CHAPTER 15.4
BRIDGE CIRCUITS, DETECTORS, AND AMPLIFIERS

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PRINCIPLES OF BRIDGE MEASUREMENTS

Bridge circuits are used to determine the value of an unknown impedance in terms of other impedances of known value. Highly accurate measurements are possible because a null condition is used to compare ratios of impedances.

The most common bridge arrangement (Fig. 15.4.1) contains four branch impedances, a voltage source, and a null detector. Galvanometers, alone or with chopper amplifiers, are used as null detectors for dc bridges; while telephone receivers, vibration galvanometers, and tuned amplifiers with suitable detectors and indicators are used for null detection in ac bridges. The voltage across an infinite-impedance detector is

$$V_d = \frac{(Z_1Z_3 - Z_2Z_x)E}{(Z_1 + Z_2)(Z_3 + Z_x)}$$

If the detector has a finite impedance Z_5, the current in the detector is

$$I_d = \frac{(Z_1Z_3 - Z_2Z_s)E}{Z_5(Z_1 + Z_2)(Z_3 + Z_x) + Z_1Z_2(Z_3 + Z_x) + Z_3Z_x(Z_1 + Z_2)}$$

where E is the potential applied across the bridge terminals.

A null or balance condition exists when there is no potential across the detector. This condition is satisfied, independent of the detector impedance, when $Z_1Z_3 = Z_2Z_x$. Therefore, at balance, the value of the unknown impedance Z_x can be determined in terms of the known impedances Z_1, Z_2, and Z_3:

$$Z_x = Z_1Z_3/Z_2$$

Since the impedances are complex quantities, balance requires that both magnitude and phase angle conditions be met: $|Z_x| = |Z_1| \cdot |Z_3|/|Z_2|$ and $\theta_x = \theta_1 + \theta_3 - \theta_2$. Two of the known impedances are usually fixed impedances, while the third impedance is adjusted in resistance and reactance until balance is attained.

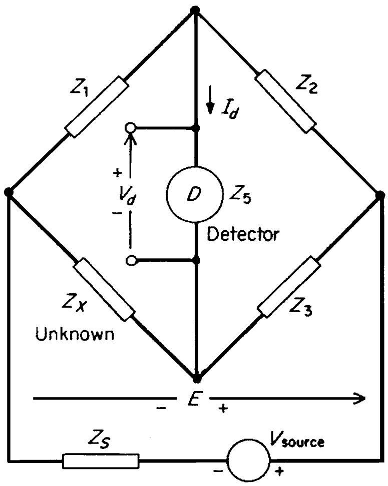

FIGURE 15.4.1 Basic impedance bridge.

The sensitivity of the bridge can be expressed in terms of the incremental detector current ΔI_d for a given small per-unit deviation δ of the adjustable impedance from the balance value. If Z_1 is adjusted, $\delta = \Delta Z_1/Z_1$ and

$$\Delta I_d = \frac{Z_3 Z_x E\delta}{(Z_3 + Z_x)^2[Z_5 + Z_1Z_2/(Z_1 + Z_2) + Z_3Z_x/(Z_3 + Z_x)]}$$

when Z_5 is the detector impedance.

If a high-input-impedance amplifier is used for the detector and impedance Z_5 can be considered infinite, the sensitivity can be expressed in terms of the incremental input voltage to the detector ΔV_d for a small deviation from balance

$$\Delta V_d = Z_3 Z_x E\delta/(Z_3 + Z_x)^2 = Z_1 Z_2 E\delta/(Z_1 + Z_2)^2$$

where $\delta = \Delta Z_1/Z_1$ and ΔZ_1 is the deviation of impedance Z_1 from its balance value Z_1. Maximum sensitivity occurs when the magnitudes of Z_3 and Z_x are equal (which for balance implies that the magnitudes of Z_1 and Z_2 are equal). Under this condition, $\Delta V_d = E\delta/4$ when the phase angles θ_3 and θ_x are equal; $\Delta V_d = E\delta/2$ when the phase angles θ_3 and θ_x are in quadrature; and ΔV_d is infinite when $\theta_3 = -\theta_x$, as is the case with lossless reactive components of opposite sign. In practice, the value of the adjustable impedance must be sufficiently large to ensure that the resolution provided by the finest adjusting step permits the desired precision to be obtained. This value may not be compatible with the highest sensitivity, but adequate sensitivity can be obtained for an order-of-magnitude difference between Z_3 and Z_x or Z_1 and Z_2, especially if a tuned-amplifier detector is used.

Interchanging the source and detector can be shown to be equivalent to interchanging impedances Z_1 and Z_3. This interchange does not change the equation for balance but does change the sensitivity of the bridge. For a fixed applied voltage E higher sensitivity is obtained with the detector connected from the junction of the two high-impedance arms to the junction of the two low-impedance arms.

The source voltage must be carefully selected to ensure that the allowable power dissipation and voltage ratings of the known and unknown impedances of the bridge are not exceeded. If the bridge impedances are low with respect to the source impedance Z_s, the bridge-terminal voltage E will be lowered. This can adversely affect the sensitivity, which is proportional to E. The source for an ac bridge should provide a pure sinusoidal voltage since the harmonic voltages will usually not be nulled when balance is achieved at the fundamental frequency. A tuned detector is helpful in achieving an accurate balance.

Balance Convergence. The process of balancing an ac bridge consists of making successive adjustments of two parameters until a null is obtained at the detector. It is desirable that these parameters do not interact and that convergence be rapid.

The equation for balance can be written in terms of resistances and reactances as

$$R_x + jX_x = (R_1 + jX_1)(R_3 + jX_3)/(R_2 + jX_2)$$

Balance can be achieved by adjusting any or all of the six known parameters, but only two of them need be adjusted to achieve the required equality of both magnitude and phase (or real and imaginary components). In a ratio-type bridge, one of the arms adjacent to the unknown, either Z_1 and Z_3, is adjusted. Assuming that Z_1 is adjusted, then to make the resistance adjustment independent of the change in the corresponding reactance, the ratio $(R_3 + jX_3)/(R_2 + jX_2)$ must be either real or imaginary but not complex. If this ratio is equal to the real number k, then for balance $R_x = kR_1$ and $X_x = kX_1$. In a product-type bridge, the arm opposite the unknown Z_2

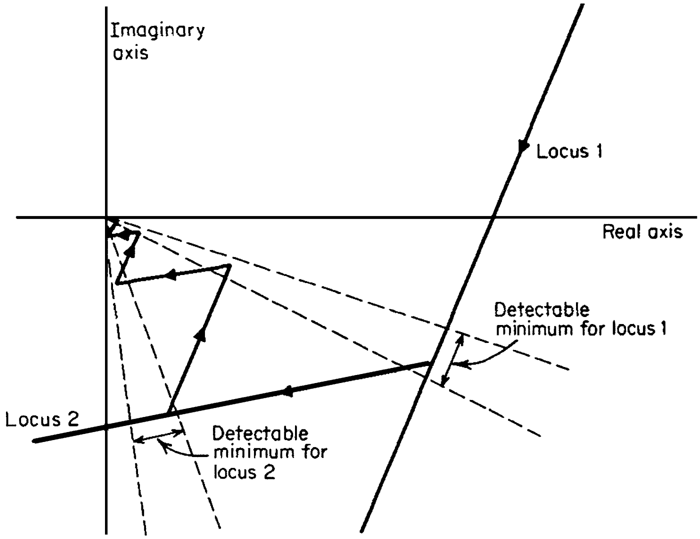

FIGURE 15.4.2 Linearized convergence locus.

is adjusted for balance, and the product of Z_1Z_3 must be either real or imaginary to make the resistance adjustment independent of the reactance adjustment.

Near balance, the denominator of the equation giving the detector voltage (or current) changes little with the varied parameter, while the numerator changes considerably. The usual convergence loci, which consist of circular segments, can be simplified to obtain linear convergence loci by assuming that the detector voltage near balance is proportional to the numerator, $Z_1Z_3 - Z_2Z_x$. Values of this quantity can be plotted on the complex plane. When only a single adjustable parameter is varied, a straight-line locus will be produced as shown in Fig. 15.4.2. Varying the other adjustable parameter will produce a different straight-line locus. The rate of convergence to the origin (balance condition) will be most rapid if these two loci are perpendicular, slow if they intersect at a small angle, and zero if they are parallel. The cases of independent resistance and reactance adjustments described above correspond to perpendicular loci.

RESISTANCE BRIDGES

The Wheatstone bridge is used for the precise measurement of two-terminal resistances. The lower limit for accurate measurement is about 1 Ω, because contact resistance is likely to be several milliohms. For simple galvanometer detectors, the upper limit is about 1 MΩ, which can be extended to 10^{12} by using a high-impedance high-sensitivity detector and a guard terminal to substantially eliminate the effects of stray leakage resistance to ground.

The Wheatstone bridge (Fig. 15.4.3) although historically older, may be considered as a resistance version of the impedance bridge of Fig. 15.4.1, and therefore the sensitivity equations are applicable. At balance

$$R_x = R_1R_3/R_2$$

Known fixed resistors, having values of 1, 10, 100, or 1000 Ω, are generally used for two arms of the bridge, for example, R_2 and R_3. These arms provide a ratio R_3/R_2, which can be selected from 10^{-3} to 10^3. Resistor R_1, typically adjustable to 10,000 Ω in 1- or 0.1-Ω steps, is adjusted to achieve balance. The ratio R_3/R_2 should be chosen so that R_1 can be read to its full precision. The magnitudes of R_2 and R_3 should be chosen to maximize the sensitivity while taking care not to draw excessive current.

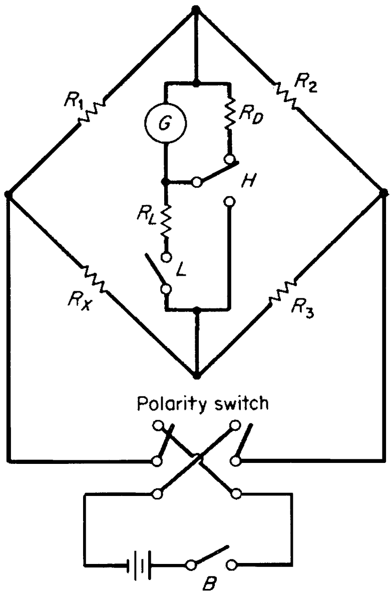

FIGURE 15.4.3 Wheatstone bridge.

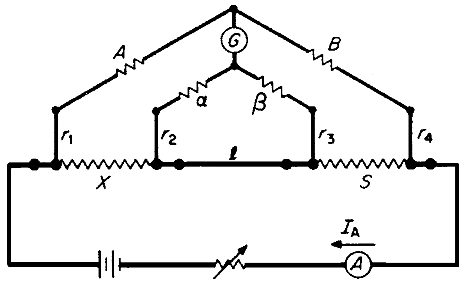

FIGURE 15.4.4 Kelvin double bridge. $A + B$ is typically 1000 Ω and $\alpha + \beta$ is typically 1000 Ω.

An alternate arrangement using R_1 and R_2 for the ratio resistors and adjusting R_3 for balance will generally provide a different sensitivity.

The battery key B should be depressed first to allow any reactive transients to decay before the galvanometer key is depressed. The low-galvanometer-sensitivity key L should be used until the bridge is close to balance. The high-sensitivity key H is then used to achieve the final balance. Resistance R_D provides critical damping between the galvanometer measurements. The battery connections to the bridge may be reversed and two separate resistance determinations made to eliminate any thermoelectric errors.

The Kelvin double bridge (Fig. 15.4.4) is used for the precise measurement of low-value four-terminal resistors in the range 1 $\mu\Omega$ to 10 Ω. The resistance to be measured X and a standard resistance S are connected by means of their current terminals in a series loop containing a battery, an ammeter, an adjustable resistor, and a low-resistance link l. Ratio-arm resistances A and B and α and β are connected to the potential terminals of resistors X and S as shown. The equation for balance is

$$X = S\frac{A}{B} + \frac{\beta l}{\alpha + \beta + l}\left(\frac{A}{B} - \frac{\alpha}{\beta}\right)$$

If the ratio α/β is made equal to the ratio A/B, the equation reduces to $X = S(A/B)$.

The equality of the ratios should be verified after the bridge is balanced by removing the link. If $\alpha/\beta = A/B$, the bridge will remain balanced. Lead resistances r_1, r_2, r_3, and r_4 between the bridge and the potential terminals of the resistors may contribute to ratio imbalance unless they have the same ratio as the arms to which they are connected. Ratio imbalance caused by lead resistance can be compensated by shunting α or β with a high resistance until balance is obtained with the link removed.

In some bridges a fixed standard resistor S having a value of the same order of magnitude as resistor X is used. Fixed resistors of 10, 100, or 1000 Ω are used for two arms, for example, B and β, with B and β having equal values. Bridge balance is obtained by adjusting tap switches to select equal resistances for the other two arms, for example, A and α, from values adjustable up to 1000 Ω in 0.1-Ω steps. In other bridges, only decimal ratio resistors are provided for A, B, α, and β, and balance is obtained by means of an adjustable standard having nine steps of 0.001 Ω each and a Manganin slide bar of 0.0011 Ω.

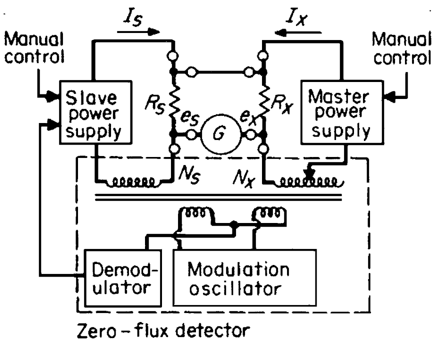

FIGURE 15.4.5 Comparator ratio bridge.

The battery connection should be reversed and two separate resistance determinations made to eliminate thermoelectric errors.

The dc-comparator ratio bridge (Fig. 15.4.5) is used for very precise measurement of four-terminal resistors. Its accuracy and stability depend mainly on the turns ratio of a precision transformer. The master current supply is set at a convenient fixed value I_x. The zero-flux detector maintains an ampere-turn balance, $I_xN_x = I_sN_s$, by automatically adjusting the current I_s from the slave supply as N_x is manually adjusted. A null reading on the galvanometer is obtained when $I_sR_s = I_xR_x$. Since the current ratio is precisely related to the turns ratio, the unknown resistance $R_x = N_xR_s/N_s$. Fractional turn resolution for N_x can be obtained by diverting a fraction of the current I_x as obtained from a decade current divider through an additional winding on the transformer. Turns ratios have been achieved with an accuracy of better than 1 part in 10^7. The zero-flux detector operates by superimposing a modulating mmf on the core using modulation and detector windings in a second-harmonic modulator configuration. The limit sensitivity of the bridge is set by noise and is about 3 μA turns.

Murray and Varley bridge circuits are used for locating faults in wire lines and cables. The faulted line is connected to a good line at one end by means of a jumper to form a loop. The resistance r of the loop is measured using a Wheatstone bridge. The loop is then connected as shown in Fig. 15.4.6 to form a bridge in which one arm contains the resistance R_x between the test set and the fault and the adjacent arm contains the remainder of the loop resistance. The galvanometer detector is connected across the open terminals of the loop, while the voltage supply is connected between the fault and the junction of fixed resistor R_2 and variable resistor R_3. When balance is attained

$$R_x = rR_3/(R_2 + R_3)$$

where r is the resistance of the loop. Resistance R_x is proportional to the distance to the fault. In the Varley loop of Fig. 15.4.7, variable resistor R_1 is adjusted to achieve balance and

$$R_x = \frac{rR_3 - R_1R_2}{R_2 + R_3}$$

where r is the resistance of the loop.

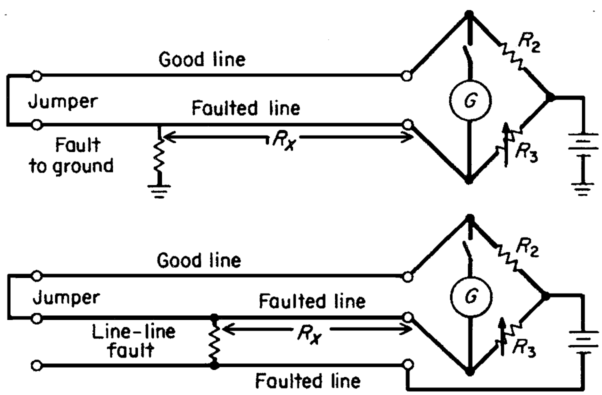

FIGURE 15.4.6 Murray loop-bridge circuits.

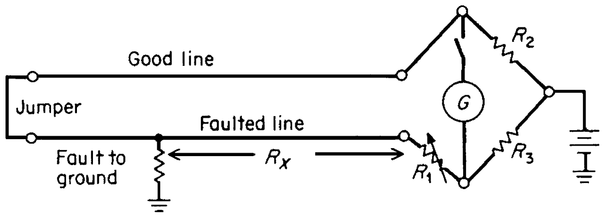

FIGURE 15.4.7 Varley loop circuit.

INDUCTANCE BRIDGES

General. Many bridge types are possible since the impedance of each arm may be a combination of resistances, inductances, and capacitances. A number of popular inductance bridges are shown in Fig. 15.4.8. In the balance equations L and M are given in henrys, C in farads, and R in ohms; ω is 2π times the frequency in hertz. The Q of an inductance is equal to $\omega L/R$, where R is the series resistance of the inductance.

The symmetrical inductance bridge (Fig. 15.4.8*a*) is useful for comparing the impedance of an unknown inductance with that of a known inductance. An adjustable resistance is connected in series with the inductance having the higher Q, and the inductance and resistance values of this resistance are added to those of the associated inductance to obtain the impedance of that arm. If this series resistance is adjusted along with the known inductance to obtain balance, the resistance and reactance balances are independent and balance convergence is rapid. If only a fixed inductance is available, the series resistance is adjusted along with the ratio R_3/R_2 until balance is obtained. These adjustments are interacting, and the rate of convergence will be proportional to the Q of the unknown inductance. Care must be taken to avoid inductive coupling between the known and unknown inductances since it will cause a measurement error.

The Maxwell-Wien bridge (Fig. 15.4.8*b*) is widely used for accurate inductance measurements. It has the advantage of using a capacitance standard which is more accurate and easier to shield and produces practically no external field. R_2 and C_2 are usually adjusted since they provide a noninteracting resistance and inductance balance. If C_2 is fixed and R_2 and R_1 or R_3 are adjusted, the balance adjustments interact and balancing may be tedious.

Anderson's bridge (Fig. 15.4.8*c*) is useful for measuring a wide range of inductances with reasonable values of fixed capacitance. The bridge is usually balanced by adjusting r and a resistance in series with the unknown inductance. Preferred values for good sensitivity are $R_1 = R_2 = R_3/2 = R_x/2$ and $L/C = 2R_x^2$. This bridge is also used to measure the residuals of resistors using a substitution method to eliminate the effects of residuals in the bridge elements.

Owen's bridge (Fig. 15.4.8*d*) is used to measure a wide range of inductance values in terms of resistance and capacitance. The inductance and resistance balances are independent if R_3 and C_3 are adjusted. The bridge can also be balanced by adjusting R_1 and R_3.

This bridge is useful for finding the incremental inductance of iron-cored inductors to alternating current superimposed on a direct current. The direct current can be introduced by connecting a dc-voltage source with a large series inductance across the detector branch. Low-impedance blocking capacitors are placed in series with the detector and the ac source.

Hay's bridge (Fig. 15.4.8*e*) is similar to the Maxwell-Wien bridge and is used for measuring inductances having large values of Q. The series $R_2 C_2$ arrangement permits the use of smaller resistance values than the parallel arrangement. The frequency-dependent $1/Q_x^2$ term in the inductance equation is inconvenient since the dials cannot be calibrated to indicate inductance directly unless the term is neglected, which causes a 1 percent error for $Q_x = 10$.

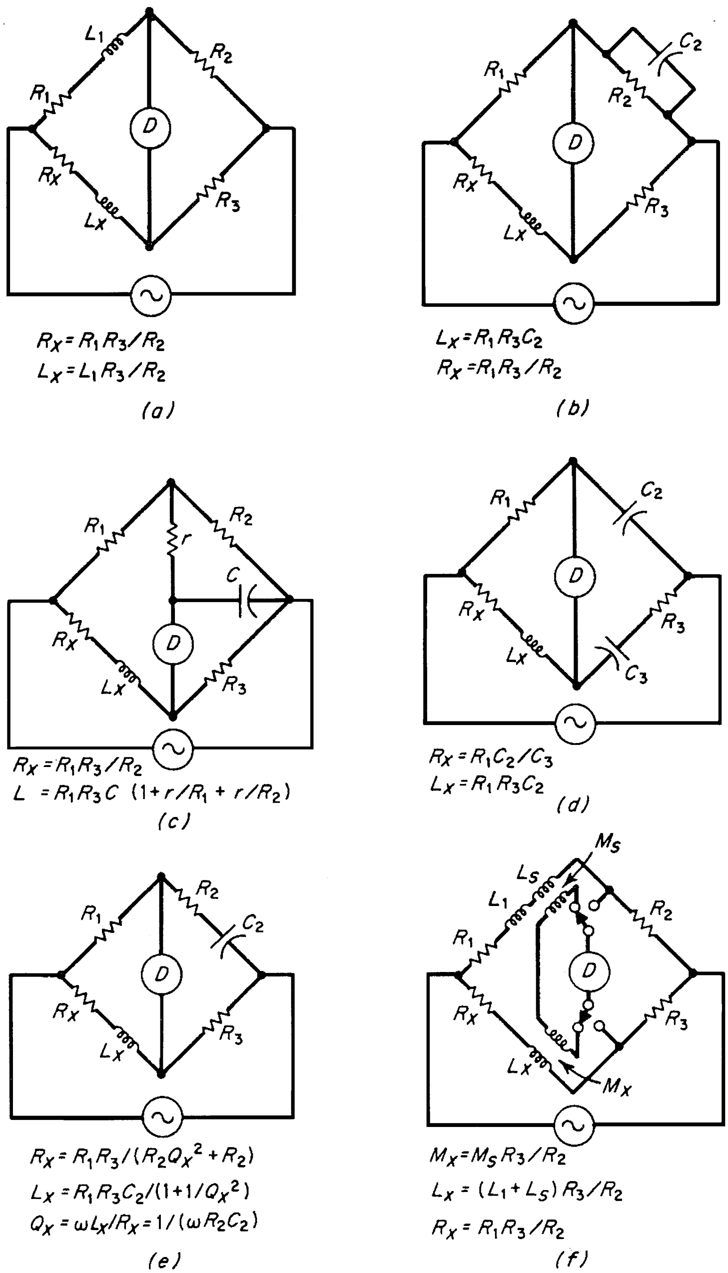

FIGURE 15.4.8 Inductance bridges: (*a*) symmetrical inductance bridge; (*b*) Maxwell-Wien bridge; (*c*) Anderson's bridge; (*d*) Owen's bridge; (*e*) Hay's bridge; (*f*) Campbell's bridge.

This bridge is also used for determining the incremental inductance of iron-cored reactors, as discussed for Owen's bridge.

Campbell's bridge (Fig. 15.4.8*f*) for measuring mutual inductance makes possible the comparison of unknown and standard mutual inductances having different values. The resistances and self-inductances of the primaries are balanced with the detector switches to the right by adjusting L_1 and R_1. The switches are thrown to the left, and the mutual-inductance balance is made by adjusting M_s. Care must be taken to avoid coupling between the standard and unknown inductances.

CAPACITANCE BRIDGES

Capacitance bridges are used to make precise measurements of capacitance and the associated loss resistance in terms of known capacitance and resistance values. Several different bridge circuits are shown in Fig. 15.4.9. In the balance equations R is given in ohms and C in farads, and ω is 2π times the frequency in hertz. The loss angle δ of a capacitor may be expressed either in terms of its series loss resistance r_s, which gives $\tan \delta = \omega C r_s$, or in terms of the parallel loss resistance r_p, in which case, $\tan \delta = 1/\omega C r_p$.

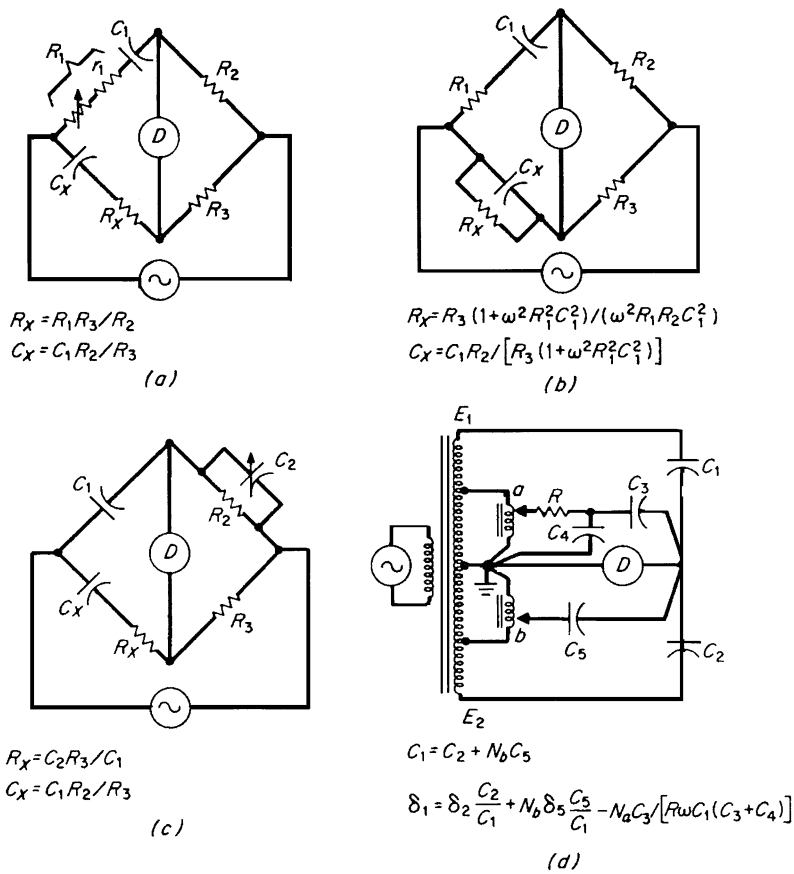

FIGURE 15.4.9 Capacitance bridges: (*a*) series-resistance-capacitance bridge; (*b*) Wien bridge; (*c*) Schering's bridge; (*d*) transformer bridge.

The Series* RC *bridge (Fig. 15.4.9*a*) is a resistance-ratio bridge used to compare a known capacitance with an unknown capacitance. The adjustable series resistance is added to the arm containing the capacitor having the smaller loss angle δ.

The Wien bridge (Fig. 15.4.9*b*) is useful for determining the equivalent capacitance C_x and parallel loss resistance R_x of an imperfect capacitor, e.g., a sample of insulation or a length of cable.

An important application of the Wien bridge network is its use as the frequency-determining network in *RC* oscillators.

Schering's bridge (Fig. 15.4.9*c*) is widely used for measuring capacitance and dissipation factors. The unknown capacitance is directly proportional to known capacitance C_1. The dissipation factor $\omega C_x R_x$ can be measured with good accuracy using this bridge. The bridge is also used for measuring the loss angles of high-voltage power cables and insulators. In this application, the bridge is grounded at the R_2/R_3 node, thereby keeping the adjustable elements R_2, R_3, and C_2 at ground potential.

The transformer bridge is used for the precise comparison of capacitors, especially for three-terminal shielded capacitors. A three-winding toroidal transformer having low leakage reactance is used to provide a stable ratio, known to better than 1 part in 10^7. In Fig. 15.4.9*d* capacitors C_1 and C_2 are being compared, and a balance scheme using inductive-voltage dividers *a* and *b* is shown. It is assumed that $C_1 > C_2$ and loss angle $\delta_2 > \delta_1$. In-phase current to balance any inequality in magnitude between C_1 and C_2 is injected through C_5 while quadrature current is supplied by means of resistor *R* and current divider $C_3/(C_3 + C_4)$. The current divider permits the value of *R* to be kept below 1 MΩ. Fine adjustments are provided by dividers *a* and *b*. N_a is the fraction of the voltage E_1 that is applied to *R*, while N_b is the fraction of the voltage E_2 applied to C_5, δ_1 is the loss angle of capacitor C_1 and $\tan \delta_1 = \omega C_1 r_1$, where r_1 is the series loss resistance associated with C_1. The impedance of C_3 and C_4 in parallel must be small compared with the resistance of *R*.

The substitution-bridge method is particularly valuable for determining the value of capacitance at radtio frequency. The *shunt-substitution method* is shown for the series *RC* bridge in Fig. 15.4.10. Calibrated adjustable standards R_s and C_s are connected as shown, and the bridge is balanced in the usual manner with the unknown capacitance disconnected. The unknown is then connected in parallel with C_s, and C_s and R_s are readjusted to obtain balance. The unknown capacitance C_x and its equivalent series resistance R_x are determined by the rebalancing changes ΔC_s and ΔR_s in C_s and R_s, respectively: $C_x = \Delta C_s$ and $R_x = \Delta R_s(C_{s1}/C_x)^2$, where C_{s1} is the value of C_s in the initial balance.

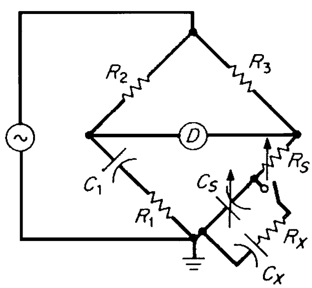

FIGURE 15.4.10 Substitution measurement.

In *series substitution* the bridge arm is first balanced with the standard elements alone, the standard elements having an impedance of Z_{s1}, and then the unknown is inserted in series with the standard elements. The standard elements are readjusted to an impedance Z_{s2} to restore balance. The unknown impedance Z_x is equal to the change in the standard impedance, that is, $Z_x = Z_{s1} - Z_{s2}$.

Measurement accuracy depends on the accuracy with which the changes in the standard values are known. The effects of residuals, stray capacitance, stray coupling, and inaccuracies in the impedances of the other three bridges arms are minimal, since these effects are the same with and without the unknown impedance. The proper handling of the leads used to connect the unknown impedance can be important.

FACTORS AFFECTING ACCURACY

Stray Capacitance and Residuals. The bridge circuits of Figs. 15.4.8 and 15.4.9 are idealized since stray capacitances which are inevitably present and the residual inductances associated with resistances and connecting leads have been neglected. These spurious circuit elements can disturb the balance conditions and

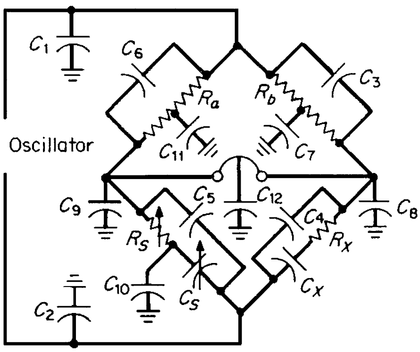

FIGURE 15.4.11 Stray capacitances in unshielded and ungrounded bridge.

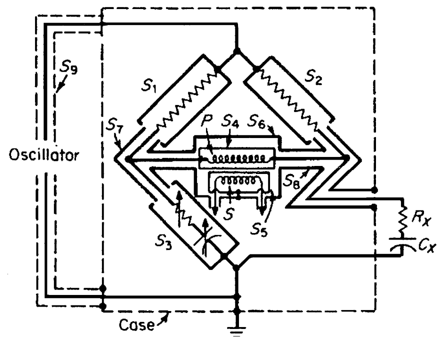

FIGURE 15.4.12 Bridge with shields and ground.

result in serious measurement errors. Detailed discussions of the residuals associated with the various bridges are given in Souders.

Shielding and grounding can be used to control errors caused by stray capacitance. Stray capacitances in an ungrounded, unshielded series *RC* bridge are shown schematically by C_1 through C_{12} in Fig. 15.4.11. The elements of the bridge may be enclosed in the grounded metal shield, as shown schematically in Fig. 15.4.12. Shielding and grounding eliminate some capacitances and make the others definite localized capacitances which act in a known way as illustrated in Fig. 15.4.13. The capacitances associated with terminal *D* shunt the oscillator and have no adverse effect. The possible adverse effects of the capacitance associated with the output diagonal *EF* are overcome by using a shielded output transformer. If the shields are adjusted so that $C_{22}/C_{21} = R_a/R_b$, the ratio of the bridge is independent of frequency. Capacitance C_{24} can be taken into account in the calibration of C_s, and capacitance C_{23} can be measured and its shunting effect across the unknown impedance can be calculated. Shielding, which is used at audio frequencies, becomes more necessary as the frequency and impedance levels are increased.

Guard circuits (Fig. 15.4.14) are often used at critical circuit points to prevent leakage currents from causing measurement errors. In an unguarded circuit surface leakage current may bypass the resistor *R* and flow through the detector *G*, thereby giving an erroneous reading. If a guard ring surrounds the positive terminal

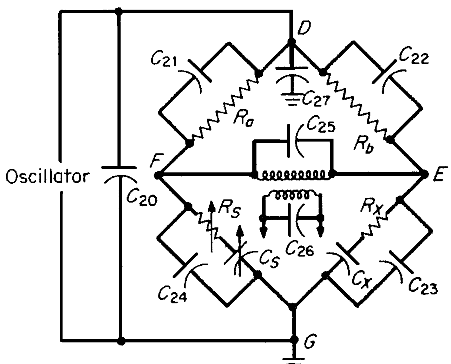

FIGURE 15.4.13 Schematic circuit of shielded and grounded bridge.

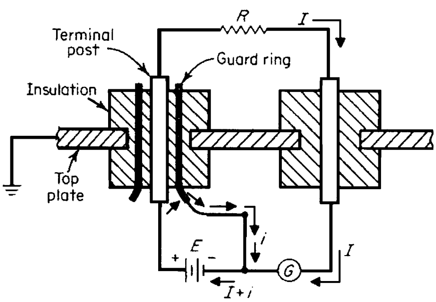

FIGURE 15.4.14 Leakage current in guarded circuit. (*Leeds and Northrup*)

post (as in the circuit of Fig. 15.4.14), the surface leakage current flows through the guard ring and a noncritical return path to the voltage source. A true reading is obtained since only the resistor current flows through the detector.

Coaxial leads and twisted-wire pairs may be used in connecting impedances to a bridge arm in order to minimize spurious-signal pickup from electrostatic and electromagnetic fields. It is important to keep lead lengths short, especially at high frequencies.

BRIDGE DETECTORS AND AMPLIFIERS

Galvanometers are used for null decision in dc bridges. The permanent-magnet moving-coil d'Arsonval galvanometer is widely used. The suspension provides a restoring torque so that the coil seeks a zero position for zero current. A mirror is used in the sensitive suspension-type galvanometer to reflect light from a fixed source to a scale. This type of galvanometer is capable of sensitivities on the order of 0.001 μA per millimeter scale division but is delicate and subject to mechanical disturbances. Galvanometers for portable instruments generally have indicating pointers and use taut suspensions which are less sensitive but more rugged and less subject to disturbances. Sensitivities are typically in the range of 0.5 μA per millimeter scale division. Galvanometers exhibit a natural mechanical frequency which depends on the suspension stiffness and the moment of inertia. Overshoot and oscillatory behavior can be avoided without an excessive increase in response time if an external resistance of the proper value to produce critical damping is connected across the galvanometer terminals.

Null-detector amplifiers incorporating choppers or modulators (see Chap. 15.3) are used to amplify the null output signal from dc bridges to provide higher sensitivity and permit the use of rugged, less-sensitive microammeter indicators. Null-detector systems such as the L&N 9838 are available with sensitivities of 10 nV per division for a 300-Ω input impedance. The Guideline 9460A nanovolt amplifier uses a light-beam-coupled amplifier that can provide 7.5-mm deflection per nV when used with a sensitive galvanometer. The input signal polarity to this amplifier may be reversed without introducing errors because of hysteresis or input offset currents. This reversal-capability is useful to balance out parasitic or thermal emfs in the measured circuit.

Frequency-selective amplifiers are extensively used to increase the sensitivity of ac bridges. An ac amplifier with a twin-T network in the feedback loop provides full amplification at the selected frequency but falls off rapidly as the frequency is changed. Rectifiers or phase-sensitive detectors are used to convert the amplified ac signal into a direct current to drive a dc microammeter indicator. The General Radio 1232-A tuned amplifier and null detector, which is tunable from 20 Hz to 20 kHz with fixed-tuned frequencies of 50 and 100 kHz, provides a sensitivity better than 0.1 μV. Cathode-ray-tube displays using Lissajous patterns are also used to indicate the deviation from null conditions. Amplifier and detector circuits are described in Chap. 15.3.

MISCELLANEOUS MEASUREMENT CIRCUITS

Multifrequency LCR meters incorporate microprocessor control of ranging and decimal-point positioning, which permits automated measurement of inductance, capacitance, and resistance in less than 1 s. Typical of the new generation of microprocessor instrumentations is the General Radio 1689 Digibridge, which automatically measures a wide range of L, C, R, D, and Q values from 12 Hz to 100 kHz with a basic accuracy of 0.02 percent. This instrument compares sequential measurements rather than obtaining a null condition and therefore is not actually a bridge.

Similar performance is provided by the Hewlett-Packard microprocessor-based 4274A (100 Hz to 100 kHz) and 4275A (10 kHz to 10 MHz) multifrequency *LCR* meters, which measure the impedance of the device under test at a selected frequency and compute the value of L, C, R, D, and Q as well as the impedance, reactance, conductance, susceptance, and phase angle with a basic accuracy of 0.1 percent.

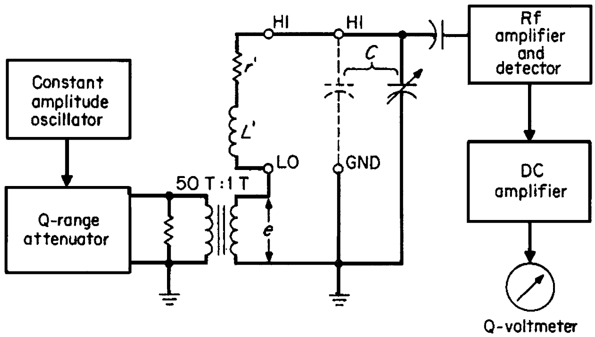

FIGURE 15.4.15 *Q* meter. (*Hewlett-Packard*)

The Q meter is used to measure the quality factor Q of coils and the dissipation factor of capacitors; the dissipation factor is the reciprocal of Q. The Q meter provides a convenient method of measuring the effective values of inductors and capacitors at the frequency of interest over a range of 22 kHz to 70 MHz.

The simplified circuit of a Q meter is shown in Fig. 15.4.15, where an unknown impedance of effective inductance L' and effective resistance r' is being measured. A sinusoidal voltage e is injected by the transformer secondary in series with the circuit containing the unknown impedance and the tuning capacitor C. The transformer secondary has an output impedance of approximately 1 mΩ. Unknown capacitors can be measured by connecting them between the HI and GND terminals while using a known standard inductor for L'.

Either the oscillator frequency or the tuning-capacitor value is adjusted to bring the circuit to approximate resonance, as indicated by a maximum voltage across capacitor C. At resonance $X_{L'} = X_c$ where $X_{L'} = 2\pi f L'$, $X_c = 1/2\pi f C$, L' is the effective inductance in henrys, C is the capacitance in farads, and f is the frequency in hertz. The current at resonance is $I = e/R$, where R is the sum of the resistances of the unknown and the internal circuit. The voltage across the capacitor C is $V_C = IX_C = eX_C/R$, and the indicated circuit Q is equal to V_C/e. In practice, the injected voltage e is known for each Q-range attenuator setting and the meter is calibrated to indicate the value of Q. Corrections for residual resistances and reactances in the internal circuit become increasingly important at higher frequencies (see Chap. 15.2). For low values of Q, neglecting the difference between the resonance and the approximate resonance achieved by maximizing the capacitor voltage may result in an unacceptable error. Exact equations are given in Chap. 15.2.

The rf impedance analyzer is a microprocessor-based instrument designed to measure impedance parameter values of devices and materials in the rf and UHF regions. The basic measurement circuit consists of a signal source, an rf directional bridge, and a vector voltage ratio detector as shown in Fig. 15.4.16. The measurement source produces a selectable 1-MHz to 1-GHz sinusoidal signal using frequency synthesizer techniques. The unknown impedance Z_x is connected to a test port of an rf directional bridge having resistor values Z equal to the 50-Ω characteristic impedance of the measuring circuit. The test-channel and reference-channel signal frequencies are converted to 100 kHz by the sampling i.f. converters in order to improve the accuracy of vector ratio detection. The vector ratio of the test channel and reference channel i.f. signals is detected for both the real and imaginary component vectors. The vector ratio, which is equal to e_2/e_1, is proportional to the reflection coefficient Γ, where $\Gamma = (Z - Z_x)/(Z + Z_x)$. The microprocessor computes values of L, C, D, Q, R, X, θ, G, and B from the real and imaginary components Γ_x and Γ_y of the reflection coefficient Γ. The basic accuracy of the magnitude of Γ is better than 1 percent, while that of other parameters is typically better than 2 percent.

The twin-T measuring circuit of Fig. 15.4.17 is used for admittance measurements at radio frequencies. This circuit operates on a null principle similar to a bridge circuit, but it has an advantage in that one side of the oscillator and detector are common and therefore can be grounded. The substitution method is used with this circuit, and therefore the effect of stray capacitances is minimized. The circuit is first balanced to a null condition

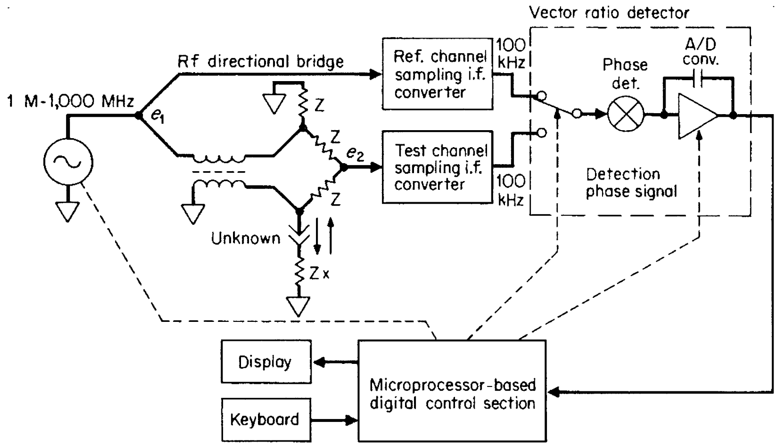

FIGURE 15.4.16 Rf impedance analyzer. (*Hewlett-Packard*)

with the unknown admittance $G_x + jB_x$ unconnected.

$$G_L = \omega^2 RC_1C_2(1 + C_o/C_3)$$
$$L = 1/[\omega^2(C_b + C_1 + C_2 + C_1C_2/C_3)]$$

The unknown admittance is connected to terminals *a* and *b*, and a null condition is obtained by readjusting the variable capacitors to values C'_a and C'_b The conductance G_x and the susceptance B_x of the unknown are proportional to the changes in the capacitance settings:

$$G_x = \omega^2 RC_1C_2(C'_a - C_a)/C_3 \qquad B_x = \omega(C_b - C'_b)$$

Measurement of Coefficient of Coupling. Two coils are inductively coupled when their relative positions are such that lines of flux from each coil link with turns of the other coil. The mutual inductance *M* in henrys can be measured in terms of the voltage *e* induced in one coil by a rate of change of current *di*/*dt* in the other coil; $M = -e_1/(di_2/dt) = -e_2/(di_1/dt)$. The maximum coupling between two coils of self-inductance L_1 and L_2 exists when all the flux from each of the coils links all the turns of the other coil; this condition produces the maximum value of mutual inductance, $M_{max} = \sqrt{L_1L_2}$. The *coefficient of coupling k* is defined as the ratio of the actual mutual inductance to its maximum value; $k = M/\sqrt{L_1L_2}$.

The value of mutual inductance can be measured using Campbell's mutual-inductance bridge. Alternately, the mutual inductance can be measured using a self-inductance bridge. When the coils are connected in series with the mutual-inductance emf aiding the self-inductance emf (Fig. 15.4.18*a*), the total inductance $L_a = L_1 + L_2 + 2M$ is measured. With the coils connected with

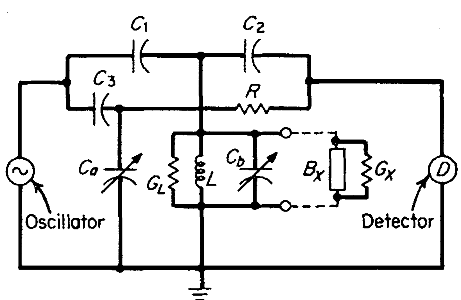

FIGURE 15.4.17 Twin-T measuring circuit. (*General Radio Co.*)

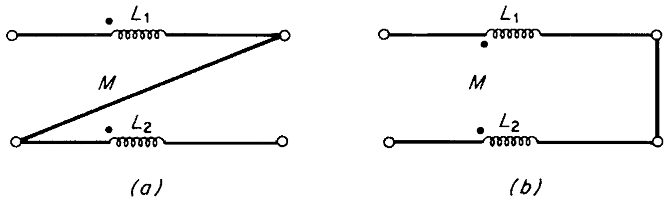

FIGURE 15.4.18 Mutual inductance connected for self-inductance measurement: (*a*) aiding configuration; (*b*) opposing configuration.

the mutual-inductance emf opposing the self-inductance emf (Fig. 15.4.18*b*), inductance $L_b = L_1 + L_2 - 2M$ is measured. The mutual inductance is $M = (L_a - L_b)/4$.

Permeameters are used to test magnetic materials. By simulating the conditions of an infinite solenoid, the magnetizing force H can be computed from the ampere-turns per unit length. When H is reversed, the change in flux linkages in a test coil induces an emf whose time integral can be measured by a ballistic galvanometer. The *Burrows permeameter* (Fig. 15.4.19) uses two magnetic specimen bars, S_1 and S_2, usually 1 cm in diameter and

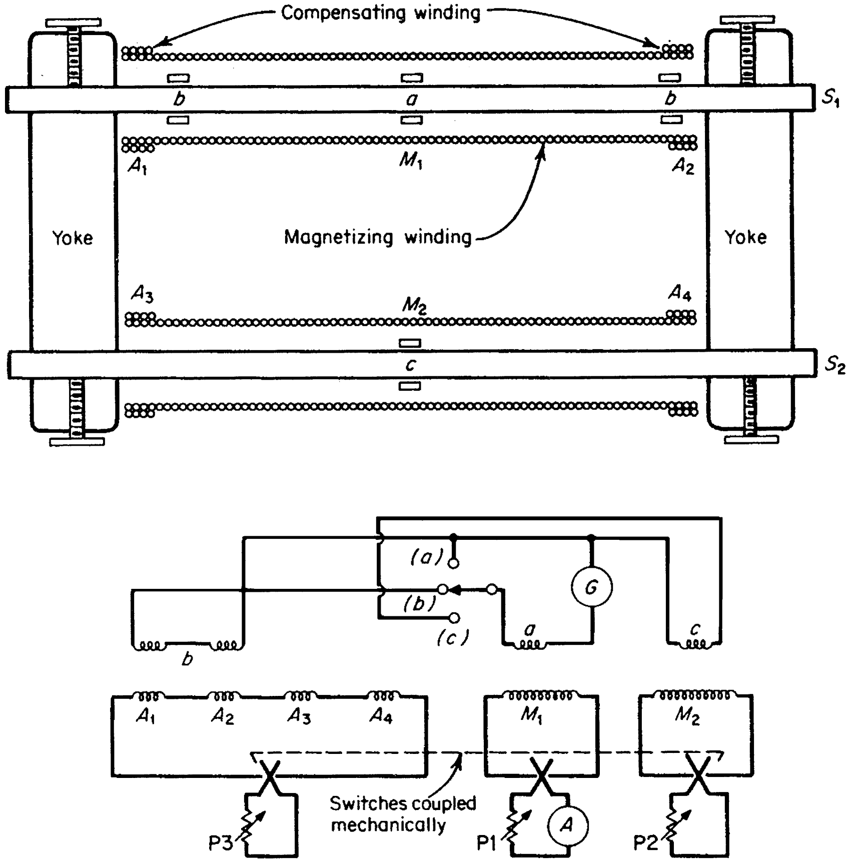

FIGURE 15.4.19 Burrows permeameter.

30 cm long, joined by soft-iron yokes. High precision is obtainable for magnetizing forces up to 300 Oe. The currents in magnetizing windings M_1 and M_2 and in compensating windings A_1, A_2, A_3, and A_4 are adjusted independently to obtain uniform induction over the entire magnetic circuit. Windings A_1, A_2, A_3, and A_4 compensate for the reluctance of the joints. The reversing switches are mechanically coupled and operate simultaneously. Test coils *a* and *c* each have *n* turns, while each half of the test coil *b* has *n*/2 turns. Coils *a* and *b* are connected in opposing polarity to the galvanometer when the switch is in position *b,* while coils *a* and *c* are opposed across the galvanometer for switch position *c*.

Potentiometer P1 is adjusted to obtain the desired magnetizing force, and potentiometers P2 and P3 are adjusted so that no galvanometer deflection is obtained on magnetizing current reversal with the switches in either position *b* or *c*. This establishes uniform flux density at each coil. The switch is now set at position *a* and the galvanometer deflection *d* is noted when the magnetizing current is reversed.

The values of *B* in gauss and *H* in oersteds can be calculated from

$$H = \frac{0.4\pi NI}{l} \qquad B = 10^8 \frac{dkR}{2an} - \frac{A-a}{a} H$$

where N = turns of coil M_1
I = current in coil M_1 (A)
l = length of coil M_1 (cm)
d = galvanometer deflection
k = galvanometer constant
R = total resistance of test coil *a* circuit
a = area of specimen (cm^2)
A = area of test coil (cm^2)
n = turns in test coil *a*

The term $(A - a)H/a$ is a small correction term for the flux in the space between the surface of the specimen and the test coil.

Other permeameters such as the Fahy permeameter, which requires only a single specimen, the Sandford-Winter permeameter, which uses a single specimen of rectangular cross section, and Ewing's isthmus permeameter, which is useful for magnetizing forces as high as 24,000 G, are discussed in Harris.

The frequency standard of the National Institute of Standards and Technology (formerly the National Bureau of Standards) is based on atomic resonance of the cesium atom and is accurate to 1 part in 10^{13}. The second is defined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyper-fine levels of the ground state of the atom of cesium 133. Reference frequency signals are transmitted by the NIST radio stations WWV and WWH at 2.5, 5, 10, and 15 MHz. Pulses are transmitted to mark the seconds of each minute. In alternate minutes during most of each hour, 500- or 600-Hz audio tones are broadcast. A binary-coded-decimal time code is transmitted continuously on a 100-Hz subcarrier. The carrier and modulation frequencies are accurate to better than 1 part in 10^{11}.

These frequencies are offset by a known and stable amount relative to the atomic-resonance frequency standard to provide "Coordinated Universal Time" (UTC), which is coordinated through international agreements by the International Time Bureau. UTC is maintained within ±0.9 s of the UT1 time scale used for astronomical measurements by adding leap seconds about once per year to UTC, depending on the behavior of the earth's rotation.

Quartz-crystal oscillators are used as secondary standards for frequency and time-interval measurement. They are periodically calibrated using the standard radio transmissions.

Frequency measurements can be made by comparing the unknown frequency with a known frequency, by counting cycles over a known time interval, by balancing a frequency-sensitive bridge, or by using a calibrated resonant circuit.

Frequency-comparison methods include using Lissajous patterns on an oscilloscope and heterodyne measurement methods. In Fig. 15.4.20, the frequency to be measured is compared with a harmonic of the 100-kHz reference oscillator. The difference frequency lying between 0 and 50 kHz is selected by the low-pass filter and

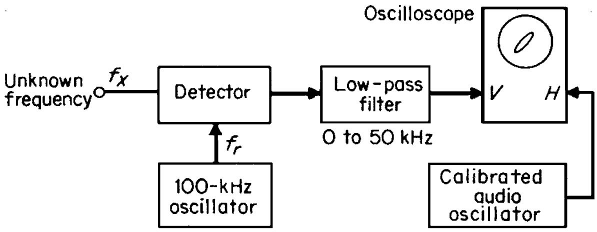

FIGURE 15.4.20 Heterodyne frequency-comparison method.

compared with the output of a calibrated audio oscillator using Lissajous patterns. Alternately, the difference frequency and the audio oscillator frequency may be applied to another detector capable of providing a zero output frequency.

Digital frequency meters provide a convenient and accurate means for measuring frequency. The unknown frequency is counted for a known time interval, usually 1 or 10 s, and displayed in digital form. The time interval is derived by counting pulses from a quartz-crystal oscillator reference. Frequencies as high as 50 MHz can be measured by using scalers (frequency dividers). Frequencies as high as 110 GHz are measured using heterodyne frequency-conversion techniques. At low frequencies, for example 60 Hz, better resolution is obtained by measuring the period $T = 1/f$. A counter with a built-in computer is available, which measures the period at low frequencies and automatically calculates and displays the frequency.

A frequency-sensitive bridge can be used to measure frequency to an accuracy of about 0.5 percent if the impedance elements are known. The Wien bridge of Fig. 15.4.21 is commonly used, R_3 and R_4 being identical slide-wire resistors mounted on a common shaft. The equations for balance are $f = 1/2\pi\sqrt{R_3R_4C_3C_4}$ and $R_1/R_2 = R_4/R_3 + C_3/C_4$.

In practice, the values are selected so that $R_3 = R_4$, $C_3 = C_4$, and $R_1 = 2R_2$. Slide wire r, which has a total resistance of $R_1/100$, is used to correct any slight tracking errors in R_3 and R_4. Under these conditions $f = 1/2\pi R_4C_4$. A filter is needed to reject harmonics if a null indicator is used since the bridge is not balanced at harmonic frequencies.

Time intervals can be measured accurately and conveniently by gating a reference frequency derived from a quartz-crystal oscillator standard to a counter during the time interval to be measured. Reference frequencies of 10, 1, and 0.1 MHz, derived from a 10-MHz oscillator, are commonly used.

Analog frequency circuits that produce an analog output proportional to frequency are used in control systems and to drive frequency-indicating meters. In Fig. 15.4.22, a fixed amount of charge proportional to $C_1(E - 2d)$, where d is the diode-voltage drop, is withdrawn through diode D_1 during each cycle of the input. The current

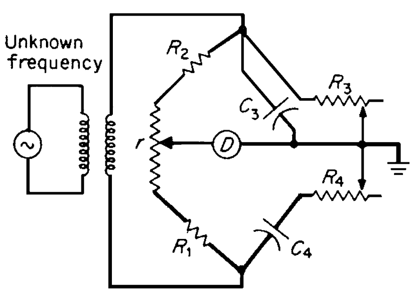

FIGURE 15.4.21 Wien frequency bridge.

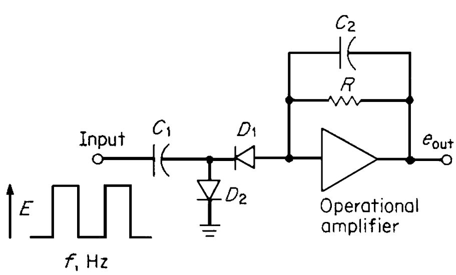

FIGURE 15.4.22 Frequency-to-voltage converter.

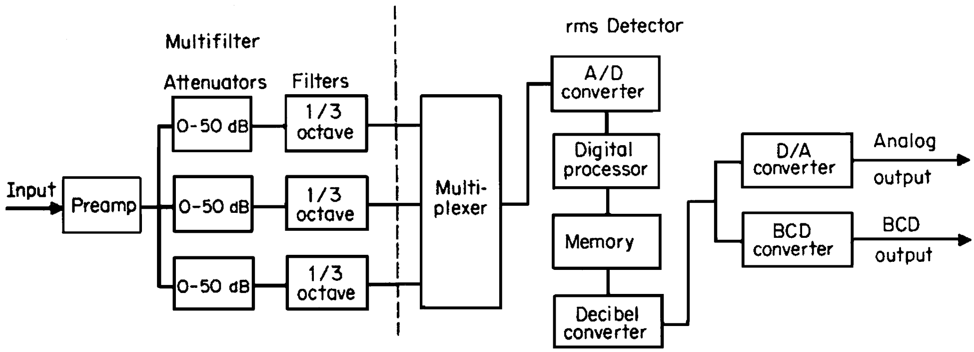

FIGURE 15.4.23 Real-time analyzer using 30 attenuators and filters. (*General Radio Co.*)

through diode D_1, which is proportional to frequency, is balanced by the current through resistor R, which is proportional to e_{out}. Therefore, $e_{out} = fRC_1(E - 2d)$. Temperature compensation is achieved by adjusting the voltage E with temperature so that the quantity $E - 2d$ is constant.

Frequency analyzers are used for measuring the frequency components and analyzing the spectra of acoustic noise, mechanical vibrations, and complex electric signals. They permit harmonic and intermodulation distortion components to be separated and measured. A simple analyzer consists of a narrow-bandwidth filter, which can be adjusted in frequency or swept over the frequency range of interest. The output amplitude in decibels is generally plotted as a function of frequency using a logarithmic frequency scale. Desirable characteristics include wide dynamic range, low distortion, and high stop-band attenuation. Analog filters that operate at the frequency of interest exhibit a constant bandwidth, for example, 10 Hz. The signal must be averaged over a period inversely proportional to the filter bandwidth if the reading is to be within given confidence limits of the long-time average value.

Real-time frequency analyzers are available which perform $^1/_3$-octave spectrum analysis on a continuous real-time basis. The analyzer of Fig. 15.4.23 uses 30 separate filters each having a bandwidth of $^1/_3$ octave to achieve the required speed of response. The multiplexer sequentially samples the filter output of each channel at a high rate. These samples are converted into a binary number by the A/D converter. The true rms values for each channel are computed from these numbers during an integration period adjustable from $^1/_8$ to 32 s and stored in the memory. The rms value for each channel is computed from 1,024 samples for integration periods of 1 to 32 s.

Real-time analyzers are also available for analyzing narrow-bandwidth frequency components in real time. The required rapid response time is obtained by sampling the input waveform at 3 times the highest frequency of interest using an A/D converter and storing the values of a large number of samples in a digital memory. The frequency components can be calculated in real time by a microprocessor using fast Fourier transforms.

Time-compression systems can be used to preprocess the input signal so that analog filters can be used to analyze narrow-bandwidth frequency components in real time. The time-compression system of Fig. 15.4.24 uses a recirculating digital memory and a D/A converter to provide an output signal having the same waveform as the input with a repetition rate which is k times faster. This multiplies the output-frequency spectrum by a factor of k and reduces the time required to analyze the signal by the same factor. The system operates as follows. A new sample is entered into the circulating memory through gate A during one of each k shifting periods. Information from the output of the memory recirculates through gate B during the remaining $k - 1$ periods. Since information experiences k shifts between the addition of new samples in a memory of length $k - 1$, each new sample p is entered directly behind the previous sample $p - 1$, and therefore the correct order is preserved. $(k - 1)/n$ seconds is required to fill an empty memory, and thereafter the oldest sample is discarded when a new sample is entered.

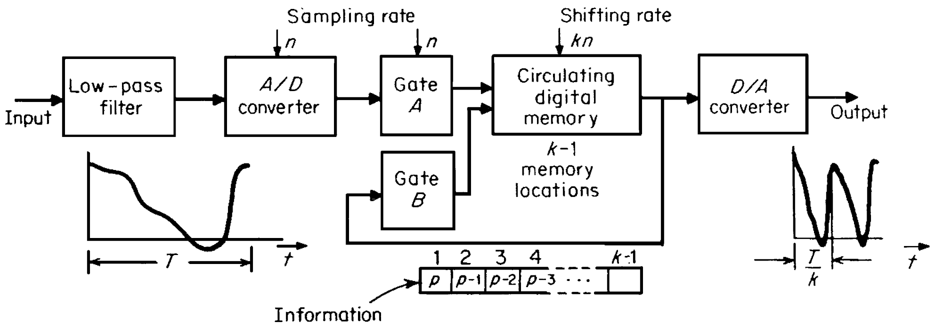

FIGURE 15.4.24 Time-compression system.

Frequency synthesizers/function generators provide sine wave, square wave, triangle, ramp, or pulse voltage outputs, which are selectable over a wide frequency range and yet have the frequency stability and accuracy of a crystal oscillator reference. They are useful for providing accurate reference frequencies and for making measurements on filter networks, tuned circuits, and communications equipment. High-precision units feature up to 11-decade digital frequency selection and programmable linear or logarithmic frequency sweep. A variety of units are available, which cover frequencies from a fraction of a hertz to tens of gigahertz.

Many synthesizers use the indirect synthesis method shown in Fig. 15.4.25. The desired output frequency is obtained from a voltage-controlled oscillator, which is part of a phase-locked loop. The selectable frequency divider is set to provide the desired ratio between the output frequency and the crystal reference frequency. Fractional frequency division can be obtained by selecting division ratio R alternately equal to N and $N + 1$ for appropriate time intervals.

Time-domain reflectometry is used to identify and locate cable faults. The cable-testing equipment is connected to a line in the cable and sends an electrical pulse that is reflected back to the equipment by a fault in the cable. The original and reflected signals are displayed on an oscilloscope. The type of fault is identified by the shape of the reflected pulse, and the distance is determined by the interval between the original and reflected pulses. Accuracies of 2 percent are typical.

A low-frequency voltmeter using a microprocessor has been developed that is capable of measuring the true rms voltage of approximately sinusoidal inputs at voltages from 2 mV to 10 V and frequencies from 0.1 to 120 Hz. A combination of computer algorithms is used to implement the voltage- and harmonic-analysis functions. Harmonic distortion is calculated using a fast Fourier transform algorithm. The total autoranging, settling, and measurement time is only two signal periods for frequencies below 10 Hz.

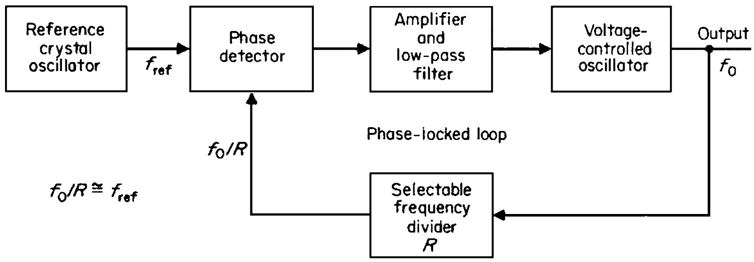

FIGURE 15.4.25 Indirect frequency synthesis.