Strength of Materials

Merriman's

STRENGTH OF MATERIALS

Revised by

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PREFACE

In revising *Merriman's Strength of Materials* I have attempted to write a basic textbook for the non-technical student and the mechanic. I have especially kept in mind the interests, needs, and backgrounds of the secondary-level practical arts and vocational school groups, who, although having a real use for this knowledge, will probably have but this one formal contact with it.

In general, I have attempted to make the book more practical and less technical than previous editions, with more explanations in the form of simple, readily observed and easily understood examples. Throughout the book there has been a shift of emphasis from the construction and civil engineering fields to the present-day industrial fields. The more abstract discussions, of less value to the practical mechanic, have been eliminated and the space has been used for explanation of the more useful aspects of the subject.

The addition of explanatory material is most evident in the earlier chapters, where the student should form his basic concepts as a foundation for the more complicated material and applications which follow. Chapter 1 is new, presenting general considerations which should aid the student in motivating and orienting his studies. Former Chapter 2, on the general properties of materials, has been moved to the appendix and is treated as supplementary text. Here is up-to-date information on such modern industrial materials as the iron alloys, plastics, and aluminum.

All the problems have been reviewed. Some have been revised, others dropped, and new ones have been added, changing the total number from 378 to 402. The number of illustrations has been increased from 60 to 71, many of the old cuts having been replaced.

I take this opportunity to acknowledge the valuable aid of Mr. George H. Bennett, metallographer and engineer, especially in the ferrous metals section of Appendix A.

EDWARD K. HANKIN

September, 1942

CONTENTS

CHAPTER 1

GENERAL CONSIDERATIONS

1. Definition of Topic. When first we think of the topic *strength of materials,* it is only natural that we expect it to deal with *strength* such as we ourselves possess; that is, the power whereby we exert or resist a force. Actually it is generally accepted that this topic includes much more than this, being the science dealing with all the properties of structures and objects of utility which enable them to resist the action of external and internal forces, and the changes which take place because of these forces.

Thus we are thinking only of the strength of a material when we say that steel is stronger than lead, or that oak is a stronger wood than pine. On the other hand, we refer to the broader meaning of strength of materials when we say that one ladder is stronger than another, or that a certain bridge will stand heavier traffic than another bridge. In these last comparisons, the materials used might be of equal strengths; but, because of the sizes, shapes, and nature of construction, the strength of the material is made more effective and useful.

Of course there are many properties of materials, such as hardness, ductility, density, and texture, which are not included in this topic. In this book, some general properties of the more common industrial materials are discussed in the appendix.

QUESTIONS

1a. Give two additional comparisons involving merely the strength of a material.

1b. What properties or factors, in addition to strength, are suggested as being within the scope of this topic?

1c. List as many as you can of the ways in which one ladder might be made stronger than another of the same type.

2. Basic Concept of Principles. Strength of materials deals largely with the relations among the forces acting on a body, the material of which the body is made, together with its size and shape, and the changes which are brought about in the body because of the forces in action. These relationships occur as natural phenomena, real alterations of position, shape, and structure, which occur as a result of changes outside the object under consideration. Thus we can see a plank, supported at its ends, bend as a result of someone standing on it between the supports. If the person had not stood there, the plank would not have bent, and, when he steps off, the plank will again change its shape. These related changes are typical of the phenomena with which this topic is concerned.

It is important that one does not lose sight of this fundamental concept of physical phenomena as he proceeds with a study of strength of materials. While it is not always easy to see or determine the changes or the nature of the changes which occur under the action of forces, the changes always occur.

We use formula and mathematics only as a convenient way of thinking about and making use of our knowledge of the relationships among the forces, the object, and resulting changes. The formula is merely a symbolic statement of the principle or relationship. The letters are used to designate measurements of the several physical properties involved, such as forces and dimensions, and ratios of these measurements.

For convenience we separate and simplify the many physical phenomena which usually occur in complex combinations. That is to say, while one force acting on a body might produce a number of different though related changes, it is common practice to consider one change at a time. Sometimes we consider only one or two of the several related changes, because of our special interest in that case, and our experience as to which changes are likely to be the most critical or undesirable. Care must be exercised in the separation and simplification of these phenomena to make certain that important factors are not overlooked or altered. For instance, it is not enough that the floors of most buildings be merely strong enough to withstand safely the loads on them, but they must also be sufficiently rigid to prevent the cracking of plaster attached to their underside.

QUESTIONS

2a. What is the fundamental basis of the study of strength of materials?

2b. List four factors the relationships of which are dealt with in strength of materials.

2c. What are three types of changes resulting from the action of forces?

3. Mathematical Application. All about us we see structures, machines, tools, and other objects, the size and construction of which are the direct result of the application of the principles of strength of materials. The modern automobile and airplane would not be so safe and efficient were it not for the planning and designing which are based on the principles included in this topic. The shape and size of vital parts, as well as the material of which they are made, were decided only after the forces were analyzed, and all the factors which might cause failure were considered. As reliability or minimum weight or minimum cost becomes more important, the principles of strength of materials are more closely applied—mathematically and graphically. A large suspension bridge represents a huge investment, and when it is finished it must do its job reliably. It would not be practical to proceed with construction of the bridge without having made doubly sure that the bridge would do its job well and without failing. Of course, if all the conditions were exactly the same for the new bridge as they were for another bridge which had already been built and was successful, it would be a simple matter of duplication. This situation is highly improbable, however, and so, if it were not for the application of the principles of strength of materials, we could not be sure that such a structure would be safe until it had been built and tested.

It should be noted that it is not unusual in the case of large and expensive structures to prove the correctness of design based on the principles of strength of materials by building a model and subjecting it to a close reproduction of the working forces. This is especially likely when the design is greatly different in size or type from any previous structures. In these cases of experimental design, the purpose is chiefly to prove the correctness of the analysis of physical phenomena.

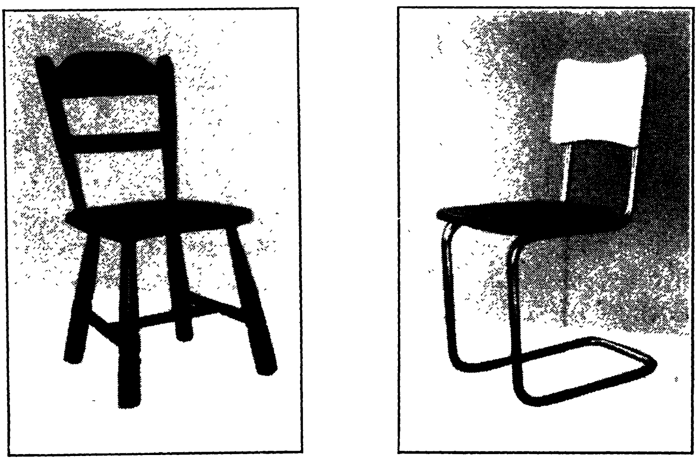
QUESTIONS

3a. List three determining conditions which justify mathematical application of the principles of strength of materials.

3b. List three additional objects which probably were designed in whole or in part with mathematical application of these principles.

3c. What is the major purpose of experimental models in designing?

4. Traditional Design. We are surrounded by a large variety of objects which owe their proportions and construction to little other than tradition. Probably the best example of these ob-



(*a*) (*b*)

FIG. 1. Traditional and Modern Design of Furniture.

jects is household furniture. Through the years we have developed basic proportions and materials which enable such things as tables and chairs to serve their purposes. We have grown used to seeing them, and only occasionally are we shocked at seeing some item of furnishing which has been constructed of a new material or in an original shape. In recent years we have witnessed the development of this "modern" design, which often owes its original appearance to a closer regard for the most effective use of new materials, particularly with application of the principles of strength of materials (Fig. 1).

The traditional design is probably the net result of many trials and failures, corrected and improved, and combined with lines of grace and beauty to produce proportions which please the eye and serve the purpose. In most cases, this resulting design is much stronger than is required by the actual forces involved, but the relative low cost of materials and the inconsequence of extra weight make this feature negligible beside such considerations as comfort and beauty.

QUESTIONS

4a. What is the usual relation between the sizes of traditionally designed objects and the actual strength requirements?

4b. For what two reasons is the relationship of 4*a* of little consequence?

4c. What is a basic difference between traditional and "modern" design?

5. Design by Approximation. We first discussed those structures requiring close application of the principles of strength of

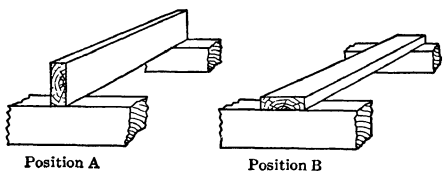


FIG. 2.

materials, and then noted the objects whose design is a matter of tradition, without regard for those principles. We now find another and very large group of structures, tools, and parts whose design is the result of only an approximate application of strength of materials. This approximation is carried on by the designers and craftsmen who constantly exercise their judgment as to what materials and dimensions to use, how a given object is to be constructed, or how a certain process is to be carried out. The basis for these judgments is often referred to as "experience" and may be the combined result of a study of the principles of strength of materials and of observation of these principles in action; of failing and successful structures in actual use. Most of us seem to "sense" unsafe or insecure construction in familiar objects probably because we become familiar with good design as the normal thing. Without having studied strength of materials, many persons would know that a rectangular timber placed in position *A*, Fig. 2, would be stronger than in position *B*. Actually this *judgment* is a conscious recognition of good design, and is explained by principles of strength of materials. In like manner many useful applications are found for a real understanding of the underlying principles of strength of materials in the regular work and daily lives of craftsmen and designers.

Figure 3 shows a floor joist with indications of four of the possible places where an electrician might drill for the passage of wires. He should know that a hole in position *A* would weaken the joist least of the four, and position *B* would weaken it most. Of positions *C* and *D*, the preference is not certain

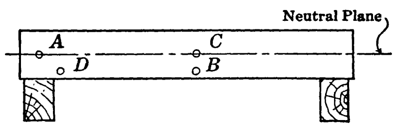


FIG. 3.

without making some calculations, but they are both much better than *B*.

QUESTIONS

5a. What are the three types of design thus far discussed?

5b. Which two types of design are based upon the principles of strength of materials?

5c. What is the name given to the basis of judgments, and of what does it consist?

5d. What groups of people carry on design by approximation? What is gained by this method? What is lost?

6. Preliminary Investigation—Organization of Data. Before applying the principles of strength of materials a certain amount of preliminary investigation and organization of data should be carried out. As a beginning, it is usually good practice to make a sketch of the arrangement under consideration, showing shape, position, and such dimensions as are known (Fig. 4*a*). Then the forces should be defined as far as possible, showing location, direction, and degree, where known (Fig. 4*b*). This calls for basic analysis with reference to direct and indirect action of primary forces, involving the application of principles of mechanics. The forces will be determined by the nature of the structure or the functions it is to perform.

We have previously mentioned that a complex problem is frequently merely a combination of several simple problems. A third series of steps, then, would be the making of separate sketches for each of the elemental problems, with clear indication of the nature of the desired answer (Fig. 4*c* and *d*).

Most problems fall into one of two classifications, investigation or design. In the investigation problem, the sizes and construction of the body are given or set, and it is required to find what loads may be safely applied, or whether certain loads are safe. In the design type problem, the loads and conditions

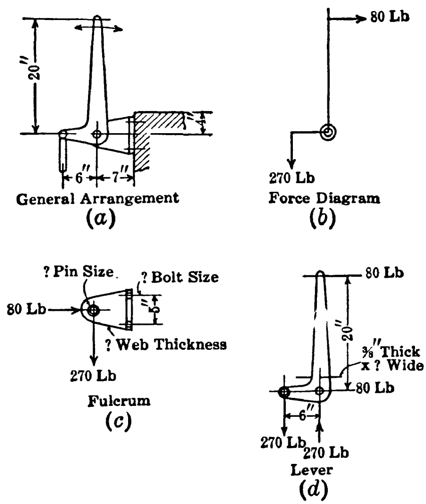


FIG. 4.

are given and it is required to find sizes or construction. Problems 21*l* and 25*o* are of the investigation type, and Problems 21*m* and 25*j* are of the design type. Of course more complex problems may be a combination of these two types.

It should be noted that single forces do not act alone upon a body, except in cases where inertia is under consideration. Bodies at rest, or under uniform motion, cannot maintain their position or condition by their own efforts. Any force acting on a body tends to move it or alter its motion, and if the body is to react to the force (resist it) there must be in action other forces opposing the acting force. As we sit on a chair, exerting a force equal to our weight on the seat, the floor exerts a resisting force on the bottom of each chair leg, the sum of which must equal our weight. If this were not so, the chair would be pushed into or through the floor.

QUESTIONS

6a. What are the two most common classifications of strength of materials problems?

6b. In addition to those already mentioned, classify four problems in this book under each of the two headings above.

6c. What determines the forces in action on a body?

6d. Of what advantage is the sketching of a problem? What things are shown or indicated?

6e. What external forces must be applied in Fig. 5 in order that the weight W will be suspended by the rope and pulley arrangement?

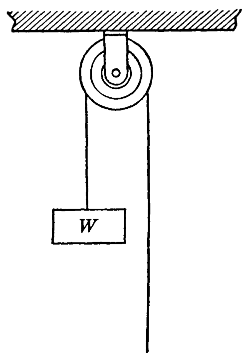


FIG. 5.

CHAPTER 2

EFFECT OF FORCES ACTING ON SOLID BODIES

7. Resisting Change in Shape. The outstanding property in the description of solids is that they tend to resist a change in shape. That is to say, some force must be exerted to change the shape of a solid body, or, in tending to maintain their given shape, the solid body will resist the action of forces. This can

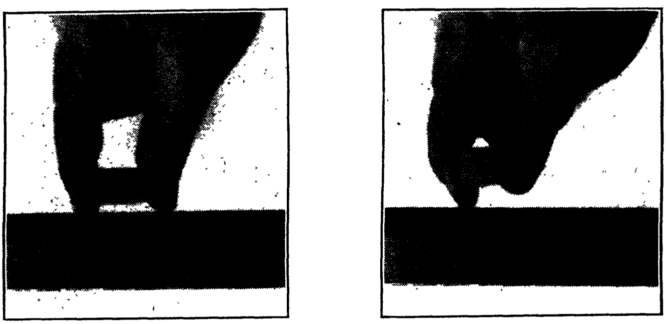


FIG. 6.

be seen in several simple experiments. If a soft rubber eraser is squeezed between the fingers it tends to become shorter and fatter in shape (Fig. 6). As greater force is exerted, this change in shape becomes more pronounced, and, as the force is released, it tends to return to its original shape.

A rubber band held at each end will increase in length only if some effort is made to stretch it. As the effort is increased, the band will become longer up to the point where it breaks, or until the force becomes inactive and the band returns to about its original shape.

If at any time in either of the above experiments, the forces in action remained unaltered, the shape of the body (eraser or band) would cease to change. That is, if the rubber band had stretched to 4 inches in length when a 5-pound force was exerted on it, it would remain at that length as long as the 5-pound force was in effect. This condition we speak of as *equilibrium.*

The property of the material which causes it to resist a change from its original shape is the development of an internal force, called *stress,* which resists and balances the action of the external force. This state of equilibrium exists in all useful structures and is a fundamental premise for further reasoning. Thus, if we are able to determine the external forces acting on a body we can accept the existence of internal resisting forces (stress) which are equal and opposed to them. In the case of the

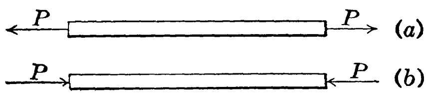


FIG. 7.

squeezed soft eraser, the fingers exerted forces tending to bring them together, while the eraser exerted a force tending to keep the fingers apart. At any time that the distance between the fingers became fixed, these forces were equal.

When the forces acting on a body tend to stretch or elongate that body along the lines of the forces (Fig. 7*a*), the body is said to be in *tension,* and the stress set up in the body is known as *tensile stress.* The stretching rubber band was an example of tension.

When the forces acting on a body tend to shorten or compress the body along the lines of the forces (Fig. 7*b*), the body is said to be in *compression* and the stress is known as *compressive stress.* The soft eraser mentioned above was in compression, as are the foundations of a house and the footing upon which any structure or body rests.

The third simple stress, not previously mentioned, is *shearing stress.* This occurs when a body tends to keep two acting forces from passing each other, as in the case of a piece of paper in the blades of a pair of scissors. The edges of the blades tend to move past each other, and the paper tends to resist their movement. If cardboard or metal is substituted for the paper, the maximum resistance may be greater than the force which can be exerted, and a state of equilibrium is reached. The stress which then occurs is a *shear stress.* At times this type of simple stress is difficult to visualize, especially when it occurs in practical industrial situations. It should be noted that the area resisting the shearing forces is in a plane parallel to the line of action of the forces and represents the surface of failure were those forces to exceed the resistance of the materials. In the case of the paper in the scissors, the resisting area was the product of the thickness of the paper times the short length of the scissors blade in action at any one position.

QUESTIONS AND PROBLEMS

7a. Name the three simple stresses and give two everyday examples of each.

7b. What is developed within a body to resist any tendency to change its shape?

7c. In the experiment of Fig. 6, what is the relation between the volume of the eraser before and after the forces are applied? Explain.

7d. If a force of 10,000 pounds is tending to stretch a steel bar, what is the value of the stress within the bar? What kind of stress would this be?

8. Summation of Effect, Unit Stresses. It is not difficult to see that two ropes of a given size should hold about twice as much load as one of them, three ropes three times as much, and so forth. This is recognizing that the cross section area (perpendicular to the line of action of the force) has a direct relation to the ability to resist tensile forces.

Thus we can say that the strenth of a body under the action of direct forces is in proportion to the *area* resisting the action of those forces. In tension and compression this is the cross section area, perpendicular to the lines of force action, and in shear it is the area in a plane parallel to the lines of force action, which would rupture were those forces to pass.

The strength of any given material is generally uniform; a half-inch bar of a certain kind of steel will always hold about the same amount of load. In order that we may conveniently think in terms of the strength of a material, we usually state it in reference to a unit area, like 60,000 pounds per square inch. This means that a bar of this material one square inch in cross section area would withstand a force of 60,000 pounds, or would exert a stress of 60,000 pounds before breaking. If the area were 2 square inches, the breaking stress would be 120,000 pounds; and, if ½ square inch, 30,000 pounds. This *strength* for a *unit area* is commonly referred to as *unit strength,* or *unit stress.* The three direct unit stresses are: *unit tensile stress, unit compressive stress, and unit shear stress.*

From the foregoing, we can see that the direct force applied to a body is equal to the product of the cross section area and the unit stress. For convenience we can express this relationship as a formula, where P equals the external force, A equals the cross section area, and S equals the unit stress. Hence:

$$P = SA \quad \text{or} \quad S = \frac{P}{A} \qquad [1]$$

At this time it should be noted that just as both sides of an equation or formula must be numerically equal, so must the *units* balance. In the equation $P = SA$, the units inserted would read P (pounds) $= S$ (pounds per square inch) $\times A$ (square inches) or pounds $= \frac{\text{pounds}}{\text{square inch}} \times$ square inches. We see that square inches cancel out, leaving pounds = pounds, proving the equation of units. This principle holds true regardless of the complexity of the formula, and is a convenient check on the truth of the formula and the correct substitution of values.

Example Problem: If a block of iron 3½ square inches in cross section area is subjected to a compressive load of 14,000 pounds, what is the unit stress?

$$S = \frac{P}{A} = \frac{14{,}000 \text{ pounds}}{3\tfrac{1}{2} \text{ square inches}} = 4000 \text{ pounds per square inch}$$

QUESTIONS AND PROBLEMS

8a. If the unit tensile stress in one piece were twice that in a second piece when under the same external loads, what is the relations between their cross section areas? If the first piece were steel and the second were wood, how would the answer to this question be affected?

8b. What two factors control the ability of any part to resist the action of direct forces?

8c. What is the relation between the plane of resisting area and the lines of force action in tension? In compression? In shear?

8d. A steel bar which is to be subjected to a tensile force of 36,000 pounds is to be designed so that the unit stress shall be 11,000 pounds per square inch. What should be the section area in square inches? If the bar is round, what should be its diameter?

8e. If a cast-iron bar, $1\frac{1}{4} \times 2\frac{1}{2}$ inches in section area, breaks under a tension of 72,000 pounds, what tension will probably break a bar $1\frac{1}{2}$ inches in diameter?

8f. What should be the diameter of a bar of steel to carry a load of 300,000 pounds with a unit stress of 15,000 pounds per square inch? If the bar is $1\frac{1}{2}$ inches thick, what should be its width?

8g. A machine weighing 8000 pounds stands on a pedestal (Fig. 8*a*), the foot of which is a hollow square 12 inches outside and 8 inches inside. What is the unit compressive stress in the floor?

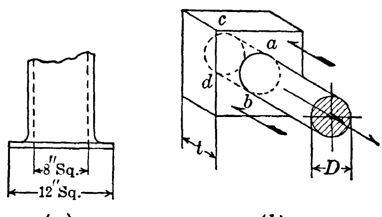


FIG. 8.

8h. A tension rod in a truss is to withstand a pull of 30,000 pounds. What must be its diameter if the unit tensile stress is not to exceed 12,000 pounds per square inch?

8i. Figure 8*b* is a schematic drawing of a bolt head showing that it may fail in shear by pulling out a pluglike cylinder *ab, cd.* The lateral area of this cylinder resists the shearing action of the tension in the bolt and the forces acting on the head. What unit shear stress is developed in a 1-inch bolt with a head $\frac{7}{8}$ inch thick (t) under a tensile load of 18,000 pounds?

8j. What head thickness (t) is required for a $\frac{3}{4}$-inch bolt in order that the unit shear stress will be one-half the unit tensile stress?

9. Ultimate Strength. When the forces acting on a body increase to the point where the material ruptures or fails, the *maximum stress* reached is called the *ultimate strength,* usually given as a unit stress. This occurs for any of the direct stresses. Table 1, page 138, shows accepted average values for common industrial materials. In this table we can see that the ultimate unit tensile strength of mild carbon steel is 65,000 pounds per square inch and of strong alloy tool steel, 120,000 pounds per square inch.

QUESTIONS AND PROBLEMS

9a. A bar of mild carbon steel 2⅜ inches in diameter ruptures under a tension of 271,000 pounds. What is its ultimate strength in pounds per square inch?

9b. What force is required to rupture in tension a cast-iron bar 6 inches in diameter, the ultimate tensile strength of the cast iron being 20,000 pounds per square inch?

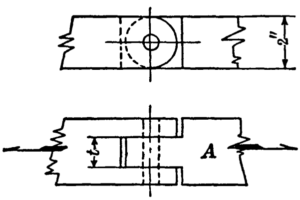


FIG. 9.

9c. A double shear pin (Fig. 9) of mild carbon steel is to fail under a load of 10,000 pounds. What should be its diameter?

9d. If, at the time of failure in shear, the tension at the weakest point of mild steel link *A*, Problem 9*c*, is to be one-sixth the ultimate strength, what should be the thickness *t*?

9e. What force is required to punch a 1½-inch hole through 3/16-inch thick mild carbon steel?

9f. A bar of wrought iron 1 square inch in section area and 1 yard long weighs 10 pounds. Find the length of a bar which will rupture under its own weight when hung from its upper end.

9g. A brick $2 \times 4 \times 8$ inches weighs about 4½ pounds. What will be the height of a pile of bricks so that the unit compressive stress on the lowest brick will be one-half its ultimate strength?

9h. Two flat structural steel bars, each 1 inch thick, 2 inches wide, and 4 feet long, are riveted together with one steel rivet 1⅛ inches in diameter. What load will this jointed bar support at the instant of its failure under a tensile load? How will it fail?

10. Elastic Limits. We have seen that the action of any force on a body changes the shape of that body (Art. 7). This change of shape is called *deformation.* In tension the deformation is *elongation* and in compression, *shortening.*

If a piece of straight wire is bent only slightly, when the bending forces are removed it will return to its original shape. If the bending forces are increased, the shape of the wire may be permanently altered or "set," and, when the forces are removed, the wire will not again become straight.

The property whereby a material tends to return to its original shape when loads are removed is known as *elasticity,* and the unit stress which occurs when the maximum temporary deformation is produced is known as the *elastic limit.* All solids possess this property in varying degree. Table 1, page 138, gives average values for common industrial materials. This property and other changes related to it are discussed in Art. 41.

In cases of direct stress the elastic limit is not readily visualized. It has been found by experimentation that within the elastic limit the deformation is in direct proportion to the stress developed. Thus, within the 35,000 pounds per square inch elastic limit, structural steel will elongate the same amount for each 5000 pounds per square inch of stress developed. Beyond

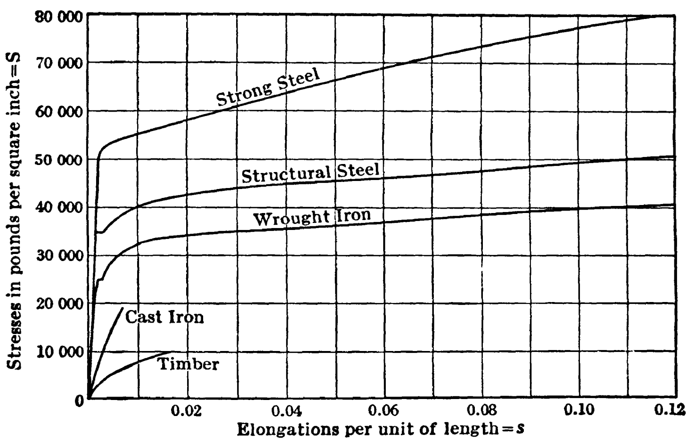


FIG. 10.

the elastic limit this direct relationship does not exist. The amount of elongation produced with each addition of unit stress beyond the elastic limit increases at a great rate. The elastic limit is determined experimentally by locating the stress point where the direct relationship between unit stress and elongation ceases to exist. Figure 10 shows a graph of this relationship between stress and deformation for several materials.

The elastic limit is always lower than the *ultimate strength* but the relationship between these values varies greatly with different materials. This condition greatly influences the uses to which various materials may be put. While we shall learn that a high elastic limit permits larger safe loads, it is also true that a relatively low elastic limit is often of great value. One limiting quality of cast iron is that its elastic limit is high with respect to its ultimate strength. This is spoken of as brittleness, generally an undesirable property. Soft steel, on the other hand, has a greater spread between its elastic limit and ultimate strength, and so is readily worked and bends without breaking.

The property of elasticity may be both of value and of detriment. Springs and many devices of rubber, such as rubber bands, are examples of the usefulness of the elastic property. On the other hand, elasticity may cause an object to be insufficiently rigid for practical purposes unless it is made much larger than its required strength would indicate.

QUESTIONS AND PROBLEMS

10a. In addition to those mentioned in the text, give three everyday examples of useful application of elasticity.

10b. What are the two direct deformations?

10c. What percentage of the ultimate strength is the elastic limit of cast iron? Of structural steel?

10d. A steel tie rod in a bridge is $1\frac{1}{8}$ inches in diameter. What load or tensile force will this rod carry if the unit tensile stress is not to exceed one-half the elastic limit?

10e. A short square stick of timber is to carry a compressive load of 91,000 pounds. What should be its size in order that the unit stress will be one-third the elastic limit?

10f. A solid cast-iron block is 12 inches in diameter and 12 inches high. What compressive load will it carry when stressed to its elastic limit?

11. Working Unit Stress. It is not practical to make most articles just strong enough to do the job for which they were designed. For one thing, the exact usage and resulting stresses which a certain piece is to withstand cannot usually be predicted with certainty. We shall see that just the presence of fluctuating or moving forces will cause stresses far in excess of those same forces at rest. Then, again, we do not always adhere to our original plans for the use of the object, and it is helpful if it is able to serve this added purpose. All these things we can classify as *abnormal* and *unpredictable use.*

Another condition which must be considered is *abnormal construction.* The very materials of which the object is made may vary in strength to a great degree. The figures used for average values of permissible stress do not indicate the wide variance which may exist, especially in certain materials like wood. The presence of knots or weather and shrinkage cracks may so reduce the strength that failure will occur if there has not been sufficient oversize allowance. Workmanship, too, must be looked upon as a cause of abnormal construction and allowed for in the design. A faulty rivet or weld, the over- or under-tightening of a bolt, or some other point in the construction where the human element has entered might cause an otherwise good structure to fail if it has been "stressed up" in its design. These variations in construction do not always act directly but often cause unequal loading on some other part.

The common and very simple way of allowing for abnormal usage and construction is by adjusting the stress at which the material is used. This adjusted stress we call *working stress,* being the stress at which the object "works" when the usage and construction are as planned. In industry the selection of safe working stresses is a matter of judgment involving experience and research with the actual conditions concerned.

QUESTIONS AND PROBLEMS

11a. What are the two general conditions which must be allowed for in good design?

11b. What relationship must always exist between the working stress and the elastic limit? Why?

11c. Cite four cases of abnormal, unpredictable use and four cases of abnormal construction.

11d. A ¾-inch square cold-rolled steel key is to withstand a shearing force of 45,000 pounds, with a working stress of 10,000 pounds per square inch. What is its minimum length?

12. Factor of Safety. In determining the proper working stress it is convenient to consider it with relation to the ultimate strength and elastic limit. The *ratio* of the ultimate strength to the working stress is called *factor of safety.* This prime number represents the number of times the working forces may be increased before the ultimate strength of the material is passed. It should be borne in mind, however, that usually long before the ultimate strength is reached the elastic limit has been passed and the object is *permanently changed* in shape and probably as much harmed as if it were broken. Hence the relation of the working stress to the elastic limit must be considered, and the working stress must always be made the lesser of the two.

Table 3, page 139, shows factors of safety for some common conditions and materials. In general the relations shown should persist. Higher factors should be used when the loads are varying or shock, or when the quality of the materials or workmanship is less predictable.

Modern engineering is the art of economic construction. In most instances the best economy will be obtained by making all parts of a structure approximately the same strength, for, if one part is much stronger than the rest, it will contain a useless excess of material. At the same time it is often good design to make some one inexpensive or easily replaced part slightly weaker than the rest so that it will act much as a fuse in an electric circuit. In such a case, the factor of safety for the whole machine is identical with the factor of safety of the weakest part.

QUESTIONS AND PROBLEMS

12a. What ratio is called factor of safety? In what units is it expressed? (See Art. 8.)

12b. The factors of safety for the several parts of one machine are 7, 7.5, 6, 6.8, and 8. Which part will fail under an extreme load? Comparing this with a similar machine having factors of 6, 6.2, 6, 12, 11, which do we find to be the better design?

12c. A steel strut having a section area of 5.07 square inches carries a vertical load of 25 tons. What is its factor of safety if the ultimate compressive strength of the steel is 65,000 pounds per square inch?

12d. What should be the diameter of a mild steel bar so as to carry a tension of 400,000 pounds with a factor of safety of 4? If the bar is cast iron, what should be its diameter?

12e. What should be the size of a round bar of structural steel to carry a tension of 225,000 pounds with a factor of safety of 6?

12f. A short cast-iron post is 12 inches in outside diameter and 10 inches in inside diameter. Compute its factor of safety when carrying a load of 180,000 pounds.

12g. What load, in tons, may be imposed on 1 square foot of sandstone foundation, the limiting factor of safety being 5?

12h. A mild steel bolt $1\frac{5}{8}$ inches in diameter has a head $1\frac{3}{8}$ inches long. When a tension of 17,500 pounds is applied to the bolt, find the tensile unit stress and the factor of safety for tension. Also find the unit stress tending to shear off the head of the bolt and the factor of safety against shear.

12i. What compressive unit working stress in the cast-iron frame of a machine is indicated by a factor of safety of 20? What is the corresponding tensile unit working stress if the factor of safety is 10?

12j. The working tensile stress in a structural steel rod is to be 54 tons. Find its diameter when it is to be used in a building and also when it is to be used in a bridge.

12k. The total shear on each rivet of a lap-riveted joint is 2500 pounds. If the rivet is $\frac{3}{4}$ inch in diameter, find the factor of safety against shearing.

CHAPTER 3

PRINCIPLES OF BEAMS

13. Theory of Moments. When we pull on the rim of an automobile steering wheel, we cause the wheel to turn; and the harder we pull, the more it tends to turn. This tendency to rotate is called a *turning moment,* and is caused by a force acting on a body at some distance from a point about which the turning might occur. Other forces acting on the body resist this turning, and this may be called the *resisting moment.* When the turning moment is greater than the resisting moment, the body will rotate. When the two moments are equal, a state of equilibrium exists and there is no rotation.

We have seen that if the acting force is increased, there will be a greater tendency for rotation; the moment will be increased directly. In like manner it can be shown that if the distance between the acting force and the center of rotation, called *lever arm* or *moment arm,* is increased, the moment will be increased directly. From this relationship we can deduce that the *moment* is equal to the product of the *lever arm* (perpendicular to the force) and the *force.* Expressed as a formula this would be $M = Pp$, where M is the moment, P the force, and p the lever arm. When more than one force acts on a body to produce rotation, the total effect is the algebraic sum of the moments of all forces. Moments of forces in a plane about a given point can be in one of two directions. Those moments causing rotation tendency in the *same* direction as clock hands travel are called *positive,* and those in the *opposite* direction, *negative.*

Figure 11 shows the three forces acting on a lever. C is the point about which the lever can rotate.

If the lever is to be in *equilibrium,* meaning that there is no tendency for rotation, the algebraic sum of the three moments must be zero. We see that force P_1 acting on lever arm p_1 produces a negative moment, as does P_3 acting on p_3. P_2 acting on

p_2 produces a positive moment. Therefore, $P_1p_1 + P_3p_3$ must equal P_2p_2. It should be noted that the lever arm is always measured from the center of rotation, *perpendicular* to the line of force action.

From the foregoing it can be seen that a force acting directly on the center of rotation has a moment arm of zero value, and thus produces no moment. It should also be pointed out that for the purposes of investigation, *any point* may be assumed as the center of possible rotation; and, when the state of equilibrium exists, the algebraic sum of all moments of forces about that point must be equal to zero.

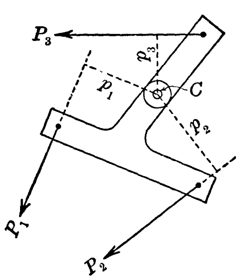


FIG. 11.

Every force producing a moment has a second effect which often must be considered. If a force is applied to a lever, the center of rotation of which is not rigidly positioned, a definite tendency for the center to move in the direction of the force will be observed. Figure 12 shows a piece of apparatus arranged in this fashion. Here the rotation tendency is resisted in such a way as not to influence the position of center A. All the force required to hold A in position is indicated on scale B, when a force is applied through scale C.

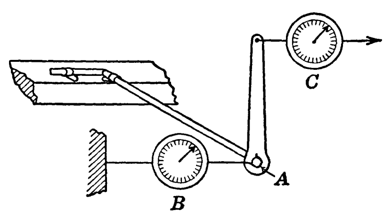


FIG. 12.

As the illustration shows, forces at B and C are equal, and so far as *rectilinear equilibrium* is concerned, force C might just as well be applied at center A. Thus we have shown that forces produce moments and at the same time have rectilinear effects, and, if a state of equilibrium is to exist, *both effects must be balanced* by the action of other forces.

These two effects are usually investigated separately. For the investigation of rectilinear effect, the force is considered to act in its proper direction but directly on the center of rotation.

QUESTIONS AND PROBLEMS

13a. What two things are necessary for the development of a moment?

13b. What are the names given to the two directions of moments of forces acting in a plane?

13c. If a turning moment is 340 inch-pounds, what would be the value of the resisting moment in equilibrium?

13d. If, in Fig. 11, $P_1 = 60$ pounds, $P_2 = 150$ pounds, $p_1 = 3$ inches, $p_2 = 6$ inches, and $p_3 = 2\frac{1}{2}$ inches, what is the required value of P_3 to produce equilibrium? What moment is produced by force P_1?

13e. If force *C*, in Fig. 12, were 120 pounds, and its lever arm were 20 inches, what would be the value of the resisting moment? What would be the value of the moment of force *B* about center *A*?

13f. What two effects might a force have acting on a body at some point other than its point of support?

13g. If a bicycle pedal in its lowest position is pushed towards the rear, in what direction will the bicycle move? What direction will the pedal move? Show how this illustrates the rectilinear effect of a force acting on a lever arm.

13h. A lever 10 feet long is supported 3 feet from one end on a fulcrum. What force acting on the long end is required to balance a 600-pound weight on the short end?

13i. A yardstick is suspended at the 9-inch point. What weight is required at the left end to balance 2-pound weights hung every 3 inches on the right-hand side, neglecting the weight of the yardstick? If the yardstick weighs $\frac{3}{4}$ pound, what additional weight would be required to balance it?

13j. A lever is 6 feet long and the fulcrum is placed 4 inches from one end. What force will be required at the longer end to lift a load of 1200 pounds at the shorter end?

14. Forces Acting on Beams. A *beam* is one useful type of body under the influence of a number of forces acting in some direction other than parallel to its length, thereby causing it to bend. Figure 13 shows several of the more common types of beams and beam loadings. In actual practice, beams may take a great variety of forms and the loading become much more complex. Basically the principles are no different, and herein only the simpler cases with parallel forces perpendicular to the beam will be considered.

In general the *supports* have the function of holding the beam in position by exerting the necessary forces to resist the action of the loading. For investigation purposes, the supports are considered to be rigidly fixed as to position. The forces exerted by the supports are called *reactions.* Beam I, Fig. 13, shows a simple beam on two supports under a single load P. The reactions are R_1 and R_2. The *span* of a beam is the distance between the supports, L.

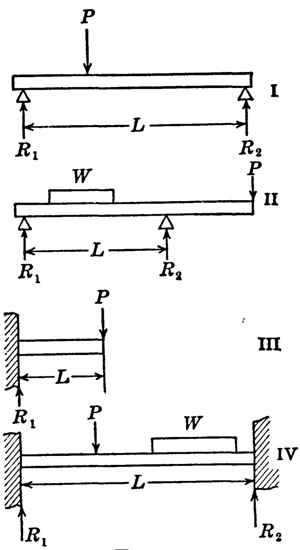


Fig. 13.

The *loads* are the applied forces, the action of which the beam is to resist. One type, the *concentrated load,* is shown on beam I, Fig. 13, as P and the other type, the *uniform load,* is shown on beam II as W. Uniform and concentrated loads commonly occur in combination.

Figure 14 shows a simple beam with one concentrated load. If the support at R_2 is taken as a center of rotation, $20R_1$ must equal 600 pounds $\times$ 15 feet, or $20R_1 = 9000$ foot-pounds.

R_1 then must equal 450 pounds. By taking the center of rotation at the support, the rotation effect of R_2 acting on a zero moment arm is zero, and need not be considered.

In like manner, taking support at R_1 as the center of rotation, $20R_2$ must equal 600 pounds $\times$ 5 feet, or $20R_2 = 3000$ foot-pounds, and $R_2 = 150$ pounds. Note that $R_1 + R_2 = 600$ pounds, which is the value of the load, as it should be.

The reactions caused by the weight of a beam itself may be found in a similar manner, the uniform load being supposed to be concentrated at its center of gravity in stating the equations of moments. Thus, if the weight of the beam is W, the two equations of moments are found to be $R_1 \times 20 - W \times 10 = 0$ and $-R_2 \times 20 + W \times 10 = 0$, from which $R_1 = \frac{1}{2}W$ and $R_2 = \frac{1}{2}W$.

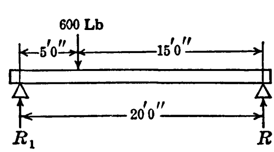


Fig. 14.

The values of reactions from each of several forces on the beam may be added together to obtain the total reaction.

The reactions due to both uniform and concentrated loads on a simple beam may also be computed in one operation. As an example, there is a simple beam 12 feet long between the supports and weighing 35 pounds per linear foot, its total weight being 420 pounds (Fig. 15). There are three loads of 300, 60, and 150 pounds, placed 3, 5, and 8 feet, respectively, from the left support. To find the left reaction, R_1, the center of moments is taken at the right support and the weight of the beam re-

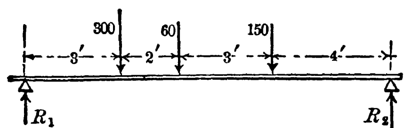


FIG. 15.

garded as concentrated at its middle. Then the equation of moments is

$$R_1 \times 12 - 420 \times 6 - 300 \times 9 - 60 \times 7 - 150 \times 4 = 0$$

from which $R_1 = 520$ pounds. In like manner, to find R_2 the center of moments is taken at the left support; then

$$-R_2 \times 12 + 420 \times 6 + 300 \times 3 + 60 \times 5 + 150 \times 8 = 0$$

from which $R_2 = 410$ pounds. As a check, the sum of R_1 and R_2 is found to be 930 pounds, which equals the combined weight of the beam and the three loads.

By means of the principle of moments, other problems relating to reactions of beams may also be solved. For instance, if a simple beam 12 feet long weighs 30 pounds per linear foot and carries a load of 600 pounds, where should this load be put so that the left reaction will be twice as great as the right reaction? Here let x be the distance from the left support to the load; let R_1 be the left reaction and R_2 the right reaction. Then, taking the centers of moments at the right and left support in succession, we find that

$$R_1 = 180 + 50\ (12 - x), \quad R_2 = 180 + 50x$$

and, placing R_1 equal to $2R_2$, we have $x = 2.8$ feet.

QUESTIONS AND PROBLEMS

14a. Make a sketch of a simple beam 12 feet long with a uniform load of 120 pounds per foot for its length and a concentrated load of 2000 pounds 3 feet from the left support. Label supports, span, loads, reactions, and find value of each reaction.

14b. What is the name of the forces exerted by the total load and the sum of the forces exerted by the supports?

14c. For what reason do we take moments about the points of support in determining beam reactions?

14d. A beam weighing 40 pounds per linear foot rests upon two supports 18 feet apart. A weight of 400 pounds is placed 5 feet from the left end, and one of 600 pounds is placed 8 feet from the right end. Find the reactions due to the total loading.

14e. A wooden beam, 10 × 12 inches in section area and 18 feet between supports, carries a uniformly distributed load of 400 pounds per linear foot for a distance of 8 feet from the left end. The remaining 10 feet carry a uniformly distributed load of 800 pounds per linear foot. Find the reactions at the supports.

14f. Where on the beam of Problem 14*e* must a concentrated load be placed so that the two reactions will be equal? What must be the magnitude of this load?

15. Perpendicular Shear Stresses in Beams. When a beam is short and heavily loaded it may fail by shearing in a perpendicular section near one of the supports. The force that produces this shearing is the resultant of all the perpendicular forces on one side of the section. Thus, in the simple beam of the first diagram (Fig. 16), this resultant is the reaction minus the weight of the beam between the reaction and the section $A'A$; in the cantilever beam of the second diagram it is the sum of the loads and the weight of the beam on the left of the section $B'B$.

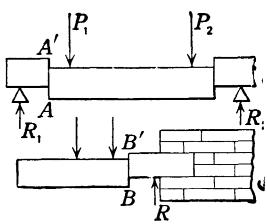


FIG. 16.

Perpendicular shear is the name given to the algebraic sum of all the perpendicular forces on the left of the section which is under consideration. Thus in the first diagram of Fig. 16, if the reaction R_1 is 6000 pounds, the perpendicular shear V just at the right of the support is 6000 pounds. If the beam weighs 100 pounds per linear foot, the perpendicular shear at a section one foot from the support and on the left of the single load P_1

is 5900 pounds. Again, in the second diagram of Fig. 16, if the beam weighs 100 pounds per linear foot and if each concentrated load is 800 pounds, and the distance from the left end of the beam to the section $B'B$ is 4 feet, the perpendicular shear in that section is 2000 pounds.

It is thus seen from these illustrations that in a simple beam the greatest perpendicular shear is at the supports, and in a cantilever beam it is at the wall. Only these sections, then, need be investigated in a solid beam. For a simple beam of length l and carrying a uniform load of w pounds per linear unit, the greatest perpendicular shear is equal to the reaction $\frac{1}{2}wl$. For a cantilever beam of length l, the greatest perpendicular shear due to a uniform load is the total weight wl.

The perpendicular shear V produces in cross section where it occurs an equal shearing stress. If A is the section area and S the shearing unit stress acting over that area,

$$V = AS, \quad S = \frac{V}{A}, \quad A = \frac{V}{S} \qquad [2]$$

are the equations similar to Equation 1 of Art. 8; these are used in computations regarding shear in solid beams.

For example, consider a steel I beam weighing 250 pounds per yard and 12 feet long, over which roll three locomotive wheels 4 feet apart and each bearing 14,000 pounds. The greatest shear will occur when one wheel is almost at the support as shown in Fig. 17. In Art. 14 the reaction is found to be 28,500 pounds, and this is equal to the greatest perpendicular shear V. The area of the cross section being 24.5 square inches, the shearing unit stress in the section is

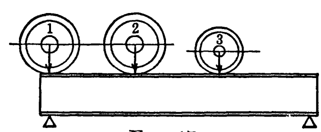


FIG. 17.

$$S = \frac{28{,}500}{24.5} = 1160 \text{ pounds per square inch}$$

which is a low working unit stress for steel.

As a second example, consider a wooden cantilever beam which projects out from a bridge floor and supports a sidewalk. Suppose it to be 6 inches wide, 8 inches deep, and 7 feet long, and the maximum load that comes upon it to be 7500 pounds. The perpendicular shear at the section where it begins to project is then 7590 pounds, or the load that it carries plus its own weight. As the section area is 48 square inches the shearing unit stress is a little less than 160 pounds per square inch. The factor of safety against shearing is hence about 19 (Art. 12), so that the security is ample.

It is indeed only in rare instances that solid beams of uniform cross section are subject to dangerous stresses from shearing. Beams almost universally fail by tearing apart under the horizontal tensile stresses, and hence the following articles will be devoted to the consideration of these bending stresses.

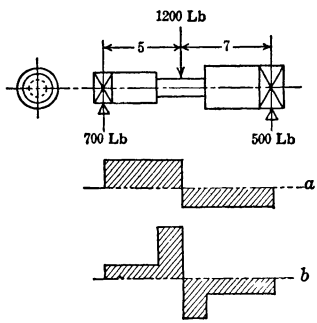


FIG. 18.

These data of shearing forces and shear stresses are often plotted graphically in a *shear diagram*, as in Fig. 18*a*. This diagram is constructed by plotting the forces vertically to a scale, with due regard to the directions of the forces, usually working from left to right. If the beam is of uniform cross section for its entire length, the diagram of shear forces is sufficient in most cases, for the shear stress diagram would have the identical shape. If, however, the cross section is not uniform, a second diagram (Fig. 18*b*) showing shear stresses might be of value. Note the difference in shapes of diagrams *a* and *b* due to the influence of changing cross sections.

QUESTIONS AND PROBLEMS

15a. Is the shear stress in beams usually critical? Explain.

15b. What two factors control the value of shear unit stress in a beam?

15c. In computing shear on a beam what is the preferred side from which to take the algebraic sum of forces?

15d. Explain the difference in shape between graphs *a* and *b*, Fig. 18.

15e. In moving loads such as are shown in Fig. 17, why does the maximum shear occur just before the first wheel reaches the support?

15f. What is the magnitude of the vertical shear under wheels 2 and 3 in Fig. 17?

15g. A timber cantilever l feet long carries a uniform load of w pounds per linear foot together with a concentrated load of P at $\frac{1}{2}l$. What is the vertical shear at the support and at the extreme end? What is the vertical shear under the concentrated load and at each of the quarter points?

15h. A simple beam of cast iron is 4×4 inches in section and 6½ feet long between supports. Besides its own weight, it is to carry a load of 5000 pounds at the middle and a load of 1000 pounds at 1½ feet from the left end. Find the factor of safety against shearing.

15i. On a simple beam 12 feet long there are two loads, each 800 pounds, one at 3 feet from the left end and one at 3 feet from the right end. Find the vertical shear due to these loads for a section near one of the supports, and also for any section between the loads.

16. Bending Moments in Beams. We have seen that a beam is merely a special body under the influence of several forces tending to produce rotation. Also, that the algebraic sum of moments of all forces acting about any point on the beam must be equal to zero, or that the sum of the moments of all forces on one side of any point on the beam must be equal to the sum of moments of all forces on the other side. The function of the beam in this case is to transmit the action of one set of forces to the other set. In so doing the beam is put under bending action, and the moment producing this bending is called the *bending moment.* Thus the bending moment at any section of a beam is equal to the algebraic sum of the moments of all the forces on either side of the section in question. Which group of forces to be used is a matter of choice, depending upon the amount of calculation required or the data available.

If the moment of each force is given its proper sign, as previously explained, the resulting sign of the bending moment will have real significance. In a position on a beam where a positive bending moment exists, the beam is bent so that its upper side is concave. If the bending moment is negative, the bending is such that the upper side is convex (bowed up). Thus, knowing the shape of the beam under loading, one can predict the sign of the bending moment or, knowing the sign of the bending moment, predict the shape of bending of the beam.

For example, let a beam 30 feet long have three loads of 100 pounds each, situated at distances of 8, 12, and 22 feet from the left support (Fig. 19). By the method of the previous article the left reaction R_1 is 160 pounds and the right reaction R_2 is

140 pounds. For a section 4 feet from the left support the bending moment is $160 \times 4 = 640$ pound-feet, and for a section at 8 feet from the left support the bending moment is $160 \times 8 = 1280$ pound-feet. For a section 10 feet from the left support there are two vertical forces on the left of the section, 160 acting up and 100 acting down, so that the bending moment is $160 \times 10 - 100 \times 2 = 1400$ pound-feet. For a section at the middle of the beam the bending moment is $160 \times 15 - 100 \times 7 - 100 \times 3 = 1400$ pound-feet. For a section at the third load the bending moment is, in like manner, 1120 pound-feet, and for a section at 3 feet from the right support it is 420 pound-feet. The vertical ordinates plotted in the diagram under the beam in Fig. 19 represent the values of these bending moments, and the diagram thus formed shows how the bending moments vary throughout the length of the beam.

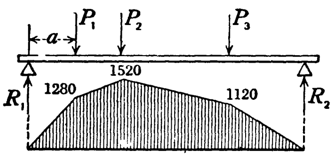


FIG. 19.

For a simple beam of span l and uniformly loaded with w pounds per linear unit, each reaction is $\frac{1}{2}wl$. For any section distant x from the left support (Fig. 20) the bending moment is $\frac{1}{2}wl \times x - wx \times \frac{1}{2}x$, where the lever arm of the reaction is x and the lever arm of the load wx is $\frac{1}{2}x$. If w is 80 pounds per linear foot and l is 30 feet, the bending moment at any section is then $1200x - 40x^2$. For $x = 10$ feet, the bending moment is 8000 pound-feet; for $x = 15$ feet, it is 9000 pound-feet; for $x = 20$ feet, it is 8000 pound-feet; and so on. The diagram shows the distributions of moments throughout the beam, and it can be demonstrated that the curve joining the ends of the ordinates is the common parabola.

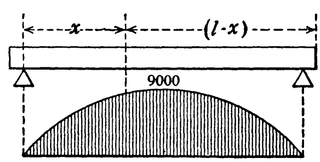


FIG. 20.

When a beam is loaded both uniformly and with concentrated loads, the bending moments for all sections may be found in a similar manner. The maximum bending moment indicates the point where the beam is under the greatest horizontal stresses; this will usually be found near the middle and often under one of the concentrated loads. For simple beams resting on two supports at their ends all the bending moments are positive. It may further be noted that, if the vertical forces on the right of the section are used, the same numerical values will be found for the bending moments.

QUESTIONS AND PROBLEMS

16a. In calculating the bending moment for a given section of a beam, what is the basis of choice between using the sum of moments on one side or the other?

16b. If a beam is bent so that its upper side is concave, what is the direction of the existing bending moment?

16c. Compare the type of line bounding the bending moment diagrams in Figs. 19 and 20. What type of line is associated with uniform loading? With concentrated loading? What type of line would you expect to find bounding a bending moment diagram for a combination of concentrated and uniform loading?

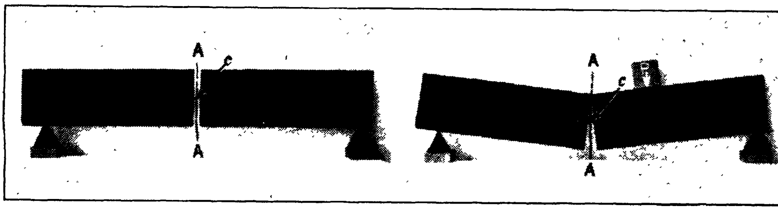
16d. A simple beam of yellow pine has a section area of 1½ square feet. Its length is 18 feet. At 6 feet from the left end there is a load of 1000 pounds. At what point on the beam must a load of 2000 pounds be placed so that the reaction at the left support will be twice that at the right?

16e. Two locomotive wheels, 6 feet apart, each carrying 20,000 pounds, roll over a beam of 27-foot span. Find the greatest reaction which can be caused by these wheels.

16f. A simple beam of 20-foot span weighs 100 pounds per linear foot and has a load of 500 pounds at 8 feet from the left end. Compute the bending moments for sections distant 2, 4, 6, 8, 10 feet from the left support and construct the diagram of bending moments.

17. Resistance to Bending. In transmitting the action of one set of forces to another set, which produces what we have called bending moment, the beam must develop internal forces which will tend to maintain the beam shape, resist bending. This internal resistance to bending we call *resisting moment* and it follows that at any point *resisting moment* = *bending moment.*

The precise nature of the resisting moment stresses must be determined so that design and investigation can be performed. Figure 21*a* shows a model beam, cut away at section *AA* so that the bending action will be concentrated and exaggerated when under load as in Fig. 21*b*. The pieces removed were of the same length, for the full depth, as shown by the shape of the cut-outs in Fig. 21*a*. When a bending moment is produced, however, we see that this gap is reduced at the top and increased at the bottom. The amount of change in the gap width varies between these extremes and there is no change at the center point C. Close examination and comparison of these two illustrations reveal that the change in gap width on either side of the center point C is in direct proportion to the distance from that point. That is, at a point 2 inches above the center, the change in gap is twice as large as at a point 1 inch above, and half as large as a point 4 inches above. It follows that if sufficiently elastic



(*a*) (*b*)

FIG. 21.

material were connected between the two sides of the gap it would be stretched (tension) or shortened (compressed) just as the gap varied in width.

In Art. 10 we saw that within the elastic limit of a material, the stress is in direct proportion to the deformation. Thus we can safely say that the stress is greatest at the very top or very bottom of the section whichever is farthest from the unchanging or *neutral* axis. Figure 22 is a diagram showing the stress relationships at a line of bending AA, as deduced above, arrows pointing left indicating tension and those pointing right indicating compression.

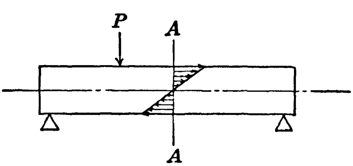


FIG. 22.

The line passing through a cross section at the point of zero deformation and stress we call the *neutral axis*. Since in every vertical cross section of a beam there exists a neutral axis, there is formed a *neutral plane* passing horizontally through the beam for its entire length.

The horizontal stresses resist the bending by acting, in effect, upon small lever arms which are equal in length to the distance to the neutral plane. Thus, if in Fig. 21 there had been in effect one area element at the top and another at the bottom, and if the deformation had produced a total stress of 60 pounds in each, the resisting moment would be $(60 \times 5) + (60 \times 5)$ or 600 inch-pounds. The resistance of any additional area element could be added to this sum to produce the total resisting moment. Hence we can say that in any cross section the resisting moment is equal to the summation of each area element stress multiplied by its distance from the neutral plane. Expressed in an equation, this is

$$M_r = a_1S_1z_1 + a_2S_2z_2 + a_3S_3z_3 + a_4S_4z_4, \text{ etc.} \qquad [3]$$

where M_r is the resisting moment, a is fiber area, S is the unit fiber stress, and z is the distance to neutral plane from points 1, 2, 3, 4, etc. But we have seen that the stress in an individual area element is in proportion to its distance from the neutral plane so that, if S is the unit stress in the extreme area element and c is half the depth of a rectangular beam,

$$S_1 = S\frac{z_1}{c}, \qquad S_2 = S\frac{z_2}{c}, \text{ etc.}$$

Substituting these values in Equation 3, we get

$$M_r = a_1S\frac{{z_1}^2}{c} + a_2S\frac{{z_2}^2}{c} + a_3S\frac{{z_3}^2}{c} + a_4S\frac{{z_4}^2}{c}, \text{ etc.}$$

or

$$M_r = \frac{S}{c}(a_1{z_1}^2 + a_2{z_2}^2 + a_3{z_3}^2 + a_4{z_4}^2, \text{ etc.})$$

or

$$M = \frac{S}{c}(\Sigma az^2)$$

Here the notation Σaz^2 is used to denote the quantity $a_1{z_1}^2 + a_2{z_2}^2 + \cdots$. The letter Σ (sigma) is not a factor but a symbol which indicates the process of summation, and it should be read "summation of all the values of."

This quantity Σaz^2 is called the *moment of inertia* of the cross section of the beam. How its value is found is shown in Art. 19. The moment of inertia is designated by I; hence

$$\text{Resisting moment} = \frac{SI}{c}$$

$$5 \times 6\tfrac{5}{8} + 6 \times 3 - 11 \times c = 0$$

from which the value of c is found to be 4.65 inches. For the channel section, shown on the right of Fig. 24, the same method is to be followed as for the ⊥.

The method of moments may thus be applied to areas as well as to forces. If a is any area and z the distance of its center of gravity from an axis, the product az is called the static moment of the area. The algebraic sum of the static moments of all parts of the figure is represented by Σaz which is the summation of the values a_1z_1, a_2z_2, a_3z_3, etc. If A is the total section area,

$$c = \frac{\Sigma az}{A}$$

is a general expression of the method of finding the distance c. If the axis is taken within the section, some of the z's are negative; and, if the axis passes through the center of gravity of the section, the quantity Σaz is zero.

When the cross section is bounded by curved lines, as in a railroad rail, it is to be divided up into small rectangles and the value of a found for each. The sum of all the a's is A, and then by the above method the value of c is computed. For the various rolled shapes found in the market the values of c are thus determined by the manufacturers and published for the information of engineers.

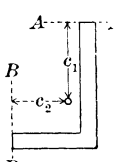


FIG. 25.

Triangular beams are seldom used, but it is often convenient to remember that for any triangle whose depth is d the value of c is $\tfrac{2}{3}d$.

For the angle section, shown in Fig. 25, the center of gravity usually lies outside the section; and there are two values of c, called c_1 and c_2, to be determined.

Let the thickness of each leg be $\tfrac{3}{4}$ inch and the length of the long leg 6 and that of the short leg 4 inches. The area of the long leg, including the lower corner, is $6 \times \tfrac{3}{4} = 4.5$ square inches, and its center of gravity is 3 inches below the axis AA and $3\tfrac{5}{8}$ inches to the right of the axis BB. The area of the short leg, excluding the corner, is $3\tfrac{1}{4} \times \tfrac{3}{4} = 2.4375$ square inches, and its center of gravity is $5\tfrac{5}{8}$ inches below the axis AA and $1\tfrac{5}{8}$ inches to the right of the axis BB. Then, as the total area of the section is 6.9375 square inches, the equation of moments with respect to the axis *AA* is

$$6.9375c_1 = 4.5 \times 3 + 2.4375 \times 5.625,$$

and then $c_1 = 3.92$ inches. Also the equation of moments with respect to the axis *BB* is

$$6.9375c_2 = 4.5 \times 3.625 + 2.4375 \times 1.625$$

from which $c_2 = 2.92$ inches.

QUESTIONS AND PROBLEMS

18a. Find the center of gravity of a trapezoid the upper and lower parallel edges of which are 6 inches apart. The upper edge is 10 inches long; the angle between the right-hand and top edges is 135 degrees, and the angle between the top and left-hand edges is 86 degrees.

18b. For Fig. 24 let $c = 6$ and $c_1 = 3$ inches. If the unit stress S at the top of the web is 6000 pounds per square inch, what is the unit stress S_1 on the lower side of the flange?

18c. A deck beam used in buildings has a rectangular flange $4 \times \frac{3}{4}$ inches, a rectangular web $5 \times \frac{1}{2}$ inches, and an elliptical head which is 1 inch in depth and whose area is 1.6 square inches. Find the distance of the center of gravity from the top of the head.

18d. Find the center of gravity of a T section 12 inches deep by 5 inches wide with a vertical web $\frac{3}{4}$ inches thick and flanges 1 inch thick. Cut out paper model and check for balance on a pin held horizontally.

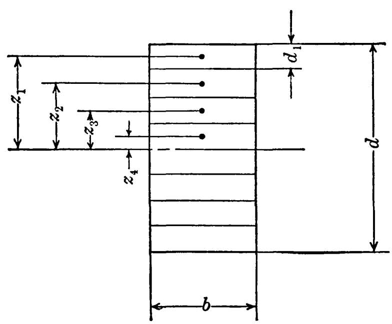


FIG. 26.

19. Moment of Inertia. In Art. 17 it is shown that a beam's resistance to bending is dependent upon the distribution of the cross section area expressing the influence of each elemental unit of area with respect to an axis passing through the center of gravity. The sum of the products of all the individual elements of area multiplied by the square of their respective distances from the neutral axis we call moment of inertia, designated by the letter *I*. Expressed as an equation, $I = \Sigma az^2$, where Σ is used to indicate the process of summation,

a represents the elements of area, and z their distances from the neutral axis. This equation is in a general form and must be further developed in order to apply to any specific section.

If a rectangular section b wide by d deep is broken up into elements of area b by d_1, as in Fig. 26, an approximate value of its moment of inertia may be obtained by other than higher mathematics, as follows:

In the equation, $I = a_1z_1^2 + a_2z_2^2 + a_3z_3^2$, etc.,

$$a_1,\ a_2,\ a_3, \text{ etc.} = bd_1, \text{ and } z_1 = \frac{d}{2} - \frac{d_1}{2},\ z_2 = \frac{d}{2} - \frac{3d_1}{2},$$

$$z_3 = \frac{d}{2} - \frac{5d_1}{2},\quad z_4 = \frac{d}{2} - \frac{7d_1}{2}$$

Then

$$I = 2bd_1\left(\frac{d}{2} - \frac{d_1}{2}\right)^2 + 2bd_1\left(\frac{d}{2} - \frac{3d_1}{2}\right)^2$$

$$+2bd_1\left(\frac{d}{2} - \frac{5d_1}{2}\right)^2 + 2bd_1\left(\frac{d}{2} - \frac{7d_1}{2}\right)^2$$

Or

$$I = 2bd_1(d^2 - 8dd_1 + 21d_1^2)$$

But, in this case,

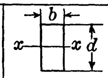
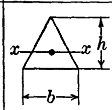
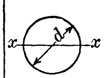
$$d_1 = \frac{d}{8} \quad\text{and}\quad d_1^2 = \frac{d^2}{64}$$

$$I = \frac{2bd}{8}\left(d^2 - 8d\frac{d}{8} + 21\frac{d^2}{64}\right) = \frac{bd}{4} \times \frac{21d^2}{64}$$

$$I = \frac{21bd^3}{256}$$

If the size of the elements is reduced to the infinitesimal, and summated by the use of calculus, an exact value for this case is obtained, $I = \frac{1}{12}bd^3$, which is slightly larger than our approximate value.

By the use of calculus, formulas for other common shapes are obtained as follows:

Moments of Inertia About Gravity Axis x-x		
Rectangle		$I = \frac{bd^3}{12}$
Triangle		$I = \frac{bh^3}{36}$
Circle		$I = \frac{\pi d^4}{64}$

By examining these formulas we see that I is in units of cross section measure to the fourth power. If the depth and width of a beam are expressed in inches, the moment of inertia will be in units of inches to the fourth power.

Moments of inertia, when referred to the same axis, can be added or subtracted like any other qualities which are of the same kind. Thus, in a hollow rectangular section whose outside depth and breadth are b and d, the thickness of the metal being the same throughout (Fig. 27), the moment of inertia is found by subtracting the moment of inertia of the inner rectangle from that of the outer one; or

$$I = \tfrac{1}{12}bd^3 - \tfrac{1}{12}b_1 d_1^3$$

is the moment of inertia for the rectangular section whose area is $bd - b_1 d_1$.

For an I beam the flanges of which are equal the same method applies. Let b be the width of the flanges and d the total depth of the section shown on the left of Fig. 27; also let t be the thickness of the web and t_1 the thickness of the flanges. The moment of inertia of the area $(b - t)\ (d - 2t_1)$ is then to be subtracted from the moment of inertia of the area bd, or

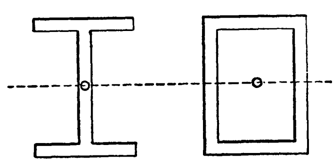


FIG. 27.

$$I = \tfrac{1}{12}bd^3 - \tfrac{1}{12}\ (b - t)\ (d - 2t_1)^3$$

is the moment of inertia for the I section.

19c. A steel I beam weighing 80 pounds per linear foot is 24 inches deep, its flanges being 7 inches wide and ⅞ inch mean thickness, and the web 0.5 inch thick. The moment of inertia stated by the manufacturer is 2088 inches^4. Compute it by the equation given in this article.

19d. Calculate the moment of inertia of a built-up section similar to Fig. 29*b*, made up of two 6-inch channels and one plate $8 \times 5/16$ inches. The area of each channel is listed as 2.39 square inches and about its gravity axis, $I = 13$ inches^4.

19e. Calculate the moment of inertia of a section similar to Fig. 29*c* except that it is made solid, without the two cavities shown.

19f. What would be the moment of inertia of a rectangular section 4×10 inches with a 2½-inch circular cavity ½ inch in from a 4-inch edge? Use the gravity axis perpendicular to the 10-inch side.

19g. Calculate the moment of inertia of a section through hole *D*, Fig. 3, if the beam were 5×12 inches and the hole 1½ inches in diameter, located 2 inches from its center line to the bottom of the beam.

20. Section Modulus. Beams of several shapes and sizes might be made of the same material and have a cross section with the same moment of inertia, and yet not be equally strong. This we can see by examining Equation 4, $M = SI/c$, which shows that the strength varies not only with the moment of inertia, but also inversely as the distance from the neutral axis to the extreme fiber. Therefore, the actual measure of the strength of a given cross section is the ratio I/c, which we call *section modulus.* This value is commonly used, and is frequently given in tables of beam section properties, such as Table 4, page 140.

Examining the section modulus of a rectangular section $I/c = bd^2/6$, we see that the reason for using shapes deeper than they are wide, as in floor joists, is that this value increases with the square of the depth, and only directly as the width. Thus a beam twice as wide as another is twice as strong, whereas one twice as deep is four times as strong. A 3- × 10-inch joist, standing on the 3-inch side has an I/c of 50 inches^3, whereas lying on its 10-inch side its I/c is only 15 inches^3.

PROBLEMS

20a. Compute the section modulus of the T section of Problem 18*d*. (Be sure to use the greater value of *c*.)

20b. Two symmetrical beams, one 10 inches deep and the other 16 inches deep, are so proportioned that they have the same moment of inertia, 300 inches^4. What is the section modulus of each; which is stronger; and how much stronger (in percentage) is it?

Art. 21. REVIEW PROBLEMS

21a. A plain concrete beam is 14 inches deep and 10 inches wide. What is its section area? Locate its neutral axis. How much does it weigh per linear foot? What is the moment of inertia of its cross section? If the beam is laid on its side, what will the moment of inertia then be?

21b. Compare the moment of inertia of a circle 86 inches in diameter with the moment of inertia of the circumscribing square?

21c. A piece of hemlock 2×4 inches in section lies flatwise and spans an opening 6 feet wide. A man weighing 150 pounds stands at its center. Is he in a safe place? How much safer would he be if the piece were turned so that its 4-inch dimension would be vertical?

21d. Three men carry a stick of timber, two taking hold at a common point and one at one of the ends. Where should the common point be so that each man may carry one-third of the weight?

21e. Compute the bending moments under each concentrated load for Fig. 15, assuming the beam to weigh 100 pounds per linear foot.

21f. The two bases of a trapezoid are 8 and 5 inches, and its height is 5 inches. Find the center of gravity.

21g. For a solid circular section the moment of inertia with respect to an axis through the center is $\frac{1}{64}\pi d^4$. Find the moment of inertia for a hollow circular section with outside diameter d_1 and inside diameter d_2.

21h. A simple beam of 20-foot span weighs 160 pounds per linear foot and has a concentrated load of 5000 pounds at a distance of 4 feet from the left end. Compute the bending moments for several sections throughout the beam and construct the diagram of moments.

21i. Locate both gravity axes of a steel channel 8 inches deep, the average thickness of the web being 0.25 inch, average thickness of flange 0.42 inch, and width of flanges 2.32 inches.

21j. Compute the moments of inertia of the beam section given in Problem 18*c* with respect to each of the gravity axes.

21k. Find the moment of inertia of a circle 5 inches in diameter. Also the moment of inertia of that circle with respect to another axis in the same plane, the shortest distance from the center of the circle to that axis being 9 inches.

21l. A timber cantilever 4×8 inches in section projects 5 feet out of a wall. What load must be put upon it so that the greatest shearing stress will be 140 pounds per square inch?

21m. Show that the moment of inertia of a rectangle with respect to an axis passing through its base is $\frac{1}{3}bd^3$.

21n. A temporary grandstand is to be erected. The seats are to be of hemlock boards 12 inches wide and 2 inches thick. What is the maximum permissible distance between the supports under the seats if they are to carry safely a solid row of people each weighing 150 pounds and each occupying a longitudinal space of 14 inches?

21o. What is the maximum resisting moment of the beam of Problem 21*e*? What are the reactions of its supports?

CHAPTER 4

APPLICATION OF BEAM PRINCIPLES

22. Investigation of Beam Strength. To determine the maximum safe load for a given beam, the following data must be obtained:

1. Span and nature of supports.
2. Shape and dimensions of the cross section.
3. Properties of the material of which the beam is made.
4. Essential nature and location of loads.

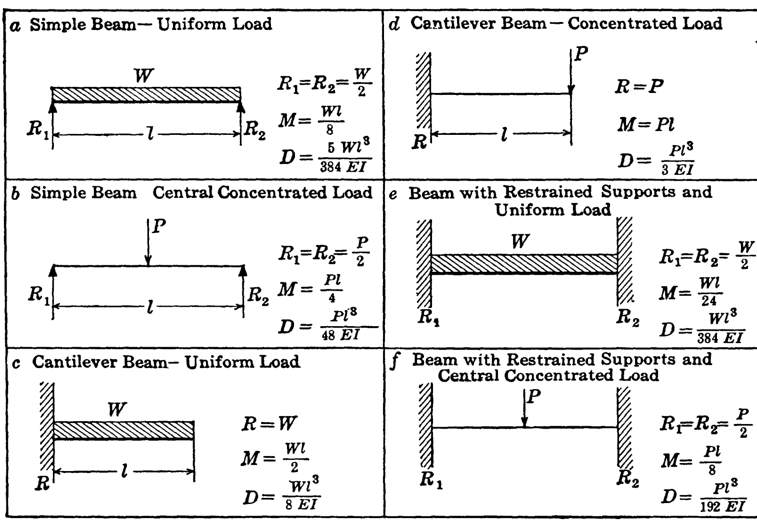


FIG. 30.

The *supports* may be either *restrained* or *simple,* the latter being the more common. Beams with restrained supports, such as shown in Fig. 30*e,* are stronger than equal beams with simple supports; but secondary forces must be resisted in the supports themselves, and this is often undesirable. The analysis of the factor relationships for beams with restrained supports involves higher mathematics, and hence the formulas given in Fig. 30 are merely for reference. For beams with simple supports the reactions and bending moments are calculated as shown in Arts. 14 and 16.

We have seen that the section modulus of a beam is based upon the shape and dimensions of its cross section. For materials, such as steel, equally strong in tension and compression, the calculations given will be correct. For materials such as cast iron, which are not equally strong in tension and compression, a special analysis must be made. In Art. 17, it is noted that under a positive bending moment, the top area elements are in compression and the bottom area elements are in tension. In a symmetrical section like an I beam, there would be produced equal stresses at the top and bottom, and the comparison would be with the lower strength value (tension). Frequently the beam section of these materials is designed to take advantage of this unequal strength, sometimes using a ⊥ shape as in Fig. 31. Here the center of gravity and neutral axis are nearer the bottom, producing higher stresses at the top than at the bottom because c_1 is greater than c_2. In such a case, the beam must be investigated for both stresses, being limited by the stress permitting the smallest load.

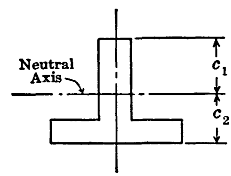


FIG. 31.

In general, loads are either concentrated as in Fig. 30*a* or distributed as in Fig. 30*b*. Both kinds appear in combination as in Fig. 30*c*. The way the loads are applied, such as steady loads, or varying or shock loads, must also be considered, but this commonly is compensated for by adjusting the working stress value to give a higher factor of safety (Art. 12).

The position and value of the loads determine the value of maximum bending moment and the place where it occurs. For beams of constant cross section, only the maximum bending moment must be considered, and its position can be determined in several ways. In general, maximum bending moments occur under concentrated loads, or at the supports for cantilever beams or overhung loads. Maximum bending moments in any one direction always occur at the point where the shear diagram crosses the zero axis. This can be proved by higher mathematics and demonstrated by experimental calculation. Figure 32 shows a beam with more or less complex loading, together with its shear and bending moment diagram. It can be seen that the maximum bending moment between the supports (positive) occurs at a point, x, 6.9 feet from the left end, and another maximum moment occurs over the right support (negative). In this case the

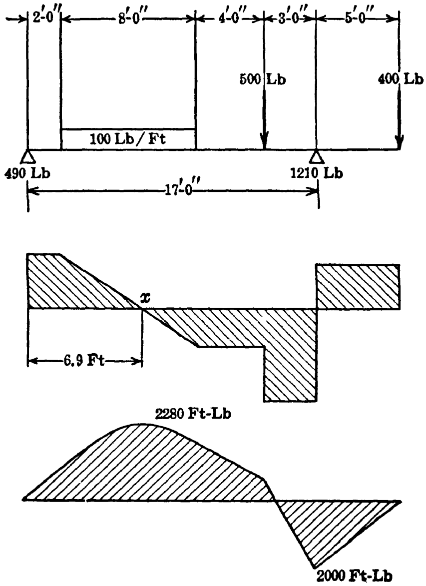


FIG. 32.

positive moment is greater, but each case must be thoroughly investigated to show which predominates.

Where more than one load is in effect, and their maximum allowable values are to be determined, their effect must be considered in combination, along the lines of the preceding analysis and with experimental values being used.

It should be noted that, as in all formulas, all measurements of a given type must be expressed in the same units. Thus, S, I, and c are normally expressed in terms of inches, and so the bending moment must also be expressed in inch-pounds, which means that distances and spans used in calculating the bending moment must be in inches, or its whole value converted before substitution.

As a final precaution, the beam should be checked for maximum shearing stress. On beams of constant cross section, this occurs at point of maximum shear strain, determined from the shear diagram and calculated as in Art. 15. When the beam has cross sections of more than one size, the shear stress should be checked at the point of maximum shear strain for each section.

QUESTIONS AND PROBLEMS

22a. What two types of supports does the text mention? Which type is found with cantilever beams?

22b. Why would a cast-iron T section cantilever beam be made with the flanges up (T) rather than down (⊥)?

22c. How do we commonly compensate for shock or moving loads on beams?

22d. Where, in relation to points on the shear diagram, do maximum bending moments occur?

22e. An I beam which is 20 feet long weighs 700 pounds and the area of its cross section is 10.29 square inches. What is the kind of material?

22f. An advertising sign weighing 1500 pounds and hung from two points 6 feet apart is to be supported by a cantilever beam 8 feet long and weighing 30 pounds per lineal foot. Sketch the arrangement and determine both the maximum shear and the maximum bending moment in the beam. What is the stress in the suspenders?

22g. What is the bending moment at the quarter point of a simple beam of length l weighing w pounds per lineal foot when it carries two concentrated loads of two tons each at its third points?

22h. A cantilever beam has a load of 900 pounds at its end and is also uniformly loaded with 150 pounds per linear foot; its length is 5 feet. Compute the bending moments for five sections, one foot apart, and construct the diagram of bending moments.

22i. A simple beam weighing 80 pounds per linear foot is 13 feet in span and has a load of 2000 pounds at the middle. Compute the maximum bending moment.

22j. The wooden girders of a floor are 10×14 inches in cross section, 25 feet span, and 12 feet apart. The floor carries a load of 100 pounds per square foot. Find the maximum unit stress at the middle of the girders.

22k. A steel pin, 8 inches long and 3 inches in diameter, is arranged like a simple beam to carry a load of 10,000 pounds at the middle. Find the maximum tensile and compressive unit stresses.

23. Design of Beams. The design of a beam consists in determining its size when the loads it is to carry and its length are given. The allowable working unit stress S is first assumed according to the principles of Art. 11. From the given loads the maximum bending moment M is then computed. Thus in Equation 4 everything is known except I and c, and

$$\frac{I}{c} = \frac{M}{S}$$

is an equation which must be satisfied by the dimensions to be selected.

For a rectangular beam of breadth b and depth d the value of c is $\frac{1}{2}d$, and the value of I is $\frac{1}{12}bd^3$. Thus the equation above becomes

$$bd^2 = \frac{6M}{S}$$

and if either b or d is assumed the other can be computed. For example, let the requirements be to design a rectangular wooden

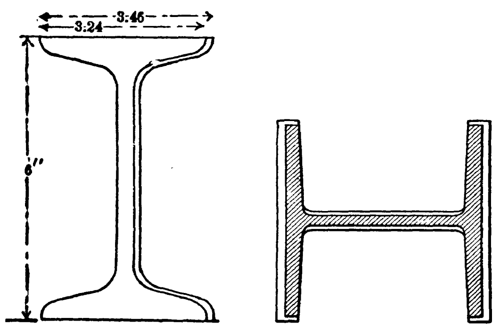


Fig. 33.

beam for a total uniform load of 80 pounds, the beam to be used as a cantilever with a length of 6 feet, and the working value of S to be 800 pounds per square inch. Here the maximum value of M is $80 \times 3 = 350$ pound-feet $= 2880$ pound-inches. Thus $bd^2 = 21.6$ inches^3. If b is taken as 1 inch, $d = 4.65$ inches; if b is 2 inches, $d = 3.29$ inches; if b is 3 inches, $d = 2.68$ inches. With due regard to sizes readily found in the market 2×4 inches are perhaps good proportions to adopt.

For rolled sections, such as channels and I beams, the required section modulus may be compared with tables of values, such as Table 4, page 140, and a section selected which will do the job most economically as far as strength is concerned, and at the same time meet other existing conditions, such as space limitations or special manufacturing problems. These sections are normally made of a mild steel having an ultimate strength of about 60,000 pounds per square inch and an elastic limit of 35,000 pounds per square inch. In actual practice, reference should be made to more complete tables as published by manufacturers, or by engineering societies. Various weights of a section of given size are obtained by varying certain roll spacings which results in a shape change like that shown in Fig. 33.

QUESTIONS AND PROBLEMS

23a. Distinguish between design and investigation of beams.

23b. A steel I beam of 25-foot span is to carry a uniformly distributed load of 900 pounds per linear foot. In addition there is a concentrated load of 8000 pounds at 5 feet from the left end. Find the proper size of the beam.

23c. A steel I beam weighing 70 pounds per foot is placed as a cantilever 19 feet long to support a load of 2000 pounds at its extreme end. What should be the value of I/c so that the maximum fiber stress will not exceed 16,000 pounds per square inch?

23d. A simple cast-iron beam of 16-foot span carries a load of 4000 pounds at the middle. If its width is 6 inches, find its depth for a factor of safety of 10; also find its width for a depth of 12 inches.

23e. A yellow pine beam of 18-foot span is to carry a uniformly distributed load of 600 pounds per linear foot with a factor of safety of 9. The depth of the beam is to be $1\frac{1}{2}$ times the breadth. Find the dimensions of the beam.

24. Modification of Beam Shapes. Thus far in our discussion, we have considered beams whose section is constant for their entire lengths. This is not necessary, for we see by examining a bending moment diagram that the strength requirements vary greatly along the beam. Thus, if there is some need for saving material, or for reducing the section for any reason whatsoever, we may do so by eliminating what is surplus material.

In the case of a simple beam, Fig. 34*a*, the bending moment varies from zero at the supports to a maximum at the load. If the section is strong enough at the load, certainly it is stronger than necessary at every other point, more so as the supports are approached. If shear were disregarded, a rectangular beam of *constant strength* and *uniform width* would have a shape like Fig. 34*b*, where the depth at any point would be $d = \sqrt{6M/bs}$.

At the supports this shape would not be sufficiently strong in shear, for the minimum section for this consideration would be $d = P/2Sb$ (see Art. 15) and the beam shape would be similar to Fig. 34*c*. Hence it can be seen that the surplus material is distributed as shaded in Fig. 34*c*. Unless the space is needed, or the weight or cost of the material is an unusually important factor, such modification of shape is usually not economically practical.

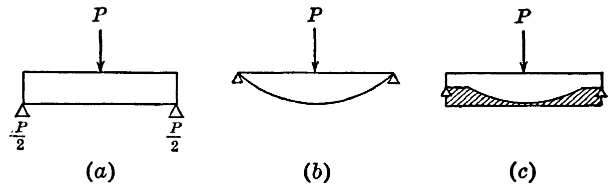


FIG. 34.

QUESTIONS AND PROBLEMS

24a. On the basis of the above discussion, explain why the statements in Art. 5 concerning drilling of holes in the beam, Fig. 3, are correct. If that beam were a 3×10 fir timber with a 14-foot span, and the holes were 1 inch in diameter, determine whether or not hole *C* on the neutral plane in beam center would be safer than hole *D*, 1 foot from the support and 3 inches below the neutral plane.

24b. A simple timber beam is 20 feet long and has a depth of 18 and a width of 12 inches. Taking no account of the shearing stresses, indicate how much of the depth of this beam might safely be cut away at its quarter point. How much at a point 2 feet from the left end?

24c. A simple beam of uniform strength is to be designed to carry a heavy load *P* at the middle. If d_1 is the depth at the middle, show that the depths at distances $0.1l$, $0.2l$, $0.3l$, and $0.4l$ should be $0.45d_1$, $0.63d_1$, $0.77d_1$, and $0.89d_1$.

24d. A cast-iron cantilever beam is to be 4 feet long, 3 inches wide, and is to carry a load of 10,000 pounds at the end. Find the proper depths for every foot of length, using 3000 pounds per square inch for the horizontal unit stress and 4000 for the vertical shearing unit stress.

Art. 25. REVIEW PROBLEMS

25a. How does the strength of a simple rectangular beam vary with its length? Its depth? Its breadth?

25b. A rectangular section area has a width of *b* and a depth of *d*. What is the value of its moment of inertia in terms of *b* and *d* when referred to an axis parallel to its width and one inch below its bottom?

25c. An elevator is suspended by a steel cable 1¼ inches in diameter.

What load will it safely carry if its own weight, including the elevator, is 6000 pounds? What horsepower must the hoisting elevator develop if this load is to be raised at the uniform rate of 400 feet per minute?

25d. Locate the neutral axis for a T section which is 3×3 inches and ¾ inch thick.

25e. A timber 4×6 inches in section projects 6 feet out of a wall. What load must be put upon it so that the greatest shearing stress will be 100 pounds per square inch?

25f. A simple wooden beam, 8 inches wide, 10 inches deep, and 12 feet in span, carries two equal loads, one being 2.5 feet at the left and the other 2.5 feet at the right of the middle. Find these loads so that the factor of safety of the beam will be 8.

25g. A simple wooden beam, 3 inches wide, 4 inches deep, and 10 feet in span, has a load of 500 pounds at the middle. Compute its factor of safety.

25h. A simple beam of structural steel, 4 inches deep and 16 feet in span, is subject to a rolling load of 500 pounds. What must be its width in order that the factor of safety may be 6?

25i. Compare the strength of a joist, 4×10 inches, when laid with long side vertical with that when it is laid with short side vertical.

25j. Compare the strength of an 8-inch 18-pound steel I beam with that of a wooden beam 9 inches wide and 13 inches deep, the span being the same for both.

25k. Show that a beam 3 inches wide, 6 inches deep, and 4 feet long is nine times as strong as a beam 2 inches wide, 4 inches deep, and 10⅔ feet long.

25l. Compute the reactions for an overhanging beam where the distance between the supports is 10 feet and the overhanging arm is 4 feet, the beam weighing 60 pounds per linear foot.

25m. A wooden beam, 10×12 inches in section area, projects 6 feet from the wall of a building. What load can be suspended from the end of the beam so that the factor of safety will be 10?

25n. A piece of wooden scantling 2¼ inches square and 18 feet long is hung horizontally by a rope at each end and a student weighing 200 pounds stands upon it. Is it safe?

25o. A floor is supported by 4- $\times$8-inch wooden joists of 16-foot span spaced 18 inches apart center to center. When this floor carries a total load of 250 pounds per square foot, what is the factor of safety of the joists?

25p. What must be the depth of a wooden beam, the cross section of which has a moment of inertia of 16 inches^4 when the maximum unit fiber stress is 1900 pounds per square inch and the maximum bending moment is 3500 pound-inches?

25q. A steel I beam 7 inches deep and weighing 22 pounds per foot has for the moment of inertia of its cross section 52.05 inches. It is to be used as a simple beam with a span of 18 feet. What uniform load will it carry when the maximum unit stress S is to be 16,000 pounds per square inch?

25r. What uniformly distributed load can be safely carried by an oak beam, 4 inches wide and 6 inches deep, on a span of 18 feet, if the maximum unit stress is not to exceed 1200 pounds per square inch?

25s. If the moment of inertia of a 20-inch I beam is 1240 inches^4, what is the value of its section modulus? Do not refer to Table 4.

25t. A 20-inch steel I beam 10 feet long carries a uniform load of 1000 pounds per foot. It is to be replaced by two beams of the smallest possible depth. How deep must these two beams be so that S will not be greater than 16,000 pounds per square inch?

25u. What does the ratio between the section area of an I beam and its weight signify?

25v. A 15-inch 42-pound steel I beam of simple span carries a uniform load of 42 net tons. Find its factor of safety if the span is 7 feet; also if the span is 10 feet.

CHAPTER 5

BODIES UNDER COMPRESSIVE FORCE

26. Short Bodies in Compression. When a body whose length is short in relation to its weakest cross section is subjected to opposed forces, direct compressive stresses are developed as discussed in Art. 8: $S = P/A$.

27. Slender Bodies in Compression. The above becomes less true as the relative length becomes greater, approaching the condition of slenderness classed as *columns* and *struts*. The body is thus classified when the length is more than about eight or ten times the smallest cross section distance. Under such conditions, failure usually occurs from stress produced by a combination of *bending* and *direct compression.* This may readily be observed by pushing endwise on a yardstick or steel scale. It does not take much force acting in this manner to cause them definitely to bow. Such bending, it should be noted, takes place about the axis of least moment of inertia.

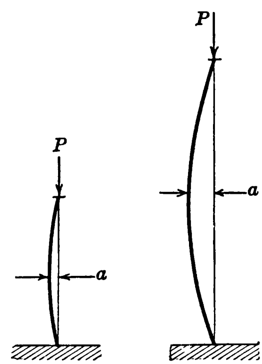


FIG. 35.

These bending stresses, in all probability, would not occur were the column perfectly straight and the forces applied directly opposite and parallel to each other. Such ideal conditions cannot be expected, and so the column must be made strong enough to withstand both the above stresses. As the bending will normally occur about the cross section axis of least moment of inertia, it follows that an economically designed column will have a cross section whose moments of inertia about any axis will be almost equal. Square or round columns most nearly meet this condition, and H columns and built-up sections should approximate it.

It can be readily demonstrated on models that the longer a column of a given cross section, the greater its tendency to bend under compressive loading and the smaller the load it can safely withstand. It can also be observed, as in Fig. 35, that the moment arm *a* is greater for the long column even though carrying equal loads *P*. This would also be true if the flexural stress developed in each column were equal, for the equal unit deflection would produce greater total deflection from the straight line in the longer column.

The summation of the direct compressive stress and the flexural stress is algebraic. The maximum compressive stress is $P/A + F$, where F represents the flexural stress. The maximum tensile stress is $F - P/A$, when F is greater than P/A. This distinction is only of concern where the strength of the material is not equal in tension and compression, such as cast iron.

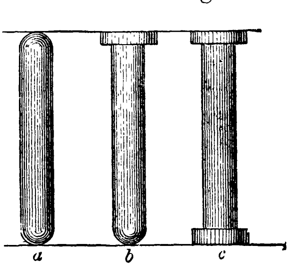


FIG. 36.

There are three ways of arranging the ends of columns (Fig. 36). Class *a* includes those with "round ends" or those having their ends hinged on pins. Class *b* includes those with one end round and the other fixed; the piston rod of a steam engine is of this type. Class *c* includes those having fixed ends; these are used in bridge and building constructions. The figure here given is a symbolical representation and is not intended to imply that the ends of the columns are necessarily enlarged in practice. It is found by experiment that class *c* is stronger than *b* and that *b* is stronger than *a*.

QUESTIONS AND PROBLEMS

27a. What two stresses are combined through the action of forces on columns and struts?

27b. What is the relationship of the direction of these two stresses?

27c. What two measurements of a column can be said to indicate its weakness or strength?

27d. Why would an H section make a better column than an I section of the same weight per foot?

27e. An I beam 20 inches deep is used as a column. Its section area is 22.1 square inches. What is its moment of inertia with respect to an axis through its center of gravity and parallel to its web? Should this value of the moment be used in computations relating to columns?

28. Radius of Gyration. The *radius of gyration* is the distance from the axis of a plane area to a point where the total area might be considered as being concentrated, without changing its moment of inertia about that axis. By Art. 19, I is equal to Σaz^2, so if A is the effective total area and its distance z from the axis is the radius of gyration, r, $I = Ar^2$ or $r = \sqrt{I/A}$.

As will be shown, in some column theories the radius of gyration about the axis of least moment of inertia is used as one factor in determining column strength. In general, all other factors remaining constant, the greater the value of r, the stronger the column. A value commonly used as a measure of column weakness is the slenderness ratio l/r, where l is the length of the column in the same units as r. Thus, the greater the value of l/r, the smaller the load the column can withstand, other conditions being constant. A column is said to be "short" if its l/r is greater than 35 but less than 150. A long column is one in which this ratio is greater than 150.

Values of r for various standard shapes can be found in tables, or can be found by substituting for I and A in the above formula. For simple symmetrical sections r may be solved for directly by the use of the following derived formula:

Solid rectangle or square	$r^2 = \frac{1}{12}d^2$
Solid circle	$r^2 = \frac{1}{16}d^2$
Hollow circle	$r^2 = \frac{1}{16}(d^2 + d_1{}^2)$

PROBLEMS

28a. What is the value of r for an H column 16 inches deep? Is r dependent on the material of which the beam is made? Why?

28b. What is the value of the radius of gyration for a hollow square when $d = 12$ inches and the wall thickness is 1½ inches?

28c. Compute the radius of gyration for a circular ring of 10 inches outer and 8 inches inner diameter.

29. Column Formula. The phenomena of the development of bending stresses in columns are so complex that, while the foregoing discussion is basic and essentially true, no purely theoretical formula will fully represent all cases. The following more commonly used formulas are presented without derivation other than the fundamental relationships which have been discussed.

Probably the most rational and generally most satisfactory formula is Rankine's, which applies to cases for which the ratio l/r lies between 20 and about 150.

$$S = \frac{P}{A}\left(1 + q\frac{l^2}{r^2}\right) \quad \text{or} \quad \frac{P}{A} = \frac{S}{1 + q\frac{l^2}{r^2}} \qquad [5]$$

where q has values which depend on the kind of material and the arrangement of the ends as shown in Table 5, page 141. The mean values shown have been derived by consideration of numerous experiments on the rupture of columns and struts.

The value S is the maximum compressive stress. Examining the first form of the formula we see that it is the sum of the direct compressive P/A and the maximum flexural stress $(P/A) \cdot q \cdot (l^2/r^2)$. Within the limiting conditions it is possible that the flexural stress will be greater than the direct and thus produce flexural tensile stresses which might be critical for certain materials such as cast iron.

Another formula for columns is the "straight-line formula," because the relation between P/A and l/r is the same as that between y and x in the equation of a straight line. This formula is

$$\frac{P}{A} = S - C\frac{l}{r} \qquad [6]$$

in which S is the unit stress on the concave side of the column and C is a quantity which varies with the material and the condition of the ends. For columns with fixed ends which are used in buildings under steady loads the following are used in cases of design:

For cast iron $\quad \dfrac{P}{A} = 10{,}000 - 40\dfrac{l}{r}$

For wrought iron $\quad \dfrac{P}{A} = 12{,}000 - 60\dfrac{l}{r}$

For structural steel $\quad \dfrac{P}{A} = 16{,}000 - 70\dfrac{l}{r}$

These formulas apply only when P is in pounds, A in square inches, and when the value of l/r is less than 120. They do not have the same degree of reliability as Rankine's formula, since they are wholly empirical. When a specification requires that they should be used, this must be done, but otherwise Rankine's formula (Equation 5) should be employed. The ratio l/r was defined and explained in Art. 28.

For example, find the safe load for a hollow cast-iron column 6×6 inches in outside dimensions and 5×5 inches in inside dimensions, the length being 18 feet and the ends fixed. Here $A = 11$ square inches, $r^2 = \frac{1}{12}(36 + 25) = 5.08$, whence $r =$ 2.252 inches, $l/r = 95.9$, and then, from the formula, $P = 67{,}800$ pounds. In this solution no use is made of the unit stress S on the concave side of the column. By Rankine's formula, using $S = 15{,}000$ pounds per square inch, we find $P = 58{,}100$ pounds, which is a more reliable value.

Again, let the requirement be to find the diameter of a solid cast-iron strut 6 feet long to carry safely a steady load of 64,000 pounds. Here for a very short strut, where $l = 0$, the area required is $A = 64{,}000/10{,}000 = 6.4$ square inches, which corresponds to $d = 2.85$ and $r = 0.71$ inch. Assume then $d = 4$ inches; whence $A = 12.57$ square inches, $r = 1$ inch, and $l/r =$ 72. Inserting these in the formula, we find that $P = 89{,}000$ pounds, which, being greater than the given value, shows that 4 inches is too large a diameter. Assume again that $d = 3.5$ inches; whence $A = 9.62$ square inches, $r = 0.875$, and $l/r =$ 84.6. Inserting these in the formula, we find that $P = 63{,}600$ pounds, which is very close to the given value, so that $d = 3.5$ inches is a satisfactory solution of the problem by the straight-line formula (Equation 6).

QUESTIONS AND PROBLEMS

29a. It has been shown that $S = P/A + F$. What is the value of F in terms of the quantities in Equation 5? What does F represent?

29b. A steel H column with both ends fixed is to be 20 feet long and its depth is 16.5 inches. What load will it carry if the factor of safety is 5?

29c. If $P/A = 500$ pounds per square inch for a timber column with fixed ends, find from Equation 5 the values of S when $l/r = 0$, $l/r = 50$, and $l/r = 100$.

29d. When the length l becomes very small, show that Equation 5 reduees to Equation 1.

30. Safe Loads for Columns. To find a safe load for a column of given size and material the working value of S is to be assumed from the considerations presented in Art. 11. The value of r is determined by Art. 28, and q from the table in Art. 29. Then from Equation 5

$$P = \frac{AS}{1 + q\frac{l^2}{r^2}}$$

which gives the safe load P for the column.

For example, let the requirement be to find the safe load for a timber strut 3×4 inches in section and 5 feet long, having both ends fixed, so that the greatest compressive unit stress S will be 800 pounds per square inch. Here $b = 4$ inches, $d = 3$ inches, $r^2 = 1/12d^2 = 3/4$ inches^2, $l^2 = 3600$ inches^2, $l^2/r^2 = 4800$, $q = 1/3000$, $ql^2/r^2 = 1.6$. Then

$$P = \frac{12 \times 800}{1 + 1.6} = 3690 \text{ pounds}$$

which is the safe load for the strut. If the length is only about one foot, the safe load will be simply $P = 12 \times 800 = 9600$ pounds. If the length is 12 feet, P will be found by the formula to be only 940 pounds. The influence of the length on the safe load is hence very great.

PROBLEMS

30a. As between columns of the same material, equal section areas, the same arrangement of ends, and the same loading how does the unit compressive stress vary with the length?

30b. Why is a hollow or a built-up column preferred over a solid one of equal section area?

30c. A hollow cast-iron column to be used in a building is 6×6 inches outside dimensions and 4×4 inches inside dimensions, the length being 18 feet and the ends fixed. Find its safe load.

30d. Find the safe load for the piston rod of a steam engine, its diameter being 1⅞ inches and its length 36 inches, when the allowable value of S is 6000 pounds per square inch.

31. Investigation of Columns. The investigation of a column under a given load consists in computing the unit stress S from Equation 5 and then comparing this with the ultimate strength and elastic limit of the material, having due regard to whether the stresses are steady, variable, or sudden (Art. 12). The value of S is

$$S = \frac{P}{A}\left(1 + q\frac{l^2}{r^2}\right)$$

and the given data will include all the quantities in the second member.

For example, a wrought-iron tube used as a column with fixed ends carries a load of 38,000 pounds. Its outside diameter is 6.36 inches, its inside diameter 6.02 inches, and its length 18 feet. It is required to find the unit stress S and the factor of safety. Here $P = 38{,}000$ pounds, $A = \frac{1}{4}\pi(6.36^2 - 6.02^2) = 3.31$ square inches, $q = 1/35{,}000$, $l = 18 \times 12 = 216$ inches, $r^2 = \frac{1}{16}\ (6.35^2 + 6.02^2) = 4.79$ inches^2. Then by the formula,

$$S = \frac{38{,}000}{3.31}\left(1 + \frac{216 \times 216}{35{,}000 \times 4.79}\right)$$

or $S = 14{,}700$ pounds per square inch. The factor of safety is thus about 4, which is a safe value if the column is used under steady stress, but too small if sudden stresses or shocks are liable to occur. If the length of this column is 36 feet, the unit stress S will become about 25,000 pounds per square inch, so that its factor of safety will be only 2.2, a value far too low for proper security.

As a second example, let a heavy 10-inch steel I beam, 25 feet long, be used as a strut in a bridge truss, the ends being hinged on pins. Let the load on it be 5900 pounds. Here, from the tabie in Art. 23, is found $A = 11.8$ square inches and $I' = 9.50$ inches^4, whence $r^2 = 0.80$ inch^2; also $q = 4/25{,}000$, $l = 300$ inches, $P = 5900$ pounds. Then, from the formula, S is found to be 9500 pounds per square inch, which is about one-third of the elastic limit of the material, and hence a safe value.

PROBLEMS

31a. A circular cast-iron column 12 inches in outside diameter and 12 feet long has a uniform thickness of one inch. What compressive unit stress will be caused by a load P of 17.5 tons? Assume that both ends are round.

31b. A pine stick 4×4 inches and 12 feet long is used in a building as a column with fixed ends. Find its factor of safety under a load of 3500 pounds. If its length is only one foot, what is the factor of safety?

31c. A rectangular wooden column, 12×12 inches in outside dimensions and 8×8 inches in inside dimensions, is 12 feet long. Compute the unit stress S when the load P is 12,000 pounds and the ends are fixed.

32. Design of Columns. When the length of a column is known and the load to be carried by it is also given, the design consists in selecting the proper material and then finding the dimensions so that the unit stress S in Equation 5 will have the proper value. This is often done by trial, dimensions being assumed and inserted in 5; and, if these do not fit, changes are made in them until a satisfactory agreement is found. For example, it is required to find the size of a square wooden column with fixed ends and 24 feet long to carry a load of 100,000 pounds with a unit stress S of 800 pounds per square inch. If the column is very short, the area A should be 100,000/800 – 125 square inches, and the side of the square about 11 inches. The column 24 feet long must be larger than this; assume it is 16 inches. Then, from the formula of the last article, find the value of S; this being a little larger than 800 shows that 16 inches is too small. Again, trying 17 inches, S is found to be a little smaller than 800. Hence $16\frac{1}{2}$ inches is an approximate solution of the problem.

Equations can be derived, however, for finding the size of solid square and round columns by placing for A and r^2 in Equation 5 their values in terms of the side or diameter d. Thus, for a solid square column, $A = d^2$ and $r^2 = \frac{1}{12}d^4$; then Equation 5 becomes

$$d^4 - \frac{P}{S} d^2 = \frac{P}{S} 12ql^2$$

and for a solid round column

$$d^4 - \frac{4P}{\pi S} d^2 = \frac{4P}{\pi S} \cdot 16ql^2$$

As an example, take the data of the last paragraph, where $P = 100{,}000$, $S = 800$, $q = 1/3000$, and $l = 24 \times 12$. Inserting these in the first equation, we have $d^4 - 125d^2 = 41{,}472$, and, solving, we find that $d^2 = 275.5$; whence $d = 16.6$ inches is the side of the square column.

For hollow square and round columns, equations can be derived in a similar way for finding the inner side or diameter d_1 when the outer side or diameter d is given. Thus for a hollow square column

$$d_1^4 + \frac{P}{S} d_1^2 = d^4 - \frac{P}{S}(d^2 + 12ql^2)$$

and for a hollow round column

$$d_1^4 + \frac{4P}{\pi S} d_1^2 = d^4 - \frac{4P}{\pi S}(d^2 + 16ql^2)$$

For example, it is required to find the inner diameter d for a cast-iron hollow round column with fixed ends, which is 18 feet long and 10 inches outer diameter, and which is to carry a steady load of 240,000 pounds. Here the working value of S is 15,000 pounds per square inch and $q = 1/5000$. Then the last equation gives $d_1^2 = 60.7$; hence $d_1 = 7.8$ inches for the inner diameter.

PROBLEMS

32a. A solid round column of cast iron, with fixed ends, is 14 feet long. What must be its diameter if S is to be 15,000 pounds per square inch under a load of 400 tons?

32b. Find what steel H column 12 feet long may be used to carry a load of 100,000 pounds, taking the working value of S at 16,000 pounds per square inch.

32c. A hollow square column of wood with fixed ends and 14 feet long has outside dimensions of 14×14 inches and carries a load of 5 tons. Find the inside dimensions so that S will be 900 pounds per square inch.

33. Eccentric Loads. Thus far it has been supposed that the load is applied to the end of a column so that its line of action coincides with the axis of the column. In many instances, however, this is not the case. Let Fig. 37 represent a short post where the load P is applied at a distance e from the vertical axis passing through the center of gravity of the cross section. The distribution of the internal compressive unit stresses in every section is then not uniform. The mean unit stress on the area A is P/A, but the actual stress is increased on the side nearest P and decreased on the opposite side by that unit stress which is due to the eccentricity of the application of the load and the consequent bending. Let CC be the neutral axis of the cross section and c the distance to the side; let I be the moment of inertia and r the radius of gyration of the cross section with respect to the axis CC; let F be the flexural unit stress at the side of the column. Then from the flexure formula (Equation 4), $F = Mc/I$. But the bending moment M is Pe; hence $F = Pec/I = Pec/Ar^2$. Adding this to the mean unit stress, P/A, there results

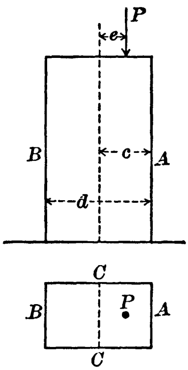


FIG. 37.

$$S = \frac{P}{A}\left(1 + \frac{ce}{r^2}\right) \qquad [7]$$

which is the compressive unit stress on that side of the column nearest P. On the other side of the column the unit stress is found by changing the $+$ sign to $-$.

A small eccentricity e causes the unit stress S to deviate much from the mean value P/A. For a rectangular section, $r^2 = \frac{1}{12}d^2$ and $c = \frac{1}{2}d$, so that

$$\text{For the side } A \text{ of the prism} \quad S_1 = \frac{P}{A}\left(1 + 6\frac{e}{d}\right)$$

$$\text{For the side } B \text{ of the prism} \quad S_2 = \frac{P}{A}\left(1 - 6\frac{e}{c}\right)$$

When $e = \frac{1}{6}d$, then $S_1 = 2P/A$, which is double the mean value, and $S_2 = 0$. When $e = \frac{1}{3}d$, then $S_1 = 3P/A$ and $S_2 = -P/A$; hence the side B is under tension instead of compression. It is thus seen that, in placing loads on a column, eccentricity of application should be avoided.

Equation 7 applies to a short column or to one in which l/r does not exceed 40. For longer columns it is customary to add the quantity ce/r^2 to the denominator in Rankine's formula (Equation 5), which thus becomes

$$\frac{P}{A} = \frac{S}{1 + q\frac{l^2}{r^2} + \frac{ce}{r^2}} \qquad [7a]$$

This equation may be used for finding the safe load on a column having an eccentric load, for investigating an existing column, or for designing a section for a proposed column.

QUESTIONS AND PROBLEMS

33a. Draw a diagram showing how the unit stress varies with the eccentricity. Plot the values of S_1 for $e = \frac{1}{6}d$; $e = \frac{1}{4}d$; $e = \frac{1}{3}d$; $e = \frac{1}{2}d$; and $e = d$.

33b. How are the stresses in the foundation of a short column affected by an eccentricity in its loading?

33c. Using Equation 7*a*, find the safe load for the data given in Problem 30*c*, taking the eccentricity of the load as ¾ inch.

33d. Using Equation 7*a*, find the factor of safety for the data given in Problem 31*b*, taking the eccentricity of the load as ⅜ inch.

Art. 34. REVIEW PROBLEMS

34a. What does S in Rankine's formula represent? Solve this formula for q and for l^2/r^2.

34b. How much greater is the radius of gyration of a solid square steel column having a side of d than that of a solid circular column whose diameter is d? If d is 10 inches, what are the numerical values?

34c. If the moment of inertia of an I beam is 2400 inches^4 and its section area is 30 square inches, what is the value of its radius of gyration with respect to an axis passing through its center of gravity and perpendicular to the axis of the given moment of inertia?

34d. If the square of the radius of gyration is the average of all the values of z^2 for a cross section, why cannot it be computed directly from the given dimensions of that section? If so, write the expression for r^2 in the case of a solid circle.

34e. A 1–2–4 concrete pier is 6 feet high, 4 feet square at the base, and 2 feet square at the top. What steady load will it safely carry at 6 months (*a*) if the load is uniformly distributed over its top and (*b*) if the eccentricity of the loading is 4 inches?

34f. Find the safe steady load for a hollow short cast-iron column which is 12 inches in outside and 10 inches in inside diameter.

34g. Given $q = \frac{1}{5000}$ and $S = 10{,}000$ pounds per square inch for a cast-iron column. Plot a curve for Equation 5, taking values of l/r as abscissas and values of P/A as ordinates.

34h. Determine the loading of a fixed-end timber column 6×4 inches in section and 10 feet long, so that the greatest compressive unit stress will be 900 pounds per square inch.

34i. A cylindrical wrought-iron column with fixed ends is 12 feet long, 6.46 inches in outside diameter, 6.02 inches in inside diameter, and carries a load of 52,000 pounds. Find its factor of safety.

34j. Compute the size of a square timber column with fixed ends to carry a load of 55 tons with a factor of safety of 10, its length being 12 feet.

34k. A beam 20 feet long carries a uniform load of 3500 pounds per linear foot and is supported at its ends by two round cast-iron columns 15 feet long. The columns have fixed ends and are 6 inches in outer diameter. Find the inner diameter of the columns so that the unit stress S will be 10,000 pounds per square inch.

34l. A 12-inch I beam weighing 30 pounds per linear foot is used as one of the compression members in a small bridge. The column is fixed-ended and is 20 feet long. Will it be safe under a load of 30 tons?

34m. The steel piston rod of an engine is circular in shape and its stroke is 3 feet. The maximum load upon the piston is 20,000 pounds. Find the proper diameter of the rod, using S as 8000 pounds per square inch.

CHAPTER 6

BODIES IN TORSION

35. Phenomena of Torsion. *Torsion* is that condition of loading which tends to twist a body on its axis. This twisting is a strain which is resisted by *torsional stresses* set up in the material. A shaft which transmits power is twisted by the forces applied to the pulleys, and thus all its cross sections are brought into stress. Torsion is somewhat akin to *shear,* but the forces which induce it do not act in parallel planes.

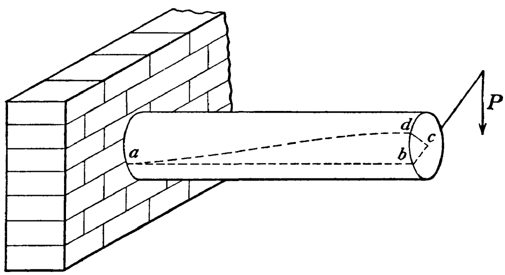


Fig. 38.

Let one end of a horizontal bar be rigidly fixed, and to the free end attach a lever at right angles to its axis (Fig. 38). A weight P hung at the end of this lever will twist the shaft so that a line ab on the bar which originally was horizontal will assume a spiral form ad, while the radial line cb will move to the position cd. It has been shown by experiment that, if the material is not stressed beyond its elastic limit, the angles bcd and bad are proportional to the applied weight P and that on the removal of this weight the lines cd and ad will return to their original positions. If the elastic limit is exceeded, this proportionality does not hold; and, if the stress induced by the load P is great enough, the bar will be ruptured.

Let p be the lever arm of P with respect to the axis c. Then experience has also shown that the amount of twist is proportional to p. The product Pp is the moment of P with respect to the axis, and it is called the *twisting moment.* If there are several forces P_1, P_2, etc., acting on the shaft with lever arms p_1, p_2, etc., the total twisting moment Pp is the algebraic sum of the separate moments P_1p_1, P_2p_2, etc., those being positive which tend to turn in the direction of the hands of a watch and those negative which turn in the opposite direction.

For example, let the three lever arms be applied to a bar at the points B, C, and D, whose distances from A are 5, 8, and 12 feet. Let the forces in Fig. 39 be $P_1 = 30$ pounds, $P_2 = 60$

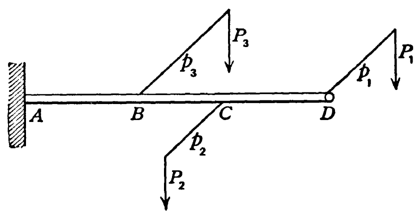


FIG. 39.

pounds, and $P_3 = 100$ pounds, their lever arms being $p_1 = 2.5$ feet, $p_2 = 2.0$ feet, and $p_3 = 3.5$ feet. Then for all sections between D and C the twisting moment is $+30 \times 2.5 = +75$ pound-feet; for all sections between C and B the twisting moment is $+30 \times 2.5 - 60 \times 2.0 = -45$ pound-feet; and for all sections between B and A the twisting moment is $+30 \times 2.5 - 60 \times 2.0 + 100 \times 3.5 = +305$ pound-feet. Thus the tendency to twisting between B and C is seen to be in the opposite direction to that in the other parts of the bar.

QUESTIONS AND PROBLEMS

35a. If a bar in torsion is of uniform section, what can be said of the amount of deformation over any unit of its length?

35b. What is the relation between the length of such a bar and the total deformation?

35c. Figure 39 shows three forces acting downwards to produce torsion in the bar. What other kinds of stresses are developed in the bar at *A?* Determine their values using data provided in the text. What torsional resistance must be offered by the support at *A?*

35d. A ½-inch bolt has been jammed into a nut which is held firmly in a vise. A man using a 16-inch wrench applies a load of 100 pounds before the bolt turns. What was the twisting moment in the bolt at the instant that it came free?

35e. If a force of 750 pounds acting at 5 inches from the axis twists the end of a shaft 32 degrees, what force acting at 12 inches from the axis will twist it 48 degrees?

35f. It is found by experiment that the angle *bcd* in Fig. 38 is proportional to the length of the bar when *P* and *p* are constant. If the angle *bcd* is 6°15′ for a shaft 9.2 feet long, what will this angle be for a shaft 13.8 feet long?

36. Resistance to Torsion. If two cross sections are taken in a shaft very near together, each section tends to twist with respect to the other, and *shearing stresses* are thus set up in all

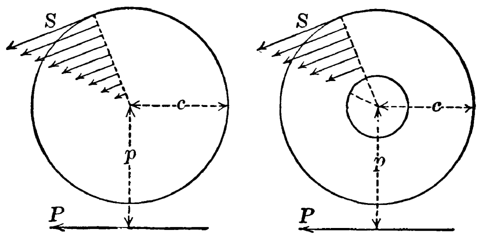


FIG. 40.

parts of each section. These stresses are zero at the center and greatest at the outside or boundary of the section. They act everywhere *perpendicular* to the lever arms drawn to them from the center. If the elastic limit is not exceeded the stresses will be proportional to their lever arms.

Let P be the force acting through the lever arm p which produces the twisting moment Pp (Fig. 40). This moment must be equal to the resisting moment of the internal stresses. Let S be the shearing unit stress at the remotest part of the section whose distance from the center is c. Then the stress at a distance $\frac{1}{2}c$ from the center is $\frac{1}{2}S$, and the stress at a distance x from the center is Sx/c. The total stress on an elementary area a at a distance x from the center is then aSx/c, and the moment of this stress with respect to the center is $(S/c)ax^2$. The resisting moment is the sum of all the values of $(S/c)ax^2$, or, since S and c are constants, this sum is $(S/c)\Sigma ax$. The value Σax^2 we call *polar moment of inertia,* represented by the letter J. Its mathematical calculation is discussed in the next article. Accordingly, the resisting moment of the internal shearing stresses is SJ/c, and, equating this to the twisting moment Pp, we have

$$\frac{SJ}{c} = Pp \qquad [8]$$

which is the fundamental equation for the torsion of shafts of circular cross sections.

This equation is analogous to Equation 4 for beams, and is used in a similar manner to investigate and design shafts. The unit stress S is here always a shearing stress, and its working values are to be determined by applying factors of safety to the ultimate shearing strengths. Shafts which transmit power are subject to variable loads, and often to shocks, and hence the values of S should be conservative. Equation 8 is subject to the same limitation as Equation 4, namely, it is true only when the unit stress S is less than the elastic limit of the material.

QUESTIONS AND PROBLEMS

36a. What line in a body under torsion corresponds to the neutral plane of a beam in bending?

36b. If the twisting moment in a cold-rolled steel shaft is 1500 inch-pounds, what value of J/c would be required to give a factor of safety of 10? Determine the units in which your answer is given, as explained in Art. 8.

37. Polar Moment of Inertia. We have seen that the sum of the products obtained by multiplying each elementary area by the square of its distance from the center of gravity of a surface is called the polar moment of inertia of that surface. Stated symbolically, $J = \Sigma ax^2$ where J is the polar moment of inertia and Σ denotes the process of summation of all the values of ax^2, in which a is any elementary area and x is its distance from the center of gravity of the total area.

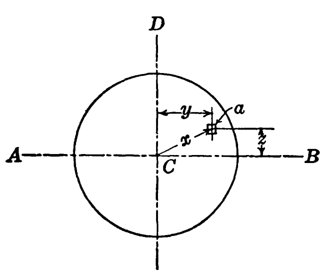


FIG. 41.

In Fig. 41 let a be any elementary area and z its distance from an axis AB passing through the center of gravity of that section; then Σaz^2, or the summation of all the values of az^2, is the moment of inertia with respect to the axis AB (Art. 19). Also, if y is the distance from a to an axis CD which is normal to AB, Σay^2 is the moment of inertia with respect to the axis CD. But since $z^2 + y^2 = x^2$, the product of Σax^2 is equal to $\Sigma az^2 + \Sigma ay^2$; that is, the *polar moment of inertia* is the *sum* of the *moments of inertia* taken with respect to any *two rectangular axes* (axes at 90 degrees to each other).

By the aid of the above principle, the value of J is readily found from the values of I given in Art. 19. Let d be the diameter of a circle; then, for a solid circle,

$$J = \tfrac{1}{32}\pi d^4$$

Also, in the case of a hollow section, let d be the outer and d_1 be the inner diameter; then, for a hollow circle,

$$J = \tfrac{1}{32}\pi(d^4 - d_1{}^4)$$

Circular sections are most frequently used for shafts, and the discussions of this chapter apply only to these. The theory of torsion in square and rectangular bars is very complicated and cannot be touched on in this book. In Equation 8,

$$\frac{SJ}{c} = Pp$$

The value J/c is *constant* for any given cross section, and hence is called the *polar section modulus* or polar measure of a section. Relative strengths of various cross sections can be compared on the basis of this value, and, in design problems, the requirements can be expressed in these same terms.

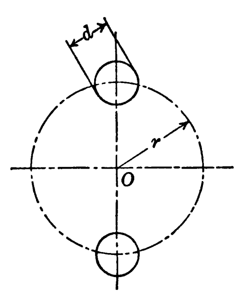


FIG. 42.

The polar moment of inertia of a section about a center other than its own is found by adding to its polar moment of inertia the product of its area times the square of its distance from that center. All polar moments of inertia about the same center may be added or subtracted as the situation requires. Figure 42 shows two circular sections whose polar moment of inertia about center O is required.

From the foregoing we see that

$$J = 2\left(\frac{\pi d^4}{32} + \frac{\pi d^2}{4} \cdot r^2\right)$$

QUESTIONS AND PROBLEMS

37a. Find the polar moment of inertia for a circular section 3 inches in diameter.

37b. Find the polar moment of inertia of a hollow circular section 4½ inches outside diameter having the same section area as that in 37*a*. Compare the results of 37*a* and 37*b*. (Express in percentage.)

37c. Compare the polar section modulus for the sections in 37*a* and 37*b*. (Express in percentage.)

37d. Show that the polar moment of inertia of a hollow circular section is $\frac{1}{8}A\ (d^2 + d_1{}^2)$, where A is the section area.

37e. Find the polar section modulus of five ½-inch pins on a 4-inch diameter center line circle.

38. Application of Principles, Investigation and Design. As in the case of previous analysis, the principles of torsion may be applied in either of two ways: determining what torsion a given body can safely resist or finding what body is required to withstand safely a given twisting moment. A special case of the first type of application is the checking of stress developed when a given twisting moment is acting on a given body.

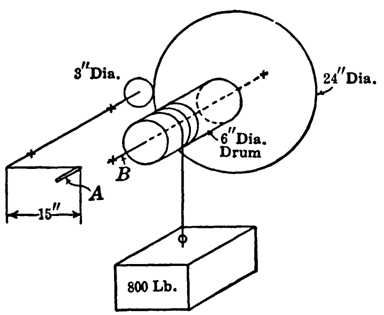


Fig. 43.

In every application, part of the problem is the analysis of the acting force and its lever arm. Care must be exercised to pair up properly the force and distance. The available data might give these values directly, but often they must be deduced. Usually the body is transmitting the action of one or more forces to the resistance of other forces, and the most convenient set may be used for the problem. For example, on the windlass shown in Fig. 43, there might be some question about the force acting on crank A to produce the moment in shaft A, but by analyzing the effect of the 800-pound force acting on the drum and through the gear reduction we see that the torsion in shaft A is

$$\frac{800 \times 3}{12} \times 1\frac{1}{2} = 300 \text{ inch-pounds} = Pp$$

Dividing this value by the allowable working stress will give the polar section modulus of the required section.

QUESTIONS AND PROBLEMS

38a. If the load in Fig. 43 were 2000 pounds, what diameter of mild carbon steel shaft would be required, allowing a safety factor of 10?

38b. If the drum and 24-inch gear were each fastened to the shaft and not to each other, what size mild carbon steel shaft would be required, using the same loading and factor as in Problem 38*a*?

38c. What is the maximum unit shearing stress in the bolt of Problem 35*d* at the instant when the bolt becomes loosened?

38d. A round steel shaft is subject to a twisting moment of 3500 pound-inches. What should be its diameter so that the greatest shear S will be 8000 pounds per square inch?

38e. A pulley 36 inches in diameter is placed on a 2½-inch mild carbon shaft, and the effective pull of the belt on the pulley is 800 pounds. What is the factor of safety of the shaft?

38f. What diameters of mild carbon steel shafting, with a safety factor of 8, would be required for each of the loadings in Problem 35*e*?

39. Shafts to Transmit Power. *Work* is the product of a force by the distance through which it is exerted. Thus, if a weight of 10 pounds is lifted vertically a distance of 5 feet 50 foot-pounds of work are performed. If this weight is moved horizontally, however, the force required is dependent only on frictional and other resistances. If to overcome these a force of 3 pounds is required and is exerted through a distance of 5 feet, 15 foot-pounds of work are performed.

Power is work performed in a given time. One unit of power is the *horsepower,* which is defined as 33,000 foot-pounds of work performed in one minute. Thus, if 99,000 foot-pounds of work are performed in one minute, the power exerted is 3 horsepower; if 99,000 foot-pounds of work are performed in two minutes, the power exerted is 1½ horsepower.

Power from a motor is usually transmitted to a shaft either directly or by belts or gears and the shaft then transmits the power to the places the work is to be performed. In doing this the material of the shaft is brought under stress. Let H be the power transmitted through a belt to a pulley, P the tangential force in pounds brought by the belt on the circumference of the pulley, p the radius of the pulley in inches, and n the number of revolutions made by the shaft and pulley in one minute. In one revolution the force P pounds acts through $2\pi p$ inches, and the work of $P \times 2\pi p$ pound-inches, or $\frac{1}{6}\pi Pp$ pound-feet, is performed. In one minute the work performed is $\frac{1}{6}n\pi Pp$ pound-feet. The number of horsepowers exerted is found by dividing this work by 33,000, or

$$H = \frac{n\pi Pp}{198{,}000}$$

and

$$Pp = \frac{198{,}000H}{n\pi}$$

The twisting moment Pp may be replaced by the resisting moment SJ/c, and hence,

$$\frac{SJ}{c} = \frac{198{,}000H}{n} \qquad [9]$$

which is the formula for the design and investigation of round shafts when the horsepower and speed are factors. It should be noted that, other factors remaining constant, as the speed decreases, a larger shaft will be required to transmit a given horsepower.

For round solid shafts of diameter d, the polar moment of inertia is $\frac{1}{32}\pi d^4$, the value of c is $\frac{1}{2}d$, and Equation 9 then reduces to

$$Sd^3 = 321{,}000\,\frac{H}{n}$$

in which d must be taken in inches and S in pounds per square inch. From this formula S may be found for a given shaft which transmits power, or d may be computed when it is required to design a shaft for that purpose.

For example, let the requirement be to find the factor of safety of a round solid shaft of mild carbon steel, $2\frac{1}{2}$ inches in diameter, when transmitting 25 horsepower at 100 revolutions per minute. Here $d = 2.5$ inches, $H = 25$, $n = 100$, and the formula gives

$$S = \frac{321{,}000 \times 25}{2.5^3 \times 100} = 5140 \text{ pounds per square inch}$$

so that the factor of safety is about 6.

As an example of design, let the requirement be to find the diameter of a mild steel shaft when transmitting 90 horsepower at 250 revolutions per minute. Here the factor of safety will be taken at 8, or the allowable unit stress S at 4000 pounds per square inch. Then, from the formula,

$$d^3 = \frac{321{,}000 \times 90}{4000 \times 250} = 28.89$$

and hence the diameter d should be $3\frac{1}{8}$ inches.

Where power is to be transmitted with a minimum of shaft weight, hollow steel shafts are used. Their polar section modulus is greater than that of solid shafts of the same cross section area, or, for the same polar section modulus, the hollow shaft is lighter than an equal length of solid shaft.

If d is the outside and d_1 the inside diameter, the value of J is $\frac{1}{32}\pi(d_4 - d_1^4)$ and c is $\frac{1}{2}d$. These inserted in Equation 9 give

$$S\frac{d^4 - d_1^4}{d} = 321{,}000\frac{H}{n}$$

which is the formula for investigation and discussion of hollow shafts.

For example, a nickel steel shaft of 17 inches outside diameter is to transmit 16,000 horsepower at 50 revolutions per minute. What should be the inside diameter so that the unit stress S will be 25,000 pounds per square inch? Here everything is given except d_1, and from the equation its value is found to be nearly 11 inches. The area of the cross section of this shaft will be about 132 square inches, and its weight per linear foot about 449 pounds.

QUESTIONS AND PROBLEMS

39a. In this article, what stresses obviously in the shafting have not been considered? Explain.

39b. If the shaft in the above example were solid, what horsepower would it transmit and how much would it weigh per linear foot?

39c. Compare the horsepower per pound per foot of the shaft of Problem 39*b* with the similar value for the shaft of 17 inches outside and 11 inches inside diameter.

39d. If a hollow shaft has the same area of cross section as the solid one, and if the inside diameter of a hollow shaft is one-half the outside diameter, show that the hollow shaft is 44 per cent stronger than the solid one.

39e. The tail shaft of a marine engine is 15 inches outside and 10 inches inside diameter. What horsepower is being transmitted when the shaft is making 250 revolutions per minute and the unit stress S is 9000 pounds per square inch?

39f. What should be the diameter of a solid round steel shaft to transmit safely 10,000 horsepower at 90 revolutions per minute?

39g. Find the horsepower that can be transmitted by a solid round steel shaft of 7½ inches diameter when making 150 revolutions per minute, S being 7500 pounds per square inch.

Art. 40. REVIEW PROBLEMS

40a. A circular concrete fence post 4½ inches in diameter is firmly set in a concrete base. Four longitudinal fence wires bring to it a load of 300 pounds each applied at 2¼ inches from the center of the post. What is the twisting moment at the base of the post? What is the unit shearing stress?

40b. A steel gate stem is operated by a 30-inch diameter gear. If the stem is 4 inches in diameter and the load on the gear is 600 pounds, what is the unit shearing stress in the stem?

40c. What is the polar moment of inertia of a hollow shaft 6 inches square with walls 2 inches thick?

40d. A twisting moment of 1200 inch-pounds produces an angle of twist on one shaft of 3°10′ whereas on another shaft of the same diameter the twist is only 1°27′. What inference do you draw from these results?

40e. If a force of 180 pounds, acting at 17 inches from the axis, twists the end of a shaft through 10 degrees, what force will produce the same result when acting at 4 feet from the axis?

40f. Compute the polar moment of inertia for a hollow shaft with outside diameter 17 inches and inside diameter 11 inches.

40g. Compute the shearing unit stress for the shaft of the last problem when it is subject to a twisting moment of 250,000 pound-inches.

40h. Find the horsepower that can be transmitted by a mild carbon steel shaft 3½ inches in diameter when making 60 revolutions per minute, the value of S being 8000 pounds per square inch.

40i. Find the diameter of a solid mild carbon steel shaft to transmit 90 horsepower at 250 revolutions per minute, the value of S being 7000 pounds per square inch.

40j. Find the ratio of the strength of a hollow shaft to that of a solid one, the section areas being equal and the outer diameter of the hollow section being three times as great as the inner.

40k. The crank of an engine is 10 inches long, and the maximum tangential thrust brought upon it by the connecting rod is 6000 pounds. Find the diameter of a steel shaft to resist safely the above twisting moment when the allowable stress S is 6000 pounds per square inch.

40l. What horsepower will be transmitted by a hollow shaft of 8 inches outside and 5 inches inside diameter when running at 200 revolutions per minute, the value of S being 7000 pounds per square inch? Find the diameter of a solid steel shaft to transmit the same horsepower with the same speed and unit stress.

40m. Express the polar moment of inertia of a rectangular steel shaft in terms of its depth d, its breadth b, and its thickness t. If this shaft is of bronze what will be its value of J?

CHAPTER 7

ELASTIC DEFORMATIONS

41. Modulus of Elasticity. It was explained in Chapter 2 that. when a bar is subjected to stresses produced by gradually applied forces, the elongations will increase proportionately with the stresses, provided the elastic limit is not exceeded. This law of elasticity enables the computation of the elongations of bars and the deflections of beams, in all cases when the stresses are within the elastic limit of the material.

The *modulus of elasticity* in tension is the ratio of the unit stress to the unit elongation. Thus, if a bar one inch long and one square inch in cross section is under the stress S an elongation s is produced, and

$$E = \frac{S}{s} \qquad [10]$$

is the modulus of elasticity. If the bar has a section area A which is acted on by the pull P, then the unit stress S is P/A; if the bar has the length l, an elongation e is produced and the unit elongation s is given by e/l.

For compression, E is the ratio of the unit stress to the unit shortening accompanying that stress, and in general E is the ratio of the unit stress to the unit deformation. Since s is an abstract number, E is expressed in the same unit as S, that is, in pounds per square inch or kilos per square centimeter.

Within the elastic limit S increases at the same rate as s, and thus E is a constant; beyond the elastic limit there is no proper modulus of elasticity. For different materials under the same unit stress S, the value of E increases as s decreases; thus E is a measure of the stiffness of materials. Equation 10 may be written

$$s = \frac{S}{E}$$

and the change in one unit of length of a bar under a given unit stress S may thus be determined when E is known.

The values of the moduli of elasticity for tension and compression are practically the same, and their mean values for different materials are given in Table 6, page 141. For shear the moduli of elasticity are about one-third of those stated in the table. These values show that, within the elastic limit, steel is the stiffest of the seven materials, over twice as stiff as cast iron and twenty times as stiff as timber. In other words, a given stress within the elastic limit will elongate a timber bar twenty times as much as a steel bar, and a cast-iron bar more than twice as much.

QUESTIONS AND PROBLEMS

41a. If the modulus of elasticity of bronze is 15,000,000 pounds per square inch, how will its elongation under a load P per square inch compare with that of steel? What does this difference between these metals signify?

41b. Consult Arts. 53 and 77 and note the moduli of elasticity there stated for concrete. Why is a 1 : 2 : 4 concrete stiffer than one mixed to 1 : 3 : 6?

41c. A bar having a section area of 2 inches and 2 inches long elongates 0.0004 inch under a tension of 10,000 pounds. Compute the modulus of elasticity.

41d. When a steel bar 30 feet long was subjected to a tensile unit stress of 12,000 pounds per square inch it elongated 0.143 inch. Compute the modulus of elasticity of the steel.

42. Deformation Accompanying Direct Stresses. Let a bar whose section area is A and whose length is l be under the load P, and let e be the deformation produced. The unit stress S is P/A and the unit deformation s is e/l. Then the modulus of elasticity E is

$$E = \frac{S}{s} = \frac{Pl}{Ae}$$

Hence, if P/A is less than the elastic limit,

$$e = \frac{Pl}{AE}$$

is the total elastic deformation of the bar due to the applied load P. Where P is tensile, the deformation will be elongation and, where compression, e will be shortening.

For example, find the elongation of a wrought-iron bar 30 feet long when stressed up to its elastic limit. Here $P/A = 27{,}000$ pounds per square inch, $E = 28{,}000{,}000$ pounds per square inch, and $l = 360$ inches. Then, from the formula, $e = 0.35$ inch. This is the elastic elongation; the ultimate elongation will be about 72 inches. In all cases, as seen from Fig. 10, the elastic elongations are very small as compared with the ultimate elongations.

It should be noted that the above law of deformation cannot be applied to columns under compressive loads, as defined in Chapter 5, but is true in compression only for relatively short prisms.

QUESTIONS AND PROBLEMS

42a. What four factors influence the total deformation of a bar under direct stress? Is the relationship direct or inverse? Explain.

42b. A 1 : 2 : 4 concrete column 6 feet long and 12 inches in diameter carries a load of 250 tons. What is the total shortening?

42c. A 12.5-inch H column is 9 feet long and carries a load of 300 tons. How much longer will it be after the load has been removed?

42d. A steel bar 18 inches long weighs 48 pounds. How much will it shorten under a compression of 15,000 pounds?

42e. The piston rod of a steam engine is 4 inches in diameter and 20 inches long. The piston is 24 inches in diameter. What is the change in length of the piston rod when the steam pressure is 175 pounds per square inch?

42f. What is the difference between elastic elongation and ultimate elongation? Which one if exceeded may cause immediate loss of life?

42g. A steel eye bar 30 feet long has a section of $1\frac{3}{4} \times 7$ inches. How much does it elongate under a pull of 120,000 pounds?

42h. What is the ultimate elongation of a steel bar 2 inches square and 18 feet long?

43. Beam Deformation. In Chapter 3 we saw that the resistance offered by a beam to a bending moment resulted in the development of tensile and compressive stresses with accompanying deformation of the material. On the compressive side of the neutral axis there was shortening, and on the tensile side the deformation was elongation.

This shortening and elongation accompany a bending of the beam which can be readily demonstrated with models. The gross change in beam shape from the unloaded to the loaded condition we call *deflection.*

The best method of deriving formulas for the deflections of beams is by the help of the calculus. These methods are given in higher works on the subject; see, for instance, Merriman's *Mechanics of Materials.** The formulas applying to certain cases will be stated here without proof and be accompanied by illustrations showing their value and importance.

Cantilever Beams. When a load P is at the end of a cantilever beam whose length is l (Fig. 44), a deflection designated by f results. This deflection will evidently be greater the greater the load and the longer the length of the beam. The formula for it is

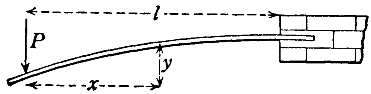


FIG. 44.

$$f = \frac{Pl^3}{3EI}$$

in which E is the modulus of elasticity of the material (Art. 41) and I is the moment of inertia of the cross section (Art. 19). The ordinate y at any distance x from the free end is given by $y = \frac{1}{2}f\ (3n - n^2)$ in which n represents x/l.

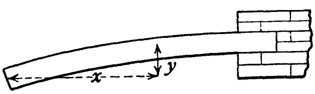


FIG. 45.

When a uniform load is on the beam let this be called W (Fig. 45). Then the deflection is

$$f = \frac{Wl^3}{8EI}$$

It is thus seen that the deflection varies as the cube of the length of the beam, so that if the length is doubled the deflection will be eight times as great. It is also seen that a uniform load produces only three-eighths as much deflection as a single load of equal intensity applied at the end. The ordinate y at any distance x from the free end is given by $y = \frac{1}{3}f(4n - n^4)$, where n represents x/l.

For example, compute the deflection of a cast-iron cantilever 2×2 inches and 6 feet long, due to a load of 100 pounds at the end. Here $P = 100$ pounds, $l = 72$ inches, $E = 15{,}000{,}000$ pounds per square inch, and $I = \frac{1}{12} \cdot 2^4 = 1\frac{1}{3}$ inches^4. Then,

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For example, find the elongation of a wrought-iron bar 30 feet long when stressed up to its elastic limit. Here $P/A = 27{,}000$ pounds per square inch, $E = 28{,}000{,}000$ pounds per square inch, and $l = 360$ inches. Then, from the formula, $e = 0.35$ inch. This is the elastic elongation; the ultimate elongation will be about 72 inches. In all cases, as seen from Fig. 10, the elastic elongations are very small as compared with the ultimate elongations.

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43. Beam Deformation. In Chapter 3 we saw that the resistance offered by a beam to a bending moment resulted in the development of tensile and compressive stresses with accompanying deformation of the material. On the compressive side of the neutral axis there was shortening, and on the tensile side the deformation was elongation.

This shortening and elongation accompany a bending of the beam which can be readily demonstrated with models. The gross change in beam shape from the unloaded to the loaded condition we call *deflection.*

The best method of deriving formulas for the deflections of beams is by the help of the calculus. These methods are given in higher works on the subject; see, for instance, Merriman's *Mechanics of Materials.** The formulas applying to certain cases will be stated here without proof and be accompanied by illustrations showing their value and importance.

Cantilever Beams. When a load P is at the end of a cantilever beam whose length is l (Fig. 44), a deflection designated by f results. This deflection will evidently be greater the greater the load and the longer the length of the beam. The formula for it is

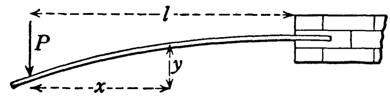

FIG. 44.

$$f = \frac{Pl^3}{3EI}$$

in which E is the modulus of elasticity of the material (Art. 41) and I is the moment of inertia of the cross section (Art. 19). The ordinate y at any distance x from the free end is given by $y = \frac{1}{2}f\ (3n - n^2)$ in which n represents x/l.

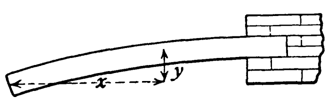

FIG. 45.

When a uniform load is on the beam let this be called W (Fig. 45). Then the deflection is

$$f = \frac{Wl^3}{8EI}$$

It is thus seen that the deflection varies as the cube of the length of the beam, so that if the length is doubled the deflection will be eight times as great. It is also seen that a uniform load produces only three-eighths as much deflection as a single load of equal intensity applied at the end. The ordinate y at any distance x from the free end is given by $y = \frac{1}{3}f(4n - n^4)$, where n represents x/l.

For example, compute the deflection of a cast-iron cantilever 2×2 inches and 6 feet long, due to a load of 100 pounds at the end. Here $P = 100$ pounds, $l = 72$ inches, $E = 15{,}000{,}000$ pounds per square inch, and $I = \frac{1}{12} \cdot 2^4 = 1\frac{1}{3}$ inches^4. Then,

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from the formula, $f = 0.622$ inch, which is the deflection at the end.

For a rectangular cross section of breadth b and depth d the value of I is $\frac{1}{12}bd^3$. Thus the deflections of rectangular beams vary inversely as b and d^3. As stiffness is the reverse of deflection, it is seen that the stiffness of a beam is directly as its breadth, directly as the cube of its depth, and inversely as the cube of its length. The laws of stiffness are hence quite different from those of strength.

Simple Beams. When a simple beam of span l has a load P at the middle (Fig. 46), each reaction is $\frac{1}{2}P$. Imagine this beam to be inverted, and we see that it is equivalent to two cantilevers of length $\frac{1}{2}l$, each having the load $\frac{1}{2}P$ at the end. Hence in the first formula for the deflection of a cantilever, given above, if l is replaced by $\frac{1}{2}l$ and P by $\frac{1}{2}P$, it becomes

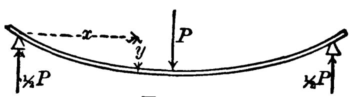


FIG. 46.

$$f = \frac{Pl^3}{48EI}$$

which gives the deflection of a simple beam due to a load at the middle.

When a simple beam is loaded with w per linear unit the total load wl is represented by W. The deflection at the middle due to this load is

$$f = \frac{5Wl^3}{384EI}$$

which is only five-eighths of the deflection caused by the same load at the middle.

The formulas of this article are valid only when the greatest horizontal stress S produced by the load is less than the elastic limit. These formulas may be expressed in terms of S by substituting the values of P and W from Equation 4 of Art. 17. Thus for the simple beam with load at the middle $\frac{1}{4}Pl = SI/c$, and for the uniform load $\frac{1}{8}Wl = SI/c$. Hence

For the single load P $\qquad f = \dfrac{Sl^2}{12Ec}$

$$\text{For the uniform load } W \quad f = \frac{5Sl^2}{48Ec}$$

which show that the deflections of beams under the same unit stresses increase directly as the squares of their lengths.

Restrained Beams. A beam is said to be restrained at one end when that end is horizontally fixed in a wall and the other end rests on a support (Fig. 47). In this case the reaction of the support is less than for a simple beam. For a uniform load of w per linear unit over the span l, it is proved in Merriman's *Mechanics of Materials* that the reaction at the support is $\frac{3}{8}wl$, provided the elastic limit is not exceeded.

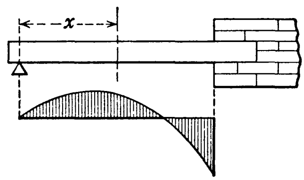


Fig. 47.

The bending moment at any section distant x from the support, then, is $\frac{3}{8}wlx - \frac{1}{2}wx^2$, and it thus appears that when $x = \frac{3}{4}l$ there is no bending moment; when $x = \frac{3}{8}l$ the greatest positive bending moment is $\frac{9}{128}wl^2$; and when $x = l$ the greatest negative bending moment is $\frac{1}{8}wl^2$. The distribution of bending moments is as shown in the figure. The maximum deflection is

$$f = \frac{wl^4}{185EI} = \frac{Wl^3}{185EI}$$

which occurs when x has the value $0.4215l$.

For a beam fixed at both ends and uniformly loaded (Fig. 48) there is a negative bending moment of $\frac{1}{12}wl^2$ at each wall and

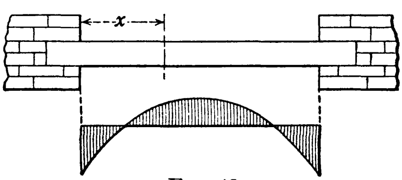


Fig. 48.

a positive bending moment of $\frac{1}{24}wl^2$ at the middle. The deflection at the middle is

$$f = \frac{Wl^3}{384EI}$$

in which W is the total uniform load wl.

In these restrained beams the lower side is thus seen to be partly in tension and partly in compression, since a positive bending moment indicates the former and a negative one the latter (Art. 22). For a simple beam the greatest bending moment is $\frac{1}{8}Wl$ and for a beam fixed at both ends the greatest bending moment is $\frac{1}{12}Wl$; hence if both are the same size the restrained beam will carry the greater load, or if both carry the same load the restrained beam may be of smaller size than the simple one. Thus if beams can be fixed horizontally at their ends the construction may be more economically made.

QUESTIONS AND PROBLEMS

43a. Which is easier to demonstrate with stronger materials, such as wood and metal, deformation from direct forces or beam deflection? Why?

43b. How does the deflection of a simple beam under constant load vary with its length?

43c. What is the effect of restraining the supports of a simple beam on its deflection?

43d. Compute the deflection of a steel I beam 8 inches deep and 12 feet long when it is loaded so that the flexural unit stress at the middle equals the elastic limit of the material.

43e. Two cantilever beams of the same length, the same material, and under the same loading show that beam A has a deflection of $2.2f$ as compared with beam B. How is this difference to be accounted for?

43f. A steel I beam 10 inches deep and 8 feet long is used as a cantilever to carry a uniform load of 240,000 pounds. What will be its deflection?

43g. A cantilever beam 6.5 feet long and loaded at the end has a deflection of 0.50 inch at that end. What is the deflection of a point half way between that end and the wall?

43h. How much will a 24-inch I beam weighing 80 pounds per foot deflect on a span of 30 feet when a load of 60,000 pounds is placed at its center?

43i. In order to find the modulus of elasticity of oak, a bar 2×2 inches, and 6 feet long, was loaded at the middle with 50 pounds, and then with 100 pounds, the corresponding deflections being found to be 0.16 and 0.31 inch. Compute the modulus of elasticity E.

43j. An 8-inch I beam weighing 18 pounds per foot carries a uniform loading of 100 pounds per lineal foot on a span of 10 feet. What is the central deflection in feet?

43k. When a beam is fixed at one end and supported at the other, the reaction of the supported end due to a load P at the middle is $\frac{5}{16}P$. Show that there is a positive bending moment of $\frac{5}{32}Pl$. under the load and a negative bending moment of $\frac{3}{16}Pl$ at the wall. Draw the diagram of bending moments.

43l. Show that the deflection of a simple beam is five times as great as that of a beam fixed at both ends, both beams being uniformly loaded.

44. Twist in Shafts. As in every other case where internal stresses are developed, and as noted in Art. 35, shafts in torsion twist for their length under load. One end is rotated about its axis, with respect to the other end, and in the direction indicated by the applied forces.

If the torsional moment is equal to force P acting on lever arm p, and l is the length, F the modulus of elasticity for shear, J the polar moment of inertia of the cross section, and all measurements of length are in the same units (say inches), then the formula for D, the angle of twist in degrees of one end of a round shaft with respect to the other end, is

$$D = 56.5 \frac{Ppl}{FJ}$$

If the equivalent value of Pp given in Art. 39 in terms of horsepower H transmitted at n revolutions per minute is substituted, then

$$D = 3{,}610{,}000 \frac{Hl}{nFJ}$$

For example, let a steel shaft 125 feet long, 17 inches outside diameter, and 11 inches inside diameter transmit 16,000 horsepower at 50 revolutions per minute. Here $H = 16{,}000$ horsepower, $l = 1500$ inches, $n = 50$, $F = 10{,}000{,}000$ pounds per square inch, $J = 6765$ inches. Then, from the formula $D = 25.3$ degrees, which is the angle through which a point on one end is twisted relative to the corresponding point on the other end. If this shaft revolves with a speed of only 25 revolutions per minute while doing the same work, its angle of twist will be twice as great and the stresses in it also twice as great as before. The formula also shows that the angle of twist varies directly as the length of the shaft.

PROBLEMS

44a. A solid steel shaft 110 feet long and 17 inches in diameter transmits 8500 horsepower at a speed of 35 revolutions per minute. Compute the angle of twist.

44b. A solid steel shaft 8 feet long and 2 inches in diameter is driven by a belt on a pulley 30 inches in diameter, the effective pull of the belt being 200 pounds. Compute the angle of twist.

Art. 45. REVIEW PROBLEMS

45a. In Problem 43*j* what is the value of the maximum unit tensile stress? What is the value of the maximum shear in that beam? What is the sum of the two reactions?

45b. A concrete telegraph pole 6 inches in diameter at the top and 12 inches at the bottom carries a cross arm 48 inches long on which two wires are strung, one on each side of the pole and 24 inches from it. Each wire is stressed to a pull of 500 pounds. One of the wires breaks on both sides of the pole, the other on one side only. What twisting moment is set up in the pole and what is the shearing stress at the top of the pole and at the bottom?

45c. Show for timber and wrought-iron bars stressed to their elastic limits that the change of length of the former is double that of the latter.

45d. Compute the tensile force required to stretch a bar of structural steel, $1\frac{3}{8} \times 9\frac{1}{2}$ inches in section area and 22 feet $3\frac{1}{4}$ inches long, so that its length may become 22 feet $3\frac{7}{8}$ inches.

45e. Show that the modulus of elasticity is that unit stress which would stretch a bar to double its original length, provided this could be done without impairing the elasticity of the material.

45f. What unit stress will shorten a block of cast iron 0.03 per cent of its length?

45g. A cast-iron bar, 2 inches wide, 4 inches deep, and 6 feet long, was balanced upon a support and a weight of 4000 pounds hung at each end, when the deflection of each end was found to be 0.401 inch. Compute the modulus of elasticity.

45h. Compute the elastic deflection of a light steel 12-inch beam of 40 feet span, due to its own weight, when resting on supports at the ends.

45i. Compute for the beam of the last problem the deflection when the beam is fixed at both ends.

45j. An alloy steel shaft, 6 feet long and $2\frac{1}{8}$ inches in diameter, is twisted through an angle of 0.5 degree when transmitting 4 horsepower at 120 revolutions per minute. Compute the shearing modulus of elasticity.

CHAPTER 8

MISCELLANEOUS APPLICATIONS

46. Internal Pressure. The pressure of water, steam, or other fluids in a pipe or tank is exerted in every direction and tends to tear the walls apart longitudinally. This strain is resisted by the internal tensile stresses which act in the walls of the pipe, normal to the radii.

If the wall thickness is not large compared with the pipe diameter, the resisting *area* is $2tl$, where t is wall thickness and

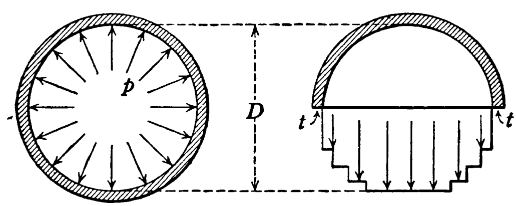


FIG. 49.

l the length, and the total resistance is $2tl \times S$, where S is the tensile unit stress. The total force P to be resisted is plD, where p is the unit pressure and D the inside diameter of the pipe. Hence

$$plD = 2Slt \quad \text{or} \quad pD = 2St$$

is the formula for discussing thin-walled pipes and tanks under internal fluid pressure.

The above derivation will become evident by a consideration of Fig. 49.

Water pipes are made of cast iron, wrought iron, or steel, the first being more common for larger diameters, whereas for steam the latter is preferable. A water pipe liable to the shock of water ram should have a high factor of safety, and in steam pipes the factors should also be high. The formula above deduced shows that the thickness of a pipe must increase with its diameter, as also with the internal pressure to which it is to be subjected.

For example, find the proper thickness of a wrought-iron steam pipe 18 inches in diameter to resist a steam pressure of 250 pounds per square inch. With a factor of safety of 10 the working unit stress is 5000 pounds per square inch. Then, from the formula,

$$t = \frac{pD}{2S} = \frac{250 \times 18}{2 \times 5000} = 0.45 \text{ inch}$$

so that a thickness of ½ inch would probably be selected.

When the end of a pipe is closed the pressure tends to push the end off as indicated in Fig. 50. Here $P = \frac{1}{4}p\pi D^2$, and this

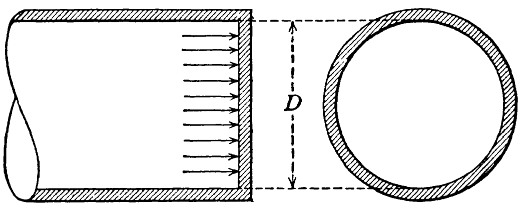


FIG. 50.

force is resisted by the metal of the pipe body which has a section area of $t\pi D$. As $S = P/A$, it follows that $pD/4t = S$ or $pD = 4St$. It is thus evident that for the same unit pressure p in any pipe the stress in the metal of the pipe due to a closure of its end is one-half that induced by the radial or bursting forces.

QUESTIONS AND PROBLEMS

46a. What is the greatest diameter to which a cast-iron water pipe may be built with walls 1 inch thick to resist safely a pressure of 125 pounds per square inch?

46b. A 72-inch steel pipe under a pressure of 70 pounds per square inch has its end closed by a drum head. What longitudinal stress is set up in the body of the pipe if t is ½ inch?

46c. Find the factor of safety of a cast-iron water pipe 16 inches in diameter and 1¼ inches thick under a pressure of 130 pounds per square inch.

46d. What internal pressure per square inch will burst a cast-iron water pipe 24 inches in diameter and 1¾ inches thick?

46e. The head of a 12-inch diameter steam engine cylinder is held on with ten ⅝-inch bolts. If the steam pressure in the cylinder has a maximum pressure of 120 pounds per square inches, and the root area of each bolt is 0.202 square inch, to what stress must the bolts be tightened to prevent opening of the joint?

47. Fastenings. *Riveted Joints.* When two plates are joined together by rivets and the plates then subjected to tension, a shear is brought upon the rivets which tends to cut them off.

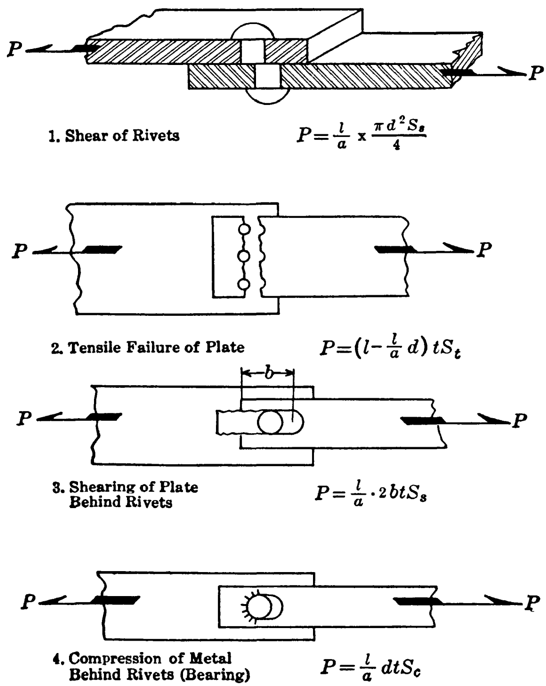


FIG. 51.

The failure of such a joint may occur in one of several ways, depending upon the proportions of the plate thickness t, rivet diameter d, rivet pitch a, the strengths of the materials, and the type of joint. In no case will a riveted joint be as strong as if the plate were in one continuous piece.

In each of the types of failure of a single-row lap joint shown in Fig. 51 the stress analysis is found by equating the total tensile force P, for the length of joint l, to the resisting area for the same length of joint, times the unit stress S for the type of stress and resisting material.

A riveted lap joint is one in which one plate laps over the other, and the two are riveted together. Figure 52 shows a single-row lap joint. In a joint where two rows of rivets are used they are generally staggered so that the rivets in one row come opposite the middle of the pitches in the other row (Fig. 53). It should be noted that in any given joint the failure will occur at the point of least resistance. Type 3 failure can be eliminated as a possibility merely by making the rivet edge distance great enough. Reducing the pitch and increasing the rivet diameter will strengthen the joint against failures 1 and 4, but will increase the likelihood of failure 2, so that a balance of values must be found which will equalize the resistance among these three failure possibilities.

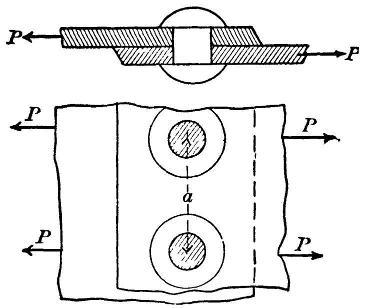


FIG. 52.

The working unit stress for shear should be about three-fourths of that for tension, or $S_s = \frac{3}{4}S_t$. Equating the above values of P under this condition gives a joint in which the security of the plates in tension is the same as that of the rivets in shear; thus

$$a = d + 0.59\frac{d^2}{t} \qquad a = d + 1.18\frac{d^2}{t}$$

the first being for single-lap riveting and the second for double-lap riveting. These are approximate rules for finding the pitch when the thickness of plates and diameter of rivets are given. In general, rivet pitch should not be less than 3 rivet diameters, to facilitate fabrication.

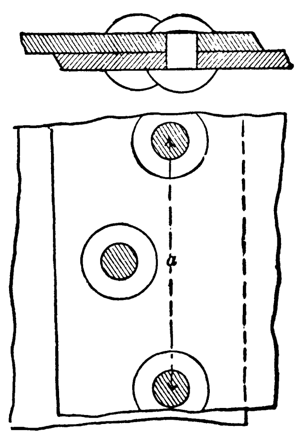


FIG. 53.

When two plates butt together, cover plates are used on one or both sides; if the covers are on both sides each is usually somewhat greater than one-half the thickness of the main plate (Fig. 54). The shear on each rivet is here divided between the upper and the lower parts of the cross section, this being called a case of double shear. Thus if P is the tension which is transmitted through one rivet, d the diameter of the rivet, a the pitch, and t the thickness of the main plate,

$$P = t(a - d)S_t \qquad P = \frac{2.1}{4\pi d^2 S_s}$$

which are the same as for two rows of lap riveting.

The *efficiency* of a riveted joint is the ratio of the strength of the joint to that of the solid plate. If the joint is designed

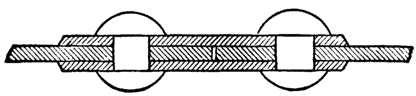


FIG. 54.

so as to be of equal strength in both tension and shear this efficiency is

$$\frac{t(a - d)S_1}{taS_1} = 1 - \frac{d}{a}$$

Thus if the rivets are ¾ inch in diameter and the pitch is 2 inches the efficiency is $1 - 3/8 = 0.625$; that is, the riveted joint has only 62.5 per cent of the strength of the solid plate. Single-lap riveting has usually an efficiency of about 60 per cent whereas double-lap riveting and common butt riveting have 70 to 75 per cent. By using three or more rows of rivets efficiencies of over 80 per cent can be secured.

When a joint is not of equal strength in tension and shear there are two efficiencies, one being the ratio of the tensile strength of the joint to that of the solid plate and the other the ratio of the shearing strength to that of the solid plate. The least of these is the true efficiency.

Welding. Recent years have seen great strides in the application of welding as a means of fabrication. Not only has it replaced riveting to a high degree, but it has been accepted in industry as a means of building up parts and sections which formerly were cast, forged, or rolled.

Welding is a process of joining two or more parts of similar materials by local fusion at a high temperature, produced by an electric arc or an oxyacetylene flame. Normally, at the time of fusion, additional metal is obtained from either the electrode, in *arc welding,* or the welding rod in *oxyacetylene welding.*

By its nature welding is rather a permanent connection forming the several pieces involved into one. Although at times this may be a disadvantage, usually it is cited as a desirable feature.

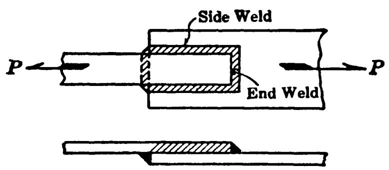


FIG. 55.

Because the strength of a weld is largely determined by the skill of the welder, and sometimes because of the lack of designer's confidence in this relatively new process, it is not at all unusual to find welded joints many times stronger than would be indicated by the other factors present. The strengths given below are for average quality welds, obtainable by most operators, and may be used without additional compensation for manufacturing variation.

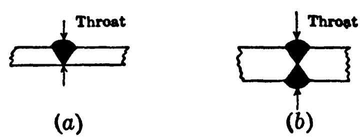


FIG. 56.

In general, welded joints fall into two categories: lap joints with fillet welds (Fig. 55) and butt joints with V-welds, single (Fig. 56*a*) or double (Fig. 56*b*) according to the plate thickness.

The resistance of side and end fillet welds is rated at so many pounds per inch of weld of a given base size (Fig. 57). Butt welds are usually rated on the unit tensile stress of the throat area (Fig. 56), an allowable working stress for steel being 12,000 pounds per square inch, or 70 to 90 per cent of the strength of the plates being joined, the higher value for double-V welds.

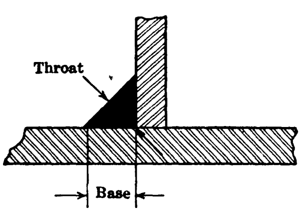


FIG. 57.

For the purpose of solving problems the strengths of steel welds at top of page 89 may be used.

Normally the stated size is the base size, and this is usually at least equal to the thickness of the thinnest of the two plates being joined.

Base, Inches	Allowable Load in Pounds per Inch of One Fillet in Tension or Direct Shear
1/8	1000
3/16	1500
1/4	2000
3/8	3000
1/2	4000

Bolts and Machine Screws. In general, bolts and machine screws fail in one of three ways: (1) by shearing across their body, (2) in tension of the body, and (3) shearing of the head.

The shearing across the body occurs at the point where the two bodies being fastened together meet.

Failure in tension normally occurs at a point of least cross section, frequently at the root of threads.

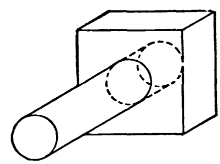


Fig. 58.

Shearing of the head takes place much as shown in Fig. 58, and it should be noted that this is an alternate failure to failure of the bolt body in tension. The failure area is equal to πdt, and the total resistance is $P = \pi dtS_s$. This type of failure, it can be seen, can be prevented by making the bolt head sufficiently thick, which is the case with bolts and machine screws of standard proportions.

QUESTIONS AND PROBLEMS

47a. What can be said of the influence of rivet diameter on joint strength with regard to the several possibilities of failure?

47b. Discuss similarly the effect of rivet pitch.

47c. Two plates each 3/4 inch thick are to be riveted together by a single row of rivets. What is the proper pitch of the rivets? If two rows of rivets are used what should the pitch be?

47d. Two strips of steel 1 1/4 inches wide and 1/2 inch thick are secured by a single rivet 3/4 inch in diameter. What weight P hung on this combination will cause failure? How will the failure occur?

47e. A steel water pipe 40 inches in diameter has rivets 3/4 inch in diameter and plates 1/2 inch thick. If double riveting is used, what should be the pitch of the rivets? What pressure will this pipe safely carry?

47f. Compute the factor of safety of a steel boiler 5 feet in diameter and 3/8 inch thick when it is subject to a steam pressure of 300 pounds per square inch, there being single longitudinal lap joints having rivets 3/4 inch in diameter with 2 1/8 inches pitch.

47g. How would you compute a riveted butt joint in which only one cover plate equal in thickness to the main plate was used?

47h. If the cover plates are thinner than one-half the main plate how is the strength of the joint affected? Assume a ½-inch plate with two covers each ⅛ inch thick and compute the stresses of both tension and shear as compared with two cover plates each ¼ inch thick.

47i. A butt joint with two cover plates has a main plate ½ inch thick, the rivets ¾ inch in diameter and the pitch of the rivets 2⅞ inches. Compute the efficiency.

47j. Show that the efficiency of a butt joint, based on the shear in the rivets, is double that of a lap joint.

47k. If the smaller plate in Fig. 55 were ⅜ inch thick and 4 inches wide and lapped the larger by 6 inches, what would be the total resistance of the welds? How does this compare to the strength of the plate?

47l. Two ½-inch plates 30 inches wide are butt-welded with a double-V weld. With a factor of safety of 6, what is the allowable pull on the joint?

47m. If the plates of Problem 47*l* were lapped, with fillet welds along each edge, what would be the allowable pull on the joint? What is the efficiency of this joint?

47n. A steam cylinder is made by welding disks into the ends of a 12-inch inside diameter tube, using ⅜-inch welds. What is the allowable steam pressure, in so far as the welds are concerned?

47o. Compare the tensile strengths of ¼–20 and 5/16–18 machine screws, their respective root diameters being 0.196 and 0.252 inch. What should be their minimum head thicknesses?

47p. If a standard ¾-inch bolt has a head height of 15/32 inch, how does its resistance to head shear compare to tensile strength? (Root diameter = 0.642 inch.)

47q. What length wrench might be used to tighten the bolt in Problem 47*p*, considering its torsional strength and using a factor of safety of 4 with a wrench pull of 60 pounds?

48. Stresses Due to Temperature. A bar which is free to move elongates when the temperature rises and shortens when it falls. But if the bar is under stress, or is fixed so that it cannot elongate or contract, the change in temperature results in producing a stress or in modifying a stress which was already existent in the bar. The unit stress so caused is that which would result from a change in length equal to that which would be occasioned in the free bar by the change in temperature.

The coefficient of linear expansion is the elongation of a bar of length unity under a rise of temperature of one degree. For the Fahrenheit degree the average values of the coefficients of expansion are as follows:

For brick and stone	$C = 0.0000050$
For concrete	$C = 0.0000060$
For cast iron	$C = 0.0000062$
For wrought iron	$C = 0.0000067$
For steel	$C = 0.0000065$

Thus a free bar of cast iron 1000 inches long will elongate 0.0062 inch for a rise of one degree, and 0.62 inch for a rise of 100 degrees.

The elongation of a bar of length unity for a change of t degrees is hence $s = Ct$. But (Art. 41) the unit stress due to the unit elongation s is $S = sE$, where E is the modulus of elasticity. Therefore

$$S = CtE$$

is the unit stress produced by a change of t degrees in a bar which is fixed. If the temperature rises, S is compression; if the temperature falls, S is tension.

For example, consider a wrought-iron rod which is used to tie together two walls of a building and is screwed up to a stress of 10,000 pounds per square inch. If the temperature falls 50 degrees there is produced a tensile unit stress,

$$S = 0.0000067 \times 50 \times 25{,}000{,}000 = 8400$$

and hence the total stress in the rod is 18,400 pounds per square inch. If the temperature rises 50 degrees the stress in the bar is reduced to 1600 pounds per square inch. In all cases the unit stresses due to temperature are independent of the length and section area of the bar.

A hoop or tire is frequently turned with the inner diameter slightly less than that of the cylinder or wheel upon which it is to be placed. The hoop is then expanded by the heat and placed upon the cylinder. Upon cooling it is held firmly in position by the radial stresses thus produced. This radial stress is one of compression and causes tension in the hoop.

Let D be the diameter of the cylinder upon which the hoop is to be shrunk and d the interior diameter to which the hoop is turned. If the thickness of the hoop is small, D will be unchanged by the shrinkage and d will be increased to D. The unit elongation of the hoop is then $s = (D - d)/d$, and hence the unit stress produced is

$$S = sE = \frac{D - d_E}{d}$$

where E is the modulus of elasticity of the material.

A common rule in steel-hoop shrinkage is to make $D - d$ equal to $\frac{1}{1500}d$; that is, the cylinder is turned so that its diameter is $\frac{1}{1500}$th greater than the inner diameter of the hoop. Accordingly, the tangential unit stress which occurs in the hoop after shrinkage is 30,000,000/1500 = 20,000 pounds per square inch.

When the hoop is thick the above rule is not correct, for a part of the stress produced by the shrinkage causes the diameter of the cylinder to be decreased. The rules for this case are complex, and cannot be developed in an elementary textbook. They will be found in the *Mechanics of Materials.*

QUESTIONS AND PROBLEMS

48a. A steel column 28 feet long carries a load which produces a unit stress of 12,000 pounds per square inch at 40° F. How far will the load be lifted when the temperature rises to 80° F., and what will then be the unit stress in the column?

48b. A cast-iron bar 5 feet long and 4 × 4 inches in section is confined between two immovable walls. What pressure is brought on the walls by a rise of 50 degrees in temperature?

48c. When steel railroad rails are improperly laid with their ends close together at a temperature of 30 degrees, what compressive unit stress occurs when the temperature rises to 80 degrees?

48d. A steel tire is to be shrunk on a wooden wagon wheel. The wheel is 5 feet in diameter. The tire is ¼ inch thick and when cold has an inner diameter of 4 feet 11½ inches. Compute the total compression in the spokes.

48e. Upon a cylinder 18 inches in diameter a thin wrought-iron hoop is to be placed. The hoop is turned to an inner diameter of 17.97 inches and then shrunk on. Compute the tensile unit stress in the hoop.

49. Shaft Couplings. A shaft is in two parts, which are connected by a flange coupling (Fig. 59). *A* shows the end view and *B* the side view of the coupling. The flanges of the coupling

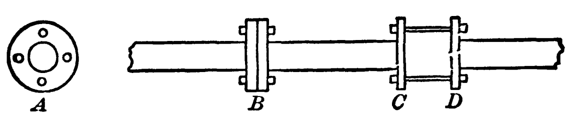


FIG. 59.

are connected by bolts which are brought into shearing stress in transmitting the torsion from one part of the shaft to the other.

The shaft is solid and of diameter D. There are n bolts of diameter d, and h is the distance from the center of the shaft to the center of a bolt. If D and d are known, as also the distance h, the resistance of the coupling, based on the polar moment of inertia as explained in Art. 37, is

$$\frac{J}{C} = \frac{n(\frac{1}{32}d^4 + \frac{1}{4}\pi d^2h^2)}{\frac{1}{2}d + h} \cdot S_s$$

and the resistance of the shaft is $\frac{J}{C} = \frac{1}{16}D^3S_s$.

Equating these values, we find

$$D^3(d + 2h) - nd^2(d^2 + 8h^2)$$

or

$$n = \frac{D^3(d + 2h)}{d^2(d^2 + 8h^2)}$$

which is a formula to determine the number of coupling bolts required to equal shaft strength, provided materials of equal strength are used. Since d is usually much smaller than h, it may be neglected within the parentheses; then the above formula becomes $n = D^3/4hd^2$, which is a simpler expression useful in approximate computations.

For example, let $D = 8$ inches, $d = 1$ inch, and $h = 12$ inches; then the second formula gives $n = 10.7$, so that eleven bolts should be used. If $D = 8$ inches, $d = 1\frac{1}{4}$ inches, and $h = 12$ inches, the formula gives $n = 6.8$ so that seven bolts should be used.

The case shown at CD in the above figure is one that should never occur in practice, because here the bolts are subject to a bending stress as well as to the shearing stresses due to the torsion. It is clear that this bending stress will increase with the distance between the flanges and that the bolts should be greater in diameter than for pure shearing.

PROBLEMS

49a. Compare the strength of a shaft coupling designed as in the first example above with the longitudinal strength of the shaft and the coupling under a tensile force.

49b. A solid steel shaft 16 inches in diameter transmits 15,000 horsepower at 40 revolutions per minute. Design a flange coupling for this shaft.

Art. 50. REVIEW PROBLEMS

50a. How many bolts 1½ inches in diameter would be required to hold a head in the end of the pipe of Problem 46*b*?

50b. What tensile strength must be developed by the joint in a 48-inch cast-iron water pipe at the place where the pipe makes a right angle turn if the internal pressure is 40 pounds per square inch?

50c. A concrete floor beam 10 feet long between two columns is placed when the temperature is 50° F. What kind of stress and of what magnitude will be set up in it when the temperature rises to 90° F.? Assume the columns to be immovable. If the beam were free how much would its length be changed?

50d. What internal pressure per square inch will burst a cast-iron sphere 24 inches in diameter and ⅜ inch thick?

50e. A wrought-iron boiler, 60 inches in diameter and 11/16 inch thick, carries a steam pressure of 180 pounds per square inch. Find the factor of safety of the metal when the efficiency of the longitudinal riveted joint is 87 per cent.

50f. Draw a figure for a double-riveted butt joint and deduce formulas for the same. Find the efficiencies for plate and rivets when the plate thickness is ½ inch, the pitch of rivets is 3¼ inches, and their diameter is 15/16 inch.

50g. Find the radial unit pressure between the rim and tire of a locomotive driving wheel when the shrinkage is 1/1500, the diameter of the tire being 62 inches and its thickness ¾ inch.

50h. A solid shaft 6 inches in diameter is coupled by bolts 1⅛ inches in diameter with their centers 4½ inches from the axis. How many bolts are necessary?

CHAPTER 9

COMPOUND BODIES

51. Discussion and Applications. We frequently find structural members fabricated of two unlike materials so combined as to obtain maximum benefit from the desirable properties of each material. Of great importance in these members are the proportions and relative positions of the parts made of unlike materials.

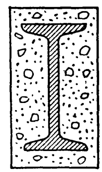


FIG. 60.

The two most common applications are *reinforced concrete* and *flitched beams*. In reinforced concrete, steel is used to supplement the strength of the concrete, especially in tension. Steel is also combined with concrete for reasons other than reinforcement. Figure 60 shows a concrete-incased beam. Here the steel beam is usually designed to carry the full load, and the concrete is used to protect the steel from corrosion or fire. Figure 61 shows a steel-incased concrete column or piling, where the concrete is designed to carry the full load and the steel to protect the concrete from cracking under the action of weather or collision with floating objects.

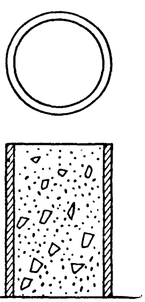


FIG. 61.

52. Principle of Combined Resistance. When two unlike materials are allied to resist the action of forces, their usefulness depends upon their acting in unison. Throughout the various stages of loading they will encounter, they must retain their original relative position and proportions.

This means that at all times the unit elongation of the two materials at any point of contact must be identical; otherwise they would separate or at least act independently of each other, thereby limiting their usefulness.

When a member made up of two materials is subjected to a longitudinal tension P, part of the tension is resisted by one material and part by the other. Let A_1 be the section area of one material and A_2 that of the other, and let S_1 be the unit stress over the area A_1 and S_2 the unit stress over the area A_2. Then A_1S_1 and A_2S_2 are the total stress on the two sections, and hence, since the resisting stresses must equal the applied tension,

$$P = A_1S_1 + A_2S_2 \qquad [11]$$

is a necessary equation of equilibrium. When P, A_1, and A_2 are given, the values of S_1 and S_2 cannot be determined from this equation, and hence a second relationship between them must be considered.

This second relationship is established from the fact that the elongation of the two parts of the member is the same. Let E_1 be the modulus of elasticity of the first material and E_2 that of the second. Then, if the elastic limit of neither material is exceeded, the elongation of a unit of length of the first material is S_1/E_1 (Art. 41) and that of the second material is S_2/E_2. Since these must be equal,

$$\frac{S_1}{E_1} = \frac{S_2}{E_2} \quad \text{or} \quad S_1 = \frac{E_1}{E_2} S_2$$

which shows that unit stresses in the two materials are proportional to their moduli of elasticity. If E_2 is ten times as great as E_1, S_2 must be ten times as great as S_1.

By help of the above formulas the values of S_1 and S_2 due to a load P on a compound rod of two materials may now be found.

$$P = A_1 \frac{E_1}{E_2} S_2 + A_2S_2 \quad \text{or} \quad S_2 = \frac{P}{A_1 \dfrac{E_1}{E_2} + A_2} \qquad [12]$$

The above reasoning applies also to compression if the length of the strut is not more than about ten times its least thickness and if the lateral flexure does not modify the uniform distribution of the stresses. For example, consider a wooden bar having wrought-iron straps fastened along two opposite sides. Here E_1 for the timber is 1,500,000 whereas E_2 for the wrought iron is 25,000,000 pounds per square inch, so that E_1/E_2 is 0.06 and hence the unit stress S_1 in the timber is equal to $0.06S_2$, so that, if S_2 is 5000 pounds per square inch, S_1 will be 300 pounds per square inch. The formula $S_1/S_2 = E_1/E_2$ cannot, however, be used when S_1 exceeds the elastic limit of the timber or when S_2 exceeds the elastic limit of the wrought iron.

The determination of the values of S_1 and S_2 for this compound rod is now easily made when A_1, A_2, and P are given. Let A_1 for the timber be 36 square inches and A_2 for the wrought iron be 4 square inches. Let the load P be 60,000 pounds. Then from Equation 12,

$$S_2 = \frac{60{,}000}{36 \times \dfrac{1{,}500{,}000}{25{,}000{,}000} + 4} = 9740 \text{ pounds per square inch}$$

and hence $S_1 = 580$ pounds per square inch for the timber. It is here seen that the total stress which comes on the wrought iron is 4×9740 or about 39,000 pounds, whereas that on the timber is about 21,000 pounds. The metal hence carries the greater part of the load, and it does this largely by virtue of its greater stiffness. In general, the higher the value of E for a material in a compound bar, the greater is the part of the load carried by it.

QUESTIONS AND PROBLEMS

52a. Derive a formula for S_1 similar to the formula of Equation 12.

52b. Upon what phenomena is based the principle of compound bodies?

52c. If a bar of low carbon steel $\frac{1}{2} \times 3$ were bonded to the lateral side of a bar of copper $\frac{1}{2} \times 3$ to form a compound bar 1×3 in cross section, what tensile load could be applied without exceeding either elastic limit? Under this load what would be the stress in each metal, and what would be the unit elongation?

52d. A 2- × 2-inch timber has two 2- × $\frac{1}{4}$-inch steel plates bolted to its opposite sides. It is 12 feet long and the timber is stressed to its elastic limit. What load hung from its lower end is it carrying?

52e. A short timber strut, 8×8 inches in section, has four steel plates fastened to its sides, each being $7 \times \frac{1}{2}$ inches in size, and it carries a load of 180,000 pounds. Compute the compressive unit stresses in the two materials.

53. Reinforced Concrete. Reinforced concrete columns and beams are made by placing freshly mixed concrete into wooden forms or boxes which surround their sides these being removed after the concrete has hardened. Steel rods are often placed in the forms and concrete poured around them. This combination is then called "reinforced concrete." The object of building in the steel is to make a safer and stronger construction than would be possible with concrete alone and to accomplish this at a lower cost than would be possible when only steel is used. Since 1895 there has been a great development in this kind of construction, steel rods being now used extensively for the reinforcement not only of columns and beams but also of walls, sewers, and arches.

The concrete generally used for this purpose is made of Portland cement, sand, and broken stone or gravel. (See Art. 76.) In each case the proportions in which the concrete is mixed are determined so as to meet the particular local requirements of the work as well as those which are indicated by the service which the structure is to perform.

The strength of concrete increases with its age (Art. 77), reaching nearly the highest value by the end of the first year. Its ultimate compressive strength is much higher than the ultimate tensile strength, and the following are average values of these in pounds per square inch for concrete twenty-eight days old:

	Compressive Strength	Tensile Strength
For 1 : 1 : 2 concrete	4200	420
For 1 : 1½ : 3 concrete	2800	280
For 1 : 2 : 4 concrete	1900	190
For 1 : 3 : 6 concrete	900	90

The ultimate shearing strength of concrete is about one-half the ultimate compressive strength.

It is seen from these values of the ultimate strengths that concrete is not well adapted to resist tension, and under tensile stresses its use is practically precluded, unless it is strengthened by reinforcing rods of metal.

Concrete suffers a greater change of shape under a given applied unit stress than does steel. In other words, the stiffness of the steel is much greater than that of the concrete. Mean values of the modulus of elasticity for the four grades of concrete are, in pounds per square inch:

For 1 : 1 : 2 concrete	$E = 3{,}000{,}000$
For 1 : 1½ : 3 concrete	$E = 2{,}500{,}000$
For 1 : 2 : 4 concrete	$E = 2{,}000{,}000$
For 1 : 3 : 6 concrete	$E = 1{,}500{,}000$

For steel the mean value of E is 30,000,000 pounds per square inch (Art. 41). Hence it is seen that concrete suffers an elastic deformation ten to fifteen times as great as that of steel when subjected to the same stress per square inch.

The elastic limit of concrete is not well defined, but as a rough average it may be taken in compression at about one-sixth and in tension at about one-fifth of the ultimate strength. The allowable working unit stress for concrete under compression is generally taken as 0.25 to 0.45 of the ultimate compressive strength at 28 days, that is, 475 to 855 pounds per square inch for 1 : 2 : 4 concrete and 225 to 400 pounds per square inch for 1 : 3 : 6 concrete. The permissible unit working stresses in shear range from 0.02 to 0.12 of the ultimate compressive strength at twenty-eight days.

The rods or bars used for reinforcement are generally of structural steel having an ultimate strength of about 60,000 and an elastic limit of about 35,000 pounds per square inch, and the allowable working stress in them ranges from 16,000 to 18,000 pounds per square inch. These may be the round, square, and rectangular shapes such as are everywhere available in the market, and many special deformed shapes are also widely used. Square bars are often twisted so that the corner lines are spirals. Other bars are rolled so as to have protuberances or swellings at intervals along their length; these forms are claimed to possess special advantages in preventing the rods from slipping in the concrete. Still others have projecting fins which are intended to reinforce the beams in which they are used against shear. See Art. 77 for a short general discussion on the principles of reinforced concrete.

QUESTIONS AND PROBLEMS

53a. What is the range of values of the allowable working stress in shear for a 1 : 1 : 2 concrete?

53b. What is the elastic limit of a 1 : 2 : 4 concrete? How does this figure compare with the elastic limit of timber?

53c. Consult the advertising columns of the engineering journals and obtain pictures of several kinds of reinforcing bars for beams.

53d. Compute the factor safety of a 1 : 2 : 4 concrete column, 12 inches square and 9 feet long, under a load of 60,000 pounds.

53e. Under a given safe load, which would compress the most, a 3-inch cube of steel or a 3-inch cube of concrete?

54. Reinforced Concrete Columns. Concrete columns are generally rectangular or round. Figure 62 shows a rectangular form having four steel rods imbedded in it near the corners. Figure 63 shows a round column having a single rod through its axis.

The investigation of a short reinforced concrete member in compression is merely a direct application of Equation 12, using values of E given in this article.

As an example of investigation, take a short reinforced column of 1 : 2 : 4 concrete which is 14 inches square and has four

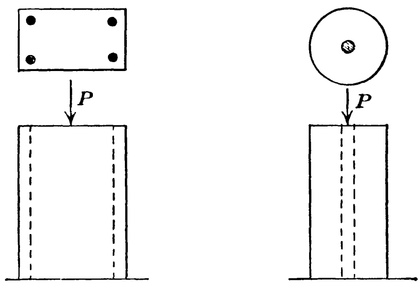


FIG. 62. FIG. 63.

steel rods, each ⅜ inch in diameter, parallel with its length near the corners, as in Fig. 62, whereas the load P is 71,000 pounds. Here the section area of the four rods is $A = 0.442$ square inch and that of the concrete is $A = 196 - 0.44 = 195.56$ square inches. Then

$$S_2 = \frac{7100}{195.56 \times \dfrac{2,000,000}{30,000,000} 0.442} = 5270 \text{ pounds per square inch}$$

for the steel, and $S_1 = 351$ pounds per square inch for the concrete. These are safe working stresses, that for the steel being very low.

When a reinforced column is to be designed, the load P and the unit stress S_1 for the concrete are known or assumed, whereas E_2/E_1 is to be taken as 10 or 15, depending on the kind of concrete. Then the above investigation shows that the section areas A_1 and A_2 of the two materials are to be determined so as to satisfy the equation

$$A_1 + \frac{E_2}{E_1} A_2 = \frac{P}{S_1}$$

In order to do this, one section area is usually assumed and the other can then be computed. Evidently many different sets of values of A_1 and A_2 can be found, and the one to be used will generally be determined by convenience and local conditions. For example, let the column in Fig. 63 be of 1 : 2 : 4 concrete for which S_1 is to be 500 pounds per square inch, and let the required diameter be 6 inches and the load P 20,000 pounds. It is now required to find the diameter d of the single steel rod. Here $A_2 = \frac{1}{4}\pi d^2$ and $A_1 = \frac{1}{4}\pi(36 - d^2)$, and then

$$\tfrac{1}{4}\pi(36 - d^2 + 10d^2) = \frac{20{,}000}{500}$$

from which d is found to be 1.28 inches, so that a rod $1\frac{5}{16}$ inches in diameter should be used.

The above examples apply only to short struts or posts. In the case of long columns or columns in which the loading is eccentrically applied the principles of Chapter 5 must be applied and the working stresses modified accordingly. If P is the total safe load applied on the axis of a column in which l/r is less than 40 and if P' is the total safe axial load on a long column, then, under the rule of the Joint Committee on Concrete and Reinforced Concrete,

$$P' = P\left(1.33 - \frac{l}{120r}\right) \qquad [13]$$

Here r is the least radius of gyration of the column core and l is the length of the column in inches. If $l/r = 40$, the $P' = P$; and if $l/r = 120$, then $P' = \frac{1}{3}P$. The influence of the length of a column on the safe load it can carry is thus made apparent.

PROBLEMS

54a. If the safe load on a short reinforced column is 200 tons, what will be the safe axial load on a similar column in which l/r is 110?

54b. A round column is to be made with ½-inch steel rods. The diameter of the concrete is 2 feet 6 inches, and the load to be carried is 500,000 pounds. Compute the number of rods required (a) when $l/r = 40$ and (b) when $l/r = 100$.

55. Reinforced Concrete Beams. Concrete beams are almost always reinforced with steel rods. Because of the low tensile strength of concrete (Art. 53), a concrete beam without this reinforcement could do little more than support its own weight. Because of this need for added resistance to tension, reinforced beams are generally built with imbedded rods only on the tensile side, as shown in Fig. 64. Let S_1 be the compressive unit stress on the upper surface of the beam, T the tensile unit stress on the lower surface, and T_2 the tensile unit stress in the steel. Let n, as before, represent the ratio E_2/E_1 found by dividing the modulus of elasticity of the steel by that of the concrete. Let b be the breadth and d the depth of the rectangular section

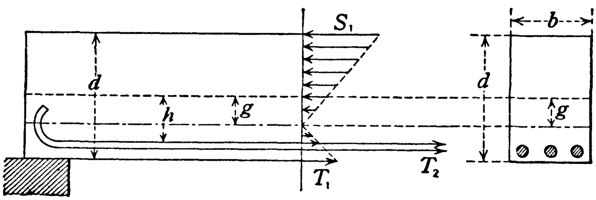


Fig. 64.

and A the section area of the steel rods, the center of which is at the distance h below the middle of the beam. Let M be the bending moment of the loads for the given section, and find the values of S_1, T_1, and T_2 due to M. The following formulas are demonstrated in *Mechanics of Materials*.

Case I. When the loads on the beam are light, so that the unit stress on the tensile side does not exceed about one-half of the tensile strength of the concrete, the distribution of stresses in the vertical section is that shown in Fig. 64. The neutral surface in this case lies at a certain distance g below the middle of the beam, and the value of g may be computed from

$$g = \frac{h}{1 + \frac{bd}{MA}}$$

Then the unit stresses for the concrete are

$$S_1 = \frac{d + 2g}{d + 12gh} \times \frac{6M}{bd} \qquad T_1 = \frac{d - 2g}{d^2 + 12gh} \times \frac{6M}{bd}$$

whereas that for the steel is

$$T_2 = \frac{2m(h - g)}{d^2 + 12gh} \cdot \frac{6M}{bd}$$

And from these formulas the beam may be investigated, provided the load does not produce a value of T_1 greater than about 100 pounds per square inch.

Case II. When a heavy load is applied to a reinforced concrete beam the tensile resistance of the concrete is first overcome, vertical cracks extending upward from the lower side, and thus a greater stress is thrown upon the steel. The theoretical analysis for this case is a difficult one unless it is assumed that the concrete below the neutral surface exerts no material resistance, and this assumption is the one usually made. Figure 65 shows

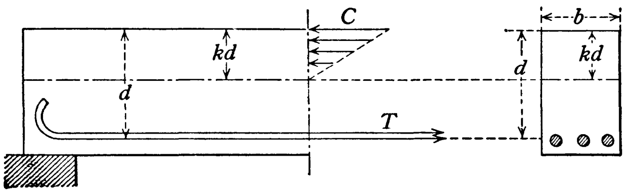


FIG. 65.

the distribution of stresses, the neutral surface being usually above the middle of the beam. Let b be the breadth of the beam, d the distance of the centers of the reinforcing rods below the top, and kd the distance of the neutral surface below the top, k being a number less than unity. Also let the ratio A/bd be called p. This ratio multiplied by 100 is the percentage of the total area of the beam's cross section which is occupied by the steel reinforcement. It is a convenient method of referring to or stating the extent to which a concrete beam is reinforced with steel. First, compute k from

$$k = -np + \sqrt{2np + (np)^2}$$

It is thus seen that the position of the neutral axis is dependent only on the percentage of steel in the beam and on n the ratio between E_2 and E_1. The unit stresses then are

$$C = \frac{2M}{k(1 - \frac{1}{3}k)bd^2} \qquad T = \frac{M}{(1 - \frac{1}{3}k)Ad} \qquad [14]$$

the first being for the maximum compression on the concrete at the extreme fiber and the second for tension on the steel.

The formulas of Case II are those usually required for the investigation of a reinforced concrete beam unless it is so lightly loaded that the unit stress T_1, found by the formula of Case I, is less than about 100 pounds per square inch.

If, in Equation 14, $(1 - \frac{1}{3}k)$ is made equal to j, it will be noted that they are identical in form with the expressions for the maximum unit compressive stress in the concrete and the maximum unit tensile stress in the steel as given by the Joint Committee on Concrete and Reinforced Concrete. These are the formulas almost universally adopted for the investigation and design of simple reinforced concrete beams.

PROBLEMS

55a. Let a reinforced beam of 1 : 2 : 4 concrete be 24 inches wide, 5 inches deep, and 8 feet span, with 2.0 square inches of steel at 2 inches below the middle. Compute the unit stresses S_1, T_1, T_2 due to the light uniform load of 1125 pounds. Also compute the unit stresses C and T due to a heavy uniform load of 4500 pounds

55b. If it were possible to make a concrete in which E_2 should equal E_1, what would then be the value of k? What would be the value of p in such case?

56. Design of Concrete Beams. When a reinforced concrete beam is to be designed the allowable unit stresses for concrete and steel are given or assumed, as also the span and width, and it is then required to compute the depth of the beam and the section area of the steel. When this is done according to the formulas applicable to Fig. 64, the steel is stressed but slightly. If T_1 is taken as 100 pounds per square inch for the concrete, the stress T_2 for the steel will be less than 1500 pounds per square inch. It is found impossible to design a beam on this theory economically because of the low tensile strength of the concrete. Nothing can be done, therefore, but to allow the concrete to crack on the tensile side and thus to bring up the stress in the steel to higher values. The formulas in Equation 14 given above for the distribution of stresses shown in Fig. 65 may be transformed so as to be applicable to cases of design.

The quantities usually given when a beam is to be designed are the allowable compressive unit stress C on the concrete, the allowable tensile unit stress T on the steel, the ratio $E_2/E_1 = n$, the bending moment M, and breadth b of the rectangular beam. It is required to find the depth d of the beam and the section area A of the reinforcing steel. Let the ratio T/C be designated by t. Then, for the case of Fig. 65,

$$d = \frac{n + t}{\sqrt{2n^2 + 3nt}} \sqrt{\frac{6M}{bC}} \qquad A = \frac{nbd}{2t(n + t)} \qquad [15]$$

are the formulas for computing d and A. The unit stress C should be taken as high as allowable by the specifications. T should not be higher than the highest allowable value, but it may be taken lower than this value if economy in cost is thereby produced.

For example, a rectangular beam of 1 : 3 : 6 concrete is to have a span of 14 feet, a breadth of 20 inches, and is to carry a uniform load of 300 pounds per square foot, including its own weight. It is required to find the depth of the beam and the section area of the reinforcing rods so that the unit stresses C and T shall be 350 and 14,000 pounds per square inch respectively. Here $n = 15$, $t = T/C = 40$, and $b = 20$ inches. The total load on the beam is $300 \times 14 \times 20/12 = 7000$ pounds, and the bending moment is $M = \frac{1}{8} \times 7000 \times 14 \times 12 = 147{,}000$ inch-pounds. Inserting these values in the first of the above formulas, d is found to equal 13.0 inches. Then $bd = 20 \times 13 = 260$ square inches, and from the second formula $A = 0.90$ square inch.

For the above case the section area of the steel is about one-third of one per cent of the section area bd of the concrete. Higher percentages of steel are frequently used, 0.60 to 1.25 per cent being common values, but these are probably not economical except for high-class concrete and low-priced steel. The depth d computed by the above formula is that from the top of the beam to the centers of the reinforcing rods. The actual depth of the beam is, however, greater than d by 1 or $1\frac{1}{2}$ inches, the extra thickness of concrete serving to protect the steel from corrosion and from the effects of fire.

The formulas and methods above presented for reinforced concrete beams are valid when the unit stresses in the concrete are proportional to their distances from the neutral surface, and this is the case only when the changes of length are proportional to the unit stresses. Concrete is a material in which this proportionality does not exist for compressive stresses higher than about 500 pounds per square inch, so that it cannot be expected that the formulas will apply to cases of rupture.

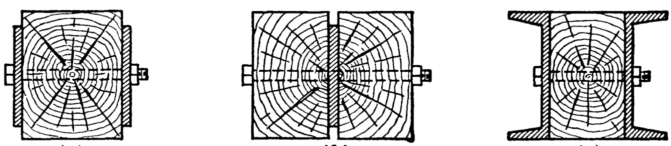
PROBLEMS

56a. A reinforced cantilever beam is to be made of 1 : 1 : 2 concrete. Its length is to be 10 feet, its breadth 12 inches, and its depth 14 inches. What percentage of steel should it contain so as to carry safely a concentrated load of 20,000 pounds 6 inches from its end?

56b. For the above numerical data except that for T, compute the depth d and the section A, taking the value of T as 12,000 pounds per square inch; also taking the value of T as 9000 pounds per square inch. If steel costs fifty times as much as concrete, per cubic unit, which of the three beams is the cheapest?

56c. Design two reinforced concrete beams 12 inches wide for a span of 12 feet 6 inches and a uniform load of 1500 pounds per linear foot, one being 1 : 2 : 4 concrete and the other 1 : 3 : 6 concrete.

57. Flitched Beams. A "flitched" beam is one made of timber with metal plates upon its sides, these being held in place by



(a) (b) (c)

FIG. 66.

bolts passing through the timber at the neutral surface. Sometimes, however, a single plate is placed between two timber beams (Fig. 66).

With the same basic conditions of distribution of load between the two materials due to differences in elasticity and the required identical deflection as we have discussed, the following formulas have been developed:

$$W_1 = \frac{W}{\left(1 + \dfrac{E_2 I_2}{E_1 I_1}\right)} \qquad W_2 = \frac{W}{\left(4 + \dfrac{E_1 I_1}{E_2 I_2}\right)} \qquad [16]$$

W is the load producing the bending moment in the beam, or its equivalent, and the subletters 1 and 2 indicate properties of the timber and steel respectively.

After W_1 and W_2 have been determined, the actual stresses may be computed by the flexure Equation 4, using moments which would be produced in each part by the values of W_1 and W_2 found above.

PROBLEMS

57a. Let a flitched beam like Fig. 66*b* consist of two timbers each 10×14 inches and a steel plate $\frac{3}{4} \times 7$ inches. When the unit stress in the timber is 900 pounds per square inch, what is the unit stress in the steel? What percentage does the metal add to the strength of the wooden beam?

57b. On a flitched beam made up as in Fig. 66*a*, with two plates $\frac{1}{2} \times 10$ and a timber 6×12, the load is placed 3 feet from one support on a 15-foot span. What is the maximum allowable load, using a factor of safety of 6 for the steel and 10 for the timber?

Art. 58 REVIEW PROBLEMS

58a. Find the safe load for a short column of 1 : 2 : 4 concrete which is 2×3 feet in section area.

58b. A short timber column, 6×8 inches in section, has two steel plates, each $\frac{1}{2} \times 8$ inches, bolted to the 8-inch sides. Compute the load P when the timber is stressed to 900 pounds per square inch.

58c. How does the modulus of elasticity of concrete vary with its ultimate compressive strength at 28 days?

58d. In the formulas for reinforced concrete beams what is the significance of n? Of p? Of k?

58e. What factors govern the position of the neutral axis in a concrete beam reinforced only against tension?

58f. How does the weight of a plain short concrete column compare with that of one of steel when the loading is 200,000 pounds? When the load is 50,000 pounds? How do the costs compare?

58g. A pier of 1 : 3 : 6 concrete, 6 feet in diameter, is surrounded by a cast-iron casing 1.25 inches thick. What part of the total load is carried by the concrete?

58h. For the dimensions in Problem 55*a*, what uniform load will probably cause the concrete to begin to fail in tension?

58i. Design a beam of 1 : 2 : 4 concrete with three reinforcing rods, the width to be 8 inches and the span 6 feet, so that it will carry safely a total uniform load of 4500 pounds.

58j. If a force of 235 pounds, acting at the end of a lever 17.5 inches long, twists the end of a shaft 6.5 feet in length through an angle of 14°45′, what force acting at the end of a lever 9.75 inches long will cause a twist of 28°31′ when the length of the shaft is 10.62 feet?

CHAPTER 10

COMBINED STRESSES

59. Discussion and Application. Different stresses commonly occur in a body at the same time, sometimes from the same set of forces acting in different ways. For instance, with a belt-driven pulley on a shaft torsional stresses are produced, and at the same time bending stresses due to the belt pull on both sides of the pulley. Again, a bolt under initial tension might act also in shear, producing a combination of shear and tensile stresses. Merely tightening a screw or bolt will develop tensile and torsional stresses in combination.

It has been found by experience and experiment that the simultaneous presence of these different stresses is more critical than any one of them alone. That is, if a body were designed to withstand safely one stress, it would not necessarily be able to withstand safely the combination with a different stress, even though that second stress alone were well within the safe limit of the material.

The common procedure in the case of combined stresses is first to analyze and determine each different stress independently. Then, by the use of derived formula, to combine the stresses into one equivalent value upon which the design or investigation can be based. This will be shown more clearly for the following types of combination.

QUESTIONS AND PROBLEMS

59a. Show in a sketch with description how the three examples of combined stress mentioned in the first paragraph might occur.

59b. Suggest two additional everyday examples of combined stress, including sketch and description.

60. Torsion and Bending (Shear and Flexure). A common type of combination is found in most machinery where rotating shafts transmit power and are driven by belts, chains, or gears.

Forces present in these drives which produce the torsional moment also produce bending, since they seldom act at the points where the shaft is supported. Figure 67 shows such a case where pull P of the chain drive, in addition to producing a torsion in the shaft, also produces a bending moment Pa at the bearing.

By an analysis involving higher mathematics, as shown in Merriman's *Mechanics of Materials*, the following formula relationship has been derived, where S is the greatest flexural unit stress, computed from Equation 4, S_s is the torsional shearing unit stress, computed from Equation 8, and S_p is the intensity of shearing stress developed.

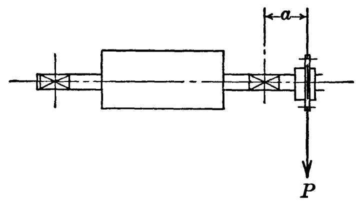


Fig. 67.

$$S_p = \sqrt{\frac{S^2}{4} + S_s^2} \qquad [17]$$

Replacing S and S_s with their equivalent values in terms of M, T, and J/c, we get

$$S_p = \frac{c}{J}\sqrt{M^2 + T^2} \qquad [18]$$

which is often a more convenient form.

For example, if in Fig. 67, P is 900 pounds, a is 8 inches, and the sprocket diameter is 10 inches,

$$M = 900 \times 8 = 7200 \text{ inch-pounds}$$

$$T = 900 \times 5 = 4500 \text{ inch-pounds}$$

Then

$$S_p = \frac{c}{J}\sqrt{(7200)^2 + (4500)^2}$$

and

$$\frac{J}{c} = \frac{8500}{S_p}$$

Using an allowable shearing stress under combined load, with allowance for keyways of 6000 pounds per square inch, we have

$$\frac{J}{c} = \frac{8500}{6000} = 1.4 \text{ inch}^3,\ 4\tfrac{1}{4}\text{-inch shaft required}$$

For a more complete discussion of this matter, reference should be made to the *Code for Design of Transmission Shafting,* published by the American Society of Mechanical Engineers.

PROBLEMS

60a. If in Fig. 67, P were 400 pounds, a 6 inches, and the sprocket radius 8 inches, what would be the required shaft size?

60b. At the time a ½-inch bolt is tightened to a tension of 1200 pounds, the pull on an 8-inch wrench is 40 pounds. What is the maximum shear stress developed? (Root area of bolt is 0.126 square inch.)

60c. If, in Problem 38*a,* the overhang of the 3-inch pinion (center line of pinion to center line of nearest bearing) were 4½ inches, what would be the required shaft diameter, using a factor of safety of 6 and the consideration of this article?

60d. In Problem 38*e,* calculate the safety factor considering the presence of a 10,000 inch-pound bending moment due to the belt pull on an overhang.

61. Shear and Tension or Compression. Combinations of shear and either of the other two direct stresses occur in many cases. The tightening of a screw and the tightened bolt acting in shear have already been mentioned. The balls and rollers in several types of anti-friction bearings are instances of combined shear and compression, though the forces involved are quite complex.

For the solution of such combinations of direct stress, Equation 17 may be used, S being the tensile or compressive stress, S_s the shear, and S_p the maximum intensity of shear stress developed.

It should be noted that, from the analysis upon which these formulas are based, the maximum shear stress is in a direction other than those of the stresses combined, much as is the vector resultant of two forces acting in different directions.

PROBLEMS

61a. If the bolt in Problem 60*b,* being tightened to the tension shown, is also to resist a direct shear force of 1500 pounds, what maximum shear unit stress will be developed in the full section area?

61b. A ¾-inch medium carbon steel ball is subjected to a 1500-pound compressive load and an 800-pound shear load on a plane through its maximum diameter. Calculate the maximum shear unit stress. What is the safety factor?

61c. A short bolt one inch in diameter is subjected to a longitudinal compression of 2000 pounds and at the same time to a cross shear of 3000 pounds. Find the maximum compressive, tensile, and shearing unit stresses which exist in the bolt.

Art. 62. REVIEW PROBLEMS

62a. An I beam, 15 inches deep and weighing 42 pounds per foot, acts as a simple beam with a span of 20 feet. Compute the flexural unit stress at the middle due to its own weight.

62b. The shank of a steel rivet one inch in diameter is under a tensile unit stress of 2000 pounds per square inch. It also resists a cross shear of 5000 pounds. What are the values of the maximum shearing unit stresses caused by the given loading?

62c. Compute the greatest shearing unit stresses due to the combination of a direct tension of 24,000 pounds with a cross shear of 7500 pounds, both acting on a bar 1¾ inches in diameter.

CHAPTER 11

RESILIENCE OF MATERIALS

63. Principle of Work and Energy. From a study of mechanics we learn that when a *force* is exerted *through* a *distance*, *work* is performed. If a force P were to act continuously for a distance e, much as though P were some weight lifted through a vertical distance e, the product Pe would be a measure of the work performed. If the force is exerted without movement, then no work is performed.

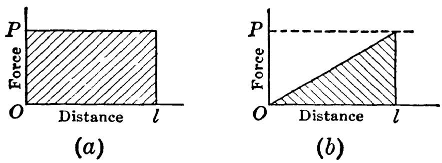


FIG. 68.

If the force P does not act continuously through a distance, but varies, then an average value must be used. When a specimen is tested in a testing machine, the load is gradually and uniformly increased from 0 up to the value P, with movement e equal to the deformation. Here the average value of the force is $\frac{1}{2}P$ and the work performed is $\frac{1}{2}P \cdot e$.

Plotting a diagram of the constant force acting through a distance, we get a rectangle as in Fig. 68*a*. For the work done in testing a specimen, we get a triangle, Fig. 68*b*. In both instances the area of the diagram is a measure of work performed.

Within the elastic limit of the material, and when the force is applied so that no work is expended in producing heat, there is stored within the deformed body an amount of energy equal to the external work mentioned above. This is called *internal energy,* a form of *potential energy,* and is in turn able to do work when the load is removed, i.e., it could lift the remainder of a gradually removed load.

When the load on a bar is increased from P_1 to P_2, within the elastic limit there is a proportionate increase of deformation from

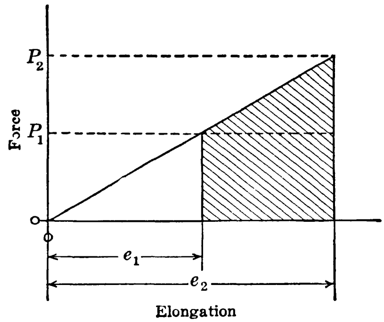


FIG. 69.

e_1 to e_2, as shown in Fig. 69. The shaded area represents the work performed to obtain this increase, and its value is

$$K = (e_2 - e_1)\left(P_1 + \frac{P_2 - P_1}{2}\right).$$

By examining the figure we can see that, if the load is doubled ($P_2 = 2P_1$), the total amount of energy stored is four times that with load P_1.

QUESTIONS AND PROBLEMS

63a. By examining the shape of the graphs in Figs. 68 and 69, within what limit can we safely say the stress remained?

63b. What two things must always be present in the doing of work?

63c. How much energy is stored in a ½-inch square steel bar, 20 inches long, under a tensile unit stress of 20,000 pounds per square inch?

64. Resilience. The term *resilience* is frequently used to designate the work that can be obtained from a body under stress when it is relieved of its load, because of its *potential energy.* We have seen, in Art. 63, that the amount of energy stored is in direct proportion to the load and the accompanying deformation. It will be shown that the resilience varies directly as the stress squared and inversely as the modulus of elasticity. In general, therefore, we can say the materials with higher elastic limits are more resilient, meaning that more energy can be stored in a given volume by loading to the elastic limit.

Strength is the capacity of a body to resist force; stiffness is the capacity of a body to resist deformation; resilience is the capacity of a body to resist work. The higher the resilience of a material the greater is its capacity to resist the work of external forces.

Elastic resilience is that internal work which has been performed up to the point where the internal stress reaches the elastic limit.

Resilience, like work, is expressed in foot-pounds, or inch-pounds, usually in the latter unit. Thus, if a bar is subject to a stress which gradually and uniformly increases from 0 up to 5000 pounds and causes an elongation of 0.5 inch, the resilience is 1250 inch-pounds.

65. Elastic Resilience of Bars. Let a bar of length l and section area A be under a tension P, which produces a unit stress S equal to the elastic limit of the material and an elongation e. The elastic resilience of the bar is then equal to $\frac{1}{2}Pe$. Now $P = SA$ and, by Art. 42, the elastic elongation is $e = Pl/AE = Sl/E$. Hence, letting K represent the *elastic resilience,* the product $\frac{1}{2}Pe$ becomes

$$K = \frac{S^2}{2E} Al$$

or the *elastic resilience* of a bar is *proportional* to its *section area* and to its *length,* that is, to its *volume.* For example, find the work needed to stress a bar of wrought iron up to 12,500 pounds per square inch, the diameter of the bar being 2 inches and its length 18 feet. Here $S = 12{,}500$ pounds per square inch, $E = 28{,}000{,}000$ pounds per square inch, $A = 3.14$ square inches, and $l = 216$ inches. Then

$$K = \frac{12{,}500^2 \times 3.14 \times 216}{2 \times 28{,}000{,}000} = 1889 \text{ inch-pounds}$$

If the bar is required to undergo this stress 250 times per minute, the work required in one minute is $250 \times 1889 = 472{,}250$ inch-pounds $= 39{,}354$ foot-pounds. The power expended in so stressing the bar is $39{,}354/33{,}000 = 1.19$ horsepower.

When a bar is under a unit stress S_1 and this is increased by an additional exterior load to S_2, the resilience due to the added load is

$$K = (S_2^2 - S_1^2)\frac{Al}{2E}$$

provided S_2 is not greater than the elastic limit.

QUESTIONS AND PROBLEMS

65a. A wrought-iron bar weighing 30 pounds per linear foot is subject to a stress of 10,000 pounds per square inch, which is accompanied by an elongation of 0.50 inch. What is the resilience in inch-pounds?

65b. A steel bar 10 feet long, weighing 490 pounds, is stressed in one second from 4000 to 10,000 pounds per square inch. What work and what horsepower are expended by the force which produced this stress?

65c. What horsepower engine is required to stress, 250 times per minute, a bar of steel 18 feet long and 2 inches in diameter from 0 up to its elastic limit?

66. Elastic Resilience of Beams. When a simple beam of span l is stressed under a load P applied gradually and uniformly at the middle, the deflection f results and the work $\frac{1}{2}Pf$ is performed. This work is equal to and is called the *resilience* of the beam. The value of f in terms of the horizontal unit stress is given in Art. 43, and the value of P in terms of the unit stress S is $\frac{1}{4}Pl = SI/c$. Accordingly, the product $\frac{1}{2}Pf$ has the value

$$K = \frac{S^2}{2E} \cdot \frac{r^2}{3c^2} \cdot Al$$

in which I, the moment of inertia of the section, has been replaced by its equivalent Ar^2, where A is the section area and r its least radius of gyration (Art. 28).

For a simple beam under a full uniform load the elastic resilience is given by

$$K = \frac{S^2}{2E} \cdot \frac{8r^2}{15c^2} \cdot Al$$

which is $1\frac{3}{5}$ times that of the simple beam with a single load at the middle.

These expressions show that the elastic resilience of beams of similar cross section is proportional to their volumes. For rectangular sections where the depth is d, the value of c is $\frac{1}{2}d$ and that of r^2 is $\frac{1}{12}d^2$; thus r^2/c^2 is $\frac{1}{3}$. Hence a rectangular bar under tensile stress has nine times the resilience of a rectangular beam loaded at the middle and $5\frac{5}{8}$ times that of the same beam under a full uniform load.

QUESTIONS AND PROBLEMS

66a. Develop an expression for the resilience of a rectangular beam fixed at both ends and uniformly loaded.

66b. Make calculations to confirm the statements in the last sentence in this article.

66c. What horsepower is required to stress in one second a heavy 20-inch steel I beam of 24-foot span from 500 up to 10,000 pounds per square inch, this being done by a load at the middle?

67. Quickly Applied Loads. When a load P is suddenly applied to a bar or beam, it produces a deformation y and is in effect for its full value for this distance. The work thus performed is Py.

The internal resistance of the bar or beam, which is initially unloaded, varies from 0 to Q as it deforms y, storing energy to the amount of $\frac{1}{2}Qy$. This stored energy must equal the work performed so that $\frac{1}{2}Qy = Py$, or $Q = 2P$. That is, the sudden load P produces the same stress as a gradually applied load of $2P$, or twice the stress of the same load gradually applied. Deformation of a given material being in direct proportion to the stress, it follows that the deformation for a suddenly applied load is twice that of the same load gradually applied, or $y = 2e$.

Having reached the maximum deformation y due to the required balance of energy, we find stored in the body an amount of energy (Pe) which is greater than the load can sustain ($\frac{1}{2}Pe$). This excess energy acts to return the body to its original shape, springing back with a series of oscillations until it finally comes to rest with the deformation e.

It is for the reason of doubled stress with suddenly applied loads that higher factors of safety are used for varying loads than for steady loads.

PROBLEMS

67a. Consider the load of Problem 21*h* to be suddenly applied. Solve that problem on that basis.

67b. A vertical mild steel bar $2\frac{1}{4}$ inches square and 13 feet long has a load of 16,000 pounds hung at its end. Compute the elongation due to this static load, and the maximum elongation which occurs when an additional load of 7500 pounds is suddenly applied. What are the maximum and final amounts of energy stored in the bar?

67c. A simple beam of structural steel, 2×2 inches and 18 inches long, is to be loaded with 3000 pounds at the middle. What will be the factor of safety if this load is suddenly applied?

68. Impact. When a load falls upon a body from some height, or otherwise accumulates *kinetic energy* before it is applied, the condition is called impact. The blows of a hammer and pile driver are effective because of impact, and designs must be made accordingly when impact is encountered. If the elastic limit is not exceeded, it is possible to deduce an expression showing the laws that govern the stresses produced by the impact. This will here be done for the case of impact on the end of a bar.

When a load P falls from a height h upon the end of a bar and produces the momentary elongation y, the work performed is $P(h + y)$. The resistance of the bar increases gradually and uniformly from 0 up to the value Q, so that the resilience or internal work is $\frac{1}{2}Qy$. Hence there results

$$\tfrac{1}{2}Qy = P(h + y)$$

Also, if e is the elongation due to the static load P, the law of proportionality of elongation to stress gives

$$\frac{y}{e} = \frac{Q}{P}$$

By solving these equations, the values of Q and y are

$$Q = P\left(1 + \sqrt{\frac{2h}{e} + 1}\right)$$

$$y = e\left(1 + \sqrt{\frac{2h}{e} + 1}\right)$$

which give the temporary stress and elongation produced by the impact.

If $h = 0$, these formulas reduce to $Q = 2P$ and $y = 2e$, and these represent the stress and the elongation caused by a load not dropped but suddenly applied (Art. 67). If $h = 4e$ they become $Q = 4P$ and $y = 4e$; if $h = 12e$ they give $Q = 6P$ and $y = 6e$. Since e is a small quantity for any bar it follows that a load P dropping from a moderate height upon the end of a bar may produce great temporary stresses and elongations. If these stresses exceed the elastic limit they cause molecular changes which result in brittleness and render the material unsafe. It thus appears that there is a great difference between the effects caused by a suddenly applied load and one that has been dropped even a short distance. The student should particularly note the import of these facts.

The above expressions for Q and y are not exact, as the resistance against motion due to the inertia of the material has not been taken into account. In Chapters XIII and XIV of Merriman's *Mechanics of Materials* (eleventh edition) the subject is discussed far more completely than has been possible here.

PROBLEMS

68a. An 8- × 8-inch timber 6 feet long spans an opening which is transverse to a highway. One wheel of an automobile truck suddenly rolls on to the center of this timber. If the wheel is carrying a total load of two tons, what unit stress and what deflection will result? If, just before reaching the timber, the wheel passes over a one-inch board and suddenly drops upon its center, what then will be the unit stress and the deflection?

68b. In an experiment upon a spring a steady weight of 16 ounces on the end produced an elongation of 0.4 inch. What temporary elongation would be produced when the same weight is dropped upon the end of the spring from a height of 8 inches?

68c. If the load in Problem 21*l* were to be dropping from a point 3 inches above the beam, how great would the load be to produce the stress given? How does this load compare with the first answer to the problem?

69. Repeated Stresses. Ultimate strength is usually understood to be that steady unit stress which causes the rupture of a bar in one application. Experience and experiment teach, however, that rupture may be caused by a unit stress less than the ultimate strength when that unit stress is applied to a bar a large number of times in succession. For example, Wohler showed that a bar of wrought iron could be broken in tension by 800 applications of 52,800 pounds per square inch and by 10,140,000 applications of 35,000 pounds per square inch, the range of stress in each application being from 0 up to the designated value.

It has been shown also by Wohler and others that the greater the range of stress, the less is the unit stress required to rupture it with a large number of applications. Also that when the range of unit stress is from 0 up to the elastic limit, rupture occurs only after an enormous number of applications.

As the permissible working unit stresses in practically all structures are always well within the elastic limit, the consideration of failure under repeated stress is seen, from the discussion presented, to be of secondary importance. In car axles and the moving parts of machinery, however, many millions of stress changes occur, and in all such cases careful study and determination of the *endurance limit* must be made. The endurance limit of a material is that unit stress to which it may be subjected an exceedingly large number of times without failing.

Failure under repeated stress was formerly considered to be due to the *fatigue* of the material. The studies of Moore and Jaspar indicate that failure in such cases is not due to a tiring of the material but to slow progressive failure which may be said to begin within the crystalline structure of the metal. The first breaking or slipping is exceedingly small and results from a concentration of stress at some small point because the material is not perfectly homogeneous. Gradually and slowly this slipping spreads and increases until failure occurs. The greater the range within which the stresses vary, the more rapid and widespread will the slipping be and hence the sooner will the metal fail.

QUESTIONS AND PROBLEMS

69a. Explain why a revolving shaft in which is produced a bending moment is a good example of repeated stresses. What is the range of variation when the value of the bending stress is F?

69b. If a countershaft runs at 1200 revolutions per minute, 8 hours a day, 5 days weekly, how many complete reversals of stress occur in 5 years? If the shaft is $1\frac{1}{2}$ inches in diameter and the maximum bending moment is 3800 inch-pounds, what is the range of stress variation?

Art. 70. REVIEW PROBLEMS

70a. If the timber beam of Problem 68*a* is replaced by a 4-inch I beam weighing $7\frac{1}{2}$ pounds per foot, solve the second part of that problem on the assumption that the wheel passes over a 2-inch brick before impinging on the beam.

70b. How many foot-pounds of work are required to stress a medium carbon steel piston rod, 2 inches in diameter and 6 feet long, from 0 up to 20,000 pounds per square inch?

70c. What horsepower is required to stress the rod of the last problem 125 times in one minute?

70d. Compute the horsepower required to deflect, 59 times per second, a structural steel cantilever beam, $2 \times 3 \times 72$ inches, so that at each deflection the unit stress S will range from 0 to 10,000 pounds per square inch.

70e. Discuss the case where a sudden load P is applied to a bar which is already under the static load P_1. What is the maximum stress?

70f. What is the height from which a weight must fall upon the end of a bar in order to produce a deformation equal to three times the static deformation?

APPENDIX A

GENERAL PROPERTIES OF INDUSTRIAL MATERIALS

71. Weight. The weight of any body may be found by multiplying its *volume* by the *unit weight* of the material, found in Table 7. Care must be taken to use volume and unit weight values in terms of the same lineal measurement, such as pounds per *cubic inches* and *cubic inches* of volume.

Where the body is uniform in cross section throughout its length, as in a pipe or beam, it is common practice to use weight for a *unit length*. Having determined how much one lineal foot of a bar weighs, merely multiplying by the length will give the total weight.

PROBLEMS

71a. A timber post 10 inches square and 8 feet long supports a floor which is made up of 100 square feet of floor boards 2 inches thick, 10 linear feet of floor beam 12 × 6 inches, and 50 linear feet of floor stringers 4 × 10 inches. What is the unit stress at the base of the post? What will be the unit stress at the base of the post when the floor is loaded with 50 pounds per square foot?

71b. What is the weight of a stone block 16 × 20 inches and 5½ feet long? How many square inches in the cross section of a steel railroad rail which weighs 105 pounds per yard?

71c. If a cast-iron water pipe 12 feet long weighs 1200 pounds, what is its section area? Find the diameter of a wrought-iron bar which is 21 feet long and weighs 2128 pounds.

71d. What is the weight per foot of a steel tube 6¼ inches outside diameter and 5½ inches inside diameter? How much would 12.7 feet of this tube weigh?

71e. What is the weight per square foot of ⅛-inch steel plate?

71f. What is the thickness of sheet copper weighing 16 ounces per square yard?

72. Testing Machines. To facilitate the application of loads, standardization of conditions, and observation of changes and results, the strengths and related properties of materials are usually determined by the use of test specimens in a testing machine. Through their use, conditions of loading and resistance to loads can be arranged so as to eliminate all but the type of stress under consideration. Thus information can be obtained for use in design, where these simplifications of actual conditions are assumed.

Testing machines vary in type, according to the properties which they are to test. Standard and special machines are

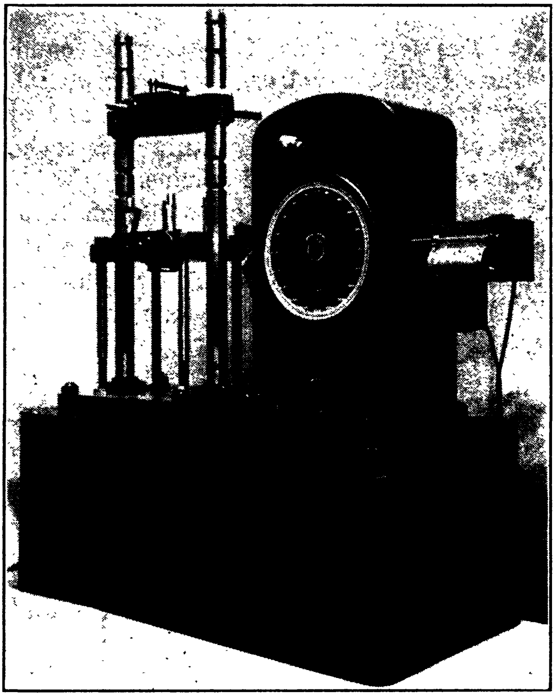


FIG. 70.

available for testing such properties as hardness, torsional resistance, shear, and resistance to impact and repeated varying stresses. Figure 70 shows a testing machine primarily for use in testing tensile and compressive strengths and determining related properties such as elastic limit, modulus of elasticity, and elongation.

In this machine the load is applied by power-driven nuts on the four screws. These cause the four screws and the platform fastened to their top to move downward. For tensile tests, specimens (Fig. 71) are mounted between this moving platform and the fixed head, either in wedge action jaws or threaded chucks. The specimens are shaped so as to present a uniform length of weaker section where the effect of loading may be observed.

For tests requiring compressive forces, the specimen is mounted in the space below the moving platform on the bed of the machine. The specimen is arranged so as to develop the stress under consideration when the platform is pulled down upon it.

The vertical reaction of the four power-driven nuts on the four screws is balanced by a series of links and levers against the action of weights. As the load varies its total value is indi-

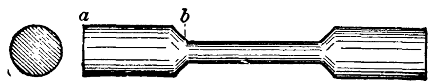


FIG. 71.

cated on the scale. The capacity of this testing machine is the maximum number of pounds it can exert in tension or compression. The required capacity varies with the size of specimens used and the material of which they are made. Smaller machines may exert only 1000 or 2000 pounds; larger machines go as high as 800,000 pounds.

During the progress of a test the elastic limit is detected by taking a number of measurements of the elongation for different loads, and then noting when these begin to vary more rapidly than the stresses. For ductile materials the change occurs rather suddenly, and is often noted by the quick jump of the scale indicator on the testing machine.

73. Timber. Good timber is of uniform color and texture, free from knots, sapwood, wind shakes, and decay. It should be well seasoned, which is best done by exposing it to the sun and wind for two or three years to dry out the sap. Much timber, however, is kiln-dried. This process often makes it brittle, but if done under careful control of temperature and moisture conditions the very best of quality may thus be secured. The heaviest timber is usually the strongest. That which has the darkest color and the closest annular rings is also generally stronger and better when all other things are equal. The strength of timber is always greatest in the direction of the grain, the sidewise or transverse resistance to tension or compression being scarcely one-fourth of the longitudinal.

Table 8, page 142, gives average values of the ultimate strength of a few of the common kinds of timber, to be used for reference. These values have been determined from tests of small specimens carefully selected and dried. Large pieces of timber such as are actually used in engineering structures will probably have an ultimate strength of 50 to 80 per cent of these values. Moreover, the figures given are liable to a range of 25 per cent on account of variations in quality and condition arising from place of growth, time cut, and method of seasoning. To cover these variations the safety factor of 8 given in Table 3, page 139, is not too high, even for steady stresses.

The shearing strength of timber is even more variable than its tensile or compressive resistance. White pine across the grain may be put at 2500 pounds per square inch, and along the grain at 500. Chestnut has 1500 and 600, respectively; yellow pine and oak perhaps 4000 and 600, respectively.

The elastic limit of timber is poorly defined. In precise tests on good specimens it has sometimes been observed at about one-half the ultimate strength, but under ordinary conditions it is more safely placed at one-third. The ultimate elongation is small, being usually between 1 and 2 per cent.

PROBLEMS

73a. A timber railroad trestle is to be built of yellow pine. Each leg is to carry a load of 100 tons. What should be the section area in square feet?

73b. An *A* frame is to be built for placing into position a piece of machinery weighing 62 tons. What should be the size of each leg if of square hemlock so that the factor of safety will be 8?

73c. What should be the size of a short piece of square yellow pine which is to carry a steady compressive load of 95,000 pounds?

73d. If a piece of white oak 2×2 inches in cross section ruptures under a compression of 34,000 pounds, what is the size of a square section that will carry a load of 75,000 pounds with a factor of safety of 10?

74. Brick. Brick is made of clay which consists mainly of silicate of alumina with compounds of lime, magnesia, and iron. The clay is prepared by cleaning, mixing with water, and tempering, which is a process of kneading and stirring. It is then molded to shape, dried, and fired in a kiln. In modern brickyards nearly all the operations are performed by machine.

Depending upon their relative positions in the kiln, three qualities of brick are thus made: arch brick, which are hard and brittle; body brick, which are of the best quality; soft brick, a weak brick suitable only for filling and "backing" purposes.

The common brick size is $2 \times 4 \times 8\frac{1}{4}$ inches, and the average weight $4\frac{1}{2}$ pounds. A pressed brick, however, may weigh nearly $5\frac{1}{2}$ pounds. Good bricks should be of regular shape, have parallel and plane faces, with sharp angles and edges. They should be of uniform texture, and when struck a quick blow should give a sharp, metallic ring. The heavier the brick, other things being equal, the stronger and better it is.

The crushing strength of brick is variable. Although a mean value of 3000 pounds per square inch has been given in Table 1, soft brick will scarcely stand 500, pressed brick may run to 10,000, and the best qualities of paving brick may run to 15,000 or more pounds per square inch. Crushing tests are usually made on whole or half bricks and are hence lacking in precision of result since the opposite surfaces are rarely truly parallel. Tensile and shearing tests of bricks are rarely made, and little is known of their behavior under such stresses; the ultimate tensile strength may be said to range from 50 to 500 pounds per square inch.

PROBLEMS

74a. A square brick and mortar pier is to replace the timber post of Problem 71*a*. What must be its dimensions?

74b. Compute the unit stress at the base of a brick wall 18 inches thick and 65 feet high. What is the factor of safety?

74c. A brick weighs 4.57 pounds when dry and 5.48 pounds after immersion for one day in water. What percentage of water has it absorbed?

75. Stone. Many different types of stone are used for building purposes, such as sandstone, limestone, granite, and trap or basalt. Their strength and other properties vary widely both with locality of source and manner of dressing and laying. Table 1 shows average values of strength as found by test. In general, larger blocks are not so strong as these values indicate.

With the exception of trap, all the above stones are fairly easy to work, which is an important factor. Availability is a factor which has much to do with the selection of building stone, and this probably has most to do with the wide use of sandstone and limestone.

The quality for a building stone cannot be safely inferred only from tests of its strength. Its durability depends largely upon its capacity to resist the action of the weather. Hence corrosion and freezing tests, impact tests, and observations of the behavior of stone under conditions of actual use are more important than the determination of crushing strength in a compression machine.

76. Portland Cement. Portland cement serves as the binder in concrete and is thus one of the most important of modern building materials. It is made by grinding together a mixture of calcareous and argillaceous materials in such proportions that the completed product will, after calcination or burning, have the desired composition. The calcareous material is usually limestone, chalk, or marl. Oyster shells, however, are sometimes used. The argillaceous elements are generally clay or shale. Certain limestones in their natural condition contain substantially the proper proportions of lime, of silica, and of alumina. From such rock Portland cement may be made directly; other "cement rock" requires the addition of limestone or of clay in order that the resulting composition will come within acceptable limits.

The cement mixture, commonly called the "raw mix," moves slowly through the kiln and on its way passes through the "hot zone," where the fuel is applied and where the maximum temperature is attained. The usual operating temperature is about 1350° C. Under this temperature the cement mix is partially fused and emerges from the lower end of the kiln as cement "clinker." During the process, which is known as "burning," the original compounds in the raw mix are broken down and new compounds of lime-silica and lime-alumina are formed. It is to these new compounds that Portland cement owes its important qualities.

After leaving the kilns the cement clinker is finely ground and is then ready for storage or for use. During the grinding of the clinker about 5 per cent of natural gypsum is usually added for the purpose of regulating and controlling the set. Cement which has been ground without gypsum will "flash" or set hard almost instantly when water is added to it.

The specifications of the American Society for the Testing of Materials set forth minimum requirements which the cement must meet, and all cements complying with these minima are generally considered as being of equal quality. The effect of this practice has been to foster the common belief that all Portland cement is possessed of identical characteristics, whereas, in reality, there are many kinds and grades.

The minimum specification above referred to requires the tensile strength of a mortar, made of three parts standard sand and one part cement, to be not less than 225 pounds at the age of 7 days while the tensile strength of such a mortar at 28 days must be greater than 325 pounds per square inch. These specifications further require that the initial or primary set must not occur within 45 minutes whereas the permanent set or hardening must take place within 10 hours. At least 78 per cent of the cement must pass through a number "200" sieve, that is, through a sieve having 40,000 meshes per square inch. Portland cement in common with all materials of construction is subject to variations in quality.

77. Concrete. Concrete is a mechanical mixture of Portland cement, sand, and broken stone or gravel tempered with water. Its strength and qualities are dependent on the cement, which, in the presence of water, forms compounds that bind the mixture into a solid mass.

Many different mixtures of concrete are made to meet the particular requirements of the situation in which they are to be used. The mixture is indicated by the proportions of cement, sand, and stone (called aggregate), in that order. Thus a mix of *1 : 3 : 6* contains 1 part cement, 3 parts sand, and 6 parts aggregate. This is a *lean mix* used for mass concrete work such as foundations and backing.

1 : 2 : 4 is a *standard mix,* extensively used for reinforced work such as floors, columns, beams, and tanks.

A *richer mix* is *1 : 2 : 3,* which can be used for highways, waterproof structures, and places where strength and durability are of greater importance.

The average weight of finished concrete is 150 pounds per cubic foot, with variations between 135 and 160 pounds per cubic foot, depending upon the materials and mix used.

Concrete is usually tested in compression in cylinders 12 inches long and 6 inches in diameter. Its strength increases with age and, for all practical purposes, its maximum is attained at about one year. Table 2, page 139, gives the average ultimate strengths for concrete of different mixes at different ages.

The figures in Table 2 are average values and in practice will be found to vary greatly, for many reasons. The quality of the cement alone, all other factors being the same, may be expected to produce variations as high as 20 per cent above and below the values given. Variations due to this cause generally become less with age but even at one year are often greater than 10 per cent.

The strength of concrete in tension is very small as compared with its compressive resistance. As an average figure the ultimate tensile strength may be assumed to be one-tenth of the ultimate compressive strength. The shearing strength of concrete averages about one-half the ultimate compressive strength. The modulus of elasticity of concrete may be considered as 2,000,000 and its coefficient of expansion per degree change in Fahrenheit temperature is 0.0000060. (See Arts. 41 and 48.) The elastic limit of concrete is very poorly defined and is not used in design or in computations of strength.

The safe working unit compressive strength of concrete may, in general, be assumed as four-tenths of its strength as shown by the 28-day test. Thus, the average allowable unit stress of 1 : 2 : 4 concrete is $0.4 \times 1900 = 760$ pounds per square inch. The factor of safety at this age is thus 2.5, whereas at the end of one year the factor of safety is $2900 \div 760 = 4$, nearly.

Concrete differs from other building materials in that it is manufactured in place and attains its strength slowly over a long period of time. This disadvantage, however, is more than offset by the ease with which it may be molded to any desired form and by its economy as compared with other classes of masonry. Concrete is literally at the foundation of nearly every engineering structure of the day.

QUESTIONS AND PROBLEMS

77a. Plot on one diagram the data given for each of the four concrete mixes in Table 2, indicating the strengths as ordinates and the ages as abscissas.

77b. If the expected variation in the strengths of concrete is 20 per cent, between what limits will range the strength of the 1 : 2 : 4 mix of Table 2 at the age of 28 days?

77c. What is the probable value for the ultimate shearing strength of a 1 : 1½ : 3 concrete at one year? What is its ultimate tensile strength?

77d. What is the safe working unit compressive strength of a 1 : 3 : 6 concrete at 7 days? Of a 1 : 1 : 2 concrete at one year? How does the age affect these values?

78. Cast Iron. Cast iron is a modern product, having been first made in England about the beginning of the fifteenth century. Ores of iron are de-oxidized and melted in a blast furnace, producing pig iron. The pig iron is remelted in a cupola furnace and poured into molds, forming castings. Pipes, braces, and machine parts of every shape required in industry are produced in this manner. The properties and strength of castings depend upon the quality of the ores and the method of their manufacture in both the blast and the cupola furnace. Long-continued fusion improves the quality of the product, as also do repeated meltings.

The percentages of carbon and silicon in cast iron are controlling factors which govern its strength, particularly those percentages which are chemically combined with the iron. There are two general classifications of cast iron, white and gray. White cast iron contains carbon in certain chemical combinations which give it its characteristic bright silvery color and make it both hard and brittle. Gray cast iron is in more general use by far. It contains a limited amount of the combined carbon, the remainder of the carbon occurring as graphite flakes. Gray cast iron has excellent casting properties, is readily machined, and has a wide range of useful qualities which make it a valuable industrial material.

In ultimate tensile strength, gray cast irons range from less than 20,000 pounds per square inch, for soft weak irons, to over 70,000 pounds per square inch for some of the high strength and heat-treated irons, especially if alloyed. Cast irons are usually specified by tensile strength, leaving the composition to the metallurgist and foundryman.

In the problems of this book, cast iron with an ultimate tensile strength of 18,000 pounds per square inch and an ultimate compressive strength of 80,000 has been assumed.

The elastic limit of cast iron is poorly defined, there being no sudden increase in deformation, as in the more ductile materials.

The high compressive strength and cheapness of cast iron render it a valuable material for many purposes; but its low ductility forbids its use in structures subject to shocks.

QUESTIONS AND PROBLEMS

78a. The specific gravity of a block of cast iron is 7.1. What is the weight of that iron in pounds per cubic foot?

78b. A cast-iron bar weighing 42 pounds per linear yard is to be subjected to tension. What load in pounds will rupture it?

78c. What must be the capacity of a testing machine to break a cast-iron block 2 inches square and 2 inches long?

79. Malleable Cast Iron. *Malleable iron* is produced by annealing white iron castings. The *combined carbon* in the casting is changed by this heat treatment into *graphite nodules* (clusters). The casting after treatment consists of iron in which nodules of graphite are more or less *uniformly distributed.*

Malleable iron having an elastic limit of 35,000 to 40,000 pounds per square inch and a tensile strength of 50,000 to 60,000 pounds per square inch can be readily produced.

80. Wrought Iron. *Wrought iron* is a ferrous material aggregated from a solidifying mass of pasty particles of highly refined metallic iron, with which, without subsequent fusion, is incorporated a minutely and uniformly distributed quantity of *slag.* The amount of slag present varies but may be said to average 2 per cent by weight. This slag contributes to the ease with which wrought iron can be welded and is largely responsible for the remarkable corrosion resistance of genuine wrought iron. A carbon content exceeding 0.10 per cent is usually considered to indicate imperfect refining. Wrought iron has an elastic limit of 25,000 to 35,000 per square inch and a tensile strength of 40,000 to 50,000 per square inch.

One method of producing wrought iron is the puddling process. Here the cast-iron pig is subjected to the oxidizing flame of a blast in a reverberatory furnace where it is formed into pasty balls by the puddler. A ball taken from the furnace is run through a squeezer to expel the cinder and then rolled into a muck bar. The muck bars are cut, laid in piles, heated, and rolled, forming what is called merchant bar. If this is cut, piled, and rolled again, a better product, called best iron, is produced. A third rolling gives "best best" iron, a superior quality, but high in price.

Wrought iron is tough and ductile, having an ultimate elongation of 20 to 30 per cent. It is malleable, can be forged and welded, and has a high capacity to withstand the action of shocks. It cannot, however, be tempered so as to be used for cutting tools.

81. Steel. Chemically, steel is a compound of iron, carbon, and other elements, generally intermediate in composition between cast and wrought iron, but having a higher specific gravity than either. The following comparison points out the distinctive differences between the three kinds of iron:

	Per Cent of Carbon	Spec. Grav.	Properties
Cast iron	5.00 to 2.00	7.2	Fusible, not malleable
Steel	1.50 to 0.10	7.8	Fusible and malleable
Wrought iron	0.10 to 0.05	7.7	Malleable, not fusible

It should be noted that the mean values of specific gravity stated are in each case subject to considerable variation.

The three principal methods of manufacture are the *crucible process*, the *open-hearth process*, and the *Bessemer process*. In the crucible process impure wrought iron or blister steel, with carbon, other alloys, and a flux, are fused in a sealed vessel to which air cannot obtain access; the best tool steels are made thus. In the open-hearth process pig iron is melted in a Siemens's furnace, scrap being added until the proper degree of carbonization is secured. In the Bessemer process pig iron is completely decarbonized in a converter by an air blast and then recarbonized to the proper degree by the addition of spiegeleisen. The metal from the open-hearth furnace or from the Bessemer converter is cast into ingots, which are rolled in mills to the required forms. The open-hearth process produces steel for guns, armor plates, machinery, shafts, rails, and for structural purposes; the Bessemer process is used also as an adjunct in the open-hearth process.

The physical properties of steel depend both upon the method of manufacture and upon the chemical composition, the carbon and other alloys having the controlling influence upon strength. Manganese promotes malleability and silicon increases the hardness, whereas phosphorus and sulphur tend to cause brittleness. The higher the percentage of carbon within reasonable limits the greater is the ultimate strength and the less the elongation.

Space does not permit a full discussion of the properties of the many steels available, but the following will indicate the more common types.

Medium carbon cast steels form the bulk of the steel foundry output. They are used both "as cast" and heat-treated. Ultimate tensile strengths range from 55,000 to 130,000 pounds per square inch and yield points from 25,000 to 100,000 pounds per square inch.

Low alloy cast steels have ultimate tensile strengths ranging from 70,000 to 200,000 pounds per square inch with yield points of 40,000 to 170,000 pounds per square inch. One or more of the following alloying elements are usually present: nickel, chromium, molybdenum, vanadium, copper.

Structural steels. Ordinary structural carbon steel (0.20 carbon, 0.15 silicon, 0.50 manganese) has a yield strength of 25,000–30,000 pounds per square inch and a tensile strength of 50,000–65,000 pounds per square inch.

Low alloy structural steels, sold under various trade names, have yield strengths of 50,000 pounds per square inch, and better, and tensile strengths of 70,000 pounds per square inch and better. These steels are available in welding and riveting grades. Some of these steels have definite corrosion resistance.

Machinery steels. Machinery steels are available in what are known as *plain carbon* steels and *alloy* steels. Plain carbon steels range from 0.10 to 0.50 per cent carbon with ultimate strengths of 40,000 pounds per square inch to 160,000 pounds per square inch, depending upon composition, processing, and heat treatment.

Bars and sheets are furnished both hot (black)-rolled and cold (bright)-finished. Cold-rolled or cold-drawn steel shafting is usually 0.15 or 0.20 carbon steel with an ultimate strength of about 62,000 pounds per square inch.

An approximate classification of *plain carbon steel* according to the percentage of carbon is as follows:

Medium	0.50 to 0.30% C.
Mild	0.30 to 0.05% C.

Alloy steels hardened and tempered to ultimate tensile strengths up to 220,000 pounds per square inch have elastic limits which *average* about 20,000 pounds per square inch lower than the tensile strength. Commonly used alloying elements are manganese, molybdenum, chromium, nickel, vanadium. Heat-treated alloy steels are usually specified by composition and Brinell hardness number. Thus the specification "SAE 4150 Heat Treated 285 to 321 Brinell" roughly and practically establishes the ultimate tensile strength, elastic limit, elongation, and reduction of area that may be expected.

Tool and die steels. Those steels whose carbon content ranges from 0.60 to 1.00 per cent, and certain alloy steels, all of which are capable of great hardness, are classified as tool and die steels. Although these steels are both hard and strong, they are very brittle and otherwise limited as to usefulness.

The compressive strength of steel is always higher than its tensile strength. The maximum value recorded for hardened steel is 392,000 pounds per square inch. The expense of commercial tests of compression is, however, so great that they are seldom made. The shearing strength is about one-half the tensile strength.

Steel is the most useful of all structural materials. It is made in nearly every conceivable shape and is used for practically every purpose. The development of special steels to meet the requirements of particular classes of service has done much toward solving many of the problems of modern engineering. Consider how far the roller bearing of an automobile would run if it were made of structural steel! The cables of the great Delaware River suspension bridge were only 30 inches in diameter because the steel wires of which they were made had an ultimate strength of 220,000 pounds per square inch. Each wire was

0.196 inch in diameter and each cable was made up of 18,666 wires. Without steel the civilization we have come to accept as part of our daily lives would hardly have been possible.

82. Brass and Bronze. In situations where steel or iron would rapidly rust and corrode, brass or bronze are often used. "Government bronze" is a brass composed of 88 parts copper, 10 parts lead, and 2 parts zinc. Its average ultimate tensile strength is 35,000, and its average elastic limit is 8000 pounds per square inch. Manganese bronze is a strong and non-corrodible alloy weighing about 530 pounds per cubic foot, an average ultimate tensile strength of 65,000 and an average elastic limit of 25,000 to 30,000 pounds per square inch. These strengths, it will be noted, compare favorably with those of structural steel, but unfortunately nearly all copper alloys, because of their low modulus of elasticity (about 15,000,000), lack the "rigidity" of steel. Under equal unit stresses they will, when stressed within the elastic limit, show elongations nearly twice those of structural steel. In many situations, this is not a desirable quality, though, as a practical matter, the difficulty may be overcome by reducing the working unit stress by about one-half.

83. Aluminum. In 1939 aluminum stood fourth among the commercial metals, in volume consumed. It has reached this state of usefulness in the short span of about fifty years. This rapid development can be explained only by the wide range of uses based on the diversity of properties of the metal.

The most striking quality of aluminum is its lightness. Comparing equal volumes, we see that it weighs only about one-third as much as most of the other commonly used metals. However, many other properties contribute to its industrial usefulness. Chief among these qualities, possessed by aluminum to a high degree, are resistance to the corrosive action of the atmosphere and of a great variety of chemical compounds; thermal and electrical conductivity; and ease of fabrication. Aluminum is welded by all commercial methods and is economically finished in a wide range of textures and colors.

Although commercially pure aluminum in the annealed or cast condition has a tensile strength only about one-fourth that of structural steel, the strength can be markedly increased by cold working. This gain in strength is accompanied by a loss in ductility; the ease of forming is decreased as the amount of cold working is increased.

The addition of other metals to form alloys offers another means of increasing the strength and hardness of aluminum. Even the small percentage of impurities in commercial aluminum is sufficient to increase the strength compared with that of the pure metal by about 50 per cent.

The metals most commonly used in the production of commercial aluminum alloys, either singly or in combinations, are copper, silicon, manganese, magnesium, chromium, iron, zinc, and nickel. If the alloy is to be manufactured in wrought forms, the total percentage of alloying elements is seldom more than 6 or 7 per cent, although in casting alloys appreciably higher percentages are frequently used.

Aluminum has an average weight of 0.10 pound per cubic inch and a modulus of elasticity of 10,300,000 pounds per square inch. Its ultimate strength ranges from 13,000 to 70,000 pounds per square inch, depending upon the alloy, processing, and heat treatment. Its elastic limit also varies widely between 5000 and 55,000 pounds per square inch. In shear, its strength is between 0.6 and 0.8 of that in tension.

84. Plastics. Those materials which are capable of being molded or formed to shape are plastics. The term *plastics*, in recent years, has by popular use come to denote synthetic organic chemical compounds and mixtures which are molded to shape in conforming dies by the application of heat and pressure.

These molding materials consist of two broad classes: *thermosetting* and *thermoplastic*. Each type requires a distinct technique for processing into finished molded products.

Thermosetting molding materials are hardened by applying additional heat after the mold is formed. *Phenolics* are of this type plastic, and the bond is produced by the chemical reaction of phenol (and its derivatives) with formaldehyde (and its derivatives). *Ureas* are another type, similar to phenolics, but the bond is obtained by reacting urea (and its derivatives) with formaldehyde (and its derivatives).

Thermoplastic molding materials harden by cooling after molding under heat and pressure, and will soften again if reheated.

Cellulose acetate is of this type and is formed by the chemical action of acetic acid on cellulose.

Polystyrene is another thermoplastic formed by the chemical action accompanying the heating of styrene.

The selection and application of plastics to industrial products is a problem beyond the scope of this book. The useful properties of the many varieties of plastics are specific and numerous, including such items as impact strength, color stability, inertness, flammability, and electrical properties. Average values of properties related to those given for other materials are shown below for plastics in the general categories mentioned.

Property	Type of Material			
	Thermosetting		Thermoplastic	
	Phenolics	Ureas	Cellulose Acetate	Polystyrene
Specific gravity	1.35 to 1.93	1.47 to 1.52	1.26 to 1.40	1.07
Weight, pounds per cubic inch	0.049 to .070	0.053 to .055	0.045 to .051	0.039
Tensile strength, pounds per square inch	4600 to 8500	9500 to 12,000	2500 to 9500	5500 to 7000
Modulus of elasticity, pounds per square inch	8.8×10^5 to 20×10^5	11×10^5 to 17×10^5		3.8×10^5 to 6×10^5

85. Ropes. Ropes are made of *hemp,* of *manila,* and of *iron* or *steel wire* with a hemp center. A hemp rope one inch in diameter has an ultimate strength of about 6000 pounds, and its safe working strength is about 800 pounds. A manila rope is slightly stronger. Iron and steel ropes one inch in diameter have ultimate strengths of about 36,000 and 50,000 pounds, respectively, the safe working strengths being 6000 and 8000 pounds. As a fair rough rule, the strength of ropes may be said to increase as the square of their diameters.

APPENDIX B

TABULAR DATA

The data in the following tables, in those tables scattered through the book, and other quantitative data given in the text are furnished for two purposes only. First, they are intended to provide the student with a basis for developing a sense of relative values and proportions. Part of the understanding of the subject, which it is hoped will grow out of the use of this book, is the realization and appreciation of the quantitative properties which are represented in these tabular data. It is of practical value to know in round numbers such things as the approximate strengths of everyday materials, and also that the strength of one material bears a certain relationship to that of each of the other materials.

The second purpose for providing these data is to give the student material with which to work in the solution of problems. For most of the students to whom this book is directed, the solution of problems will be largely a matter of drill in the organization of data, deduction of controlling factors, and the application of principles of strength of materials. To facilitate checking and comparison of results, and to give the problem solving some of the aspects of practical reality, the inclusion of these data is considered essential.

It should be borne in mind, however, that it is not within the scope of this book to provide the student with quantitative data sufficiently specific and reliable for use as the basis for actual construction. *This is not a reference book.* These data are rounded and averaged values, generalized and simplified for the purposes previously mentiond, but not certified for accuracy. Actual design must be based upon values furnished by testing and engineering societies, or specifications commonly provided by the various manufacturers.

TABLE 1

Strengths of Industrial Materials

For Use in Solving Problems

All Values in Pounds per Square Inch	Average Ultimate Strength			Elastic Limits		
Material	Tensile	Com-pression	Shear	Tension	Com-pression	Shear
Aluminum—cast	15,000	12,000	12,000	6,500	5,500	Estimated at Three-Fourths of Tensile Values
" —rolled	26,000	.....	.....	13,000	......	
" —hard wire	65,000	.....	.....	30,000	......	
Copper —cast	25,000	40,000	30,000	6,000	......	
" —rolled	35,000	32,000	.....	10,000	......	
Brass —17% Zn	32,500	.....	.....	8,200	......	
" —cast, common	21,000	30,000	36,000	6,000	......	
Bronze —20% Sn	33,000	78,000	.....	......	......	
—phosphorus	50,000	.....	.....	24,000	......	
Iron —cast (min.)	18,000	80,000	20,000	6,000	20,000	
—wrought	50,000	50,000	42,000	27,000	27,000	
Glass —common	3,000	30,000	.....	......	......	
Masonry —brick (hard)	.....	3,000	.....	......	......	
—granite	1,200	12,000	.....	......	......	
—limestone	800	8,000	.....	......	......	
—sandstone	150	5,000	.....	......	......	
Steel —cast (med.)	70,000	70,000	50,000	38,000	38,000	
—Structural	60,000	60,000	45,000	35,000	35,000	
—mild carbon	65,000	80,000	33,000	45,000	60,000	
—med. carbon	80,000	100,000	40,000	60,000	80,000	
—strong alloy	120,000	160,000	60,000	100,000	140,000	
Timber (parallel to grain)	10,000	6,400	.....	3,300	2,100	

TABLE 2

Concrete Strengths

Mix	Age, Days	Compressive Strength, Pounds per Square Inch
1 : 1 : 2	7	2900
	28	4200
	360	5000
1 : 1½ : 3	7	1900
	28	2800
	360	4000
1 : 2 : 4	7	1100
	28	1900
	360	2900
1 : 3 : 6	7	500
	28	900
	360	1700

TABLE 3

Factors of Safety

Material	For Steady Stress (Buildings)	For Varying Stress (Bridges)	For Shocks (Machines)
Timber	8	10	15
Brick and stone	15	25	35
Cast iron	6	15	20
Wrought iron	4	6	10
Steel, structural	4	6	10
Concrete		See Arts. 53 and 77	

TABLE 4

Properties of Rolled Sections

a. Steel I Beams

Depth Inches	Weight per Foot Pounds	Section Area A Sq. Inches	Moment of Inertia I Inches^4	Section Modulus $\frac{I}{c}$ Inches^3	Moment of Inertia I' Inches^4
24	100	29.4	2380	198	48.6
24	80	23.5	2088	174	42.9
20	75	22.1	1269	127	30.2
20	65	19.1	1170	117	27.9
18	70	20.6	921	102	24.6
18	55	15.9	796	88.4	21.2
15	55	15.9	511	68.1	17.1
15	42	12.5	442	58.9	14.6
12	35	10.3	228	38.0	10.1
12	31½	9.3	216	36.0	9.50
10	40	11.8	159	31.7	9.50
10	25	7.4	122	24.4	6.89
8	25¼	7.50	68.4	17.1	4.75
8	18	5.33	56.9	14.2	3.78
7	20	5.88	42.2	12.1	3.24
7	15	4.42	36.2	10.4	2.67
6	17¼	5.07	26.2	8.73	2.36
6	12¼	3.61	21.8	7.27	1.85
5	14¾	4.34	15.2	6.08	1.70
5	9¾	2.87	12.1	4.84	1.23
4	10½	3.09	7.1	3.55	1.01
4	7½	2.21	6.0	3.00	0.77
3	7½	2.21	2.9	1.93	0.60
3	5½	1.63	2.5	1.71	0.46

b. Steel H Columns

Depth Inches	Weight per Foot Pounds	Section Area A Sq. Inches	Moment of Inertia I Inches^4	Section Modulus $\frac{I}{c}$ Inches^3	Moment of Inertia I' Inches^4
16.5	262.5	76.9	3402	412	1096
16.0	228.5	67.0	2860	357	929
15.0	163.0	47.7	1894	252	626
14.0	100.0	29.0	1071	153	357
14.0	190.0	55.6	1773	253	579
12.0	79.0	22.9	616	103	208
11.0	100.5	29.3	607	110	202
10.0	55.0	15.9	297	59.4	100
9.0	72.0	21.0	285	63.5	94.4
8.0	35.0	10.2	122	30.4	41.1

The values I' given in the last column are with respect to an axis through the center of gravity but parallel to the web.

TABLE 5

Column Constants q

Material	Both Ends Fixed	One Fixed End and One Round End	Both Ends Round
Timber	$\frac{1}{3,000}$	$\frac{2}{3,000}$	$\frac{4}{3,000}$
Cast iron	$\frac{1}{5,000}$	$\frac{2}{5,000}$	$\frac{4}{5,000}$
Wrought iron	$\frac{1}{35,000}$	$\frac{2}{35,000}$	$\frac{4}{35,000}$
Steel	$\frac{1}{25,000}$	$\frac{2}{25,000}$	$\frac{4}{25,000}$

TABLE 6

Moduli of Elasticity

Material	Pounds per Square Inch
Aluminum	10,300,000
Brass, common	9,000,000
Iron, cast	12,000,000
wrought	28,000,000
Steel	30,000,000
Stone	7,000,000
Wood	1,500,000
Concrete	See Arts. 53 and 77

TABLE 7

Weights of Industrial Materials

(Average Values)

Material	Weight per Cu. Ft.	Weight per Cu. In.
Aluminum	165	0.096
Brass	534	0.309
Bronze	548	0.317
Copper	556	0.321
Iron, cast	450	0.260
" wrought	485	0.280
Masonry, brick	120	0.069
concrete	144	0.083
granite	165	0.096
limestone	156	0.090
sandstone	140	0.081
Steel	490	0.283
Wood, Fir	34	0.020
Oak	42	0.024
Pine	27	0.016
Sand	110	0.064
Lead	706	0.480

TABLE 8

Strength of Timber

Kind	Weight, Pounds per Cubic Foot	Pounds per Square Inch	
		Tensile Strength	Compressive Strength
Hemlock	25	8,000	5,000
White pine	27	8,000	5,500
Chestnut	40	12,000	5,000
Red oak	42	9,000	6,000
Yellow pine	45	15,000	9,000
White oak	48	12,000	8,000

APPENDIX C

ANSWERS TO PROBLEMS

Answers to a scattered selection of the problems are given below so that the student may occasionally check his own work. By so doing he may himself find whether or not he is using the right methods and formulas, without waiting until someone else can check his work.

Of course, the purpose of this appendix is likely to be defeated if the answers are consulted before the problems are worked, and the student is the loser thereby. In order that the teacher may check on this type of misuse, all the answers are not given.

Answers are given correct to three digits and units are shown. In addition to the numerical check, the student should check the units of his answers as described in Art. 8.

(Problem numbers are shown in parentheses.)

(8*c*)	40,700 pounds
(8*j*)	$\frac{3}{8}$ inch
(9*c*)	0.438 inch diameter
(9*e*)	29,200 pounds
(10*e*)	11.4 inches square
(12*e*)	5.35 inches diameter
(12*j*)	$3\frac{1}{32}$ inches diameter
	$3\frac{23}{32}$ inches diameter
(13*d*)	288 pounds, 180 inch-pounds
(13*j*)	70.6 pounds
(14*d*)	$R_1 = 916$ pounds
(15*i*)	800 pounds
	0, pounds
(16*e*)	35,600 pounds
(17*d*)	540,000 inch-pounds
(18*d*)	4.23 inches from flange top
(19*c*)	2090 inches^4
(21*n*)	5.8 feet
(22*f*)	8460 foot-pounds
(22*i*)	8190 foot-pounds
(22*k*)	7550 pounds per square inch
(23*c*)	38 inches^3
(25*c*)	(*b*) 79 horsepower
(25*i*)	66.7 to 26.7
(27*e*)	30.2 inches^4
(28*a*)	(*a*) 3.72 inches
(29*c*)	500 pounds per square inch (When $l/r = 0$)
(31*b*)	(*b*) 40
(34*c*)	8.95 inches
(34*e*)	219 tons
(40*a*)	2700 pound-inches
(40*e*)	63.7 pounds
(45*b*)	(*a*) 12,000 pound-inches
(46*a*)	$55\frac{1}{4}$ inches, when F = 8
(47*d*)	(*a*) $7\frac{1}{2}$ tons
(48*a*)	(*a*) 0.087 inch
(50*b*)	36.2 tons
(54*a*)	82 tons
(55*b*)	(*b*) One
(58*a*)	329 tons
(62*a*)	428 pounds per square inch

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