CHAPTER 10.4
SWITCHED CAPACITOR FILTERS

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Switched capacitor filters, also known as analog sampled data filters, result from a new technology that builds on the passive network theory of Darlington and implements the circuits in active-network integrated-circuit forms. Essentially, the switched capacitor replaces the resistor in operational-amplifier circuits, including the resonator. Early work was reported by Allstot, Broderson, Fried, Gray, Hosticka, Jacobs, Kuntz, and others. Huelsman[12] and Van Valkenburg[16] give additional information.

Consider the circuit of Fig. 10.4.1*a* and the two-phase clock signal of Fig. 10.4.1*b*. The circuit has two MOS switches and a capacitor C. The clock cycles the MOS switches between their low- and high-resistance states. In the analysis that follows, it is assumed that the clock speed is sufficiently high that a simplified analysis is valid. It is also assumed that the Nyquist sampling theorem is satisfied. It is possible to use discrete circuit analysis and z transforms if some of the approximations are not met.

Let the clock be such that switch A is closed and B is open. This may be modeled by Fig. 10.4.1*c*. The source will charge the capacitor to V_1. When the clock cycles so that B is closed and A is open, the capacitor will discharge toward V_2, transferring a charge

$$q_C = C(V_1 - V_2)$$

This will require a time $T_C = 1/f_C$, yielding an average current

$$i_{\mathrm{av}} = C(V_1 - V_2)/T_C$$

corresponding to a resistor $R_{\mathrm{eq}} = (V_1 - V_2)/i_{\mathrm{av}}$, or

$$R_{\mathrm{eq}} = T_C/C = 1/(Cf_C) \tag{1}$$

Figure 10.4.2 shows a conventional integrator, a damped integrator, and several sum and difference integrators, along with realizations and transfer functions of circuits implemented with switched capacitors. It is noted that the transfer functions are functions of the ratios of two capacitors, and this fact makes them useful. It is possible in integrated-circuit design to realize the ratio of two capacitors with high precision, leading to accurate designs of filters. With similar techniques it is possible to realize many of the second-order circuits given in earlier sections.

Possibly the most important application is in the realization of leapfrog filters. As discussed in the previous section, leapfrog filters use integrators to realize the resonators that are the basic building blocks. In this technology, the resistors of the resonators are replaced by switched capacitors. In essence, the technique is to realize the circuit with resonators, as was done in Fig. 10.3.21, and then to replace the resistors with switched capacitors. Though slightly less complex, the technique for low-pass filters is similar.

Consider the low-pass prototype filter of Fig. 10.4.3*a*. A simulation is shown in Fig. 10.4.3*b*. This simulation has equations that are identical with those of the prototype. While the simulation is similar to that of Fig. 10.3.16; two important differences may be noted. The first is that the termination resistors are separate,

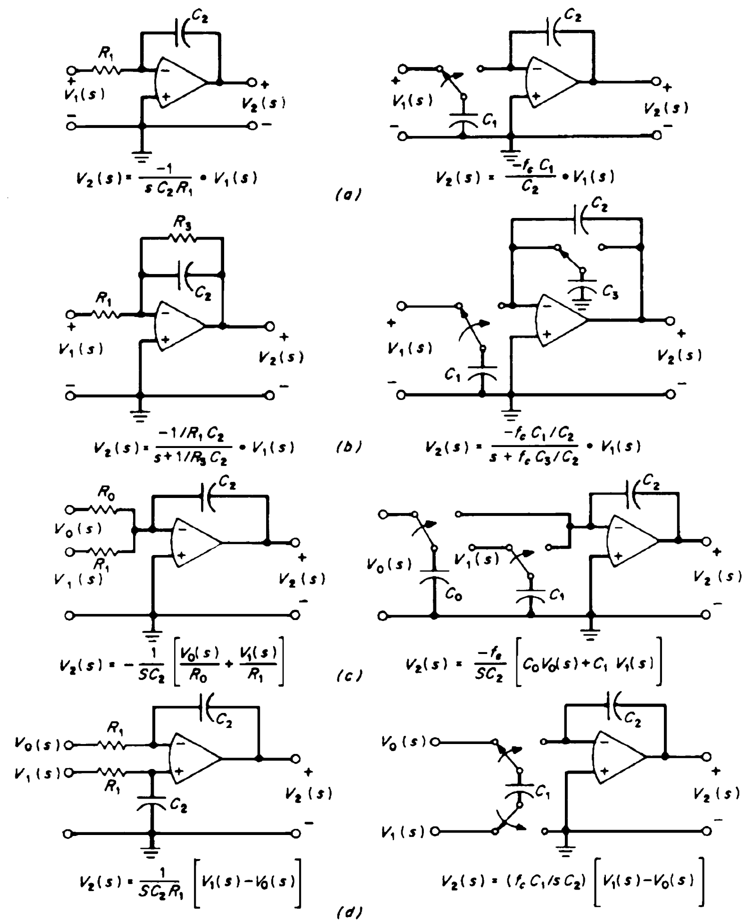

FIGURE 10.4.1 Integrators with switched capacitor realizations: (*a*) conventional integrator; (*b*) damped or lossy integrator; (*c*) summing integrator; (*d*) difference integrator.

rather than being incorporated with the input and output elements. The second is that all the elements have positive signs, and the amplifiers used are different types. This is more convenient. Figure 10.4.3*c* shows a switched capacitor equivalent for the low-pass filter. The equations for the simulation and for the development of the switched capacitor version follow.

$$
\begin{aligned}
RI_3 &= (V_1 - V_4)(R/R_1) \\
V_4 &= I_5(1/sC_3) = (I_3 - I_6)(R)(1/sC_3R) \\
RI_6 &= (V_4 - V_6)(R/sL_4) \\
V_2 &= R(I_6 - I_8)(1/sC_5R) \\
RI_8 &= V_2(R/R_2)
\end{aligned}
\tag{2}
$$

As was done previously, the current equations have been nominally multiplied by R so that all terms appear to be voltages. In practice, R may be set to 1.0 as scaling will take care of it.

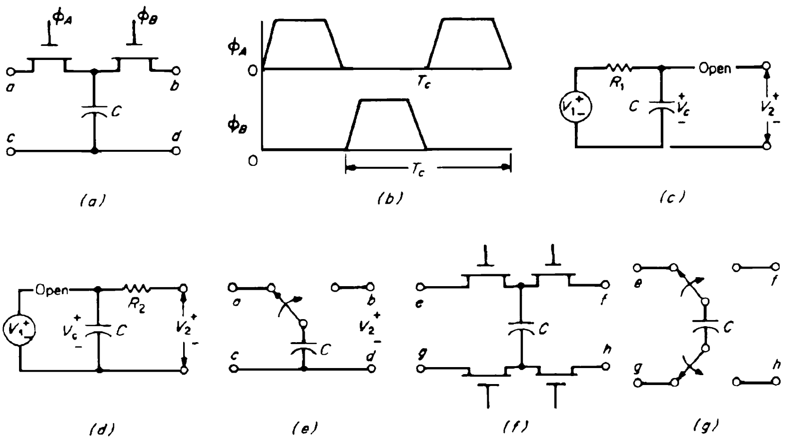

FIGURE 10.4.2 Development of equivalent circuit for a switched capacitor: (*a*) double MOS switch; (*b*) two-phase clock; (*c*) switch A closed; (*d*) switch B closed; (*e*) representation; (*f*) double-pole double-throw switch; and (*g*) representation.

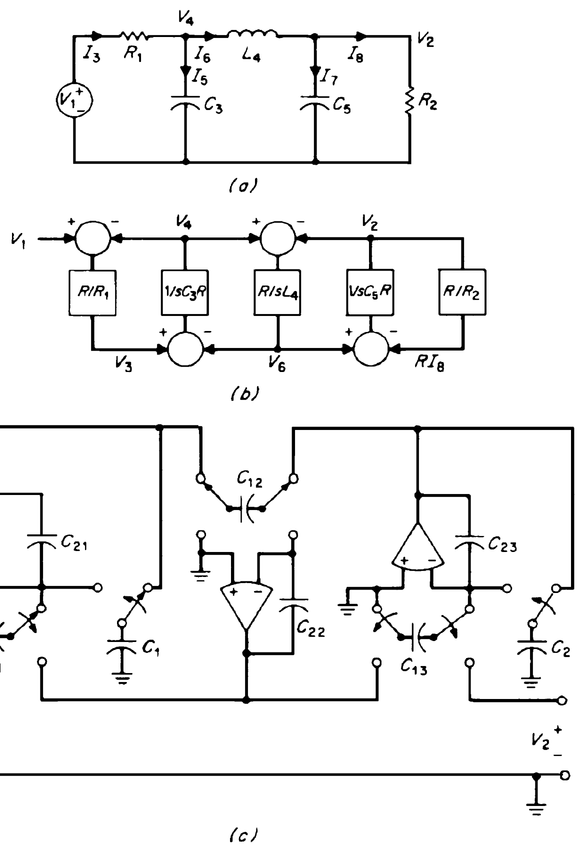

FIGURE 10.4.3 Low-pass switched capacitor filter development; (*a*) low-pass prototype and definition of equation symbols; C_3, L_4, and C_5 would be obtained from Table 10.1.10; (*b*) simulation of low-pass prototype; R may be set to 1.0; (*c*) switched capacitor equivalent network.

The next step is to determine the capacitor ratios in the final simulation. From Fig. 10.4.2*d* it may be seen that a typical integrator term is given by

$$\frac{V_2(s)}{V_1(s)-V_0(s)} = \frac{f_c C_1}{sC_2}$$

Similar results are obtained for the remaining integrators. The prototype values were obtained from Table 10.1.10, including C_3, L_4, and C_5 for this circuit. The similarity of terms then suggests that

$$\begin{aligned} C_3 &= (1/f_C)(C_{23}/C_{13}) \\ C_5 &= (1/f_C)(C_{25}/C_{15}) \end{aligned} \tag{3}$$

Extension to the inductors shows that

$$L_4 = (1/f_C)(C_{24}/C_{14}) \tag{4}$$

As used here, C_3, L_4, and C_5 are prototype values, but they may be magnitude- and frequency-scaled as desired to achieve realistic values. In Eqs. (3) and (4), the ratios C_2/C_1 are computed after the clock speed is known, and the second subscripts on both numerator and denominator denote which prototype the ratio has been derived from. In general they will differ for each element. In design, it is likely that one of these would be fixed and have a common value for all integrators, thus allowing the other capacitor to vary in each case. Figure 10.4.3*c* shows the circuit that results. It uses the difference-type integrator of Fig. 10.4.1*d*. It also shows a method for handling the terminations. It should be noted that the clock phases are adjusted so that alternate integrators have open and closed resistors at a given instant. This is necessary to avoid delay problems.

The extension of this technique to bandpass filters is a matter of combining the principles of leapfrog filters and switched capacitor implementation of resistors. From the specifications, a transformation to equivalent low-pass requirements is made, and the low-pass prototype is chosen. This prototype is transformed to a bandpass network, an appropriate simulation is developed, and the network is developed using integrators. Finally, scaling in frequency and magnitude is needed. It is desirable to test these simulations using computer programs designed for analysis of sampled data networks.