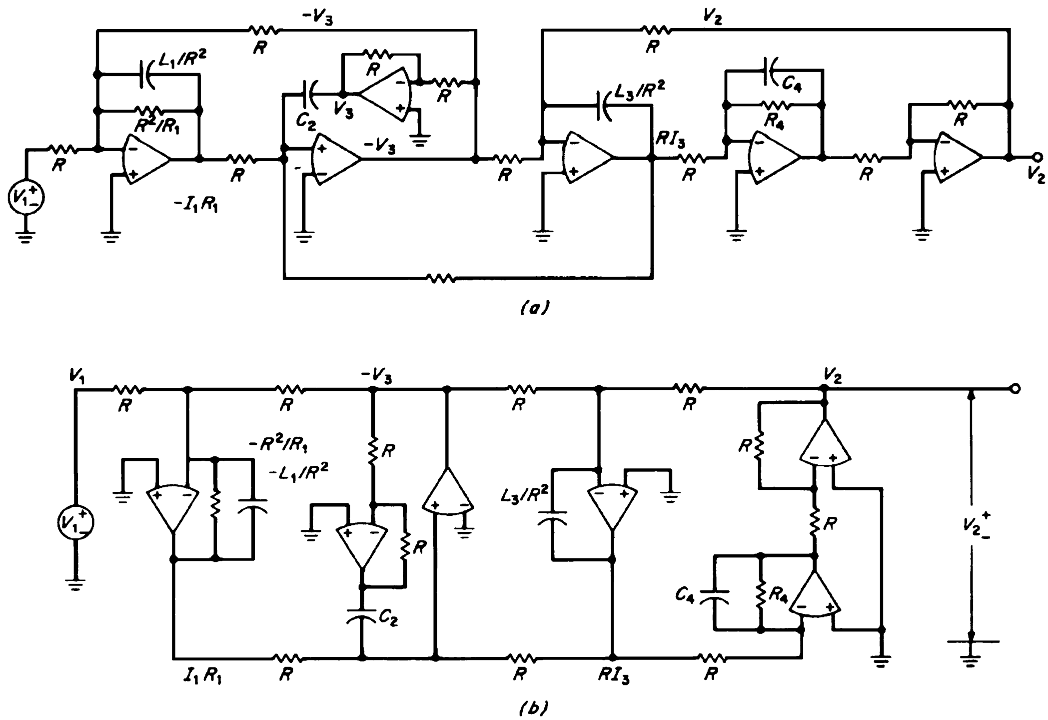


FIGURE 10.3.18 Two arrangements of a leapfrog low-pass circuit: (*a*) block-diagram arrangement; (*b*) ladder arrangement.

In Fig. 10.3.18, the combination of these blocks into a circuit is shown. In the preparation of this drawing, the integrator of Fig. 10.3.16*b* has been used, and the drawing is given in two forms. Figure 10.3.18*a* follows from the simulation, while Fig. 10.3.18*b* is a rearrangement that emphasizes the ladder structure.

The design of a low-pass leapfrog ladder may be summarized in these steps.

1. Select a normalized low-pass filter from Table 10.1.10
2. Identify the integrations represented by inductors, capacitors, and series resistor-inductor or parallel resistor-capacitor combinations. For each, determine an appropriate block diagram.
3. Connect together, using inverters, summers, and gain adjustment as needed.
4. Apply techniques of frequency and magnitude scaling to achieve a practical circuit.

BANDPASS LEAPFROG CIRCUITS

The technique of the previous section may be extended to bandpass circuits. The basic idea follows from the low-pass to bandpass transformation introduced in "Bandpass Filters" and from the recognition that it is possible to build second-order resonators from operational amplifiers, capacitors, and resistors. When the transformation is applied to an inductor, the circuit of Fig. 10.3.19*a* results. The resistor is added to allow for losses or source/load resistors that may be present. Similarly, the transformation applied to a

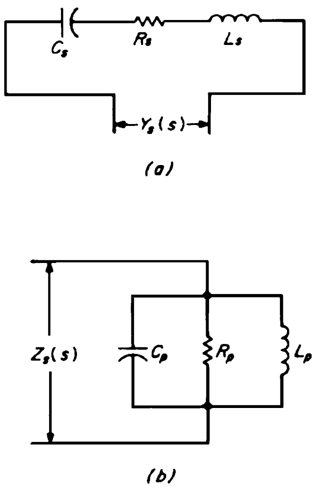


FIGURE 10.3.19 Passive resonant circuits: (*a*) series resonator; (*b*) parallel resonator.

capacitor yields Fig. 10.3.19*b*. It is to be noted that the forms of the equations are identical, and since the leapfrog technique makes use of simulation, the realizations will be similar. The necessary equations are given by Eq. (36).

Figure 10.3.20 shows an active simulation of a resonant circuit. This circuit is similar to that of Fig. 10.3.8, though the first two stages are interchanged. The new circuit has the advantage that both inverted and noninverted resonant outputs are available. Further, the input allows for summing operations, which may be needed in the leapfrog realization. Appropriate equations are given by Eq. (37).

$$Y_s(s) = \frac{(1/L_s)s}{s^2 + (R_s/L_s)s + 1/L_sC_s}$$
$$Z_p(s) = \frac{(1/C_p)s}{s^2 + (1/R_pC_p)s + 1/L_pC_p} \qquad (36)$$

$$V_{o2}(s) = -V_{o1}(s) = \frac{(1/R_3C_1)s}{s^2 + (1/R_1C_1)s + 1/R_2R_4C_1C_2}(V_{11} + V_{12})(s) \qquad (37)$$

The implementation of this circuit is substantially the same as that of Fig. 10.3.18, with the resonators being used as the blocks of the simulation. Since both inverted and noninverted signals are available, the one needed is chosen. Table 10.3.2 gives the parameters of the active resonators in terms of the transformed series or parallel resonant circuits. Frequency and magnitude scaling may be done either before or after the elements of the active resonator are determined, but it generally is more convenient to do it afterward.

An example is given to show this technique. It begins with a third-order Butterworth normalized low-pass filter, Fig. 10.3.21*a*, which has been taken from Table 10.1.10. The circuit is transformed to a bandpass network

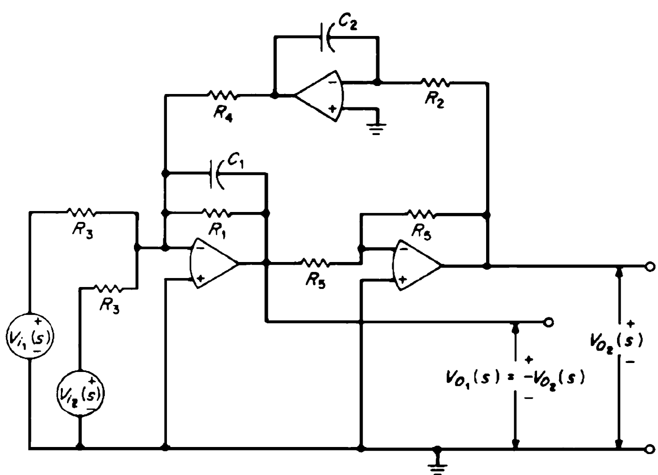


FIGURE 10.3.20 Active resonator.

TABLE 10.3.2 Resonator Design Relationships*

Circuit parameters from Fig. 10.3.20	Series circuit prototype values from Fig. 10.3.19*a*	Parallel circuit prototype values from Fig. 10.3.19*b*
$R_2 = R_4 = R$	$(1/C)\sqrt{L_s C_s}$	$(1/C)\sqrt{L_p C_p}$
R_1	$R\sqrt{L_s/C_s}$	$R\sqrt{C_p/L_p}$
R_3	$R\sqrt{L_s/C_s}$	$R\sqrt{C_p/L_p}$
R_5	Choose any convenient value	

*In this table it is presumed that $C_1 = C_2 = C$ and that this is chosen to be some convenient value. It is further presumed that $R_2 = R_4$.

for which the center frequency ω_0 is 1.0 rad/s and the bandwidth β' is 0.45 rad/s, corresponding to upper and lower half-power frequencies of 1.25 and 0.80 rad/s. This result is shown as Fig. 10.3.21*b*.

For the first and third stages of the circuit, application of the equations from Table 10.3.2 shows that $R_1 = R_3 = 20/9\ \Omega$, and that all other components have unit value. For the second stage, R_1 is infinite, as indicated by the table, and $R_3 = 9/40\ \Omega$. Other components have unit value. The complete circuit is shown as Fig. 10.3.21*c*. This circuit has been left in normalized form. Impedance and frequency denormalization techniques must be used to achieve reasonable values.

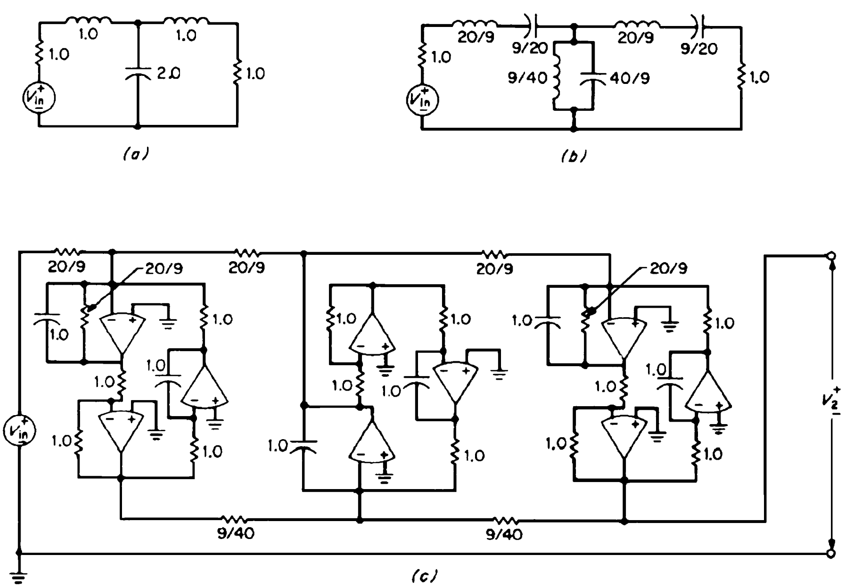


FIGURE 10.3.21 Leapfrog active resonator realization: (*a*) low-pass prototype; (*b*) bandpass transformation, $\omega_0' = 1.0\beta' = 0.45$; (*c*) complete circuit.