Course Outline Fundamentals of Thermodynamics:

- Thermodynamic System
- Thermodynamic functions
- Thermodynamic Properties
- Fundamental Laws of Thermodynamics
- Thermodynamic Relations
- Thermodynamic Processes

Thermodynamic System

 A thermodynamic system is defined as a quantity of fixed mass and identity upon which attention is focused for study. Everything external to the system is surroundings

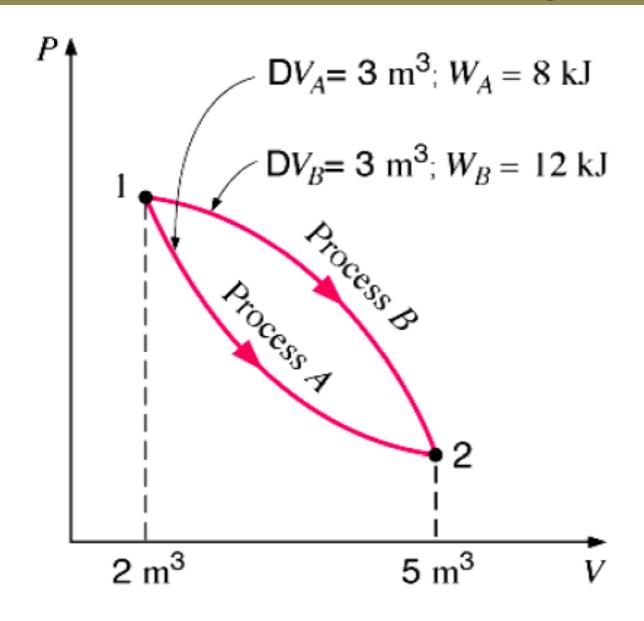
 Thermodynamic system can further be classified into closed system, open system, and an isolated system

Heat and Work

- Heat is energy transferred between a system and its surroundings by virtue of a temperature difference only.
- Heat is a way of changing the energy of a system by virtue of a temperature difference only.
 Any other means for changing the energy of a system is called work
- Sign convention for Work and Heat.

Thermodynamic Function

- There are two types of functions defined in thermodynamics: Path function and Point function
- Path function depends on the history of the system (or path by which system arrived at a given state).
 Examples for path functions are work and heat.
- Point function does not depend on the history (or path) of the system. It only depends on the state of the system. Examples of point functions are: temperature, pressure, density, mass, volume, enthalpy, entropy, internal energy etc.

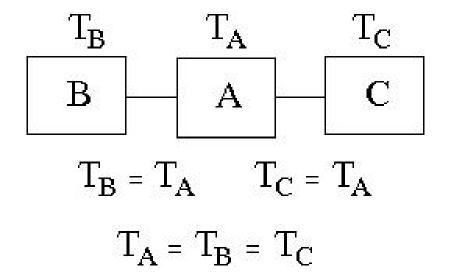


Thermodynamic Properties

- A property is any characteristic or attribute of matter, which can be evaluated quantitatively.
- A thermodynamic property depends only on the state of the system and is independent of the path by which the system arrived at the given state.
- All thermodynamic properties are point functions.
- It can be either extensive or intensive.

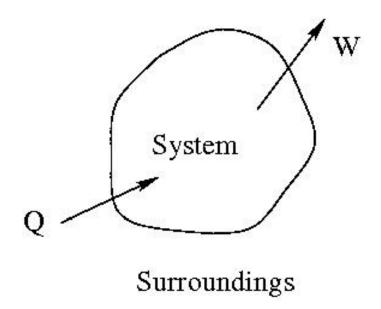
Zeroth law of Thermodynamics

Zeroth Law : Defines Temperature



First law of Thermodynamics

First Law: Conservation of Energy



First law of Thermodynamics

- First Law: defines internal energy
- For a cyclic process

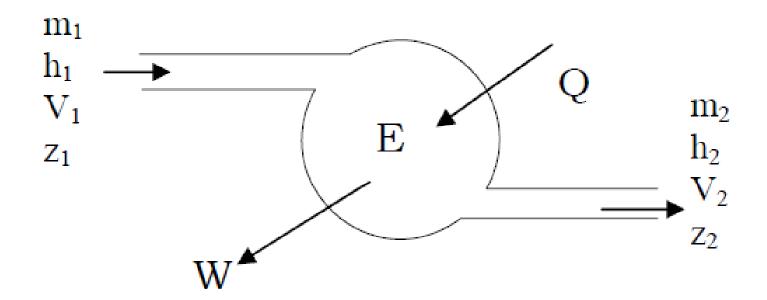
$$\oint \delta Q = \oint \delta W$$

$$\oint (\delta Q - \delta W) = 0$$

$$dU = \delta Q - \delta W$$

First law of Thermodynamics

First Law applied to an open system



$$\frac{dE}{dt} = m_2(h_2 + \frac{{V_2}^2}{2} + gz_2) - m_1(h_1 + \frac{{V_1}^2}{2} + gz_1) + W - Q$$

Second law of Thermodynamics

- Second Law : Defines Entropy
- - = 0, the cycle is reversible
 - < 0, the cycle is irreversible and possible
 - > 0, the cycle is impossible
- Mathematical expression for 2nd law: $\Delta S_{total} \geq 0$

Third law of Thermodynamics

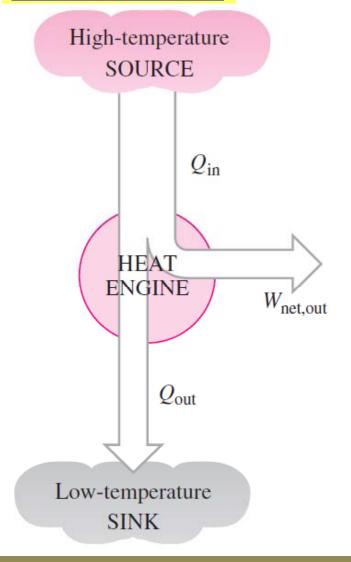
- Third Law: defines absolute zero
- It gives the definition of absolute value of entropy and also states that absolute zero cannot be achieved.
- Another version of this law is that "the entropy of perfect crystals is zero at absolute zero".

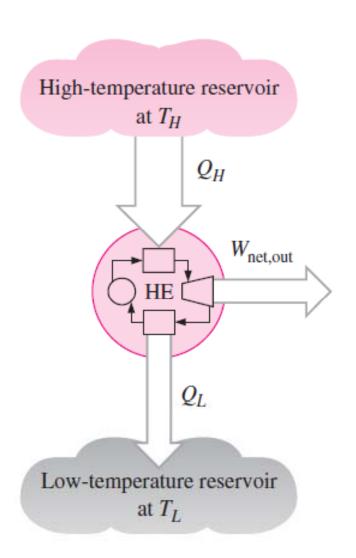
Heat Engines, Refrigerators, and Heat Pumps

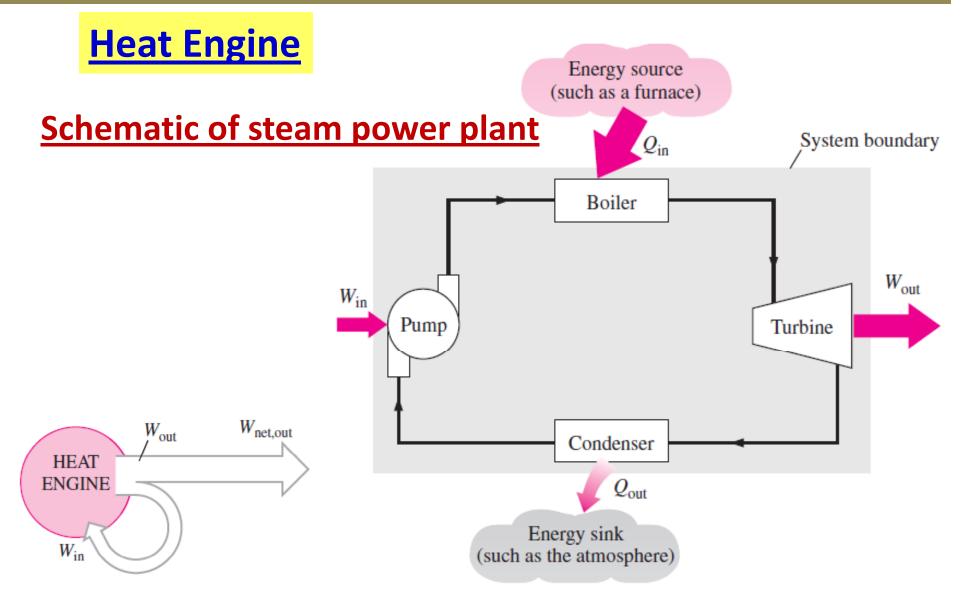
Heat Engine:

- A heat engine may be defined as a device that operates in a cycle and does a certain amount of net positive work through the transfer of heat from a high temperature body to a low temperature body.
- A steam power plant is an example of a heat engine.

Heat Engine







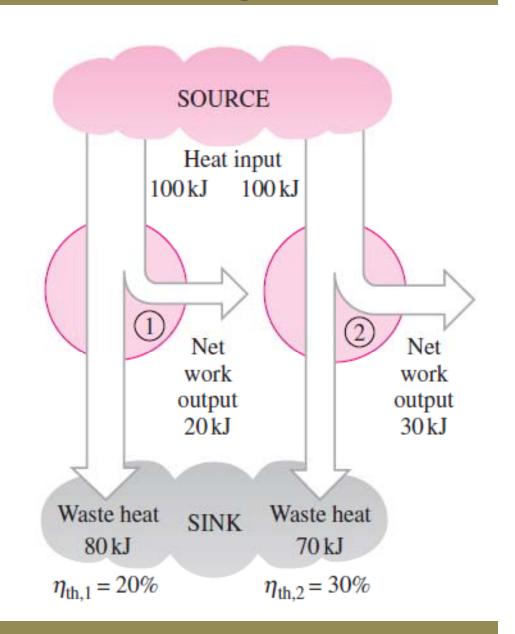
Heat Engine

Thermal efficiency

Thermal efficieny $\left(\eta_{\scriptscriptstyle th}\right)$

$$\eta_{th} = \frac{\text{Net work output}}{\text{Total heat input}}$$

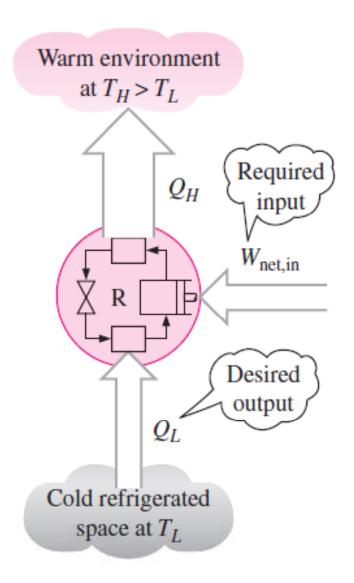
$$=\frac{W_{net,out}}{Q_{in}}=1-\frac{Q_{out}}{Q_{in}}$$



Refrigerator

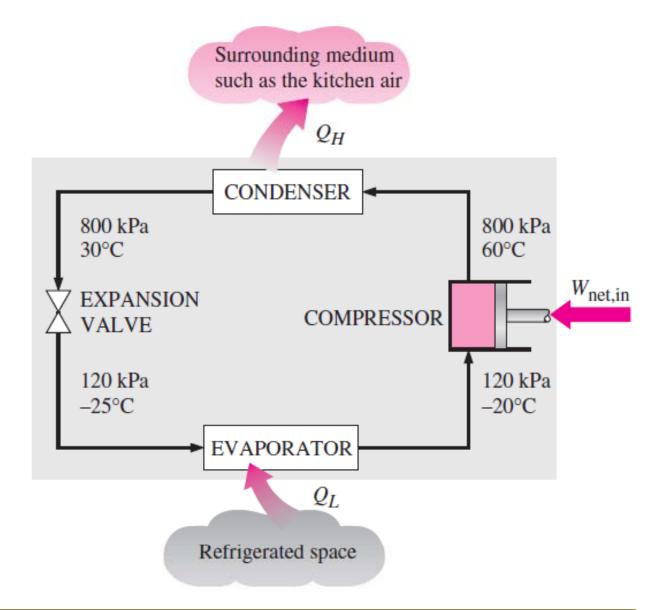
- A refrigerator may be defined as a device that operates in a cycle and transfers a certain amount of heat from a body at a lower temperature to a body at a higher temperature by consuming certain amount of external work.
- Domestic refrigerators and room air conditioners are the examples.

Refrigerator



Refrigerator

Basic components
of a refrigeration
unit



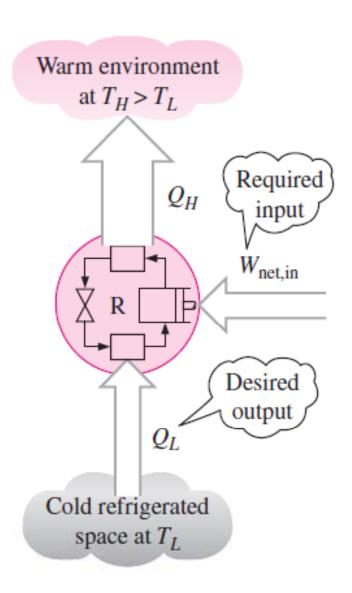
Refrigerator

Coefficient of Performance

Coefficient of Performance,

$$COP_R = \frac{\text{Desired output}}{\text{Required input}}$$

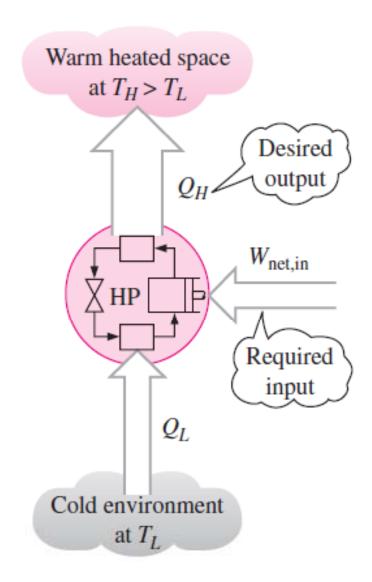
$$=\frac{Q_L}{W_{net,in}}=\frac{Q_L}{Q_H-Q_L}$$



Heat Pumps

- A heat pump is similar to a refrigerator, however, here the required output is the heat rejected to the high temperature body.
- Domestic room heaters or air conditioners are the examples.

Heat Pumps



Heat Pumps

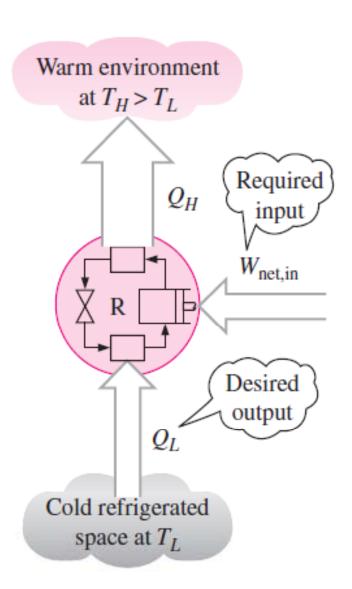
Coefficient of Performance

Coefficient of Performance,

$$COP_{HP} = \frac{\text{Desired output}}{\text{Required input}}$$

$$=\frac{Q_H}{W_{net,in}}=\frac{Q_H}{Q_H-Q_L}$$

$$COP_{HP} = COP_R + 1$$



Approximate magnitude of *COP*

Let us assume that thermal efficiency of a heat engine is 30%, i.e.

$$\eta_{th} = 0.3 = \frac{Q_H - Q_L}{Q_H}$$

Now, if the engine is reversed in operation to make it work as a refrigerator or a heat pump with operating conditions unchanged, then

$$COP_{R} = \frac{Q_{L}}{Q_{H} - Q_{L}} = \frac{1 - \eta_{th}}{\eta_{th}} = 2.33$$

Approximate magnitude of *COP*

$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{1 - \eta_{th}}{\eta_{th}} = 2.33$$

$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{\eta_{th}} = 3.33$$

For a vapour compression system, COP_R is of the order of 3 for water-cooled and 2 for air-cooled air-conditioning applications and 1 for domestic refrigerators.

For heating, which option is better: a heat pump or an electric heater?

 If W is the energy consumed by an electric resistance heater, the heat released to the space will be at most equal to W only. But, if this energy is utilised in a heat pump, the heat pumped to the space will be

$$Q_{H} = COP_{HP} \cdot W = (1 + COP_{HP}) \cdot W$$

• Therefore, Q_H will always be greater than or equal to W.

Course Outline Refrigeration – Basic Concepts:

- Unit of Refrigeration
- Thermodynamic Relations
- Thermodynamic Processes
- Thermodynamic State of a Pure Substance
- Methods of Producing Low Temperature

Unit of Refrigeration:

- The standard unit of refrigeration is ton refrigeration or simply ton denoted by the symbol TR.
- It is defined as the rate of heat extraction to convert
 1 US tonne (1 short ton = 907.185 kg = 2000 lb) of water at 32°F to ice at 32°F in one day or 24 hours.

$$1 TR = \frac{1 \times 2000 lb \times 144 Btu / min}{24 hr} = 200 Btu / min$$
$$= 50 kcal / min = 3.5167 kW$$

• 1btu = 1055.056J

Thermodynamic Relations

For a reversible process,

$$\delta Q = TdS$$

• Therefore, from first law, we can write

$$TdS = dU + \delta W$$

$$TdS = dU + pdV$$

• Also, H = U + pV

$$TdS = dH - Vdp$$

Evaluation of Thermodynamic Properties:

 To study a given system, several thermodynamic properties have to be known. The properties like enthalpy, entropy, or internal energy can not be measured directly

 These properties are evaluated with the help of mathematical relations, i.e. a relation relating them with the measurable properties such as pressure, temperature, volume etc.

Equation of State:

 The simplest equation of state is that of solids and liquids which says that they are incompressible, i.e. specific volume (v) is a constant

An ideal gas

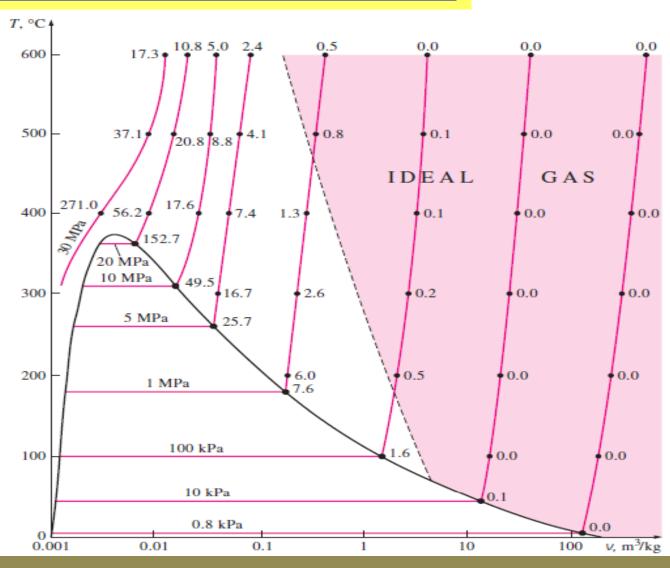
- What is an ideal gas?
- the intermolecular forces are zero and the volume of the molecules should be negligible compared to the volume of the gas

Equation of State (Ideal Gas):

$$pv = RT$$

 The above equation is valid for real gases at low pressures and high temperatures. Also, for studying the properties of moist air, it can be used without significant error

T – v diagram of pure water:



Equation of State (Ideal Gas):

For an ideal gas, internal energy (U) and enthalpy
 (H) are functions of absolute temperature only

$$U = f(T)$$
, and $H = f(T)$

$$dU = C_{v}dT$$
, and $dH = C_{p}dT$

Equation of State (Ideal Gas):

Therefore, between two states 1 and 2, we can write

$$u_2 - u_1 = c_v \left(T_2 - T_1 \right)$$

$$h_2 - h_1 = c_p (T_2 - T_1)$$

 For entropy calculation, we use the Tds relations which give

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

$$s_2 - s_1 = c_p \ln \left(\frac{T_2}{T_1}\right) - R \ln \left(\frac{p_2}{p_1}\right)$$

Equation of State (Real Gases):

 For gases at high pressures and low temperatures, we use van der Walls equation

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

 where a and b are constants that account for the intermolecular forces and volume of the gas molecules respectively

Thermodynamic Processes:

Constant Volume (Isochoric) Process

$$\begin{split} W_{1-2} &= 0 \\ Q_{1-2} &= U_2 - U_1 = mc_v \left(T_2 - T_1 \right) \\ S_2 - S_1 &= mc_v \ln \left(\frac{T_2}{T_1} \right) \end{split}$$

where m is the mass of the gas

Thermodynamic Processes:

Constant Pressure (Isobaric) Process

$$\begin{aligned} Q_{1-2} &= H_2 - H_1 = mc_p \left(T_2 - T_1 \right) \\ W_{1-2} &= p \left(V_2 - V_1 \right) \\ U_2 - U_1 &= Q_{1-2} - W_{1-2} \\ S_2 - S_1 &= mc_p \ln \left(\frac{T_2}{T_1} \right) \end{aligned}$$

where m is the mass of the gas

Thermodynamic Processes:

Constant temperature (Isothermal) Process

$$U_2 - U_1 = 0$$

$$W_{1-2} = mRT \ln \left(\frac{V_2}{V_1}\right) = mRT \ln \left(\frac{p_1}{p_2}\right)$$

$$Q_{1-2} = W_{1-2}$$

$$S_2 - S_1 = mR \ln \left(\frac{V_2}{V_1}\right) = mR \ln \left(\frac{p_1}{p_2}\right)$$

where m is the mass of the gas

Thermodynamic Processes:

Adiabatic Process

$$Q_{1-2} = 0$$

$$(U_2 - U_1) + W_{1-2} = 0$$

$$W_{1-2} = \left(\frac{\gamma}{\gamma - 1}\right) (p_2 V_2 - p_1 V_1)$$

If the process is reversible, then

$$S_2 = S_1$$

• i.e. the process is isentropic

Thermodynamic Processes:

Polytropic Process

$$(U_{2} - U_{1}) = mc_{v} (T_{2} - T_{1})$$

$$W_{1-2} = \left(\frac{n}{n-1}\right) (p_{2}V_{2} - p_{1}V_{1})$$

$$Q_{1-2} = (U_{2} - U_{1}) + W_{1-2}$$

$$S_{2} - S_{1} = \int_{1}^{2} \frac{dU}{T} + \int_{1}^{2} \frac{pdV}{T}$$

Thermodynamic Processes:

Throttling (Isenthalpic) Process

It is a flow process. It occurs when a flowing fluid suddenly encounters a restriction in its path.

$$Q_{1-2} = W_{1-2} = 0$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

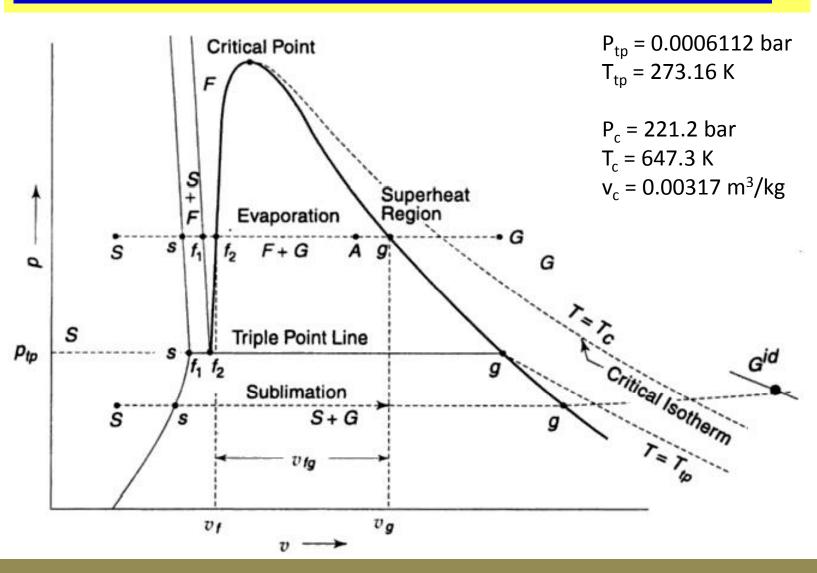
The inlet and outlet areas of a throttling device is Designed in such a way that $V_1 = V_2$.

$$\therefore h_1 = h_2$$

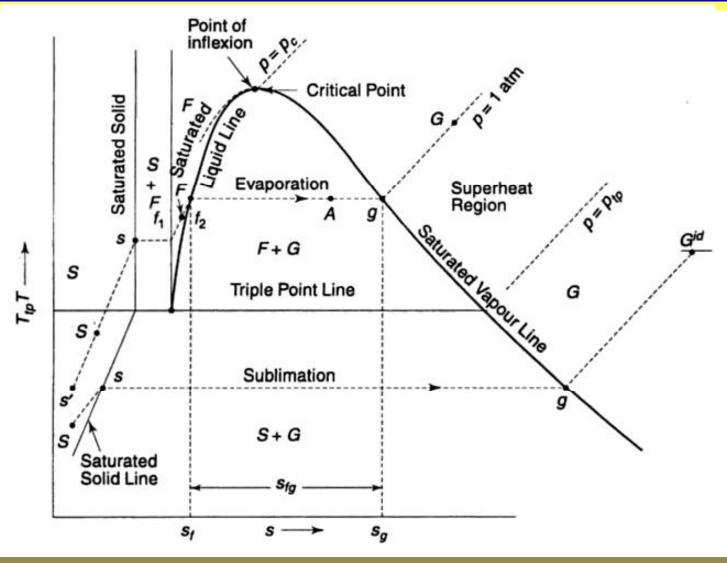
Thermodynamic State of a Pure Substance:

- To define state of a pure substance, Gibb's phase rule is used, i.e. F = C − P +2
- where, F = number of independent variables
 C = number of components
 P = number of phases
- Therefore, we require two intensive properties to fix the state of a single phase substance
- For two phase region, one property is sufficient to fix the state

Thermodynamic State of a Pure Substance:



Thermodynamic State of a Pure Substance:



Property of liquid – vapour mixture:

Let x be the quality or dryness of the mixture, then

$$v = (1 - x)v_f + xv_g = v_f + x \cdot v_{fg}$$

$$u = (1 - x)u_f + xu_g = u_f + x \cdot u_{fg}$$

$$h = (1 - x)h_f + xh_g = h_f + x \cdot h_{fg}$$

$$s = (1 - x)s_f + xs_g = s_f + x \cdot s_{fg}$$

Numerical:

• 100 kg of ice is placed in a bunker to cool some vegetables. 24 hours later, the ice has melted into water at 10° C. What is the average rate of cooling in KJ/h and TR provided by the ice? Given: $c_{ps} = 1.94 \text{ kJ/kg-K}$, $c_{pf} = 4.1868 \text{ kJ/kg-K}$, h_{sf} at 0° C = 335 kJ/kg

Ans: 1611 kJ/h and 0.127 TR

Methods of Producing Low Temperature:

- Sensible cooling by a cold medium
- Phase change process
- Expansion of liquids/gases
- Thermoelectric cooling
- Adiabatic demagnetisation

Methods of Producing Low Temperature:

Sensible cooling by a cold medium

- If a body is at lower temperature than the cooling temperature, then the body can be used to get the required cooling temperature by sensible cooling, i.e. bring the object to be cooled in contact with the body
- For example, cooling of a room with circulation of cold air

Methods of Producing Low Temperature:

- Endothermic phase change processes produce cooling effect.
- For example, sublimation, melting, and evaporation processes absorb energy from the surroundings which results in a decrease of temperature.
- Two parameters are important for these processes:
 the phase change temperature and the latent heat

Methods of Producing Low Temperature:

- For example, ice at 1 atm. pressure melts at 0°C and extracts 335 kJ/kg of heat from the surroundings.
- Similarly, dry ice (solid carbon dioxide) undergoes sublimation at -78.5°C and extracts 573 kJ/kg of heat from the surroundings.
- However, evaporation or vaporisation is preferred for obtaining refrigeration effect in a practical refrigeration cycle because it is easier to handle fluids in a cyclic device.

Methods of Producing Low Temperature:

- For all phase change processes, the refrigeration effect is proportional to the mass of the substance (m) and the latent heat of vaporisation (h_{fg}) . Therefore, substances with a large latent heat requires very less amount and vice-versa.
- Also important is the phase change temperature, which ultimately decides the refrigeration temperature.

Methods of Producing Low Temperature:

Phase change processes

 The two parameters are related by Trouton's rule, which is given as:

$$\Delta s_{fg}^n = \frac{\Delta h_{fg}^n}{T_{nbp}} = \text{a constant, in } J/mol - K$$

 Δs_{fg}^{n} is the molar entropy of vaporisation

 Δh_{fg}^{n} is the molar enthalpy of vaporisation

 T_{nbp} is the normal boiling point

Methods of Producing Low Temperature:

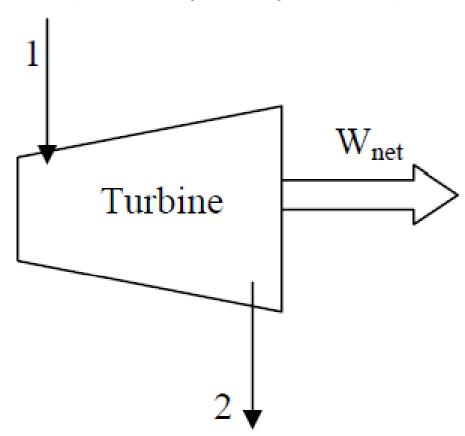
- Above equation suggests that higher the boiling point, higher will be the molar enthalpy of vaporisation.
- Also, it can be inferred that low molecular weight substances have higher specific enthalpy of vaporisation.

Methods of Producing Low Temperature:

Expansion of liquids

Expansion through a turbine (isentropic expansion)

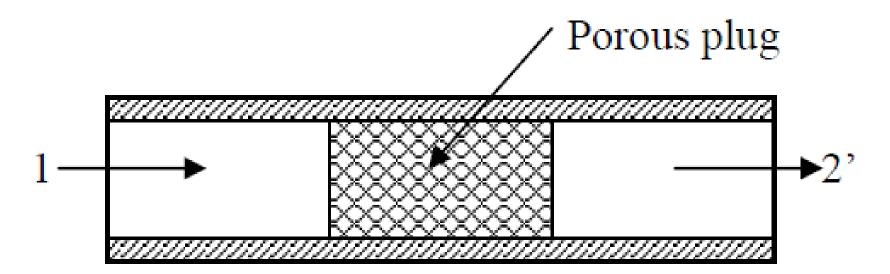
- the process is isentropic (ideally)
- the enthalpy drop is equal to the specific work output



Methods of Producing Low Temperature:

Expansion of liquids

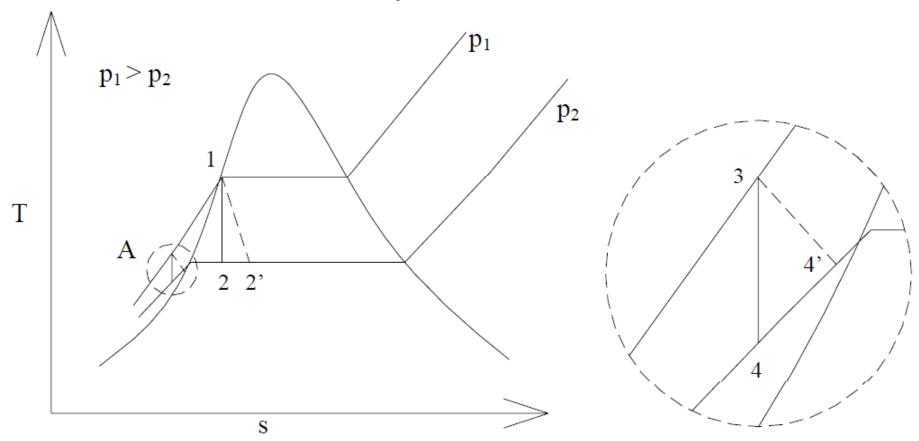
 Expansion through a porous plug or a constriction (isenthalpic expansion)



Methods of Producing Low Temperature:

Expansion of liquids

Both of the above processes can be shown as



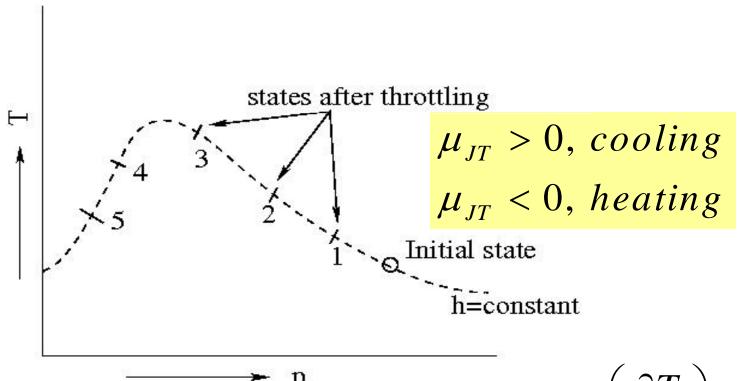
Methods of Producing Low Temperature:

Expansion of Gases

- Similar to liquids, gases can also be expanded either by using a turbine (isentropic expansion) or a throttling device (isenthalpic process).
- Since the enthalpy of an ideal gas is a function of temperature only, during an isenthalpic process, the temperature of the ideal gas remains constant.
- In case of real gases, whether the temperature decreases or increases during an isenthalpic expansion depends on the Joule-Thomson coefficient.

Methods of Producing Low Temperature:

Expansion of Gases

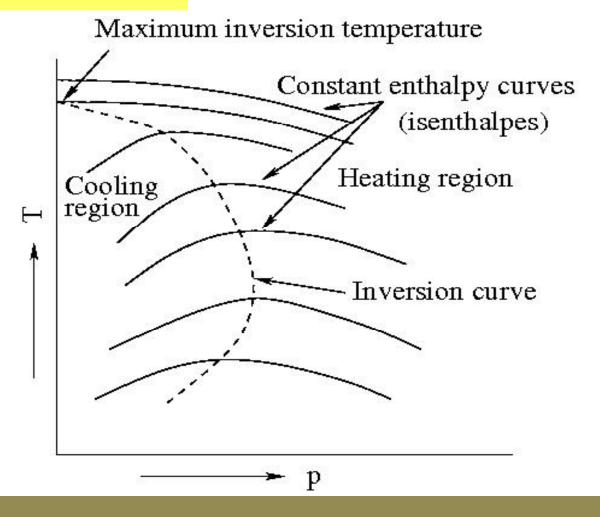


Joule – Thomson (Joule – Kelvin) coefficient, $\mu_{JT} = \left(\frac{\partial T}{\partial p}\right)_{h}$

Methods of Producing Low Temperature:

Expansion of Gases

Inversion Curve



Methods of Producing Low Temperature:

Expansion of Gases

- The temperature at the point of inflexion is known as inversion temperature.
- The locus of all the inversion point is the inversion curve.
- The point where the inversion curve intercepts the temperature axis is called as maximum inversion temperature (T_{max}) .
- For most of the gases, T_{max} is above room temperature.

Methods of Producing Low Temperature:

Show that $\mu_{JT} = 0$ for an ideal gas

$$dh = Tds + vdp$$

$$Tds = c_p dT - T \left(\frac{\partial v}{\partial T}\right)_p dp$$

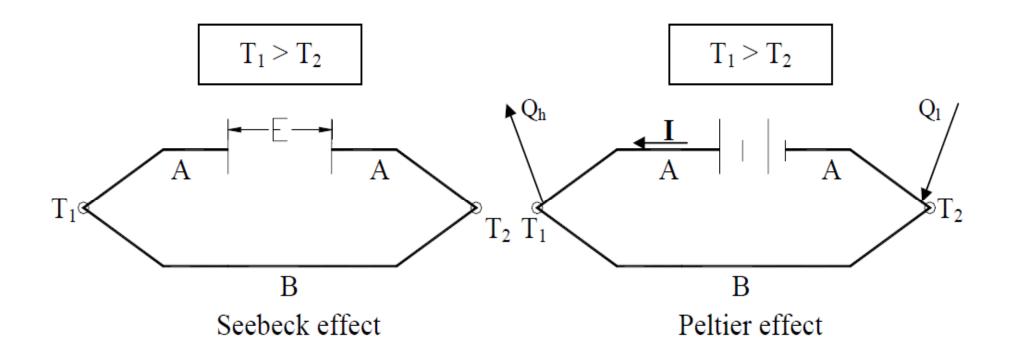
$$\mu_{JT} = \left(\frac{\partial T}{\partial p}\right)_h = \frac{1}{c_p} \left[T \left(\frac{\partial v}{\partial T}\right)_p - v \right]$$

For an ideal gas,
$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{v}{T}$$

Methods of Producing Low Temperature:

Thermoelectric Refrigeration

 It is based on the reverse Seebeck effect or the Peltier effect.



Methods of Producing Low Temperature:

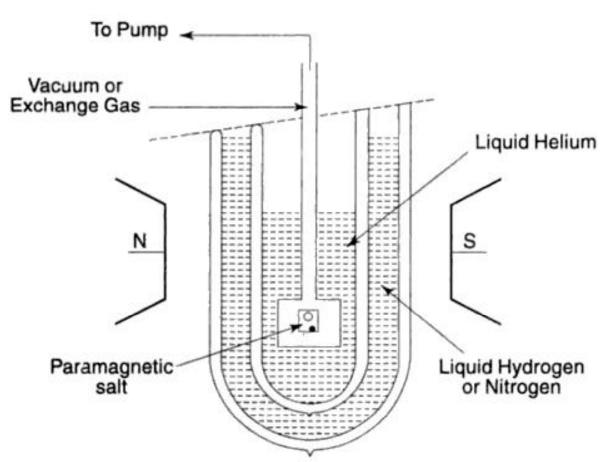
Adiabatic Demagnetisation

- It is proposed by Debye and Giaque in 1926 independently.
- The cooling effect is produced by adiabatic demagnetisation of a paramagnetic salt such as cerium magnesium nitrate
- By this method, it is possible to reach the temperature of absolute zero

Methods of Producing Low Temperature:

Adiabatic Demagnetisation

- Salt is suspended in a tube containing gaseous helium
- Salt and liquid helium is magnetised
- Vacuum is created inside the chamber
- Then suddenly it is demagnetised
- As a result cooling effect is obtained



Vapour Compression Refrigeration Systems:

The Carnot Refrigeration Cycle

Limitations of Carnot Refrigeration Cycle

Standard Vapour Compression Refrigeration System

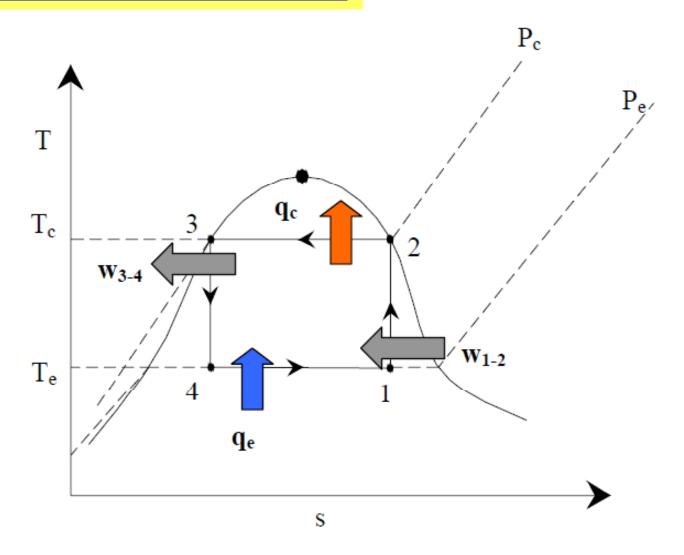
The Carnot Refrigeration Cycle:

- Carnot refrigeration cycle is a completely reversible cycle
- It is used as a reference for comparing the effectiveness of an actual refrigeration cycle
- Since it provides the maximum COP for a refrigeration cycle working between the given heat source and heat sink temperatures

The Carnot Refrigeration Cycle: Condensation Schematic of a basic 3 2 refrigeration W₃₋₄ cycle Compression **Expansion** Expansion takes place in **W**₁₋₂ an expander 4 or a turbine q_e Evaporation

The Carnot Refrigeration Cycle:

Carnot refrigeration on T – s diagram



The Carnot Refrigeration Cycle:

Some Calculations

From 1st law,

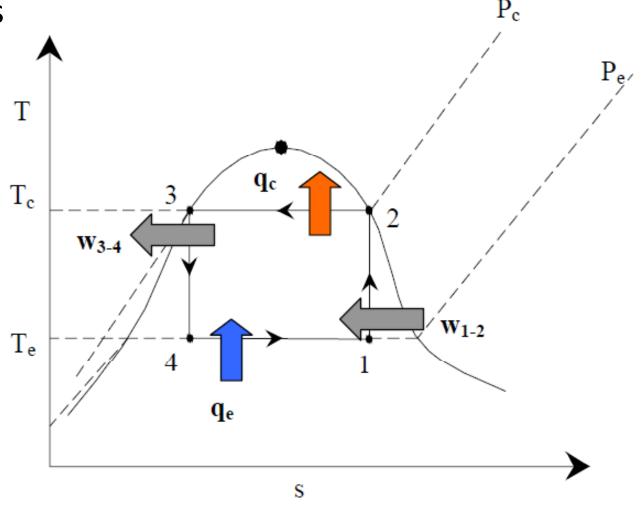
$$w_{net} = q_c - q_e$$

$$q_c = -q_{2-3} = -\int_2^3 T ds$$

$$= T_c (s_2 - s_3)$$

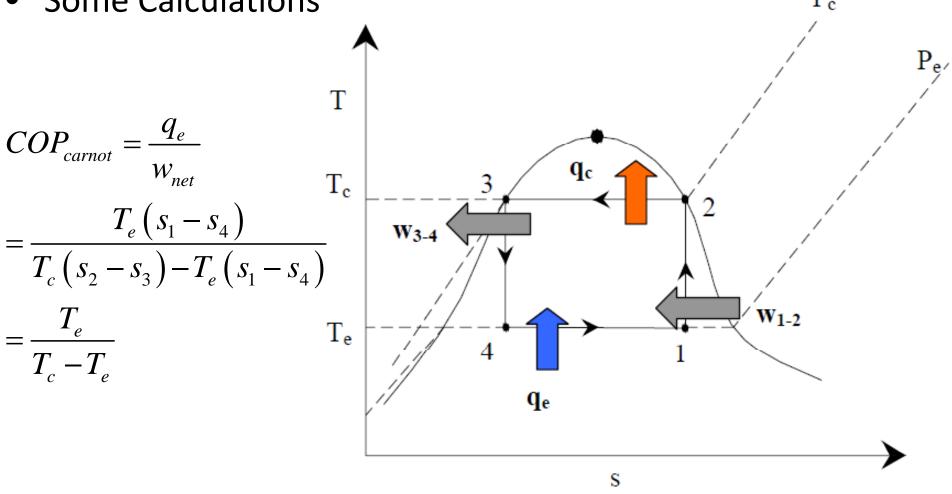
$$q_e = q_{4-1} = \int_4^1 T ds$$

$$= T_e (s_1 - s_4)$$



The Carnot Refrigeration Cycle:

Some Calculations

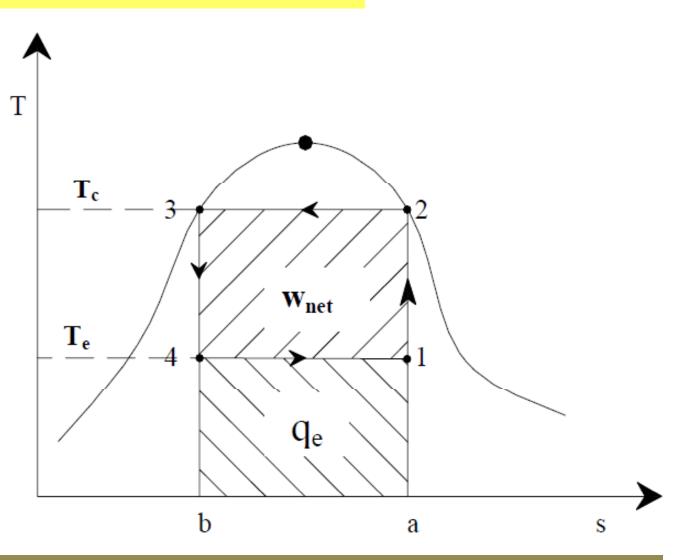


The Carnot Refrigeration Cycle:

- The COP of a Carnot refrigeration cycle is a function of evaporator and condenser temperatures only and is independent of the refrigerant properties
- This gives the maximum COP for any reversible refrigeration system working between the given temperature limits
- COP of a Carnot refrigeration system increases as the evaporator temperature increases or the condenser temperature decreases or both simultaneously

The Carnot Refrigeration Cycle:

Carnot
 refrigeration
 cycle on T – s
 diagram



The Carnot Refrigeration Cycle:

- Consider a domestic refrigerator which produces a refrigeration effect at -25°C and rejects heat at 30°C.
- Then, Carnot COP of this machine would be $COP_{carnot} = 248/(303-248) = 4.5$
- It means that the refrigerator would produce a maximum of 450 W of refrigeration for every 100 W of power consumption

The Carnot Refrigeration Cycle:

- But, in reality, the most popular size 165 L internal volume refrigerators of most manufacturers produce 89 W of refrigeration
- For this, they require an electric motor with a rating of 110 W
- Considering 75% of the running time, the power required is nearly 83 W giving a COP of 1.08 (approx.)

The Carnot Refrigeration Cycle:

- Consider an air conditioner which produces a refrigeration effect at 5°C and rejects heat at 30°C.
- Then, Carnot COP of this conditioner would be $COP_{carnot} = 278/(303-278) = 11.12$
- It means that the conditioner would produce a maximum of 1112 W of refrigeration for every 100 W of power consumption

The Carnot Refrigeration Cycle:

- The 1.5 TR (5.27 kW) air conditioner usually work with a 2kW input supply
- Considering 75% of the running time, the power required is equal to 1.5 kW
- Hence, for this case, COP would come out to be nearly 3.5, which shows that the COP decreases and the power consumption increases if we go on decreasing the refrigeration temperature

Practical Problems with Carnot Refrigeration Cycle:

- With the vapour as refrigerant, isothermal processes of condensation and evaporation are practically achievable processes. However, isentropic compression and isentropic expansion processes have some limitations
- With a reciprocating compressors, wet compression is not found suitable

Compressor

Refrigeration & Air Conditioning

Practical Problems with Carnot Refrigeration Cycle:

- The liquid refrigerant may be trapped in the head of the cylinder and may damage the compressor valves and the cylinder itself
- The liquid refrigerant droplets may wash away the lubricating oil from the walls of the compressor cylinder, thereby increasing the wear
- It is, therefore, desirable to have dry compression instead of a wet compression
- Does it mean that wet compression is not practical?

Practical Problems with Carnot Refrigeration Cycle:

- For certain cases, wet compression is desirable and also practicable with the use of a continuous flow type compressors like a centrifugal or a screw compressor without the presence of valves
- The COP of such systems utilising continuous flow type compressors is more than the systems with a reciprocating compressors
- The power consumption per ton refrigeration for this system is almost 10% less than the conventional one.

Practical Problems with Carnot Refrigeration Cycle:

 The refrigeration unit requires very small power compared to the net work output of the power plant

 Further, the positive work output due to expansion is very small in comparison to the work consumed during compression process

- Because, the concerned work is $-\int v dp$, where v is the specific volume of the substance
- $v_f << v_q$

Expander

Refrigeration & Air Conditioning

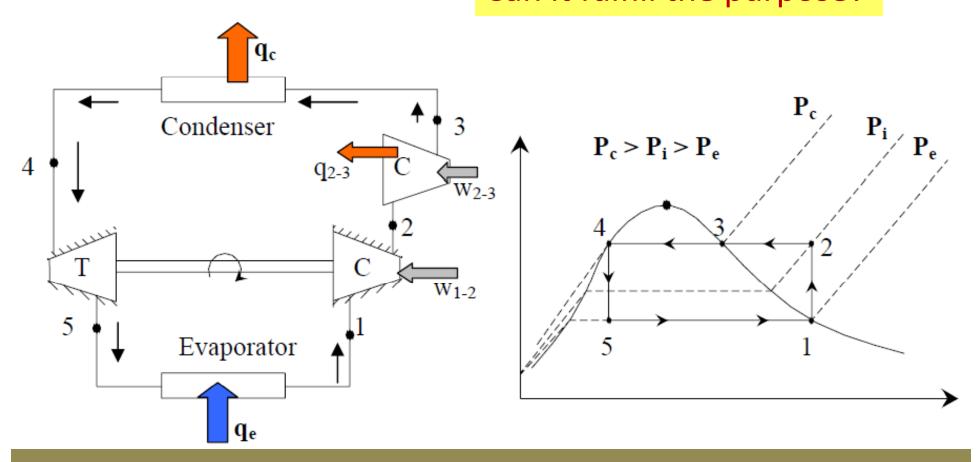
Practical Problems with Carnot Refrigeration Cycle:

 Therefore, the small positive work of turboexpander does not justify the cost of the turbine

- Moreover, considering all the losses encountered in the turbine may result in, even, work consumption for this component
- Also, there are practical difficulties in smoothly expanding a fluid having highly wet vapour
- Therefore, some modifications are required.

Remedy: <u>The Reverse Rankine Cycle</u>

Wet compression can be replaced by a dry compression
 Can it fulfill the purpose?



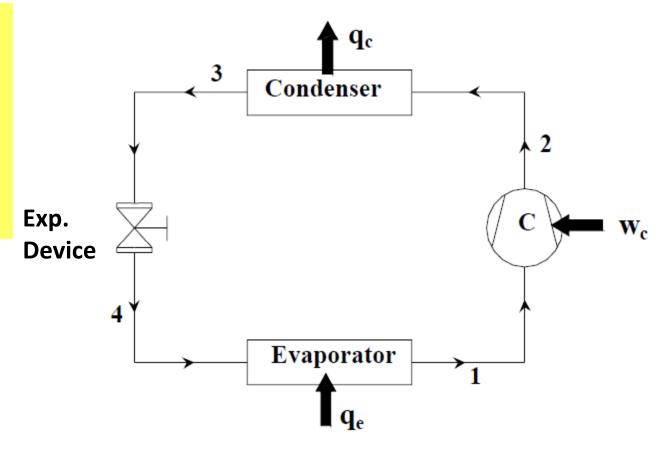
Remedy: The Reverse Rankine Cycle

- Although wet compression can be avoided by using the above method
- But, obtaining isothermal compression for high speed compressors is not practical
- In addition, the use of two compressors is not justified
- Therefore, a strategy must be adopted to get dry compression with the use of a single compressor

The Reverse Rankine Cycle:

Schematic of a standard vapour compression cycle

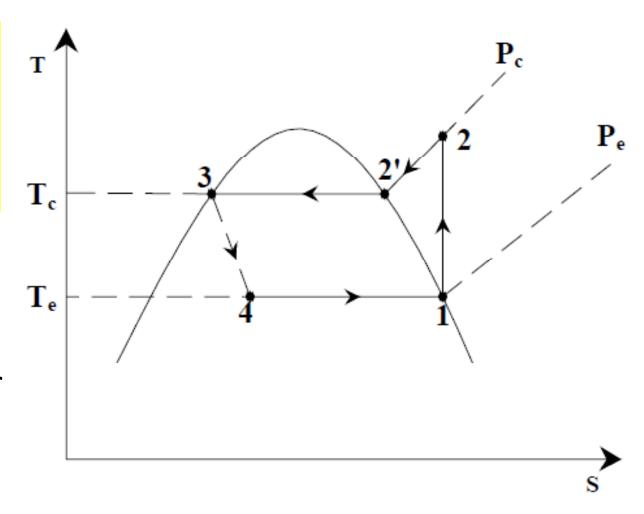
 Expansion takes place in a throttling device



The Reverse Rankine Cycle:

Reverse
Rankine cycle
on T – s
diagram

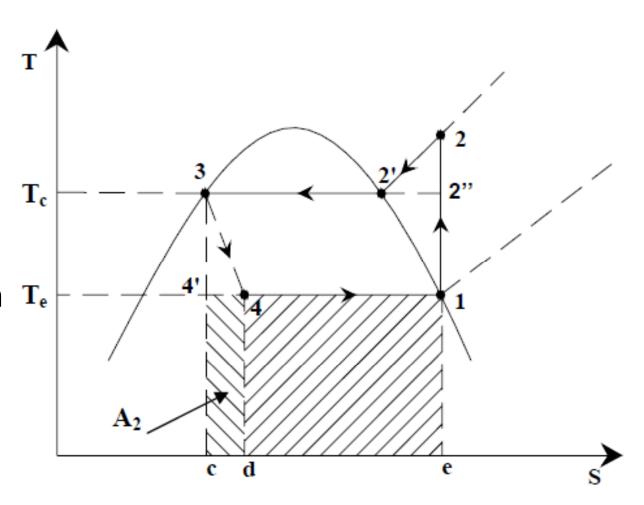
Process 2-3
 takes place in
 the condenser
 (an isobaric
 process)



The Reverse Rankine Cycle:

Comparison with the Carnot Cycle (evaporation)

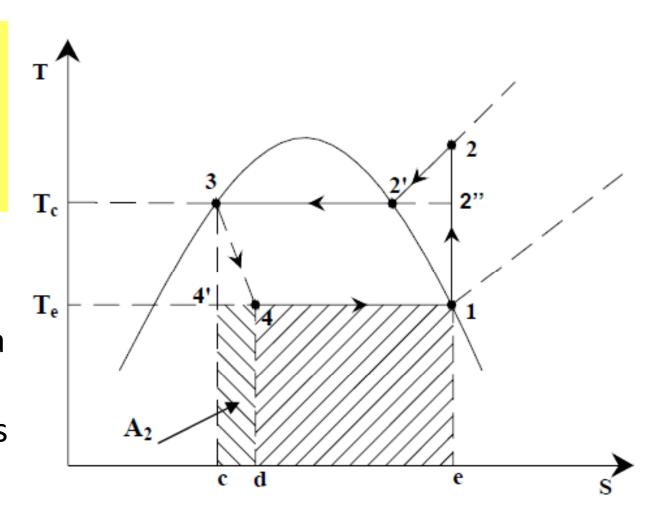
The evaporation T_e
 process is
 reversible for
 both the cycles



The Reverse Rankine Cycle:

Comparison with the Carnot Cycle (evaporation)

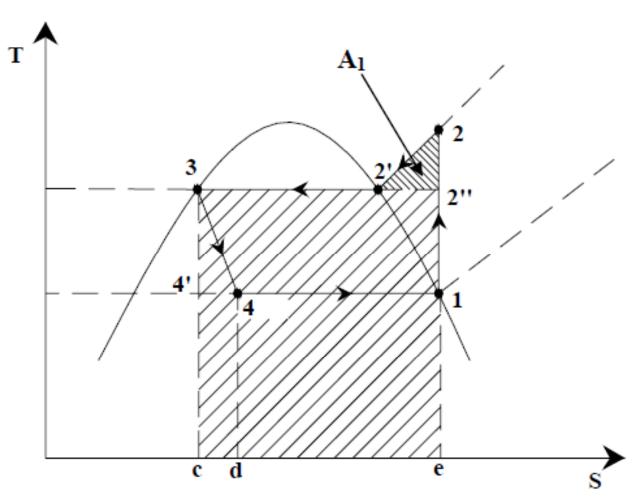
- $q_{e,Carnot} = area$ e-1-4'-c-e
- q_{e,Rankine} = areae-1-4-d-e
- area d-4-4'-c is known as throttling loss



The Reverse Rankine Cycle:

Comparison with the Carnot Cycle (condenser)

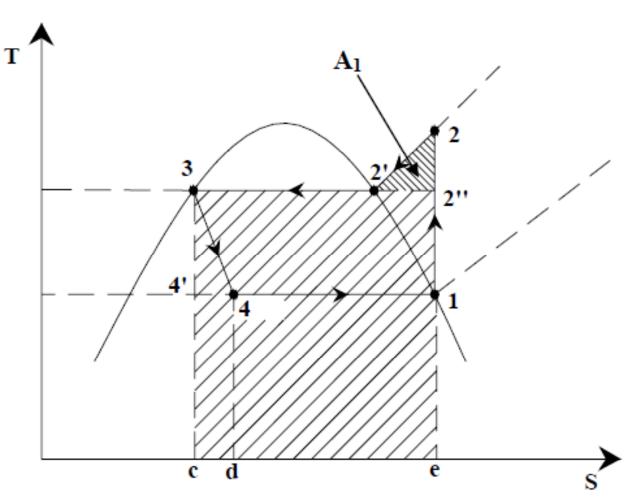
 The heat rejection in case of Rankine cycle increases in comparison to the Carnot cycle



The Reverse Rankine Cycle:

Comparison with the Carnot Cycle (condenser)

- $q_{c,Carnot} = area$ e-2"-3-c-e
- $q_{c,Rankine} = area$ e-2-2'-3-c-e
- area 2"-2-2' is known as superheat horn



The Reverse Rankine Cycle

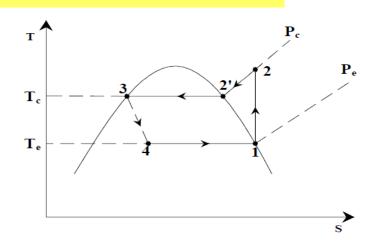
- The superheat horn is due to the replacement of an isothermal compression by an isobaric compression
- Since the heat rejection increases and the refrigeration effect decreases, the net work input increases for the standard vapour compression cycle as compared to the case of a Carnot cycle
- Therefore, COP of a Rankine cycle will be lower than the COP of a Carnot cycle

Calculations of the Reverse Rankine Cycle

Evaporator

Refrigeration capacity

$$\dot{Q}_e = \dot{m}_r \left(h_1 - h_4 \right)$$



where \dot{m}_r is the mass flow rate of the refrigerant $(h_1 - h_4)$ is known as specific refrigeration effect

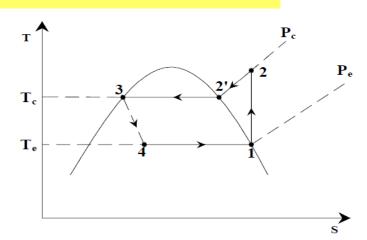
 The evaporator pressure is the saturation pressure corresponding to the evaporator temperature

Calculations of the Reverse Rankine Cycle

Compressor

Power input

$$\dot{W}_c = \dot{m}_r \left(h_2 - h_1 \right)$$

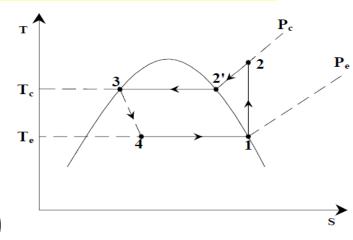


where \dot{m}_r is the mass flow rate of the refrigerant $(h_2 - h_1)$ is known as specific work of compression or simply the work of compression

Calculations of the Reverse Rankine Cycle

Condenser

Heat transfer rate



$$\dot{Q}_c = \dot{m}_r \left(h_2 - h_3 \right)$$

where \dot{m}_r is the mass flow rate of the refrigerant

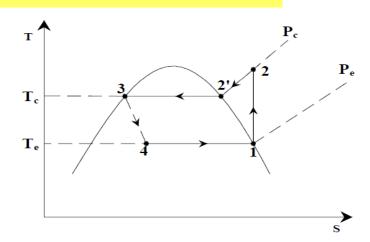
 The condenser pressure is the saturation pressure corresponding to the condenser temperature

Calculations of the Reverse Rankine Cycle

Expansion Device

For an isenthalpic expansion

$$h_3 = h_4$$



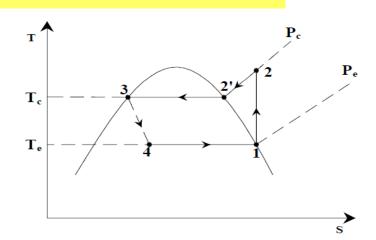
• The exit condition of the refrigerant (i.e. the dryness fraction or quality of the refrigerant, x_4) is determined as follows:

$$h_4 = (1 - x_4)h_{f,e} + x_4h_{g,e} = h_{f,e} + x_4h_{fg,e}$$

Calculations of the Reverse Rankine Cycle

COP of the system

$$COP = \frac{\dot{Q}_e}{\dot{W}_c} = \frac{h_1 - h_4}{h_2 - h_1}$$



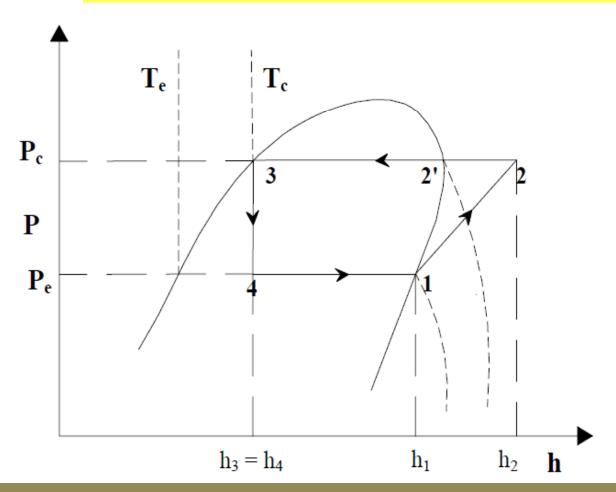
 The mass flow rate of the system can be written in terms of the volumetric flow rate and the specific volume, i.e.

 $\dot{m}_r = \dot{V}_1 / v_1$

where V_1 is the volumetric flow rate at compressor inlet and v_1 is the specific volume at that point

The Reverse Rankine Cycle:

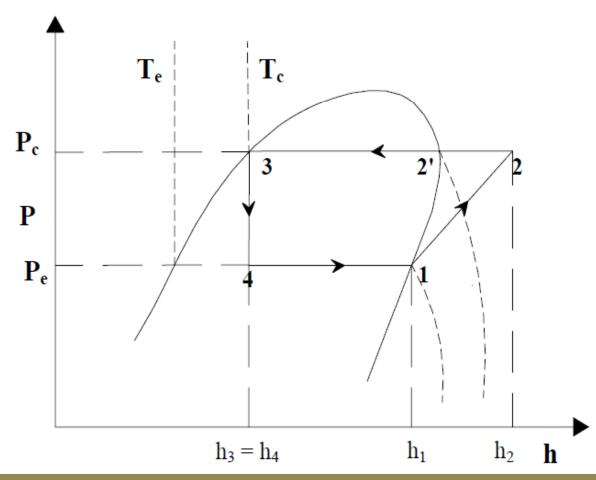
Use of p-h diagram (Mollier diagram)



The evaporator temperature T_e , corresponding to saturation pressure P_e , is also called saturated suction temperature.

The Reverse Rankine Cycle:

Use of p-h diagram (Mollier diagram)



The condensation temperature T_c, corresponding to saturation pressure P_c, is also called saturated discharge temperature. However, the actual discharge temperature from the compressor is T_2 .

Numerical:

- A Freon 12 vapour compression system operating at a condenser temperature of 40°C and an evaporator temperature of -5°C develops 15 tons of refrigeration. Using the p-h diagram for Freon 12, determine;
- a) the mass flow rate of the refrigerant circulated,
- b) the theoretical piston displacement of the compressor and piston displacement per ton of refrigeration,
- c) the theoretical horsepower of the compressor and horsepower per ton of refrigeration,
- d) the heat rejected in the condenser, and
- e) the Carnot COP and actual COP of the cycle.

Numerical:

From the p-h diagram (table) of Freon 12,

$$h_1 = 185.4 \text{ kJ/kg}, v_1 = 0.065 \text{ m}^3/\text{kg}$$

 $h_2 = 208.0 \text{ kJ/kg}$
 $h_3 = h_4 = 74.6 \text{ kJ/kg}$

Ans:

- a) refrigerating effect = $h_1 h_4 = 110 \text{ kJ/kg}$ refrigerant circulated = 15*211/110 = 28.6 kg/min
- b) Theoretical piston discharge = m^*v_1 = 1.855 m^3 /min Piston displacement per ton = 0.124 m^3 /min/TR
- c) Power consumption, $W = m (h_2 - h_1) = 10.77 \text{ kW}$ Theoretical horsepower of the compressor $HP = 10.77*10^3/746 = 14.44$ HP/TR = 0.962

Ans:

- d) Heat rejected = $m(h_2 h_3) = 63.59 \text{ kW}$
- e) Carnot COP = (273 5)/(40 (-5)) = 6
 - COP of the cycle = $(h_1 h_4)/(h_2 h_1) = 4.9$

Numerical:

A refrigeration system using R 12 as refrigerant operates between the pressures 2.5 bar and 9 bar. The compression is isentropic and there is no undercooling in the condenser.
 The vapour is in dry saturated condition at the beginning of the compression. Estimate the theoretical COP. If the actual COP is 0.65 of the theoretical value, calculate the net cooling produced per hour. The refrigerant flow is 5 kg/min.

Take c_p for superheated vapour at 9 bar = 0.64 kJ/kg-K

Numerical:

• The properties are given as:

Pressure (bar)	Saturation temp (°C)	Enthalpy, kJ/kg		Entropy of
		liquid	vapour	saturated vapour, kJ/kg-K
9	36	70.55	201.8	0.6836
2.5	-5	29.62	184.5	0.7001

Ans:

$$s_2 = s_{2'} + c_p \ln(T_2/T_{2'})$$

 $0.7001 = 0.68836 + 0.64 \ln (T_2 / 309)$
 $T_2 = 317 \text{ K}$

$$h_2 = h_{2'} + c_p (T_2 - T_{2'})$$

= 201.8 +0.64 (317-309) = 206.92 kJ/kg

Theoretical COP = $(h_1-h_4)/(h_2-h_1)$ = $(h_1-h_{f3})/(h_2-h_1)$ = 5.1 Actual COP = 0.65*5.1 = 3.315

Net cooling produced per hour = $m*(h_2-h_1)*COP_a = 371.5 \text{ kJ/min}$

Numerical:

- A Freon 12 vapour compression system operating at a condenser temperature of 40°C and an evaporator temperature of -5°C develops 15 tons of refrigeration. Using the p-h diagram for Freon 12, determine;
- a) the mass flow rate of the refrigerant circulated,
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Numerical:

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Ans:

- d) Heat rejected = $m(h_2 h_3) = 63.59 \text{ kW}$
- e) Carnot COP = (273 5)/(40 (-5)) = 6
 - COP of the cycle = $(h_1 h_4)/(h_2 h_1) = 4.9$

Modifications to Reverse Rankine Cycle:

Subcooling and Superheating:

- In actual refrigeration cycle, the temperature difference between the condensing temperature and the heat sink is quite significant, hence, to achieve the better performance, the liquid refrigerant is subcooled in the condenser by increasing the surface area for heat transfer.
- This process is known as subcooling.

Modifications to Reverse Rankine Cycle:

Subcooling and Superheating:

- On the other hand, the heat source temperature is at few degrees higher than the evaporator. Therefore, the refrigerant at the exit of the evaporator is superheated to achieve better performance.
- The heat source could be the surroundings or the refrigerated space (which is at lower temperature than the surroundings).

Modifications to Reverse Rankine Cycle:

Subcooling and Superheating:

- If the superheating is taking place due to the heat addition from the surroundings, the superheating is known as useless superheating.
- On the other hand, if it takes place due to the refrigerated space, the superheating is known as useful superheating, as it increases the refrigerating effect.

Modifications to Reverse Rankine Cycle:

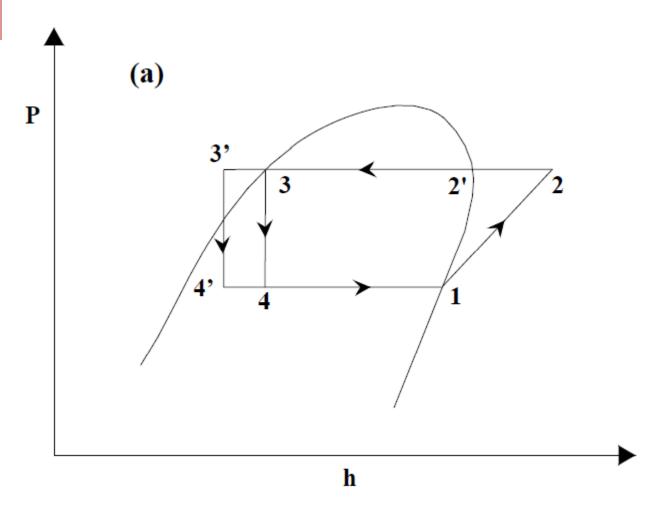
Subcooling:

- Subcooling is advantageous because it increases the refrigeration effect by reducing the throttling loss without any increase in the work input.
- Because of the subcooling, the exit of condenser will definitely be liquid which is again an added advantage for throtting device for its efficient operation.

Modifications to Reverse Rankine Cycle:

Subcooling:

- Process 3-4 is throtting process without the subcooling
- Process 3'-4' is throtting process with the subcooling

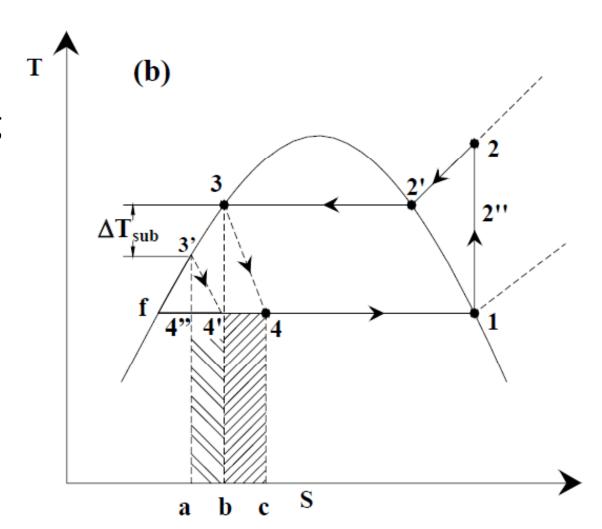


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Modifications to Reverse Rankine Cycle:

Subcooling:

- Throtting loss
 without subcooling
 is area c-4-4'-b
- Throtting loss with subcooling is area b-4'-4"-a
- Net increase in the refrigeartion effect is h_4 - $h_{4'}$ = h_3 - $h_{3'}$



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Modifications to Reverse Rankine Cycle:

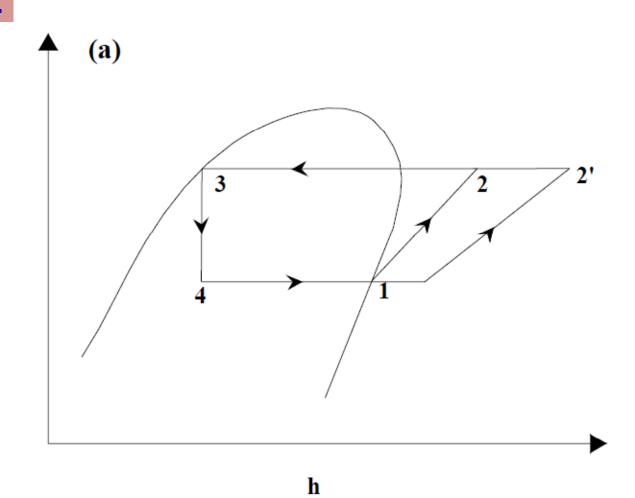
Superheating:

- Superheating may or may not increase the COP of the cycle, because it increases both the refrigeration effect as well as the specific work input to the compressor
- But, a little bit superheating is desirable to ensure that only the vapour can enter the compressor, thereby increasing the life of the compressor
- The exit temperature increases due to superheating

Modifications to Reverse Rankine Cycle:

Superheating:

- Process 1-2 is compression process
 without superheating
- Process 1'-2' is compression process with the superheating

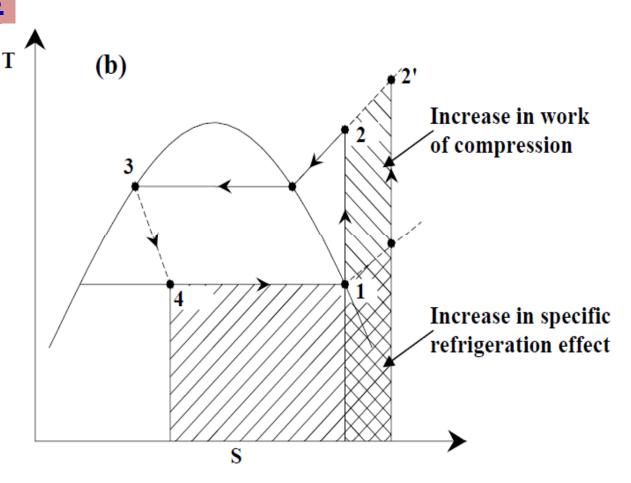


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Modifications to Reverse Rankine Cycle:

Superheating:

- Increase in work of compression is area 1-1'-2'-2
- Increase in the refrigerating effect is the crosshatched area



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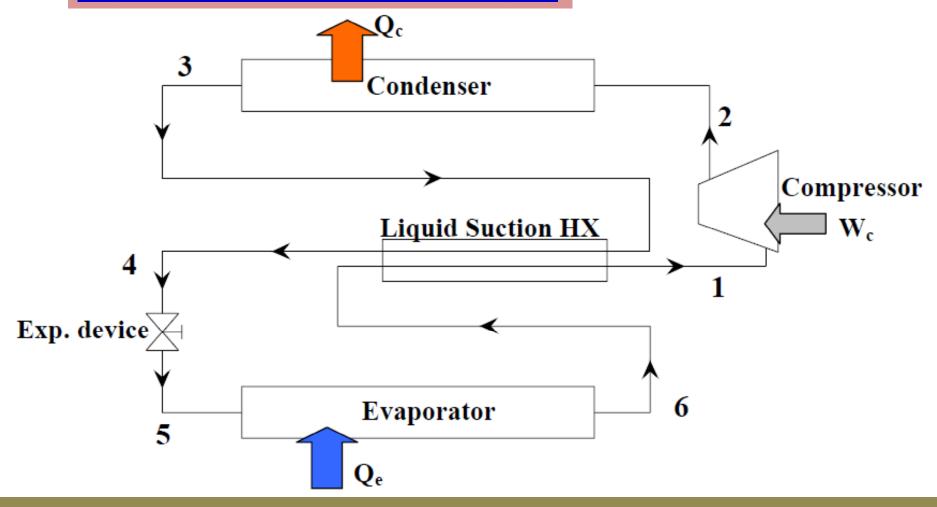
Modifications to Reverse Rankine Cycle:

Using Liquid-Vapour Regenerative Heat Exchanger:

- If we combine superheating of vapour with liquid subcooling, we have a liquid vapour regenerative heat exchanger.
- This is needed because the required superheating and subcooling may not be achieved just by exchanging heat between the refrigerant and the external heat source or sink
- The above system is also called liquid suction heat exchanger

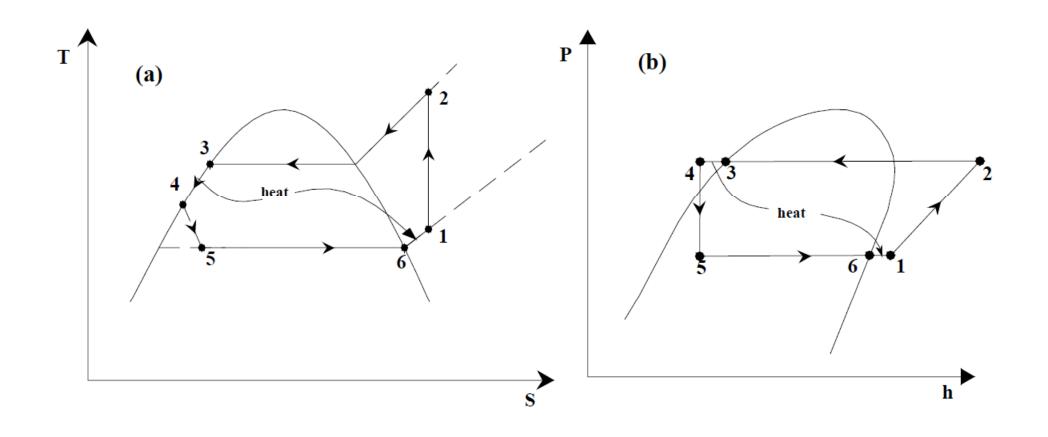
Modifications to Reverse Rankine Cycle:

Liquid-Suction Heat Exchanger:



Modifications to Reverse Rankine Cycle:

Liquid-Suction Heat Exchanger:



Modifications to Reverse Rankine Cycle:

Liquid-Suction Heat Exchanger:

• If we consider 100% heat exchange, then we can write

$$\dot{m}_r \left(h_3 - h_4 \right) = \dot{m}_r \left(h_1 - h_6 \right)$$

$$\Rightarrow \left(h_3 - h_4 \right) = \left(h_1 - h_6 \right)$$

Considering avg. sp. heat value,

$$C_{pl}\left(T_3 - T_4\right) = C_{pv}\left(T_1 - T_6\right)$$
$$\Rightarrow \left(T_3 - T_4\right) < \left(T_1 - T_6\right)$$

Numerical:

 A Freon 12 simple saturation cycle operates at temperatures of 35°C and -15°C for the condenser and evaporator respectively, develops 15 tons of refrigeration. Determine the COP and HP/TR of the system.

If a liquid-vapour heat exchanger is installed in the system, with the temperature of the vapour leaving the heat exchanger at 15°C, what will be the change in the COP and HP/TR?

Numerical:

- An ammonia refrigerating machine has working temperatures of 35°C in the condenser and -15°C in the evaporator. Assume two cases;
 - (a) dry compression
 - (b) wet compression with exit condition of saturated vapour

Calculate for each, the following;

- (i) theoretical piston displacement per ton refrigeration,
- (ii) theoretical horsepower per ton refrigeration, and
- (iii) coefficient of performance

Use only the table for ammonia.

Take
$$c_{pv} = 2.8 \text{ kJ/kg-K}$$
, $c_{pf} = 4.6 \text{ kJ/kg-K}$

Ans:

From the table for ammonia

$$h_1 = h_{g(-15C)} = 1443.9 \text{ kJ/kg and } s_1 = s_{g(-15C)} = 5.8223 \text{ kJ/kg-K}$$

 $h_4 = h_3 = h_{f(35C)} = 366.1 \text{ kJ/kg}, v_1 = v_{g(-15C)} = 0.509 \text{ m}^3/\text{kg}$

Also,

from the table, entropies and enthalpies of vapour saturated at 35°C and superheated by 50K and 100K are, respectively,

$$s_{g(35C)}$$
=5.2086, $s_{(50K)}$ =5.6466, $s_{(100K)}$ =5.9806 (all in kJ/kg-K) $s_{f(35C)}$ =0.7426 kJ/kg-K

$$h_{g(35C)}=1488.6$$
, $h_{(50K)}=1633.6$, $h_{(100K)}=1703$ (all in kJ/kg)

Ans:

- (a) For dry compression $q_e = h_1 h_4 = 1443.9 366.1 = 1077.8 \text{ kJ/kg}$
- Refrigerant circulation rate per ton of refrigeration m = (3.5167*60)/1077.8 kg/min = 0.1957 kg/min
- (i) Theoretical piston displacement per ton = $mv_1 = 0.1957*0.509 = 0.0996 \text{ m}^3/\text{min} = 5.975 \text{ m}^3/\text{h}$
- (ii) Theoretical HP per ton $w_c = h_2 h_1 = 1703 1443.9 = 259.1 \text{ kJ/kg}$ HP/ton = mw/746 = 0.1957*259.1/(0.746*60) = 1.13
- (iii) COP = $q_e/w_c = 1078.5/259.1 = 4.16$

Ans:

(b) For wet compression

$$s_{2'}=s_{g(35C)}=s_{1'}=5.2086 \text{ kJ/kg-K}$$

 $x_{1'}=(5.2086-0.7426)/(5.8223-0.7426)=0.88$
 $h_{1'}=131.4+0.88(1443.9-131.3)=1286.1 \text{ kJ/kg}$
 $v_{1'}=0.00152+0.88(0.509-0.00152)=0.448 \text{ m}^3/\text{kg}$
 $q_e=h_{1'}-h_4=1286.1-366.1=920 \text{ kJ/kg}$
 $w_c=h_{2'}-h_{1'}=1488.6-1286.1=202.5 \text{ kJ/kg}$

Ans:

- (b) For wet compressionRefrigerant circulation rate per ton of refrigerationm = (3.5167*60)/920 kg/min = 0.229 kg/min
- (i) Theoretical piston displacement per ton = $mv_1 = 0.229*0.448 = 0.1026 \text{ m}^3/\text{min} = 6.155 \text{ m}^3/\text{h}$
- (ii) Theoretical HP per ton HP/ton = mw/746 = 0.229*202.5/(0.746*60) = 1.03
- (iii) COP

$$= q_e/w_c = 920/202.5 = 4.54$$

Therefore, it can be concluded that higher COP and lower power consumption is obtained with wet compression.

Numerical:

A vapour compression refrigerator uses R 12 as refrigerant and the liquid evaporates in the evaporator at -15°C. The temperature of this refrigerant at the delivery from the compressor is 15°C when the vapour is condensed at 10°C. Find the COP if (i) there is no undercooling, and (ii) the liquid is cooled by 5°C before expansion by throttling.

Take $c_{pv} = 0.64 \text{ kJ/kg-K}$, $c_{pl} = 0.94 \text{ kJ/kg-K}$

From the table for R 12

Saturation temp (°C)	Enthalpy, kJ/kg		Entropy, kJ/kg-K	
	liquid	vapour	liquid	vapour
10	45.4	191.76	0.1750	0.6921
-15	22.3	180.88	0.0904	0.7051

Ans:

From the table for R 12

$$\begin{split} &h_{1'} = h_{g(\text{-}15\text{C})} \text{=} 180.88 \text{ kJ/kg and } s_{1'} \text{=} s_{g(\text{-}15\text{C})} \text{=} 0.7051 \text{ kJ/kg-K} \\ &h_{f(\text{-}15\text{C})} \text{=} 22.3 \text{ kJ/kg and } s_{f(\text{-}15\text{C})} \text{=} 0.0904 \text{ kJ/kg-K} \\ &h_{4} \text{=} h_{3} \text{=} h_{f(10\text{C})} \text{=} 45.4 \text{ kJ/kg,} \\ &h_{2'} \text{=} h_{g(10\text{C})} \text{=} 191.76 \text{ kJ/kg,} s_{2'} \text{=} s_{g(10\text{C})} \text{=} 0.1750 \text{ kJ/kg-K} \end{split}$$

Dryness fraction at 1,

$$s_2 = s_{2'} + c_{pv} * ln(T_2/T_{2'}); T_2 = 288 \text{ K}, T_{2'} = 283 \text{ K}$$

= 0.6921+0.64 ln(288/283) = 0.7034 kJ/kg-K
 $s_1 = s_{f(-15C)} + x_1 * (s_{g(10C)} - s_{f(-15C)}) = 0.0904 + 0.6147 x_1 \text{ K}$
 $s_1 = s_2 \text{ gives}, x_1 = 0.997$

Ans:

Without subcooling,

```
Enthalpy at 1, h_1 = h_{f(-15C)} + x_1^* (h_{g(-15C)} - h_{f(-15C)}) = 22.3 + 0.997(180.88 - 22.3) h_1 = 180.4 \text{ kJ/kg} Enthalpy at 2, h_2 = h_{g(10C)} + c_{pv}^* (T_2 - T_{2'}) = 191.76 + 0.64^* (288 - 283) h_2 = 194.96 \text{ kJ/kg} COP = (h_1 - h_3)/(h_2 - h_1) = (180.4 - 45.4)/(194.96 - 180.4) = 9.27
```

Ans:

With subcooling,

Enthalpy at 3' (subcooled point),
$$h_{3'} = h_3 - c_{pl} * (T_3 - T_{3'}) = 45.4 - 0.94 * 5$$
$$h_{3'} = 40.7 \text{ kJ/kg}$$
$$COP = (h_1 - h_{3'}) / (h_2 - h_1) = (180.4 - 40.7) / (194.96 - 180.4) = 9.59$$

Numerical:

- a) A Freon 12 simple saturation cycle operates at temperatures of 35°C and -15°C for the condenser and evaporator respectively. Determine the COP and HP/TR of the system.
- b) If a liquid vapour heat exchanger is installed in the system, with the temperature of the vapour leaving the heat exchanger at 15°C, what will be the change in the COP and HP/TR?

Take
$$c_{pv} = 0.64 \text{ kJ/kg-K}$$
, $c_{pl} = 0.94 \text{ kJ/kg-K}$

From the table for R 12

Saturation temp (°C)	Enthalpy, kJ/k	Entropy, kJ/kg-K	
	liquid	vapour	vapour
35	69.5	201.5	0.6921
-15		180.88	0.7051

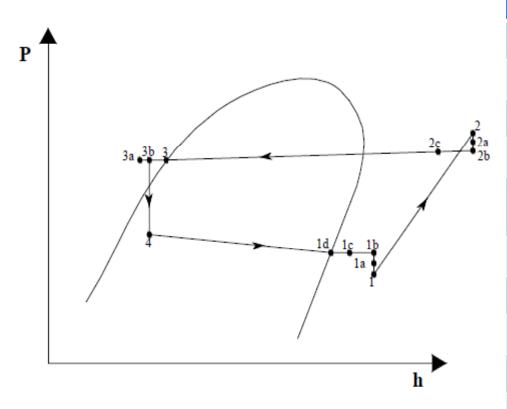
Ans:

(a)
$$COP = 4.09$$

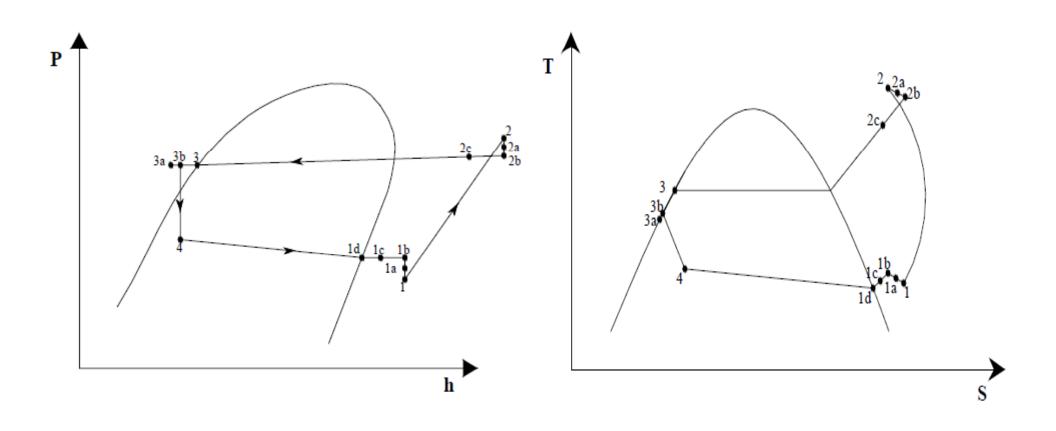
 $HP/TR = 4.761/COP = 1.16$

Remarks: Although, the increase in COP is not that much significant but superheat improves the performance by ensuring complete vaporisation of liquid refrigerant.

- Actual Refrigeration cycle differs in many ways from the reverse Carnot and reverse Rankine cycles
- There will be drops in pressures at the condenser, evaporator, and the pipelines
- Also, there could be heat losses or gains depending on the temperature difference between the refrigerant and the surroundings
- Further, the compression process will be a polytropic process with friction and heat transfer



Process	States
Evaporator pressure drop	4 - 1d
Vapour superheating (evaporator)	1d – 1c
Vapour superheating (suction line)	1c – 1b
Suction line pressure drop	1b – 1a
Pressure drop across the compressor suction valve	1a – 1
Polytropic compression	1-2
Pressure drop across the discharge valve	2 – 2a
Pressure drop (delivery line)	2a – 2b
Heat loss and desuperheating (delivery)	2b – 2c
Pressure drop (condenser)	2c – 3
Subcooling (condenser or subcooler)	3 – 3a
Heat gain (liquid line)	3a – 3b



Actual Refrigeration Cycle:

• The pressure drop in the evaporator is critical as compared to the pressure drop in the condenser

Evaporator pressure drop

- The pressure drop in the evaporator is quite large because of the two reasons: (i) frictional pressure drop and (ii) momentum pressure drop
- As the evaporation proceeds, the volume increases, and hence velocity must also increase.
- This increases kinetic energy at the cost of enthalpy which results from a pressure drop

Actual Refrigeration Cycle:

 The pressure drop in the evaporator is critical as compared to the pressure drop in the condenser

Condenser pressure drop

- The pressure drop in the evaporator is not significant because the frictional pressure drop is positive and the momentum pressure drop is negative.
- There could also be a pressure gain because of the condensation or the decrease in kinetic energy which increases the enthalpy of the system

- Due to various pressure drops, the capacity of the system decreases and the unit power consumption (per unit of refrigeration) increases.
- The decrease in pressure at the evaporator side also increases the specific volume of the vapour at the compressor inlet. This not only increases the work of compression but also affects the life of the compressor.

Numerical:

 The following data were obtained from a test on a twin cylinder, single acting 15 cm by 20 cm, 320 rpm compressor ammonia refrigeration plant.

Temperature of refrigerant:

After expansion valve, entering brine cooler	-25 ⁰ C
Leaving brine cooler	-18 ⁰ C
Entering compressor	-8 ⁰ C
Leaving compressor	140°C
Entering condenser	130°C
Leaving condenser	30° C
Entering expansion valve	32 ⁰ C

Pressure of refrigerant:

Compressor of	dischar	ge and	conde	enser	13.5	bar
---------------	---------	--------	-------	-------	------	-----

Compressor suction 1.324 bar

Brine:

Brine circulation rate	102 kg/min
------------------------	------------

Temperature drop of brine in cooler 7°C

Specific heat of brine 3.14 kJ/kg-K

Input to Motor	18.8 kW
----------------	---------

Motor efficiency at this load 92%

Compressor jacket cooling water 5 kg/min

Temperature rise of jacket water 8.9°C

- Show the thermodynamic states at various points on p-h and T-s diagrams and calculate:
- a) Refrigerating capacity in TR assuming 2% loss of useful refrigeration by heat gain from room in brine cooler.
- b) Ammonia circulated
- c) Compressor IHP and mechanical efficiency
- d) Compressor volumetric efficiency
- e) COP of the cycle

Ans:

```
The various state points are:

Point 3b: t=32^{\circ}\text{C} p=13.5 bar h=351.5 kJ/kg (at 32°C)

Point 4: t=-25^{\circ}\text{C} p=1.516 bar (sat.) h=351.5 kJ/kg

Point 1d: t=-28^{\circ}\text{C} (sat. at 1.324 bar)

Point 1c: t=-18^{\circ}\text{C} p=1.324 bar h=1451.3 kJ/kg

Point 1b: t=-8^{\circ}\text{C} p=1.324 bar h=1473.6 kJ/kg

Point 1: t=-8^{\circ}\text{C} h=1451.3 kJ/kg (=h<sub>1b</sub>)

Point 2, 2a, 2b: t=140^{\circ}\text{C} p=13.5 bar h=1751 kJ/kg

Point 2c: t=130^{\circ}\text{C} p=13.5 bar h=1751 kJ/kg
```

Point 3a: $t=30^{\circ}C$ p=13.5 bar h=341.8 kJ/kg

Point 3: $t=35^{\circ}C$ (sat. at 13.5 bar) p =13.5 bar

Ans:

- a) Refrigerating capacity (from brine)
 Q₀= 102*3.14*7*1.02(due to 2% loss) = 2287 kJ/min
 = 2287/211 = 10.84 TR
- b) Refrigerating effect

$$q_0 = h_{1c} - h_4 = 1451.3 - 351.5 = 1099.8 \text{ kJ/kg}$$

Ammonia circulated

$$m. = Q_0/q_0 = 2287/1099.88 = 2.08 \text{ kg/min}$$

c) From 2 to 2b, it is an isenthalpic process. It has been assumed that enthalpy is a function of temperature only and hence $t_{2b} = t_2$.

Enthalpy increase during compression = $h_2 - h_1$ = 1777 - 1473.6 = 303.4 kJ/kg

Ans:

Total enthalpy increase during compression

$$= m.*303.4 = 631.1 kJ/min$$

Heat to jacket water

$$Q_i = 5*4.1868*8.9 = 186.3 \text{ kJ/min}$$

Compressor work = Total enthalpy increase + Q_i

$$= 631.1 + 186.3 = 817.4 \text{ kJ/min} = 13.62 \text{ kW}$$

Compressor IHP = $13.62*10^3/746 = 18.26$

Compressor input = Power consumption of motor * motor efficiency

Compressor BHP = $17.3*10^3/746 = 23.19$

Ans:

Mechanical efficiency

$$= IHP/BHP = 0.787$$

d) Gas enters evaporator at 1b with

$$v_{1b} = v_g * (273+(-8)) / (273+(-28)) = 0.88 * (265/245) m3/kg = 0.952 m3/kg$$

Actual volume flow rate of refrigerant

$$V_{1b} = 2.08 * 0.952 = 1.99 \text{ m}^3/\text{min}$$

Compressor piston displacement

$$V_p$$
. = (3.14/4) D² L N (2)
= (3.14/4)(0.15)² (0.2) (320) (2) = 2.62 m³/min

Volumetric efficiency = V_p = 0.882 (88.2%)

e)
$$COP = Q_0./W. = 2287/817.4 = 2.798$$

Multipressure Systems

Outline of the lecture:

Limitations of a single stage system

Flash Gas Removal

Intercooling

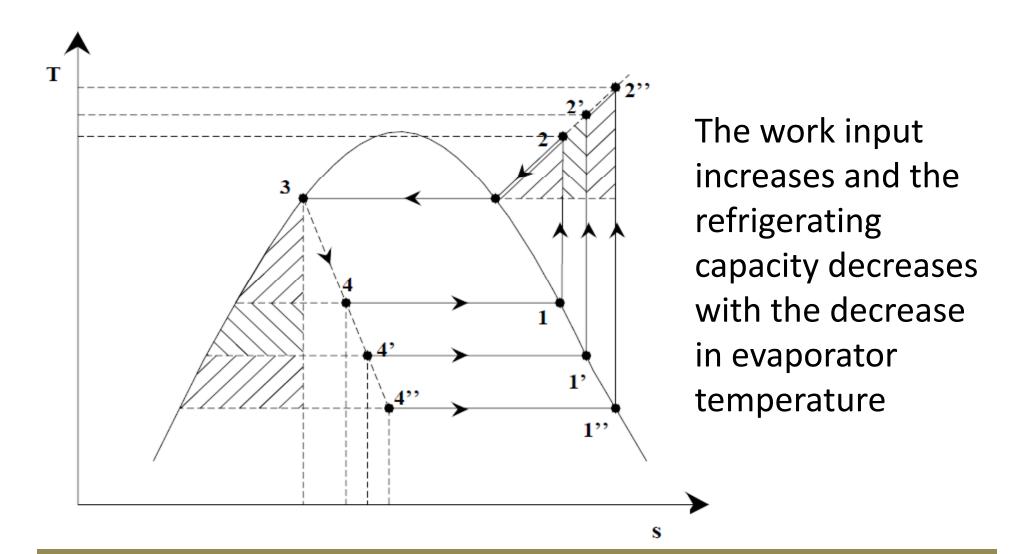
Limitations of a single stage system:

- A single stage system consists of one low side pressure (evaporator) and one high side pressure (condenser)
- For the low temperature difference between the evaporator and the condenser (temperature lift), one can use single stage system
- But, for large temperature lift, use of single stage is not justified

Limitations of a single stage system:

- For certain cases, the temperature lift can be quite high. For example, in frozen food industries the required evaporator can be as low as -40°C, while in chemical industries temperatures as low as -150°C may be required for liquefaction of gases.
- With the increase in temperature lift, the single stage system becomes inefficient and impractical for the following reasons:

Single Stage Systems with Varying Evaporator Conditions:



Necessity of multistage system:

- Therefore, it is advised to use multistage system for higher temperature lift
- Generally, for fluorocarbon and ammonia based refrigerators a single stage is used for an evaporator temperature of -30°C, two stage is used for an evaporator temperature of -60°C and more for temperature below -60°C
- Also, there are certain systems which require refrigeration at different temperatures

Necessity of multistage system:

- For example, in a dairy plant, refrigeration may be required at -30°C for making ice cream and at 2°C for chilling milk
- In a typical food processing plant, cold air may be required at -30°C for freezing and at 7°C for cooling of food products.
- For these cases, it is advantageous to use multievaporator systems, where one evaporator provides a temperature of -30°C while the second evaporator some higher temperature as needed.

Multistage systems:

- A multistage system is a system with two or more number of low-side pressures. It can be classified into:
- i. Multi-compression systems
- ii. Multi-evaporator systems
- iii. Cascade systems
- The two concepts which are used for these systems are: (i) flash gas removal and (ii) intercooling

Flash gas removal using flash tank:

- For high temperature lift, the inlet to the evaporator contains more vapour in a single stage system. This vapour is called as the flash gas and it develops during throttling process
- This flash gas has to be compressed to the condenser pressure because it does not contribute to the refrigeration effect (as it is already in a gaseous phase)
- Also, it increases the pressure drop across the evaporator. This deteriorates the performance.

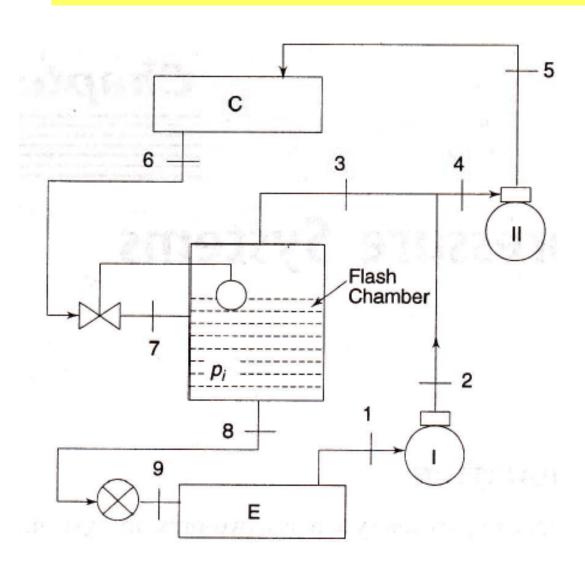
Flash gas removal using flash tank:

- One way is to remove the flash gas as soon as it forms and recompress it to condenser pressure
- However, the continuous removal of flash gas and recompressing it immediately is a difficult task

How to improve the performance of the system?

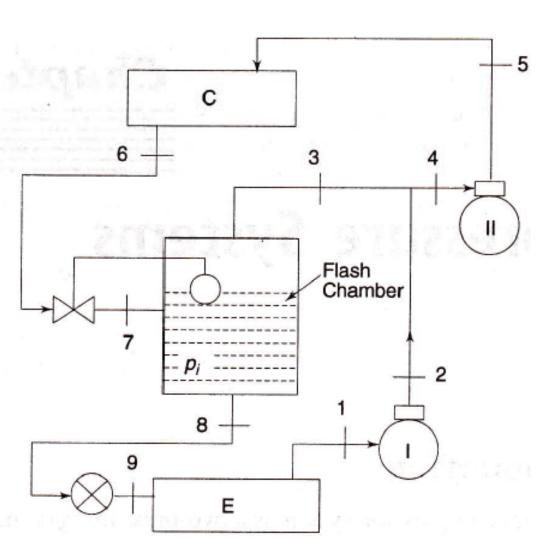
 Performance may be improved if the flash gas is removed at an intermediate pressure using flash tank

Flash gas removal using flash tank: Working principle



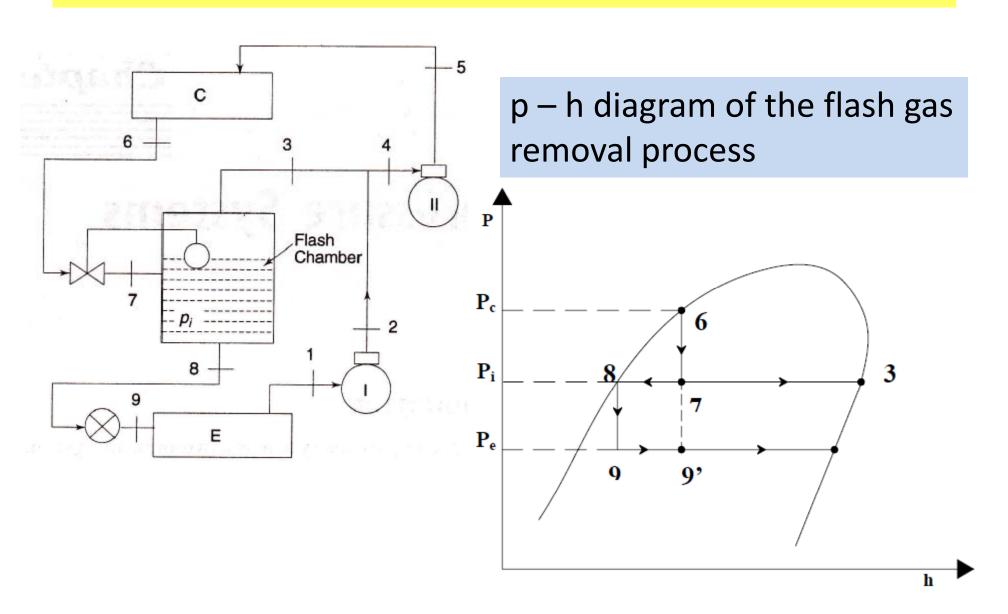
A flash tank is a
 pressure vessel
 where the liquid
 refrigerant and
 vapour are separated
 at an intermediate
 pressure

Flash gas removal using flash tank: Working principle

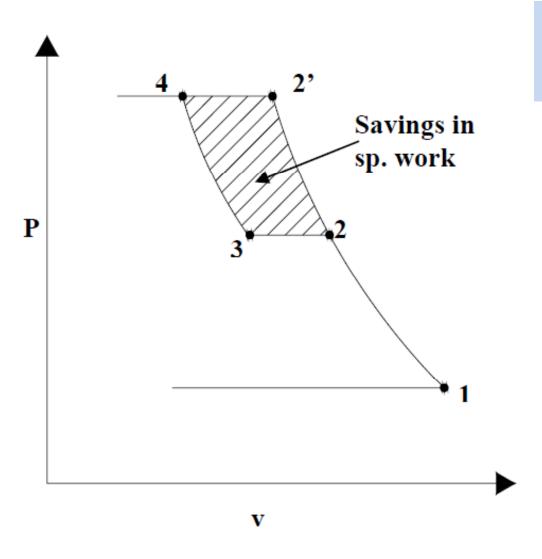


- The refrigerant from the condenser is first expanded to an intermediate pressure, p_i.
- The saturated liquid at point 8 is fed to the evaporator after throttling
- The saturated vapour is again compressed to the condenser pressure

Flash gas removal using flash tank: Working principle



Effect of Intermediate Intercooling

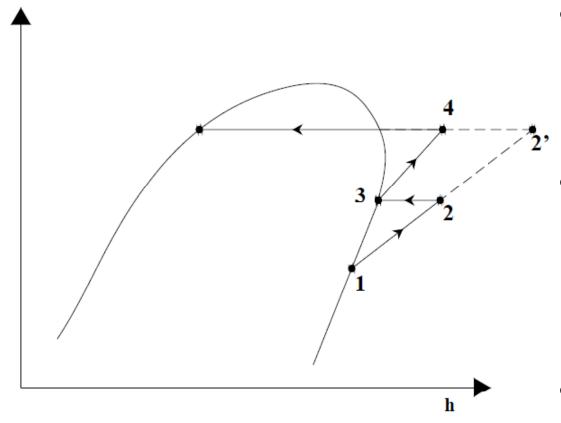


Intercooling in a two-stage compression

- The intercooler is placed between the low-stage compressor and the highstage compressor
- Instead of compressing vapour from state 1 to state 2' directly, it is compressed in stages which causes a reduction in work input

Effect of Intermediate Intercooling

p – h diagram



- The saving in work can also be noticed through p – h diagram
- As the isentropes diverge while moving away from the saturated vapour line
- Therefore,
 h₄ h₃ < h₂, h₂.

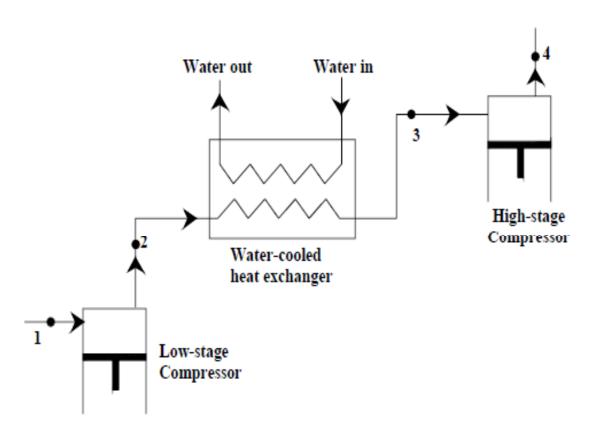
Types of Intercooling:

- Intercooling can be achieved in following ways:
- i. Using water cooled heat exchanger

ii. Refrigerant in the flash chamber

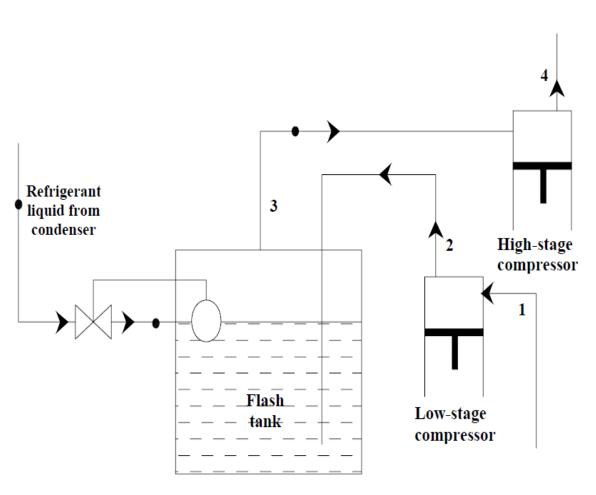
Combination of the both

Intercooling using external water cooled heat exchanger



- This method may not always be possible because it depends on the availability of sufficiently cold water to which refrigerant can reject heat
- Therefore, it is mostly used in air compressors

Intercooling using flash chamber

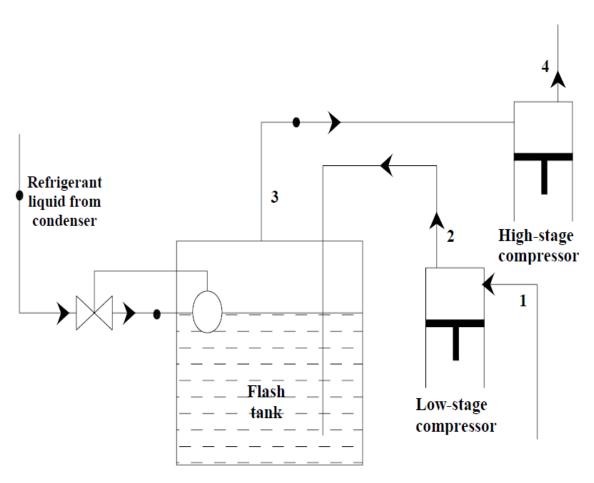


 Intercooling using the refrigerant from the condenser in the flash tank may or may not decrease the input power

Why?

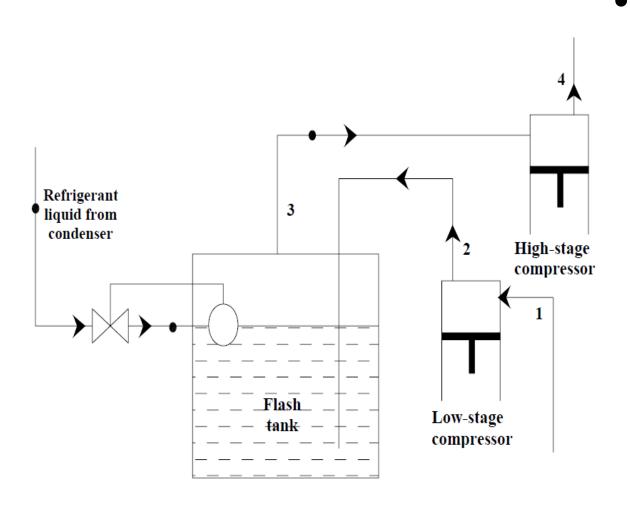
 as it depends upon the nature of the refrigerant being used

Intercooling using flash chamber



- The heat rejected to the liquid refrigerant creates more vapour which needs to be compressed in the high-stage compressor
- This increases the mass flow rate through the highstage compressor

Intercooling using flash chamber



Whether the power input to the system decreases or not depends on the relative magnitude of the increase in power consumption due to the increased mass flow rate and the decrease in work input due to intercooling

Intercooling using flash chamber

 Intercooling using flash chamber is usually effective for ammonia. However, the input power marginally increases for refrigerants R11 and R12. Hence, it is not effective for R11 and R12 refrigerants.

 But, incorporation of this method reduces the discharge temperature of the compressor which in turn enhances the compressor lubrication and increases its life.

Choice of Intermediate Pressure:

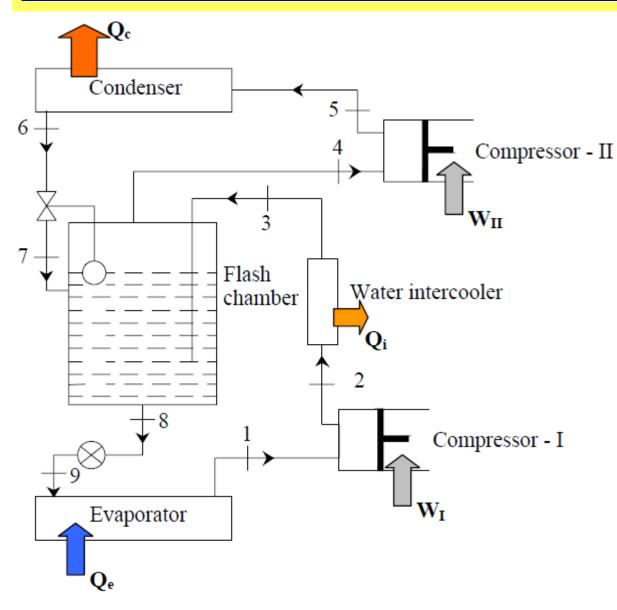
 For air compressors, the optimal intermediate pressure for minimum work input is decided by the equal pressure ratio between the stages, i.e.

$$(p_i)_{opt} = sqrt (p_e * p_c)$$

 The above relation holds good for an ideal gas. For refrigerant a correction factor is suggested which is written as

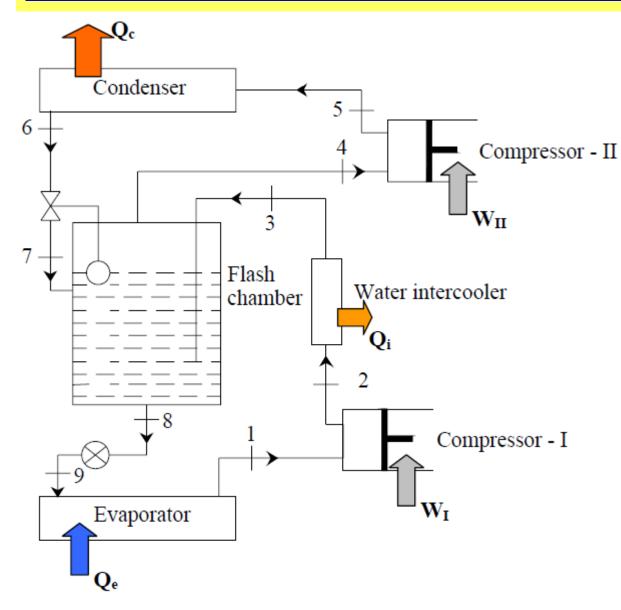
$$(p_i)_{opt} = sqrt (p_e * p_c * T_c/T_e)$$

Multi stage system with flash gas removal and intercooling



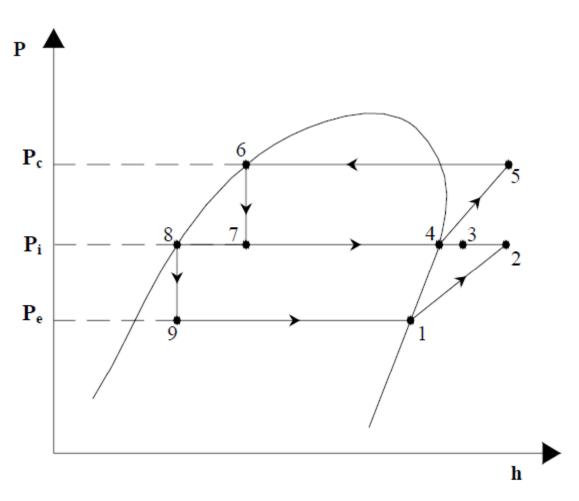
- The superheated refrigerant vapour is intercooled using water cooled heat exchanger and the flash tank.
- The superheated refrigerant after the heat exchanger bubbles through refrigerant liquid in the flash tank

Multi stage system with flash gas removal and intercooling



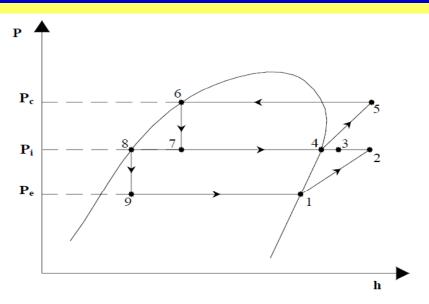
- It is assumed that the vapour gets completely desuperheated and emerges out as saturated vapour at stage 4.
- But, practically, this is not possible

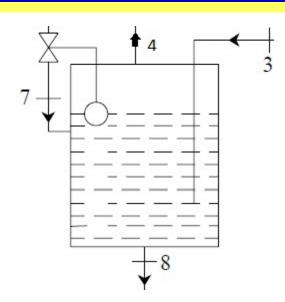
Multi stage system with flash gas removal and intercooling



- The performance of the system is worked out by balancing the mass and energy of individual components
 - It is assumed that the K.E. and P.E. changes are negligible and the flash tank is insulated

Multi stage system with flash gas removal and intercooling

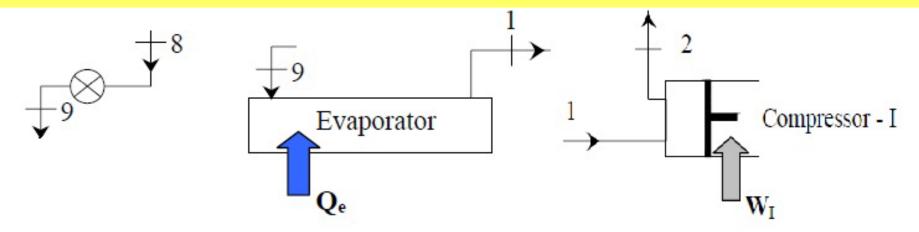




Mass and energy balance across the flash tank gives,

$$\dot{m}_3 + \dot{m}_7 = \dot{m}_4 + \dot{m}_8$$
 $\dot{m}_3 h_3 + \dot{m}_7 h_7 = \dot{m}_4 h_4 + \dot{m}_8 h_8$

Multi stage system with flash gas removal and intercooling



From mass and energy balance across expansion valve

$$\dot{m}_8 = \dot{m}_9, \qquad h_8 = h_9$$

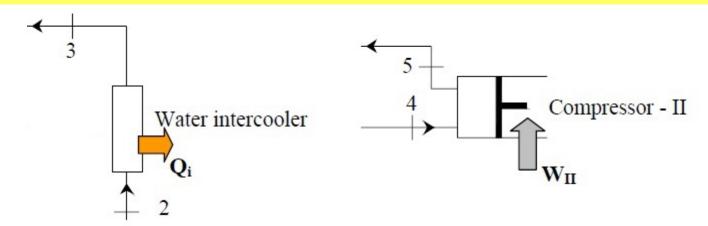
From mass and energy balance across evaporator

$$\dot{m}_9 = \dot{m}_1, \qquad \mathbf{Q}_e = \dot{m}_1 \left(h_1 - h_9 \right)$$

From mass and energy balance across compressor I

$$\dot{m}_9 = \dot{m}_1 = \dot{m}_I$$
, $W_I = \dot{m}_I (h_2 - h_1)$

Multi stage system with flash gas removal and intercooling



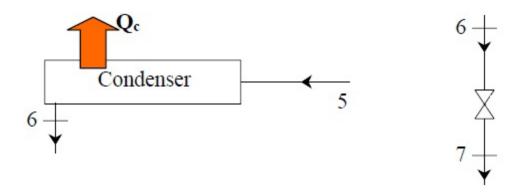
From mass and energy balance across intercooler

$$\dot{m}_2 = \dot{m}_3 = \dot{m}_I, \qquad Q_I = \dot{m}_I (h_2 - h_3)$$

From mass and energy balance across compressor II

$$\dot{m}_4 = \dot{m}_5 = \dot{m}_{II}, \qquad W_{II} = \dot{m}_{II} (h_5 - h_4)$$

Multi stage system with flash gas removal and intercooling



From mass and energy balance across condenser

$$\dot{m}_5 = \dot{m}_6 = \dot{m}_{II}, \qquad \mathbf{Q}_c = \dot{m}_{II} \left(h_5 - h_6 \right)$$

From mass and energy balance across float valve

$$\dot{m}_6 = \dot{m}_7 = \dot{m}_{II}, \qquad h_6 = h_7$$

Multi stage system with flash gas removal and intercooling

From the above analysis, we can write for flash tank

$$\dot{m}_4 = \dot{m}_7 = \dot{m}_{II}, \qquad \dot{m}_3 = \dot{m}_8 = \dot{m}_{II}$$

And, therefore

$$\dot{m}_{II} = \dot{m}_I \left(\frac{h_3 - h_8}{h_4 - h_7} \right)$$

It can be seen that the excessive vapour flow through compressor-II may be reduced by reducing the enthalpy of the refrigerant, h_3 , coming from the water intercooler

Multi stage system with flash gas removal and intercooling

The amount of additional vapour formed due to de -superheating of the refrigerant vapour from the water cooled intercooler is given by

$$\dot{m}_{gen} = \dot{m}_I \left(\frac{h_3 - h_4}{h_4 - h_8} \right)$$

Thus, the vapour generated, \dot{m}_{gen} , will be zero if the refrigerant vapour is completely de-superheated in the water cooled intercooler itself. However, this is not practical in practice.

Multi stage system with flash gas removal and intercooling

The COP of this system is given by

$$COP = \frac{Q_e}{W_I + W_{II}} = \frac{\dot{m}_I (h_1 - h_9)}{\dot{m}_I (h_2 - h_1) + \dot{m}_{II} (h_5 - h_4)}$$

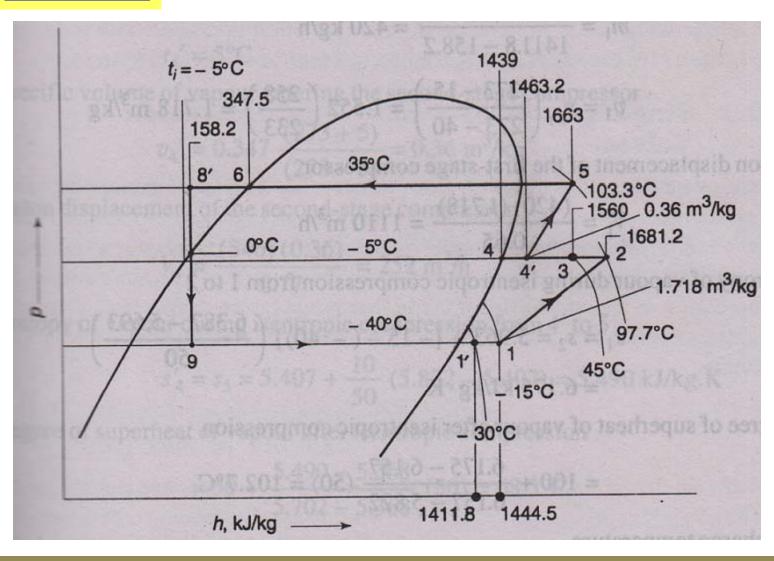
Advantages:

- higher refrigeration effect
- throttling losses are reduced
- volumetric efficiency of compressors will be high due to reduced pressure ratio
- compressor discharge temperature is reduced considerably

Numerical:

 A two stage ammonia food freezing plant with a desired capacity of 528000 kJ/h at -40°C evaporating temperature and 35°C condensing temperature has a flash intercooling system with a liquid subcooler. The vapour leaving the flash chamber is superheated by 10°C in the suction line to the second stage compressor. Water intercooling is done to cool the vapour to 45°C. Adiabatic efficiencies of both compressors are 0.75. The volumetric efficiencies of first and second stage compressors are 0.65 and 0.77 respectively. Find the piston displacemens, discharge temperatures and power requirements of the two compressors.

Answer:



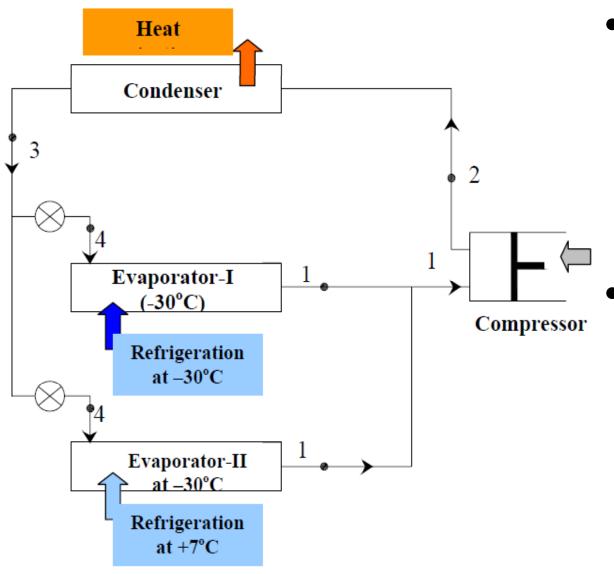
Multievaporator Systems: Need for this system

- In earlier lectures, we discussed that there are several applications which require refrigeration at different temperatures
- For example, in a dairy plant, refrigeration may be required at -30°C for making ice cream and at 2°C for chilling milk
- In a typical food processing plant, cold air may be required at -30°C for freezing and at 7°C for cooling of food products.

Multievaporator Systems: Need for this system

- Let us say, for a food freezing plant, one can have two separate refrigeration systems: one for freezing at -30°C and another for cooling of food products at 7°C
- But the above system may not be economically justified
- Another alternative will be to use one refrigeration system with one compressor and two evaporators.

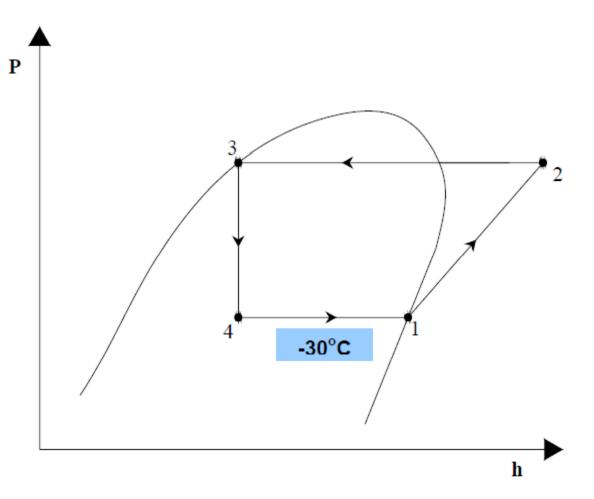
Multi evaporator system at same temperature



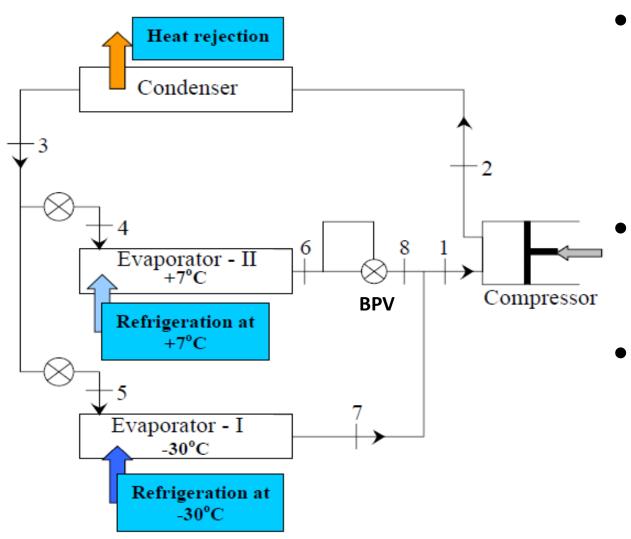
 Both evaporators operate at the same evaporator temperature of -30°C

But this system is not efficient as the refrigeration is required at 7°C while evaporation is taking place at -30°C

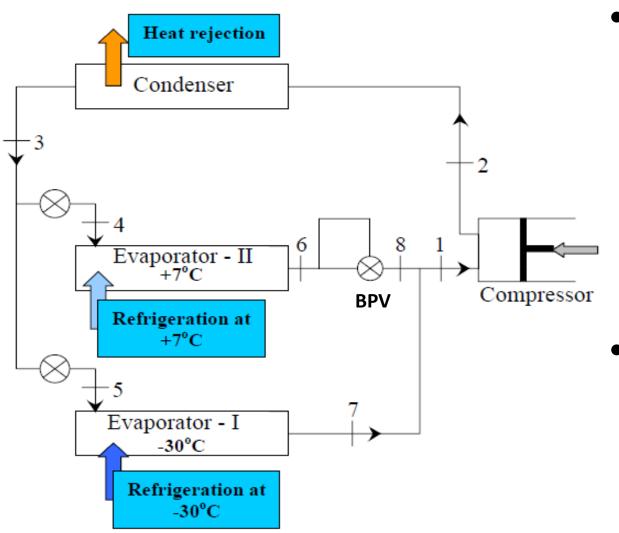
Multi evaporator system at same temperature



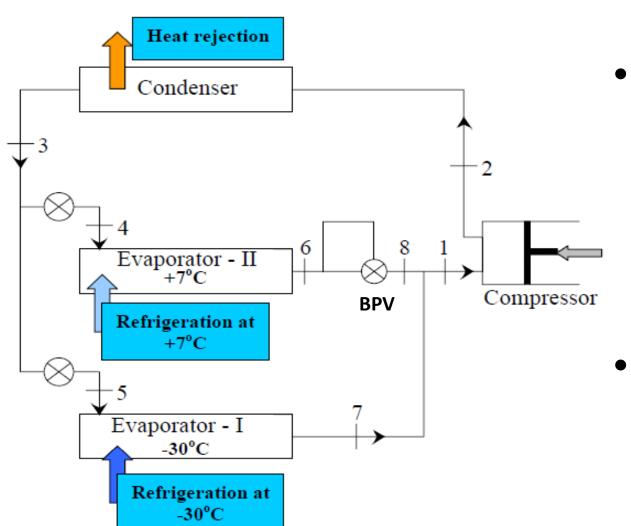
- It may also be possible that evaporator II may collect frost and finally blocks the passage.
- Therefore, we require an efficient system



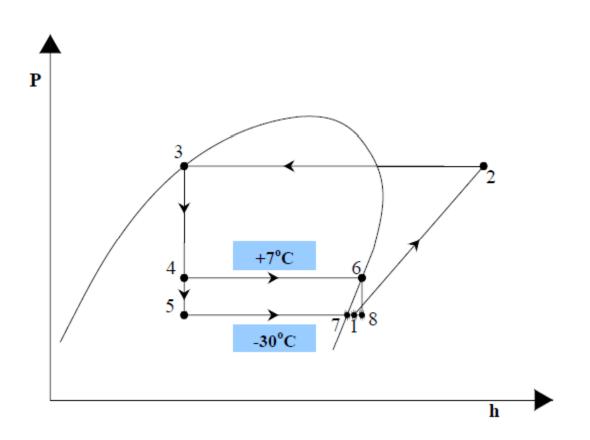
- The evaporators operate at different temperatures
 - Separate expansion valves are used
- Also, a pressure regulating valve is used known as back pressure valve (BPV)



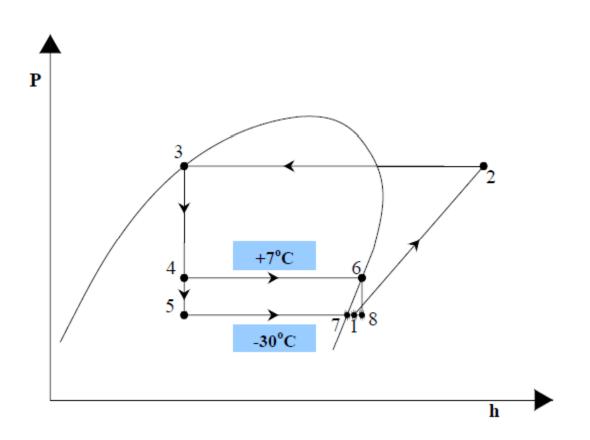
- The BPV is used to reduce the pressure at the high temperature evaporator to the compressor suction pressure
- This valve also maintains the pressure in the high temperature evaporator



- Compared to the previous system this arrangement gives higher refrigeration effect at the evaporator II.
- What about the COP?



- The higher refrigeration effect at evaporator II may be counterbalanced by the specific work input to the compressor
 - As the high pressure saturated refrigerant has to be brought back to the compressor suction pressure



- Therefore, COP of the system may or may not increase.
 - However, this system is still preferred compared to the previous one because of the proper operation of high temperature evaporator

Multi evaporator system at different temperatures with individual expansion valves and back pressure valves (BPV)

The COP of this system is given by

$$COP = \frac{Q_{e,I} + Q_{e,II}}{W_{c}} = \frac{\dot{m}_{I} (h_{7} - h_{5}) + \dot{m}_{II} (h_{6} - h_{4})}{(\dot{m}_{I} + \dot{m}_{II})(h_{2} - h_{1})}$$

where \dot{m}_I and \dot{m}_{II} are refrigerant mass flow rates through evaporator I and II which are given as,

$$\dot{m}_{I} = \frac{Q_{e,I}}{h_7 - h_5}$$
 and $\dot{m}_{II} = \frac{Q_{e,II}}{h_6 - h_4}$

Multi evaporator system at different temperatures with individual expansion valves and back pressure valves (BPV)

The enthalpy at point 1, i.e. at the compressor inlet is given by considering the mixing of two refrigerants

$$h_{1} = \frac{\dot{m}_{I}h_{7} + \dot{m}_{II}h_{8}}{\left(\dot{m}_{I} + \dot{m}_{II}\right)}$$

If the expansion across the BPV is isenthalpic then $h_8 = h_6$ Then,

$$h_{1} = \frac{\dot{m}_{I}h_{7} + \dot{m}_{II}h_{6}}{(\dot{m}_{I} + \dot{m}_{II})}$$

Numerical:

• A single compressor using R-12 as refrigerant has three evaporators of capacity 10 TR, 20 TR, and 30 TR. All the evaporators operate at -10°C and vapours leaving the evaporators are dry and saturated. The condenser temperaute is 40°C. The liquid refrigerant leaving the condenser is subcooled to 30°C. Assuming isentropic compression, find (a) the mass of refrigerant flowing through each evaporator (b) the power required to drive the compressor, and (c) the COP of the system.

Ans:

$$m_1 = 211Q_1/(h_1-h_4) = 211*10/(183.19 - 64.59) = 17.8 \text{ kg/min}$$

 $m_2 = 211Q_2/(h_1-h_4) = 211*20/(183.19 - 64.59) = 35.6 \text{ kg/min}$
 $m_3 = 211Q_3/(h_1-h_4) = 211*30/(183.19 - 64.59) = 53.4 \text{ kg/min}$

Power required

$$W = (m_1 + m_2 + m_3)(h_2 - h_1) = (17.8 + 35.6 + 53.4)(208.4 - 183.19)$$
$$= 2692.4 \text{ kJ/min} = 44.87 \text{ kW}$$

COP=
$$(h_1-h_4)/(h_2-h_1) = (183.19-64.59)/(208.4-183.19)$$

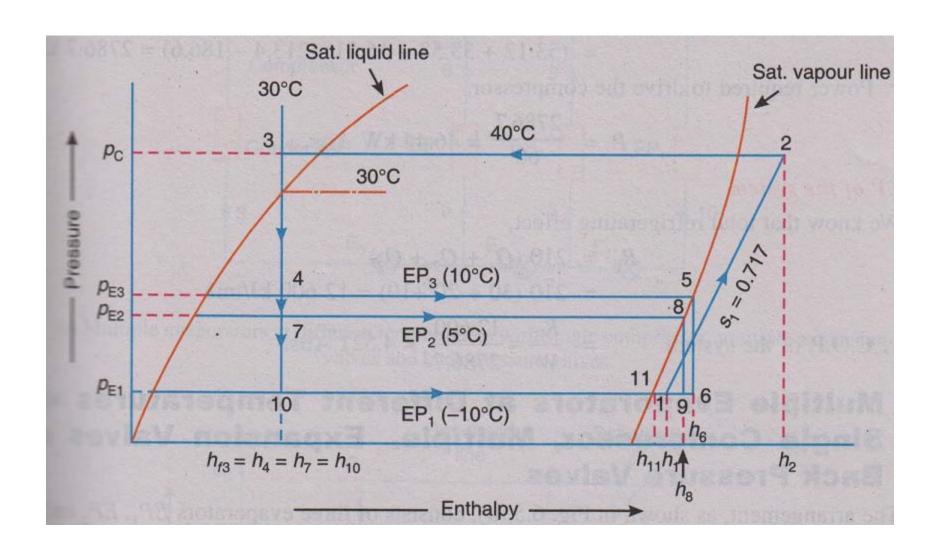
= 4.7

Numerical:

 A single compressor using R-12 as refrigerant has three evaporators of capacity 10 TR, 20 TR, and 30 TR. The temperature in the three evaporators is to be maintained at -10°C, 5°C, and 10°C. The condenser pressure is 9.609 bar. The liquid refrigerant leaving the condenser is subcooled to 30°C. The vapours leaving the evaporators are dry and saturated. Assuming isentropic compression, find (a) the mass of refrigerant flowing through each evaporator (b) the power required to drive the compressor, and (c) the COP of the system.

Numerical:

 A single compressor using R-12 as refrigerant has three evaporators of capacity 30 TR, 20 TR, and 10 TR. The temperature in the three evaporators is to be maintained at -10°C, 5°C, and 10°C respectively. The condenser pressure is 9.609 bar. The liquid refrigerant leaving the condenser is sub-cooled to 30°C. The vapours leaving the evaporators are dry and saturated. Assuming isentropic compression, find (a) the mass of refrigerant flowing through each evaporator (b) the power required to drive the compressor, and (c) the COP of the system.



Ans:

$$m_1 = 211Q_1/(h_{11}-h_{10}) = 211*30/(183.19 - 64.59) = 53.4 kg/min$$

 $m_2 = 211Q_2/(h_8-h_7) = 211*20/(189.19 - 64.59) = 33.87 kg/min$
 $m_3 = 211Q_3/(h_5-h_4) = 211*10/(191.74 - 64.59) = 16.59 kg/min$

Power required

```
h_1 = (m_1 * h_{11} + m_2 * h_8 + m_3 * h_5)/(m_1 + m_2 + m_3) = 186.51 \text{ kJ/kg}

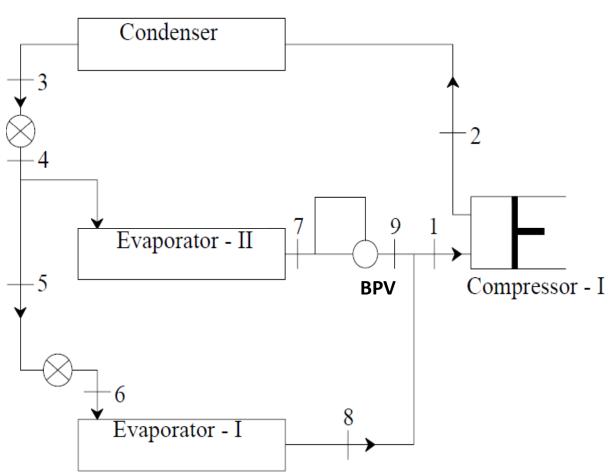
h_2 = 213.4 \text{ kJ/kg (after equating entropies)}

W = (m_1 + m_2 + m_3)(h_2 - h_1) = (53.4 + 33.87 + 16.59)(213.4 - 186.51)

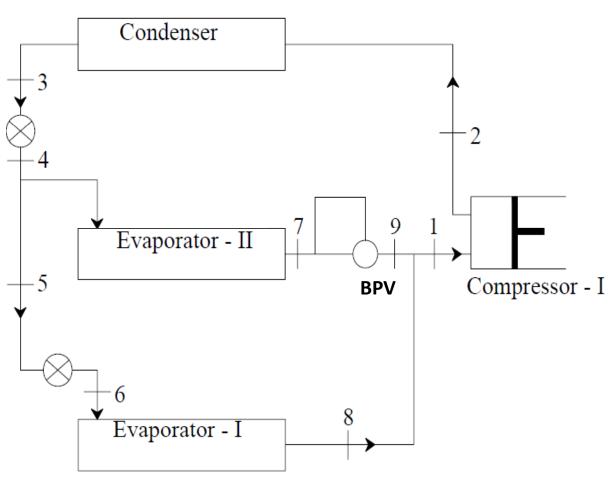
= 2792.8 \text{ kJ/min} = 46.55 \text{ kW}
```

Total refrigerating effect = $Q_E = 211*(Q_1+Q_2+Q_3) = 12660$ kJ/min

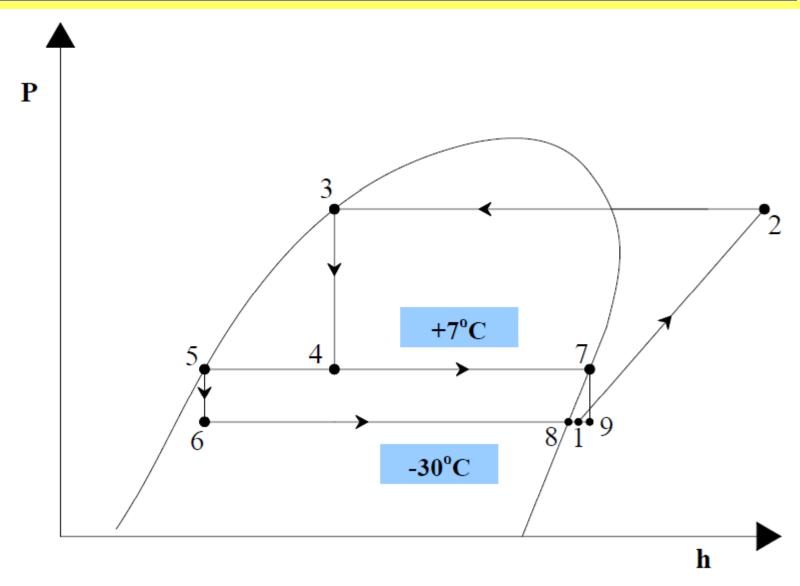
 $COP = Q_F/W = 12660/2792.8 = 4.53$



- It utilises multiple expansion valves
- This system is different than the previous case which uses the individual expansion valves
- The flash gas is removed at state 4



- Because of this the efficiency increases
- Also, this results in higher refrigeration effect at the low temperature evaporator because the saturated liquid enters the low stage expansion valve



Multi evaporator system with multiple expansion valves

The COP of this system is given by

$$COP = \frac{Q_{e,I} + Q_{e,II}}{W_c} = \frac{\dot{m}_I (h_8 - h_6) + \dot{m}_{II} (h_7 - h_4)}{(\dot{m}_I + \dot{m}_{II})(h_2 - h_1)}$$

where \dot{m}_I and \dot{m}_{II} are refrigerant mass flow rates through evaporator I and II which are given as,

$$\dot{m}_I = \frac{Q_{e,I}}{h_8 - h_6}$$
 and $\dot{m}_{II} = \frac{Q_{e,II}}{h_7 - h_4} + \dot{m}_I \left(\frac{x_4}{1 - x_4}\right)$

where second term represents the mass of vapour flashed at 4 corresponding to the mass of liquid going to the 2nd evaporator

Multi evaporator system with multiple expansion valves

The COP of this system is given by

$$COP = \frac{Q_{e,I} + Q_{e,II}}{W_c} = \frac{Q_{e,I} + Q_{e,II}}{(\dot{m}_I + \dot{m}_{II})(h_2 - h_1)}$$

where \dot{m}_I and \dot{m}_{II} are refrigerant mass flow rates through evaporators I and II which are given as,

$$\dot{m}_I = \frac{Q_{e,I}}{h_8 - h_6}$$
 and $\dot{m}_{II} = \frac{Q_{e,II}}{h_7 - h_4} + \dot{m}_I \left(\frac{x_4}{1 - x_4}\right)$

where second term represents the mass of vapour flashed at 4 corresponding to the mass of liquid going to the 2nd evaporator

Multi evaporator system with multiple expansion valves

The enthalpy at point 1, i.e. at the compressor inlet is given by considering the mixing of two refrigerants

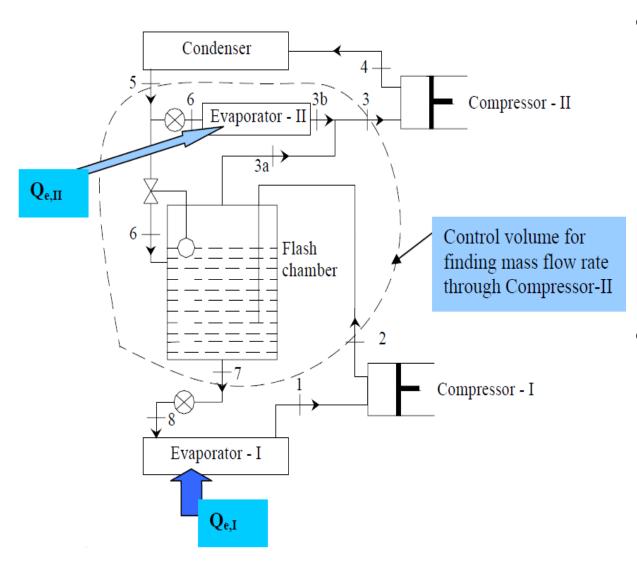
$$h_{1} = \frac{\dot{m}_{I}h_{8} + \dot{m}_{II}h_{9}}{\left(\dot{m}_{I} + \dot{m}_{II}\right)}$$

If the expansion across the BPV is isenthalpic then $h_9 - h_7$ Then,

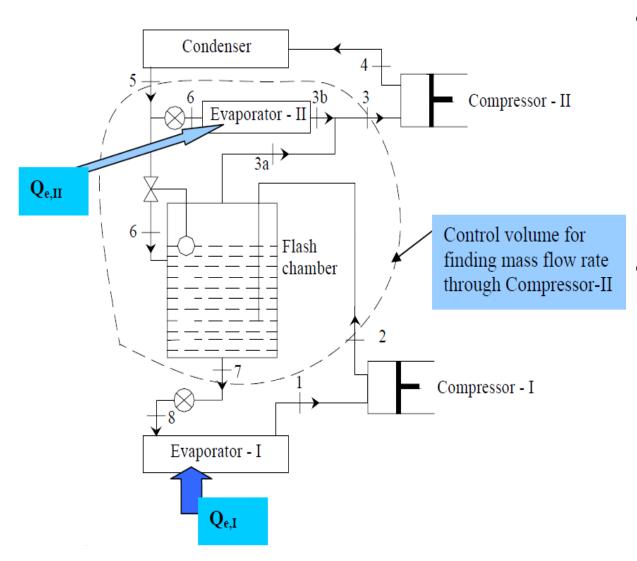
$$h_{1} = \frac{\dot{m}_{I}h_{8} + \dot{m}_{II}h_{7}}{(\dot{m}_{I} + \dot{m}_{II})}$$

 COP is not that much higher compared to the previous case because the refrigerant vapour at the intermediate stage is first throttled and then compressed

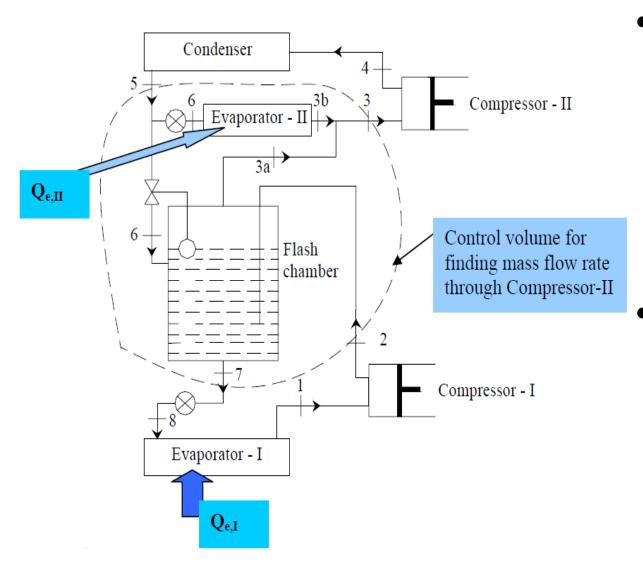
- COP is not that much higher compared to the previous case because the refrigerant vapour at the intermediate stage is first throttled and then compressed
- However, the COP can be enhanced using multi-compression system.



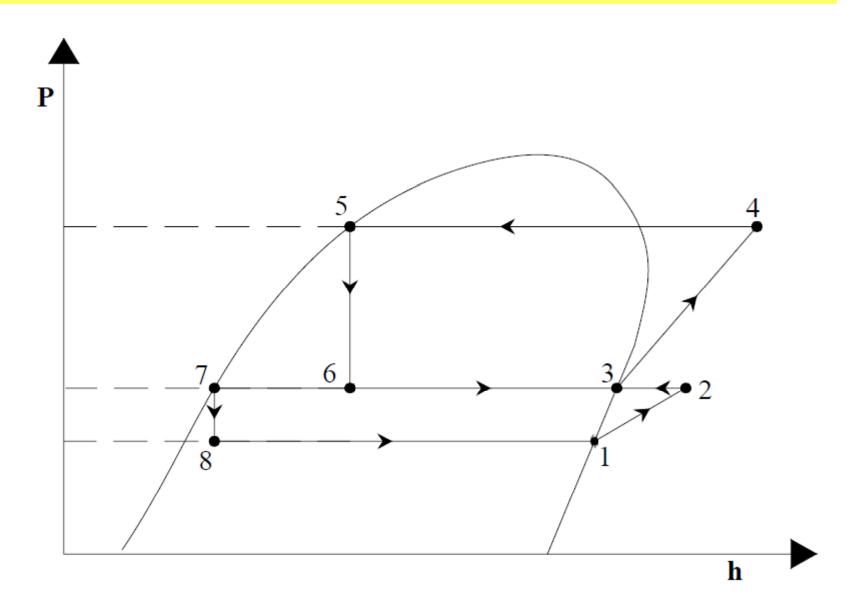
- It utilises multicompression system, a flash tank for flash gas removal and intercooling
- This system is adequate for low temperature lift with different refrigeration loads



- The pressure in the high tempeature evaporator is same as that of the flash tank
 - Superheated
 vapour from the
 low stage
 compressor is
 cooled to the
 saturation
 temperature in the
 flash tank



- The low temperature compressor works efficiently as the flash gas is removed in the flash tank
- Also, the high temperature compressor works efficiently as the suction vapour is saturated



Multi evaporator system with multi-compression

The COP of this system is given by

$$COP = \frac{Q_{e,I} + Q_{e,II}}{W_{c,I} + W_{c,II}} = \frac{Q_{e,I} + Q_{e,II}}{\dot{m}_{c,I} \left(h_2 - h_1\right) + \dot{m}_{c,II} \left(h_4 - h_3\right)}$$

where $\dot{m}_{c,I}$ and $\dot{m}_{c,II}$ are refrigerant mass flow rates through compressors I and II respectively which are given as,

$$\dot{m}_{c,I} = \frac{Q_{e,I}}{h_1 - h_8} \quad \text{and} \quad$$

$$\dot{m}_{c,II} = \dot{m}_{c,I} + \frac{Q_{e,II}}{h_3 - h_6} + \dot{m}_{c,I} \left(\frac{x_6}{1 - x_6}\right) + \frac{\dot{m}_{c,I} (h_2 - h_3)}{h_3 - h_6}$$

Multi evaporator system with multi-compression

$$\dot{m}_{c,II} = \dot{m}_{c,II} + \frac{Q_{e,II}}{h_3 - h_6} + \dot{m}_{c,I} \left(\frac{x_6}{1 - x_6} \right) + \frac{\dot{m}_{c,I} (h_2 - h_3)}{h_3 - h_6}$$

Term *i* represents the mass flow rate of the refrigerant passing through the first evaporator or the first compressor Term *ii* represents the mass flow rate of the refrigerant required to obtain the desired refrigeration effect at the second evaporator

Term *iii* represents the mass flow rate of the vapour which is formed due to flashing

Term *iv* represents the mass flow rate of the refrigerant required in flash intercooler for desuperheating the superheated vapour refrigerant coming from first compressor

Numerical:

 A single compressor using R-12 as refrigerant has three evaporators of capacity 30 TR, 20 TR, and 10 TR. The temperature in the three evaporators is to be maintained at -10°C, 5°C, and 10°C respectively. The system is provided with multiple expansion valves and back pressure valves. The condenser temperature is 40°C. The liquid refrigerant leaving the condenser is sub-cooled to 30°C. The vapours leaving the evaporators are dry and saturated. Assuming isentropic compression, find (a) the mass of refrigerant flowing through each evaporator (b) the power required to drive the compressor, and (c) the COP of the system.

Multievaporator and Compressor Systems:

Following types are important under this category:

- Multiple evaporators at the same temperature with single compressor and expansion valve
- Multiple evaporators at different temperatures with single compressor, individual expansion valves and back pressure valves
- Multiple evaporators at different temperatures with single compressor, multiple expansion valves and back pressure valves

Multievaporator and Compressor Systems:

Following types are important under this category:

- Multiple evaporators at different temperatures with individual compressors and individual expansion valves
- Multiple evaporators at different temperatures with individual compressors and multiple expansion valves
- Multiple evaporators at different temperatures with compound compression and individual expansion valves

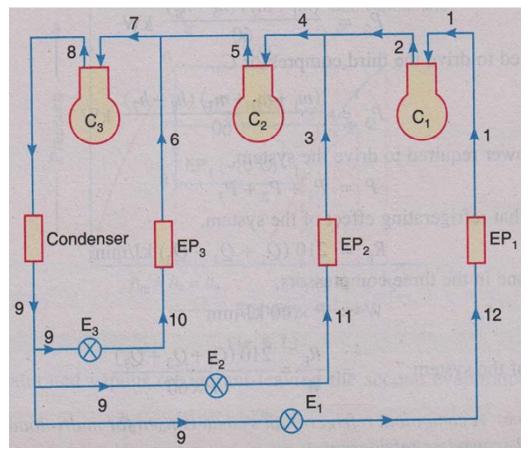
Multievaporator and Compressor Systems:

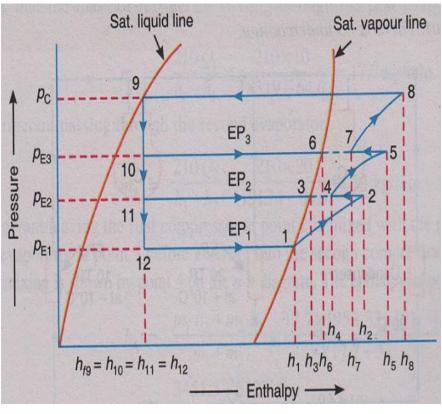
Following types are important under this category:

- Multiple evaporators at different temperatures with compound compression, individual expansion valves and flash intercoolers
- Multiple evaporators at different temperatures with compound compression, multiple expansion valves and flash intercoolers

Multievaporator and Compressor Systems:

 Multiple evaporators at different temperatures with compound compression and individual expansion valves





Numerical:

 The refrigeration system using R-12 as refrigerant has three evaporators of capacity 20 TR, 30 TR, and 10 TR with individual expansion valves and individual compressors. The temperature in the three evaporators is to be maintained at -10°C, 5°C, and 10°C respectively. The condenser temperature is 40°C. The liquid refrigerant leaving the condenser is sub-cooled to 30°C. The vapours leaving the evaporators are dry and saturated. Assuming isentropic compression in each compressor, find (a) the mass of refrigerant flowing through each evaporator (b) the power required to drive the system, and (c) the COP of the system.

Numerical:

 A compound compression refrigerating system using R-12 as refrigerant has three evaporators of capacity 30 TR, 20 TR, and 10 TR. The temperature in the three evaporators is to be maintained at -10°C, 5°C, and 10°C respectively. The condenser temperature is 40°C. The liquid refrigerant leaving the condenser is sub-cooled to 30°C. The vapours leaving the evaporators are dry and saturated. The system is provided with multiple expansion valves and flash intercoolers. Assuming isentropic compression in each compressor, find (a) the mass of refrigerant flowing through each compressor (b) the power required to drive the system, and (c) the COP of the system.

- In most of the air-conditioning system the working substance is atmospheric air
- Therefore, it is extremely important to study the properties of atmospheric air and also the different processes required for a proper design of an airconditioning system
- Atmospheric air consists of many gases, water vapour and the pollutants. The concentrations vary from place to place

- Also it decreases with the altitude, being almost dry air at an altitude of 10 km
- The composition of dry part in atmospheric air is listed below:

Components	Mol. Mass	Vol %	Mass %
N_2	28.02	78.03	75.47
O_2	32.00	20.99	23.20
Ar	39.91	0.94	1.29
CO ₂	44.00	0.03	0.05
H ₂	2.02	0.01	0.00

- For use in an air-conditioning system, the pollutant is removed from the atmospheric air
- The residue of atmospheric air after the removal of pollutants is known as the moist air, as it contains the various gases including the water vapour
- The molecular weight of the dry air is 28.966 and the characteristic gas constant is 287.036 J/kg-K
- The molecular weight of the water vapour is 18.05 and the characteristic gas constant is 461.52 J/kg-K

- Air at a given temperature and pressure can exist in two conditions: unsaturated and saturated
- At a given state, if the moisture content in dry air is less than the maximum permissible moisture corresponding to the given temperature and pressure, the air is known as unsaturated air.
- That means, it is the moisture content which decides the state of air.

Different Laws For Moist Air:

Gibbs - Dalton Law:

 According to this law, the total pressure exerted by a mixture of non-reacting ideal gases is equal to the sum of partial pressure exerted by the constituent gases, i.e.

$$p_t = p_1 + p_2 + p_3 + \dots$$

where p_t is the total pressure
 p₁ = n₁GT/V; G is the universal gas constant
 p₂ = n₂GT/V
 p₃ = n₃GT/V

Different Laws For Moist Air:

Gibbs - Dalton Law:

For moist air, it becomes

$$p_t = p_a + p_v = p_a + p_w$$

- where p_t is the total pressure, p_a is the partial pressure of dry air, p_w or p_v is the partial pressure of water vapour
- It is difficult to find the exact property of the moist air, however it is noticed that upto 3 atm. pressure, moist air behaves as an ideal gas

Different Laws For Moist Air:

Amagat Law of Partial Volumes:

 According to this law, each constituent is assumed to occupy volume corresponding to the given total pressure and the temperature. The total volume of the gas is then equal to the summation of individual volumes, i.e.

$$V = V_1 + V_2 + V_3 + \dots$$

• where V is the volume of the mixture $V_1 = n_1GT/p$; G is the universal gas constant $V_2 = n_2GT/p$, $V_3 = n_3GT/p$

Numerical:

- One cubic metre of H₂ at 1 bar and 25°C is mixed with one cubic metre of N₂ at 1 bar and 25°C. For the mixture at the same conditions, find:
 - (i) mole fractions of the components
 - (ii) partial pressures of the components
 - (iii) mass fractions of the components
 - (iv) molecular weight of the mixture
 - (v) gas constant of the mixture
 - (vi) volume of the mixture

Ans:

```
m_1 = 0.1616 \text{ kg}; subscript 1 stands for H_2 and 2 for N_2.

m_2 = 1.131 \text{ kg}
```

$$n_1 = 0.0404$$
, $n_2 = 0.0403$

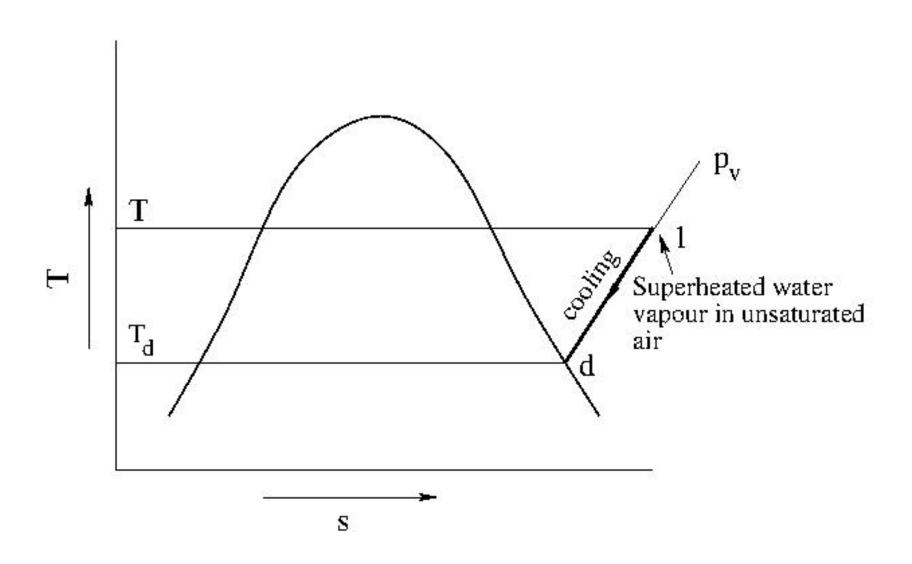
```
mole fractions, y_1 = 0.5, y_2 = 0.5
partial pressures, p_1 = 0.5 bar, p_2 = 0.5 bar
mass fractions, m_1/m = 0.125, m_2/m = 0.875
```

molecular weight = $M = y_1M_1+y_2M_2 = 16.01$ gas constant = R = G/M = 8.3143/16.01 = 0.5193 kJ/kg-K volume of mixture = mRT/p = 2 m³

Important Psychrometric Properties:

- <u>Dry Bulb Temperature (DBT)</u>: It is the temperature of the moist air as measured by a standard thermometer
- <u>Dew Point Temperature (DPT)</u>: When the unsaturated moist air is cooled isobarically, the mixture eventually reaches the saturation temperature of water vapour corresponding to its partial pressure p_v. At this point, the water vapour starts to condense. This particular temperature is known as dew point temperature

Important Psychrometric Properties:



Important Psychrometric Properties:

• Specific Humidity or Humidity Ratio (w): It is defined as the ratio of mass of water vapour to the mass of dry air in a given volume of the mixture. Thus, $\mathbf{w} = \frac{\mathbf{v}}{\mathbf{v}} = \frac{\mathbf{v}}$

where subscripts a and v stand for air and water vapour respectively

$$p_a v_a = GT/M_a$$
 and $p_v v_v = GT/M_v$

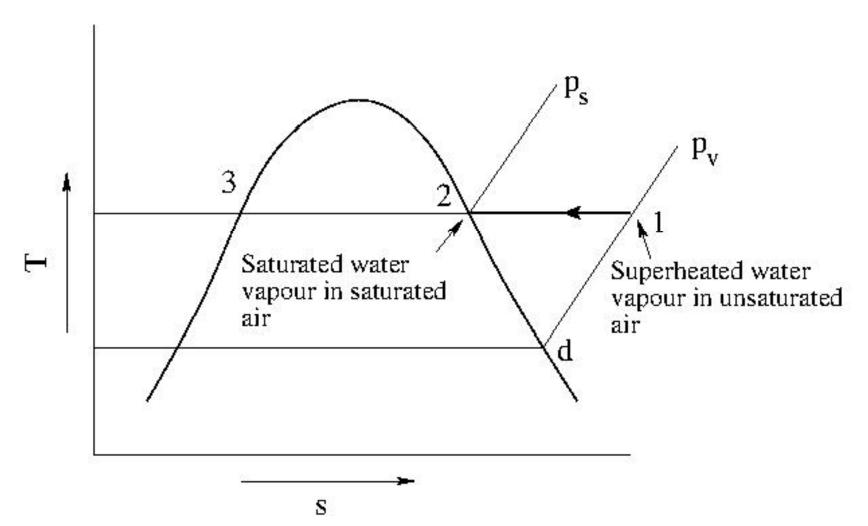
so, **w**= $M_v p_v / M_a p_a = 18.06 p_v / (28.966 p_a) = 0.622 p_v / p_a$ so, **w**=0.622 p_v / (p-p_v); Since it is a function of p_v, accordingly, if there is no change in **w** or the moisture content, the partial pressure p_v also remains constant

Important Psychrometric Properties:

- It should be noted that since p_v is very small in comparison to p, therefore, denominator remains more or less constant. Hence, the specific humidity w is approximately a linear function of p_v.
- Also, the specific humidity is not a mass fraction of water vapour. Rather, it is the a fraction of water vapour in dry air for a given volume of the mixture.

Important Psychrometric Properties:

Degree of Saturation (m):



Important Psychrometric Properties:

 <u>Degree of Saturation (m)</u>: It is defined as the ratio of actual specific humidity w to the specific humidity w_s of saturated air at the same temperature T and pressure p. Thus,

$$\mathbf{m} = \mathbf{w}/\mathbf{w}_s = (0.622p_v/(p-p_v))/(0.622p_s/(p-p_s))$$

$$\mathbf{m} = (p_v \cdot (p - p_s))/(p_s \cdot (p - p_v))$$

We, therefore, notice that degree of saturation represents the capacity of air to absorb moisture.

Important Psychrometric Properties:

• Relative Humidity (f): It is defined as the ratio of mass of water vapour in a certain volume of moist air at a given temperature (i.e. unsaturated air) to the saturated mass of water vapour in the same volume at the same temperature. Thus,

$$(phif = m_v/m_{vs} = (p_vV/GT)/(p_sV/GT) = p_v/p_s$$

where subscripts v and s stand for unsaturated and saturated conditions respectively Also,

$$f = (V/V_V)/(V/V_S) = V_S/V_V$$

Important Psychrometric Properties:

 Therefore, the relative humidity can also be defined as the ratio of partial pressure of water vapour in an unsaturated moist air at a given temperature T to the saturation pressure of water vapour at the same temperature T. Hence,

or,
$$\mathbf{w} = 0.622 \, \mathbf{f} \cdot p_s/p_a$$

or, $\mathbf{f} = \mathbf{w} \cdot p_a / (0.622 \cdot p_s)$
Also $\mathbf{m} = \mathbf{f} \cdot (1 - p_s/p) / (1 - p_v/p)$

Important Psychrometric Properties:

• Enthalpy of Moist Air: It is obtained by the summation of enthalpies of its constituents, i.e. dry air and the water vapour. Thus,

Enthalpy of moist air (h) is equal to

 $h = h_a + wh_v$, per kg of dry air

where h_a is the enthalpy of dry air and $\mathbf{W}h_v$ is the enthalpy of the water vapour

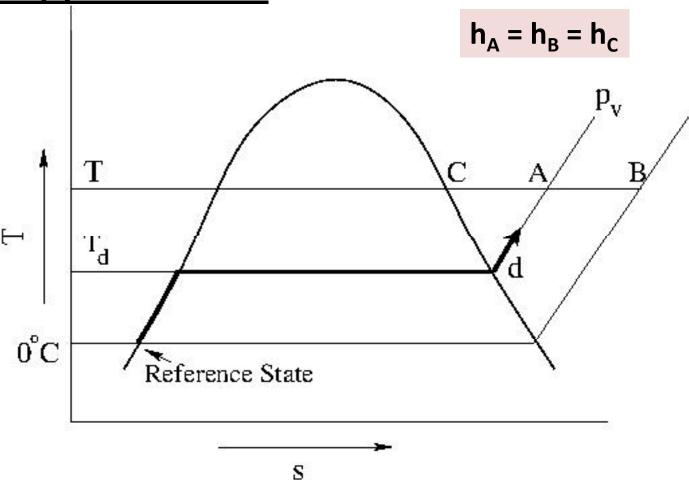
Considering enthalpy as a function of temperature only

$$h_a = C_{pa}T = 1.005 T kJ/kg d.a.$$

 $C_p = 1.005 \text{ kJ/kg-K}$ and T is the absolute temperature.

Important Psychrometric Properties:

• Enthalpy of Moist Air:



Different ways of finding enthalpy of water vapour

Important Psychrometric Properties:

Enthalpy of Moist Air:

$$h_{v} = C_{pw} \ T_{d} + (h_{fg})_{d} + C_{pv} \ . \ (T-T_{d})$$
 where C_{pw} is specific heat of liquid water
$$T_{d} \ is \ dew \ point \ temperature$$

$$(h_{fg})_{d} \ is \ latent \ heat \ of \ vaporisation \ at \ T_{d}$$

$$C_{pv} \ is \ specific \ heat \ of \ superheated \ vapour$$
 In the above expressions, datum is taken at 0°C Considering enthalpy as a function of temperature only, Taking $C_{pw} = 4.1868 \ kJ/kg-K$ and $C_{pv} = 1.88 \ kJ/kg-K$, in the range of 0°C to 60°C, we can write
$$h_{v} = 4.1868 \ T_{d} + h_{fg} + 1.88 \ . \ (T-T_{d})$$

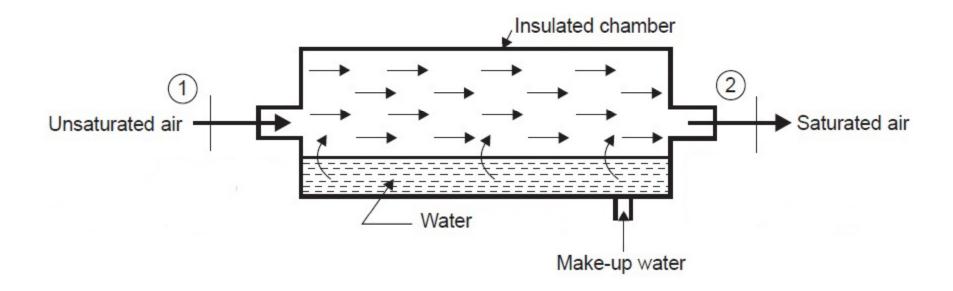
Important Psychrometric Properties:

• Wet Bulb Temperature (WBT): It is the temperature recorded by a thermometer when the bulb is completely soaked in water. This is done by putting an envelop of cotton wick saturated with water around the bulb.

As the air passes through the wick, it takes away the latent heat and some water gets evaporated. Energy is then transferred from the air to the wick. When the equilibrium condition is reached, there is a balance between the energy removed from the wick and energy supplied by air. The temperature recorded at this moment is WBT.

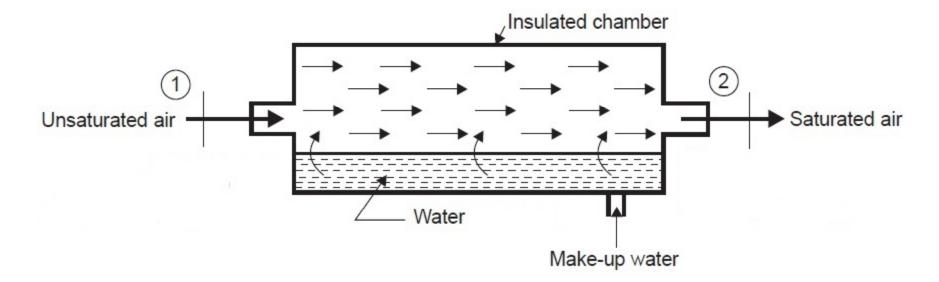
Important Psychrometric Properties:

Adiabatic Saturation Temperature (T*): When the
unsaturated air flows over a long sheet of water,
the water evaporates and the moisture content of
air increases. Because of the evaporation, both the
water and the air are cooled



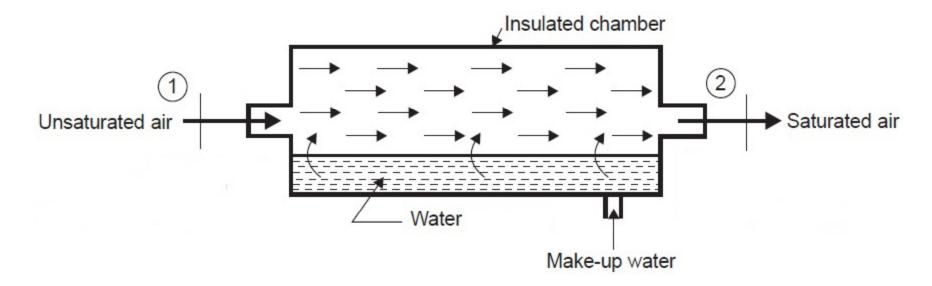
Important Psychrometric Properties:

- Adiabatic Saturation Temperature (T*): The process continues till the thermal equilibrium, i.e. energy transferred from air to water is exactly same as the energy needed to vaporise the water
- When this condition is reached, air is saturated.



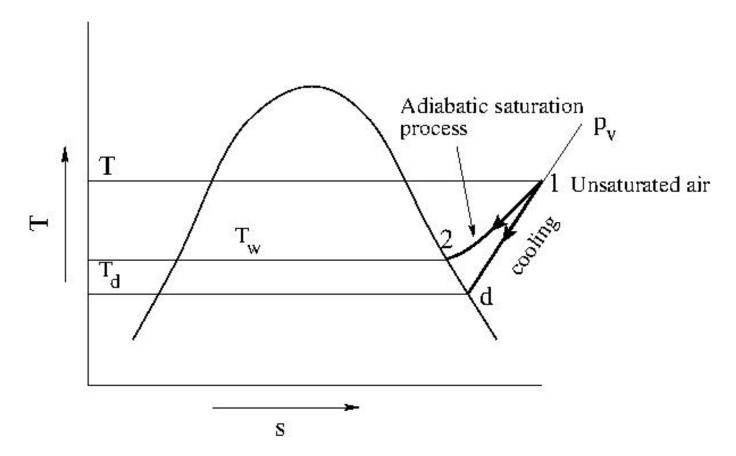
Important Psychrometric Properties:

- Adiabatic Saturation Temperature (T*):
- This equilibrium temperature is known as adiabatic saturation temperature or thermodynamic wet bulb temperature
- The adiabatic saturation temperature is taken equal to WBT for all practical purposes, i.e. WBT = T*.



Important Psychrometric Properties:

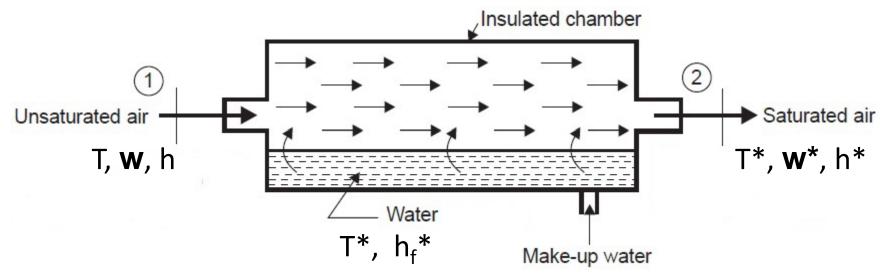
Adiabatic Saturation Temperature (T*):



Adiabatic Saturation Temperature or Thermodynamic Wet Bulb Temperature

Important Psychrometric Properties:

Adiabatic Saturation Temperature (Some calculation):



From energy balance,

$$h + (w^* - w) h_f^* = h^*$$

The enthalpy is increased from h because of the water addition. The mass of water added per kg of dry air is (w^*-w) , h_f^* is the specific enthalpy of injected water or the sensible heat of make up water

Important Psychrometric Properties:

Now,
$$h = C_{pa}T + W_{\bullet}h_{v}$$
, $h^{*} = C_{pa}T^{*} + W^{*}_{\bullet}h_{v}^{*}$

Therefore,

$$C_{pa}T + w_h_v + (w^* - w) h_f^* = C_{pa}T^* + w_h^*$$

$$\mathbf{w} = [C_{pa}(T^* - T) + \mathbf{w}^*(h_v^* - h_f^*)] / (h_v^- - h_f^*)$$

Assignment: Show that adiabatic saturation temperature is a thermodynamic property of moist air.

Numerical:

- A mixture of dry air and water vapour is at a temperature of 21°C under a total pressure of 736 mm of Hg. The dew point temperature is 15°C. Find:
 - (i) partial pressure of water vapour
 - (ii) Relative humidity
 - (iii) Specific humidity
 - (iv) Specific enthalpy of water vapour by all the methods
 - (v) Enthalpy of air per kg of dry air
 - (vi) Specific volume of air per kg of dry air

Ans:

- (i) From steam table, partial pressure of water vapour at 15° C DPT = $p_v = 1707.5$ Pa
- (ii) From steam table, partial pressure of water vapour at $21^{\circ}\text{C DBT} = p_s = 2489.81 \text{ Pa}$ Relative humidity = $p_v/p_s \times 100 = 68.58\%$
- (iii) Specific humidity = $0.622.p_v/p_a$ = $0.622.p_v/(p-p_v) = 0.011 \text{ kg w.v./kg d.a.}$
- (iv) $(h_{fg})_{21C}$ = 2452 kJ/kg, $(h_{fg})_{15C}$ = 2466.2 kJ/kg $h_C = C_{pw} .T + (h_{fg})_{21C}$ = 4.1686(21)+2452 = 2540 kJ/kg w.v. $h_A = C_{pw} .T_d + (h_{fg})_{15C} + C_{pv} . (T-T_d)$ =4.1686(15)+2466.2+1.88(21-15) = 2540.3 kJ/kg w.v. $h_B = (h_{fg})_{0C} + C_{pv} .T$ = 2501+ 1.88(21) = 2540.5 kJ/kg w.v.

Ans:

(v) Enthalpy of air

$$h = C_{pa} .T + w h_v = 1.005(21) + 0.011(2540.3)$$

= 49.0 kJ/kg d.a.

(vi) Specific volume of air is equal to the volume of 1 kg of dry air or 0.011 kg of water vapour. Based on dry air part

$$v = v_a = R_a.T/p_a = 287.3 (273+21)/(723.21*133.5)$$

=0.875 m³/kg d.a.

Calculation based on the water vapour part

$$v = v_v = R_v.T/p_v = 461.5 (273+21)/(12.79*133.5)$$

=79.463 m³/kg w.v.
=79.463 . **w** =0.874 m³/kg d.a.

Numerical:

- The humidity ratio of the atmospheric air at 28°C DBT and 760 mm of Hg is 0.016 kg/kg d.a. Determine:
 - (i) partial pressure of water vapour
 - (ii) Relative humidity
 - (iii) The DPT
 - (iv) Enthalpy of air per kg of dry air

Ans:

(i) Sp. Humidity or humidity ratio (w) is

$$W = 0.622 \cdot p_{v}/(p - p_{v})$$

$$0.016 = 0.622 \cdot p_{v}/(p - p_{v})$$

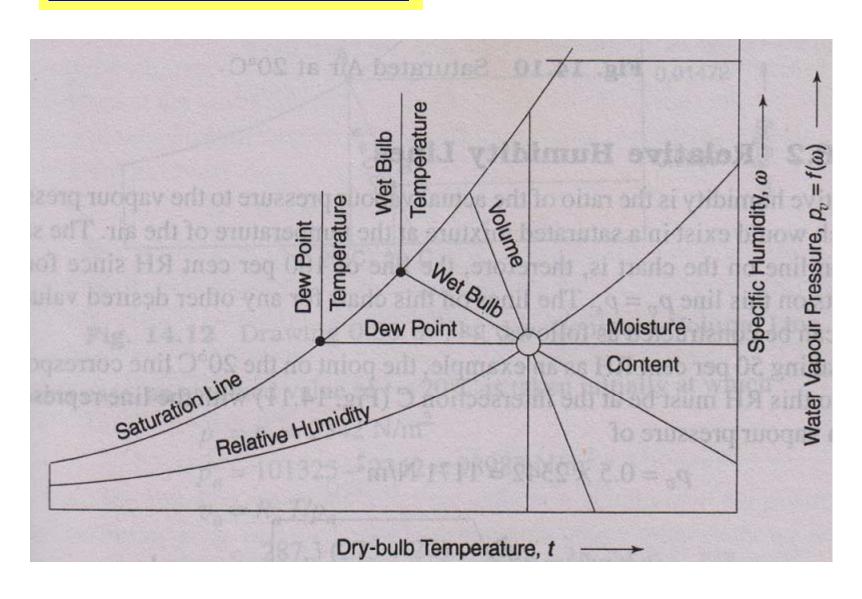
$$p_{v} = 19.06 \text{ mm of Hg} = 190.06 * 133.3 = 2540.6 \text{ N/m2}$$

- (ii) From steam table, the saturation pressure at 28°C DBT = $p_s = 3778 \text{ Pa}$ Relative humidity = $p_v/p_s \times 100 = 67.2\%$
- (iii) The DPT is the saturation temperature corresponding to the p_v , $T_d = 21.1^{\circ}C$

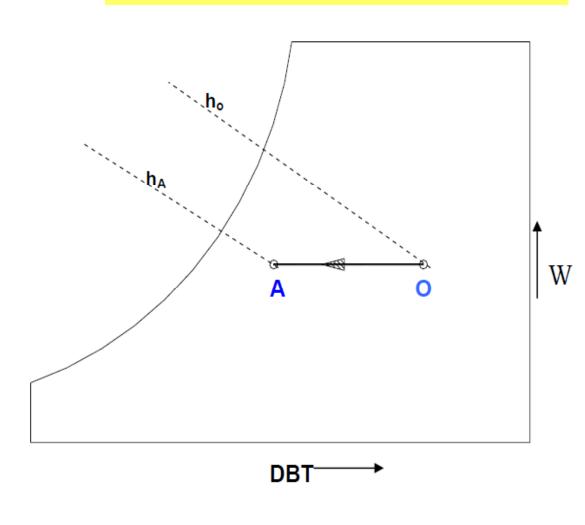
Ans:

```
(iv) Enthalpy of water vapour
    (h_{fg})_{21.1C} = 2451.76 kJ/kg
    h_w = C_{nw} . T_d + (h_{fg})_{15C} + C_{pv} . (T - T_d)
       =4.1686(21.1)+2451.76+1.88(28-21.1)
       = 88.34 + 2451.76 + 12.97 = 2553.07 \text{ kJ/kg of w.v.}
    Enthalpy of dry air = h_a = C_{pa}. T = 1.005 * 28
                        = 28.14 \text{ kJ/kg of d.a.}
     Enthalpy of moist air
       h = h_a + \mathbf{W}.h_w
          = 28.14 + \mathbf{w}.2553.07
          = 28.14 + 40.85 = 68.99 \text{ kJ/kg of d.a.}
```

Psychrometry Chart:

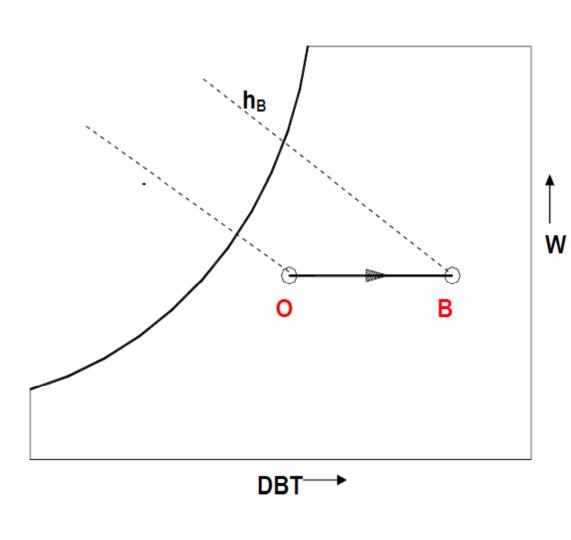


Sensible cooling process:



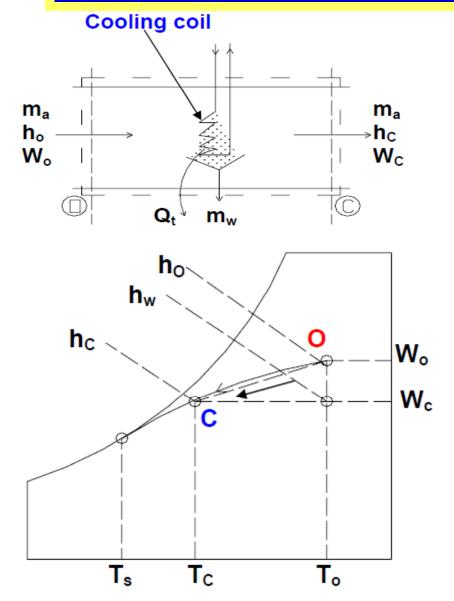
- The moisture content remains constant
- The temperature of the cooling coil should be higher than the DPT for the given pressure
- For 100% effective cooling coil, the exit air temperature will be equal to the coil temperature

Sensible heating process:



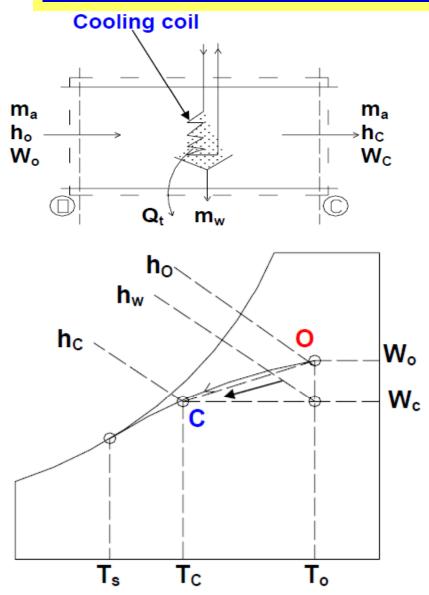
- The moisture content remains constant
- The temperature of air increases during the process
- The exit temperature will be less than the temperature of the heating coil (imperfect heating)

Cooling and Dehumidification:



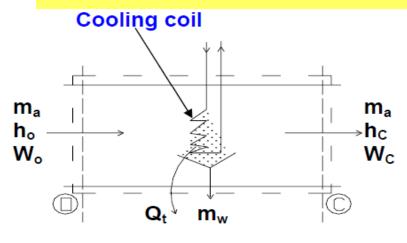
- The moisture content decreases and the temperature of air also decreases
- Some of the water vapour condenses and leaves the system as liquid
- This process is usually encountered in an air conditioner

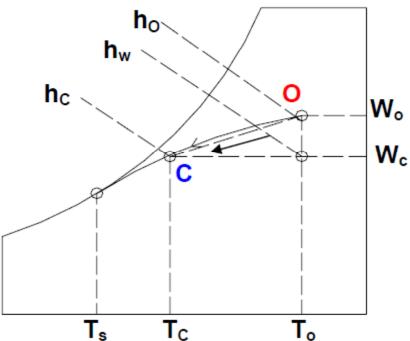
Cooling and Dehumidification:



- The mass balance of water gives m_a w_o = m_a w_c +m_w where m_w is the mass of the water vapour leaving the system
- The energy balance gives
 m_a h_o = Q_t + m_w h_w + m_a h_c
 Load on the cooling coil,
 Q_t = m_a (h_o h_c) m_w h_w.

Cooling and Dehumidification:





$$Q_{t} = m_{a} (h_{o} - h_{c})$$

 $- m_{w} h_{w}$.
 $Q_{t} = m_{a} (h_{o} - h_{c})$
 $- m_{a} (w_{o} - w_{c})h_{w}$.

 The second term is usually small in comparison to the first term. Hence,

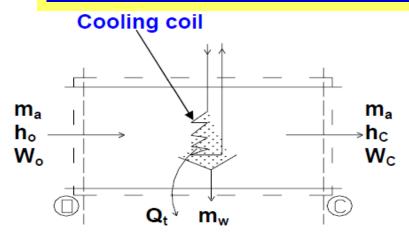
$$Q_{t} = m_{a} (h_{o} - h_{c})$$

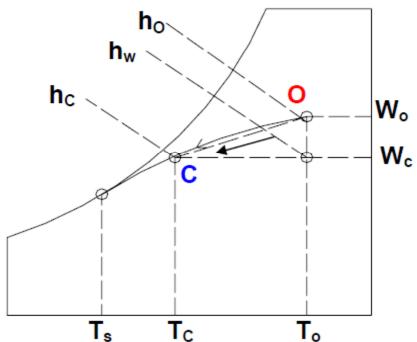
$$= m_{a} (h_{o} - h_{w} - h_{c} + h_{w})$$

$$= m_{a} (h_{o} - h_{w})$$

$$+ m_{a} (h_{w} - h_{c})$$

Cooling and Dehumidification:





$$Q_t = m_a (h_o - h_w)$$
$$+ m_a (h_w - h_c)$$
$$= LH + SH$$

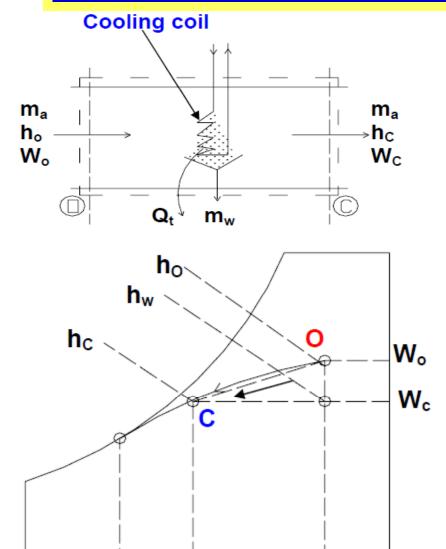
Sensible heat factor (SHF)

$$SHF = SH/(LH + SH)$$
$$= (h_w - h_c)/(h_o - h_c)$$

Hence, SHF = 0 corresponds to the condition of only latent heat transfer

Cooling and Dehumidification:

T。



Ts

 T_{C}

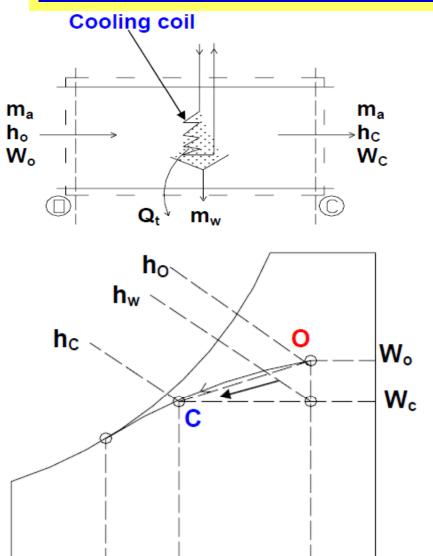
$$SHF = SH/(LH + SH)$$
$$= (h_w - h_c)/(h_o - h_c)$$

And, SHF = 1 corresponds to the condition of only sensible heat transfer

Usually, it varies between 0.75 to 0.8 in dry climate while it falls to 0.6 in a humid climate

Cooling and Dehumidification:

T。

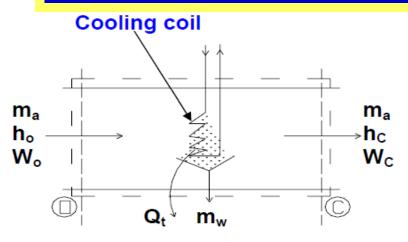


Ts

 T_{C}

- In the figure, T_s is the effective surface temperature of the cooling coil known as apparatus dew point (ADP) of the coil.
- Ideally, the exit temperature of air should be equal to the ADP. But, in practice, it is not possible.

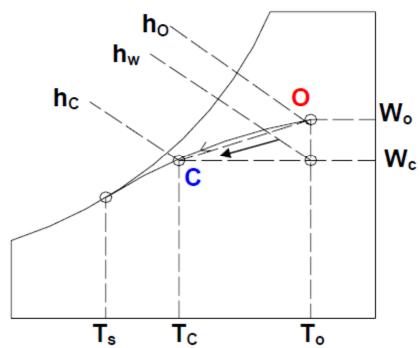
Cooling and Dehumidification:



 Therefore, a bypass factor is defined which is given as

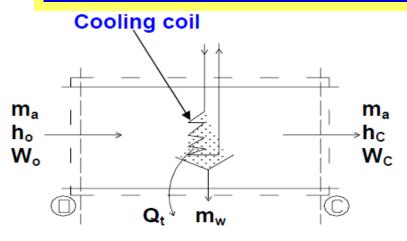
BPF =
$$(T_c - T_s)/(T_o - T_s)$$

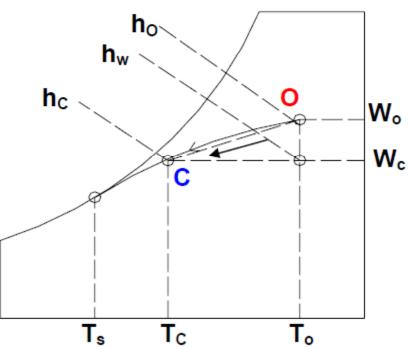
= $(w_c - w_s)/(w_o - w_s)$
= $(h_c - h_s)/(h_o - h_s)$



- A higher BPF represents a large difference between ADP and the exit air temperature
- A BPF = 1 represents that all the air bypasses the cooling coil without being cooled or dehumidified

Cooling and Dehumidification:

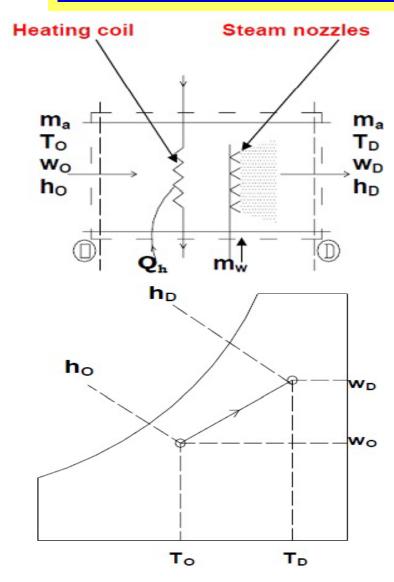




• Efficiency of the cooling coil $h_c = 1 - BPF$ $= 1 - (T_c - T_s)/(T_o - T_s)$ $= (T_o - T_c)/(T_o - T_s)$

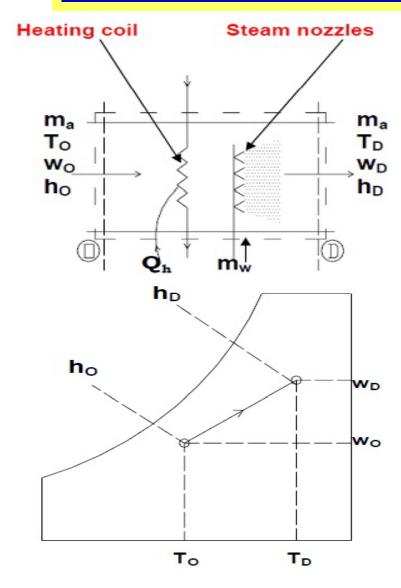
 The above efficiency is also called the contact factor

Heating and humidification:



- The moisture content increases and the temperature of air also increases
- This is usually required in winter
- The air is first heated then steam is injected into the air

Heating and humidification:

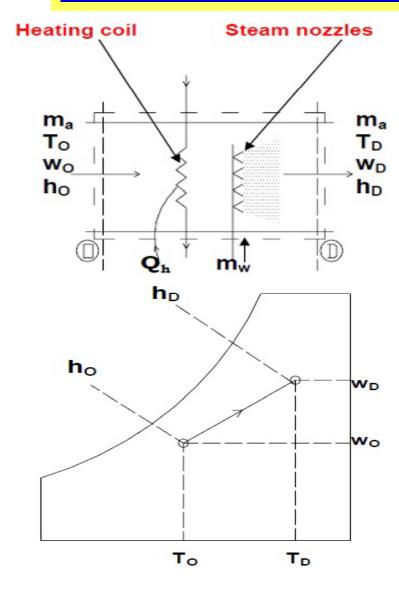


- The mass balance of water vapour gives
 m_a w_D = m_a w_o + m_w
 where m_w is the mass of the water vapour entering the system
- The energy balance gives

$$m_a h_o + Q_h + m_w h_w$$

= $m_a h_D$
Heating load of the coil,
 $Q_h = m_a (h_D - h_o)$
- $m_w h_w$.

Heating and humidification:



 Since this process also involves the latent and sensible heating, a sensible heat factor (SHF) can be defined, which is given as

SHF = SH/(LH +SH)
=
$$(h_w - h_o)/(h_D - h_o)$$

Numerical:

• 36 cubic metre per minute (cmm) of a mixture of recirculated room air and outdoor air enter a cooling coil at 31°C DBT and 18.5°C WBT. The effective surface temperature of the coil is 4.4°C. The surface area of the coil is such as would give 12.5 kW of refrigeration with the given entering air state. Determine the DBT and WBT of the air leaving the coil and the coil bypass factor.

Ans:

```
At the ADP of 4.4°C, w_s = 5.25 g/kg d.a.
                   and h_s = 17.7 \text{ kJ/kg d.a.}
State of entering air
   w_1 = 8.2 \text{ g/kg d.a.}, v_1 = 0.872 \text{ m}^3/\text{kg d.a.}
   h_1 = 52.5 \text{ kJ/kg d.a.}
Mass flow rate of dry air
   m_a = volume flow rate/sp. vol. = 39.6/0.872
       = 44.41 kg d.a./min
Cooling load per kg of d.a.
h_1 - h_2 = \text{capacity/m}_2 = 12.5 *60/44.41 = 16.89 \text{ kJ/kg d.a.}
Enthalpy of air leaving the coil = h_2 = 52.5 – 16.89
   h_2 = 35.61 \text{ kJ/kg d.a.}
```

Ans:

From equation of the BPF (or it is also called equation for the condition line)

$$(h_1 - h_2)/(h_1 - h_s) = (w_1 - w_2)/(w_1 - w_s)$$

This gives

$$w_2 = 6.77 \text{ g w.v./kg d.a.}$$

Corresponding to h_2 and w_2 , we can find DBT_2 and WBT_2 from the psychrometric chart

$$DBT_2 = 18.6$$
°C, $WBT_2 = 12.5$ °C

Coil BPF =
$$(h_2 - h_s)/(h_1 - h_s) = 0.515$$
 (Very high)

Numerical:

 In a cooling application, moist air enters a refrigeration coil at the rate of 100 kg of dry air per minute at 35°C and 50% RH. The ADP of coil is 5°C and bypass factor is 0.15. Determine the outlet state of moist air and cooling capacity of coil in TR.

Ans:

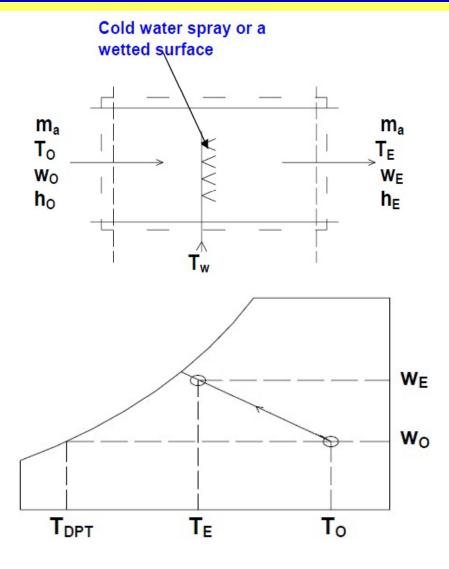
Mark the initial state of air, i.e. 35°C DBT and 50% RH on the psychrometric chart. Corresponding to this condition DPT = 23°C (obtained as the intersection point of a horizontal line passing through the above given point and the saturation line)

BPF =
$$(t_2 - t_s)/(t_1 - t_s) = (t_2 - ADP)/(DBT_1 - ADP) = 0.15$$

 $t_2 = DBT_2 = 9.5$ °C

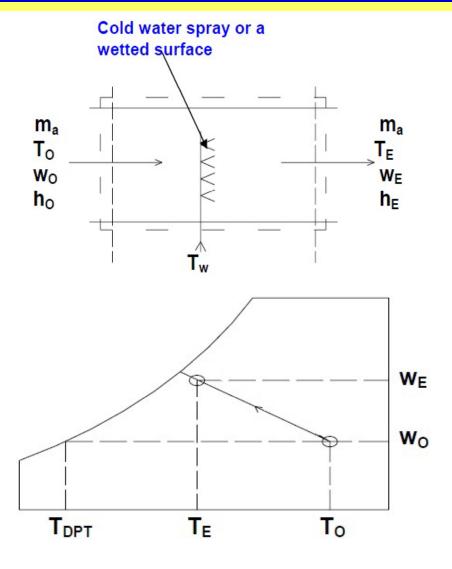
From the psychrometric chart we find that the RH corresponding to DBT_2 on line 1 - s(or ADP) is $RH_2 = 99\%$ Cooling capacity = $m_a \cdot (h_1 - h_2) = 100(81 - 28)$ kJ/min = 5300 kJ/min = 5300/211 = 25.19 TR

Cooling and humidification (Evaporative Cooling):



- The moisture content increases but the temperature of air decreases
- Water is injected into the flowing stream of air at the WBT of entering air
- The final condition of air will depend upon the amount of water evaporation

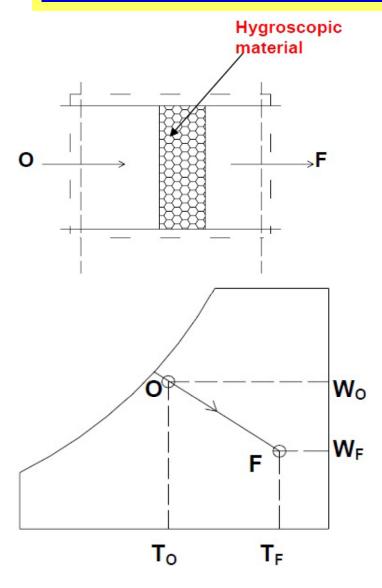
Cooling and humidification (Evaporative Cooling):



- The mass balance of water gives m_a w_E = m_a w_o +m_w where m_w is the mass of the water supplied
- The energy balance gives

 $m_a h_E = m_w h_{fw} + m_a h_o$ Since $m_w h_{fw} / m_a$ is very small, therefore $h_E = ho$, i.e. it is an isenthalpic process.

Heating and Adiabatic dehumidification:



- The moisture content of air is absorbed by a hygroscopic material
- The moisture liberates
 the latent heat during
 this process which heats
 up the air, thereby
 increasing its DBT
- This process is reverse of the adiabatic humidification process

Numerical:

 In a cooling application, moist air enters a refrigeration coil at the rate of 100 kg of dry air per minute at 35°C and 50% RH. The ADP of coil is 5°C and bypass factor is 0.15. Determine the outlet state of moist air and cooling capacity of coil in TR.

Ans:

Mark the initial state of air, i.e. 35°C DBT and 50% RH on the psychrometric chart. Corresponding to this condition DPT = 23°C (obtained as the intersection point of a horizontal line passing through the above given point and the saturation line)

BPF =
$$(t_2 - t_s)/(t_1 - t_s) = (t_2 - ADP)/(DBT_1 - ADP) = 0.15$$

 $t_2 = DBT_2 = 9.5$ °C

From the psychrometric chart we find that the RH corresponding to DBT_2 on line 1 - s(or ADP) is $RH_2 = 99\%$ Cooling capacity = $m_a \cdot (h_1 - h_2) = 100(81 - 28)$ kJ/min = 5300 kJ/min = 5300/211 = 25.19 TR

Numerical:

 The atmospheric air at 40°C DBT and 18°C WBT is flowing at the rate of 100 cmm through the system.
 Water at 18°C is injected into the air stream at the rate of 48 kg/h. Determine the specific humidity and enthalpy of the leaving air. Also, determine the DBT,
 WBT and the relative humidity of the leaving air.

Ans:

Mark the initial state of air, i.e. 40°C DBT and 18°C WBT on the psychrometric chart. Corresponding to this condition the specific volume of air is

$$v_1 = 0.89 \text{ m}^3/\text{kg d.a.}, w_1 = 4 \text{ g/kg d.a.}, h_1 = 51 \text{ kJ/kg d.a.}$$

Mass flow rate of air

```
m_a = volume flow rate/v_1
= 100/0.89 kg/min = 112.4 kg/min
w_2 = w_1 + m_w/m_a = 0.004 + 48/(112.4*60)
= 0.004 + 0.8/112.4 = 0.0111 kg/kg d.a.
```

Ans:

Since the water is injected at the WBT, therefore, this process is an isenthalpic process, i.e.

$$h_2 = h_1 = 51 \text{ kJ/kg d.a.}$$

Corresponding to w_2 and h_2 , we can mark the exit state of air on the psychrometric chart, which gives

$$DBT_2 = 22.4^{\circ}C$$

 $WBT_2 = WBT_1 = 18^{\circ}C$
 $RH_2 = 65\%$

Numerical:

 The atmospheric air at 25°C DBT and 12°C WBT is flowing at the rate of 100 cmm through the duct. The dry saturated steam at 100°C is injected into the air stream at the rate of 72 kg/hr. Determine the specific humidity and enthalpy of the leaving air. Also, determine the DBT, WBT and the relative humidity of the leaving air.

Ans:

Mark the initial state of air, i.e. 25°C DBT and 12°C WBT on the psychrometric chart. Corresponding to this condition the specific volume of air is

 $v_1 = 0.844 \text{ m}^3/\text{kg d.a.}, w_1 = 3.4 \text{ g/kg d.a.}, h_1 = 34.2 \text{ kJ/kg-d.a.}$

Mass flow rate of air

```
m_a = volume flow rate/v_1
= 100/0.844 kg/min = 118.5 kg/min
w_2 = w_1 + m_s/m_a = 0.0034 + 72/(118.5*60)
= 0.004 + 1.2/118.5 = 0.0135 kg/kg d.a.
```

Ans:

Since the steam is saturated at 100°C, therefore, enthalpy of the steam can be obtained from steam table, which gives

$$h_s$$
 = enthlapy of saturated steam at 100°C = 2676 kJ/kg

Hence, from energy balance

i.e.
$$m_a h_2 = m_a h_1 + m_s h_s$$

 $h_2 = h_1 + m_s h_s/m_a = 34.2 + 1.2*2676/118.5$
 $h_2 = 61.3 \text{ kJ/kg d.a.}$

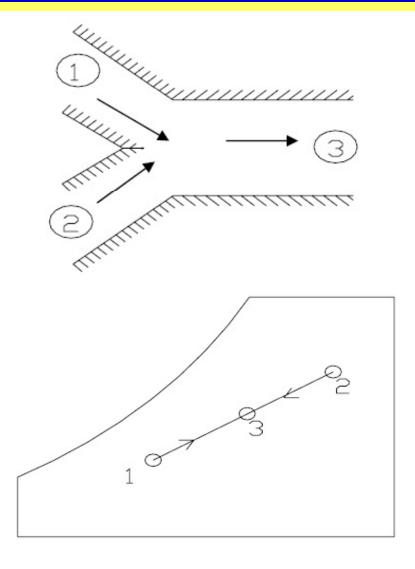
Ans:

Corresponding to w_2 and h_2 , we can mark the exit state of air on the psychrometric chart, which gives

$$DBT_2 = 26.1^{\circ}C$$

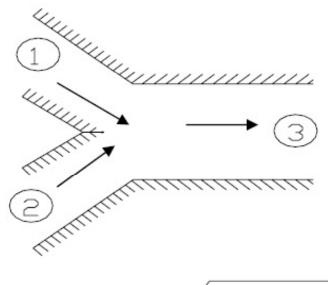
 $WBT_2 = 21.1^{\circ}C$
 $RH_2 = 62\%$

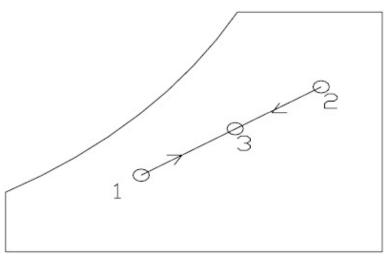
Mixing of Air Streams (Without Condensation):



- The mixing process is assumed to occur adiabatically
- This process is quite common in airconditioning systems
- Depending upon the initial states of mixing air, there could be condensation also

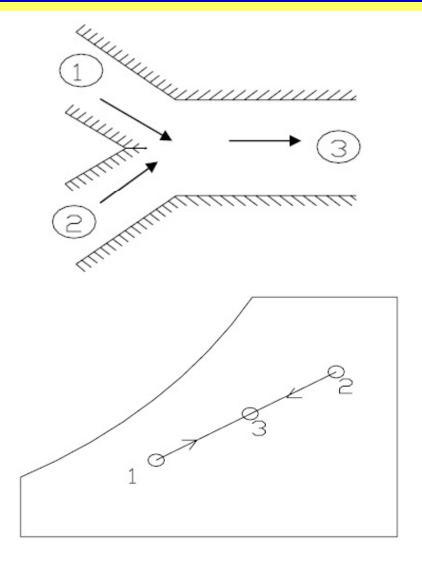
Mixing of Air Streams (Without Condensation):





- The mass balance of air gives
 m₃ = m₁ + m₂
- The mass balance of moisture gives m₃ w₃ = m₁ w₁ +m₂w₂
- The energy balance gives
 m₃ h₃ = m₁ h₁ + m₂ h₂
- No loss of energy and moisture is considered.

Mixing of Air Streams (Without Condensation):

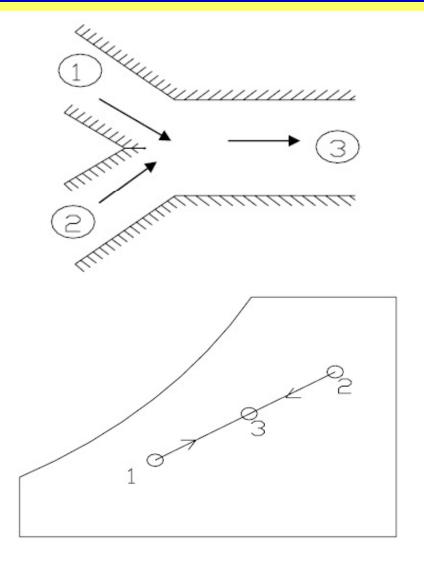


•
$$m_3 w_3 = m_1 w_1 + m_2 w_2$$

•
$$(m_1+m_2)h_3 = m_1h_1 + m_2h_2$$

From the above two
relations it can be inferred
that the final enthalpy and
humidity ratio of the
mixture are the weighted
average of inlet enthalpies
and the humidity ratios.

Mixing of Air Streams (Without Condensation):

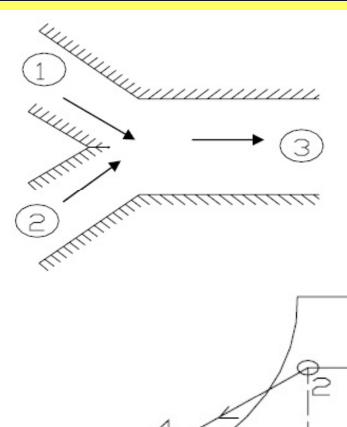


•
$$m_1/m_2 = (h_2-h_3)/(h_3-h_1)$$

•
$$m_1/m_2 = (w_2-w_3)/(w_3-w_1)$$

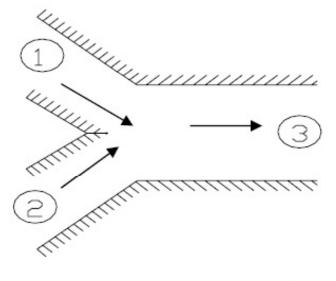
- The final state lies on the straight line 1-2 on the psychrometric chart
- The point 3 divides the line
 1-2 in the inverse ratio of the mixing masses.

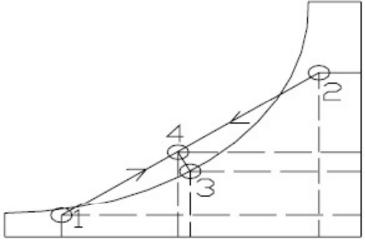
Mixing of Air Streams (With Condensation):



- The mixing process is assumed to occur adiabatically
- This process usually occurs when warm and high humidity air (point 2) is mixed with the cold air (point 1)
- The resulting mixture will lie in a two-phase region (point 4)

Mixing of Air Streams (With Condensation):





- Because of this some amount of water vapour may leave the system as liquid water
- This results in a decrease in the humidity ratio and an increase in the DBT (i.e. point 3)
- This process rarely happens in an airconditioning system

Numerical:

30 cmm of a stream of moist air at 15°C DBT and 13°C WBT are mixed with 12 cmm of a second stream at 25°C DBT and 18°C WBT. Barometric pressure is 1 atmosphere. Determine the DBT and WBT of the resulting mixture.

Ans:

Mark the initial state of air stream 1, i.e. 15°C DBT and 13°C WBT on the psychrometric chart. Corresponding to this condition

$$v_1 = 0.827 \text{ m}^3/\text{kg d.a.}, w_1 = 8.4 \text{ g/kg d.a.}, h_1 = 36.85 \text{ kJ/kg d.a.}$$

Mark the initial state of air stream 2, i.e. 25°C DBT and 18°C WBT on the psychrometric chart. Corresponding to this condition

$$v_2 = 0.859 \text{ m}^3/\text{kg d.a.}, w_2 = 10 \text{ g/kg d.a.}, h_2 = 51.1 \text{ kJ/kg d.a.}$$

$$m_1 = 30/0.827 = 36.2 \text{ kg d.a./min}$$

 $m_2 = 12/0.859 = 13.9 \text{ kg d.a./min}$

Ans:

$$m_3 = m_1 + m_2 = 50.1 \text{ kg d.a./min}$$

$$w3 = (m_1w_1 + m_2w_2)/(m_3) = 8.86 \text{ kg w.v./kg d.a.}$$

$$h3 = (m_1h_1 + m_2h_2)/(m_3) = 40.8 \text{ kJ/kg d.a.}$$

Numerical:

800 cmm of recirculated air at 22°C DBT and 10°C DPT is to be mixed with 300 cmm of fresh air at 30°C DBT and 50% RH. Determine the enthalpy, specific volume, humidity ratio, and DPT of the mixture.

Ans:

Mark the initial state of air stream 1, i.e. 30°C DBT and 50% RH on the psychrometric chart. Corresponding to this condition

 $v_1 = 0.876 \text{ m}^3/\text{kg d.a.}, w_1 = 13.4 \text{ g/kg d.a.}, h_1 = 64.6 \text{ kJ/kg d.a.}$

Mark the initial state of air stream 2, i.e. 22°C DBT and 10°C DPT on the psychrometric chart. Corresponding to this condition

 $v_2 = 0.846 \text{ m}^3/\text{kg d.a.}, w_2 = 7.6 \text{ g/kg d.a.}, h_2 = 41.8 \text{ kJ/kg d.a.}$

$$m_1 = 300/0.876 = 342.5 \text{ kg d.a./min}$$

$$m_2 = 800/0.846 = 945.6 \text{ kg d.a./min}$$

Ans:

$$m_1 / m_2 = (h_3 - h_2)/(h_1 - h_3)$$

342.5/945.6 = $(h_3 - 41.8)/(64.6 - h_3)$
 $h_3 = 47.86$ kJ/kg d.a.

Mark point 3 on the line 1-2 corresponding to the enthalpy $h_3 = 47.86 \text{ kJ/kg d.a.}$ Corresponding to this point

 $w_3 = 9.2 \text{ kg w.v./kg d.a.}$

$$v_3 = 0.855 \text{ m}^3/\text{kg d.a.}$$

$$DPT_3 = 13^{\circ}C$$

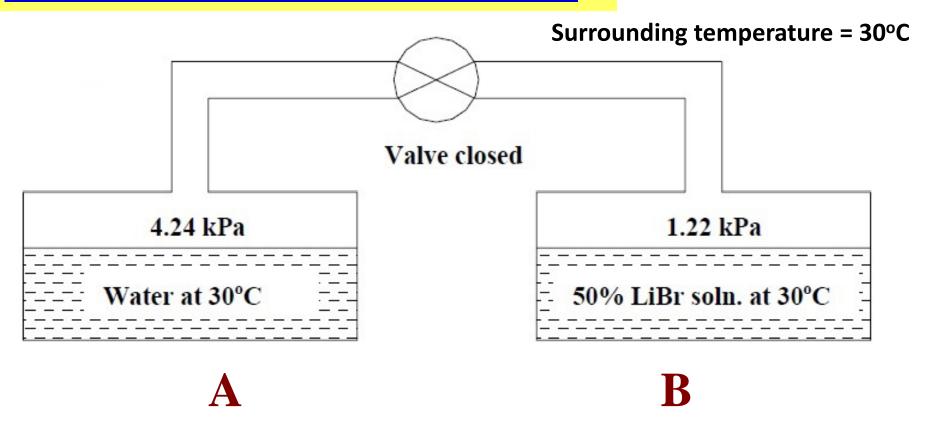
Vapour Absorption Refrigeration System (VARS):

- This system is similar to the vapour compression refrigeration system (VCRS) with the little variation in the way of compressing the refrigerant.
- An absorber, generator, and pump in the absorption refrigeration system replace the compressor of a vapour compression system
- This system utilises low grade energy like waste heat or solar energy to function which is supplied in the form of heat hence this system is also called heat operated or thermal energy driven system.

Vapour Absorption Refrigeration System (VARS):

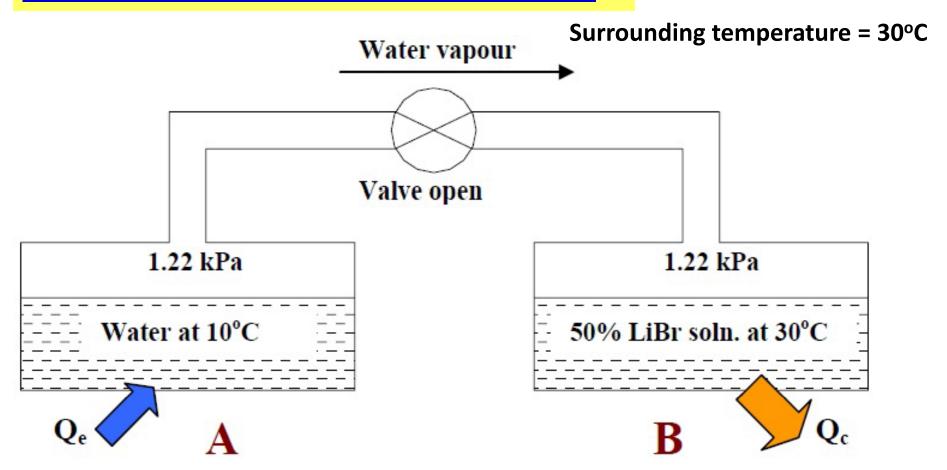
- VARS mostly relies on the absorber and the refrigerant.
 That means, to function properly it requires an absorbent and a refrigerant
- We know that water has a great affinity for absorbing large quantities of certain vapours like NH₃. Therefore, water can act as an absorbent and the NH₃ refrigerant
- Conventional absorption systems use liquid as the absorbent, therefore, these are also called wet absorption systems

Basic Working Principle of VARS:



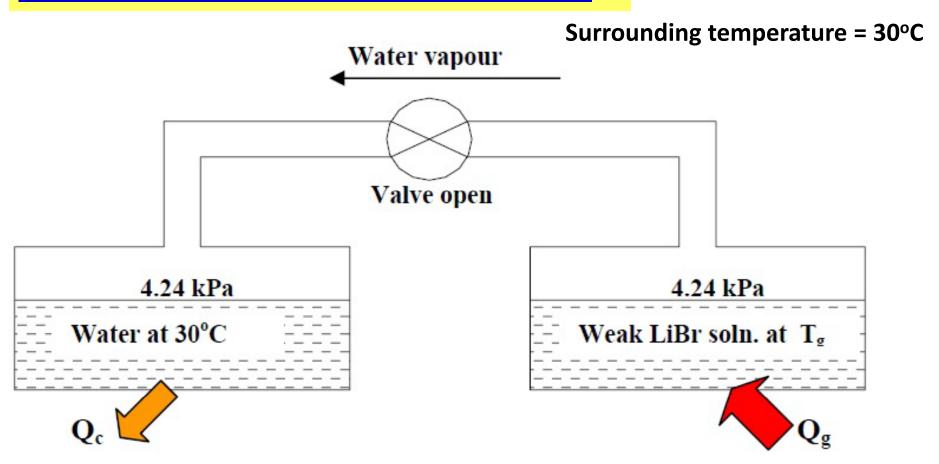
Initial Condition

Basic Working Principle of VARS:



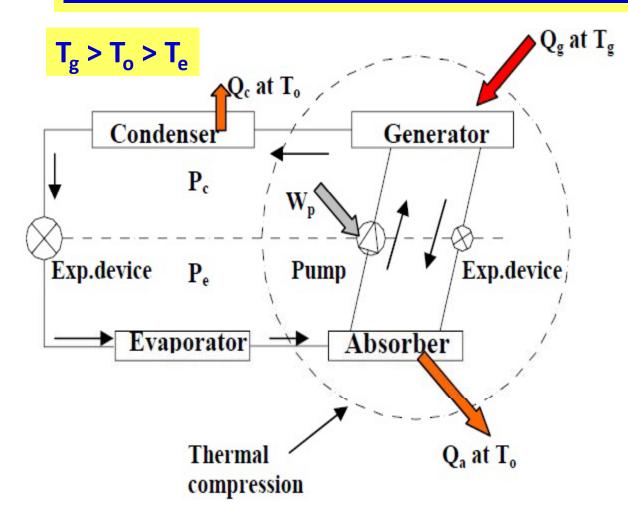
Refrigeration Process

Basic Working Principle of VARS:



Regeneration Process

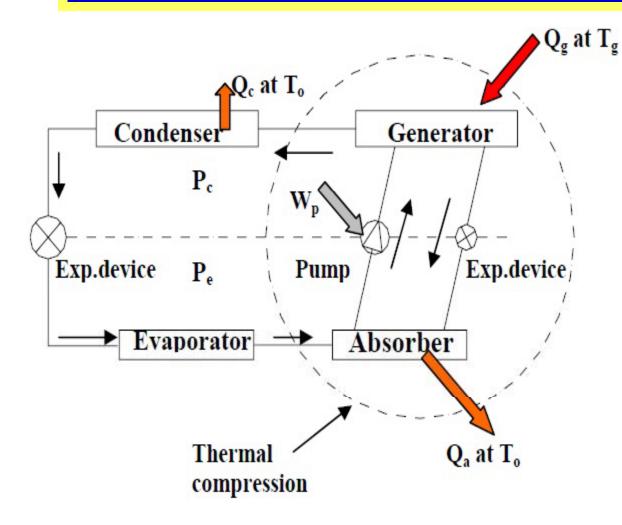
Simple Vapour Absorption Refrigeration System:



Refrigeration Cycle

- Low temperature, low pressure refrigerant enters the evaporator where it vaporises because of the refrigeration effect
- The vapour then enters the absorber where it comes in contact with the solution which is weak in refrigerant

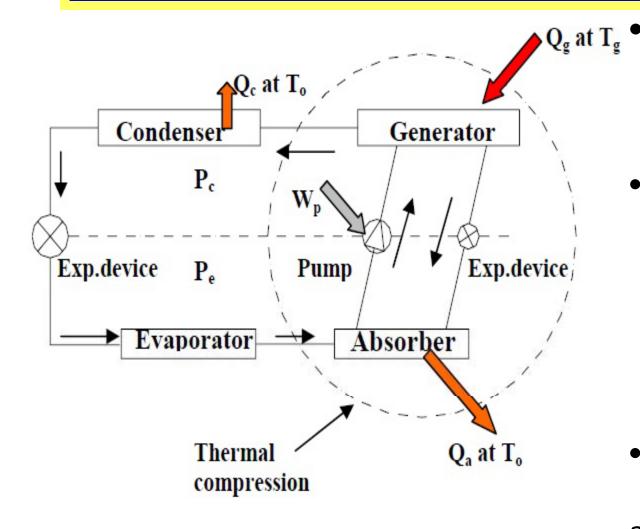
Simple Vapour Absorption Refrigeration System:



Refrigeration Cycle

- The solution becomes rich in refrigerant after absorption
- The heat is rejected to the surrounding because of the exothermic absorption
- The rich solution is now pumped to the generator thereby its pressure increases

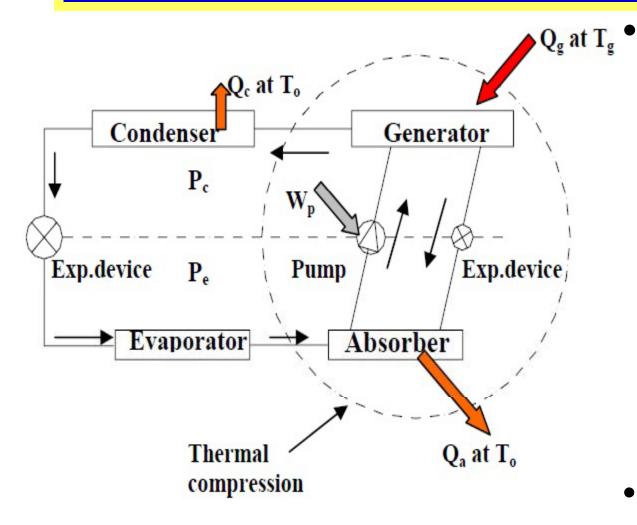
Simple Vapour Absorption Refrigeration System:



Refrigeration Cycle

- The heat is supplied in the generator at high temperature T_g
- This creates
 refrigerant vapour
 which is condensed in
 the condenser by
 rejecting heat to the
 surrounding
- The condensed refrigerant is throttled and fed to the evaporator, completing the cycle

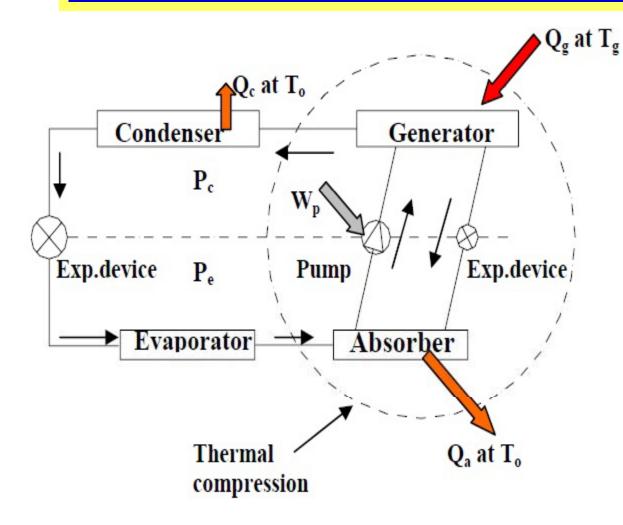
Simple Vapour Absorption Refrigeration System:



Solution Cycle

- On solution side, hot, high pressure solution which is weak in refrigerant is throttled in solution expansion valve and sent back to the absorber where it comes in contact with vapour refrigerant coming from evaporator
- This continuos process provides continuous refrigeration

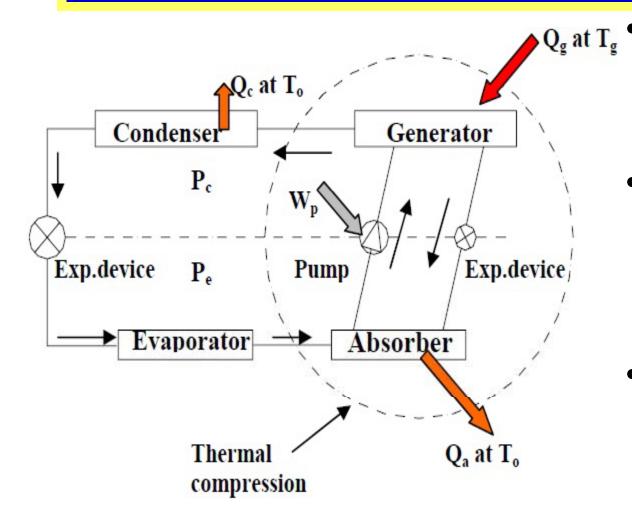
Simple Vapour Absorption Refrigeration System:



Overall Characteristics

- The refrigeration is produced continuously at the evaporator
- The heat is supplied continuously to the generator
- The heat rejection takes place at the condenser and the absorber

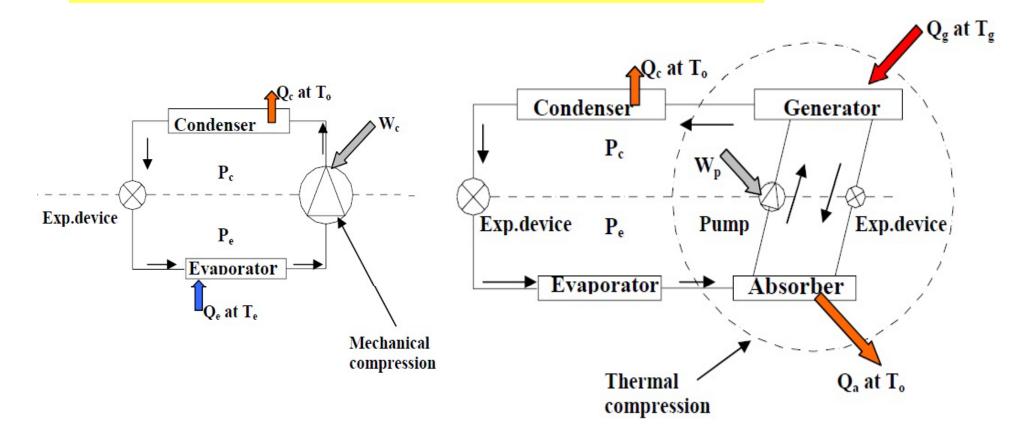
Simple Vapour Absorption Refrigeration System:



Overall Characteristics

- A small mechanical work is required to run the solution pump
- The pressure in absorber is equal to the pressure in the evaporator
- The pressure in generator is equal to the pressure in the condenser, if we neglect the pressure drop

Comparison between VCRS and VARS:



VCRS

VARS

$$COP = Q_e/W_e$$

$$COP = Q_e/(Q_g+W_p) \square Q_e/Q_g$$

Comparison between VCRS and VARS:

VARS	VCRS
Uses low grade energy like heat. Therefore, may work on exhaust systems from I.C engines, etc.	Uses high-grade energy like mechanical work.
Moving parts are only in the pump, which is a small element of the system. Hence operation is smooth.	Moving parts are in the compressor. Therefore, more wear, tear and noise.
The system can work on lower evaporator pressures also without affecting the COP.	The COP decreases considerably with decrease in evaporator pressure.
No effect of reducing the load on performance.	Performance is adversely affected at partial loads.
Liquid traces of refrigerant present in piping at the exit of evaporator constitute no danger.	Liquid traces in suction line may damage the compressor.

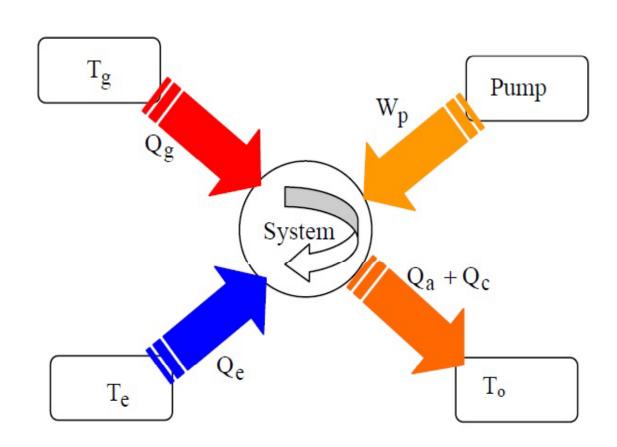
Maximum COP of An Ideal VARS:

The Carnot COP of a VCRS is given by

$$(COP)_c = T_e/(T_o - T_e)$$

- If we consider that the heat rejection from the condenser and the absorber is taking place at the same temperature T_o, then the VARS is assumed to operate between three temperature limits, i.e. T_g, T_o, and T_e.
- The COP of the above system can be derived as follows:

Maximum COP of An Ideal VARS:



Energy interactions

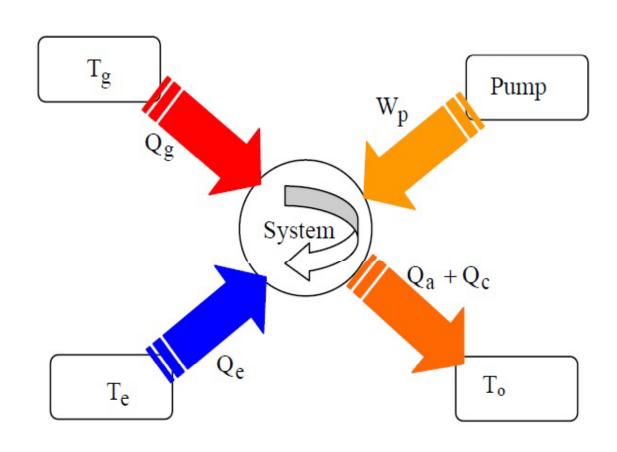
- The figure shows the interaction of energy with the system
- The first law of thermodynamics gives

$$Q_g + Q_e - Q_a - Q_c + W_p = 0$$

If we neglect W_p in comparison to other terms

$$Q_g + Q_e = Q_a + Q_c$$

Maximum COP of An Ideal VARS:



 From 2nd law of thermodynamics

$$\Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} \ge 0$$

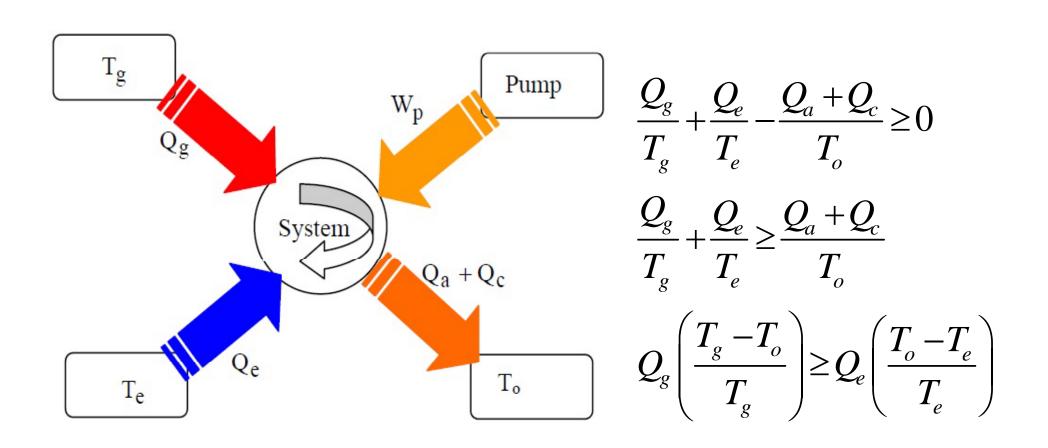
 Since the cycle operates in a closed cycle, the entropy change for the system is 0, i.e.

$$\Delta S_{sys} = 0$$

Therefore, $\Delta S_{surr} \geq 0$

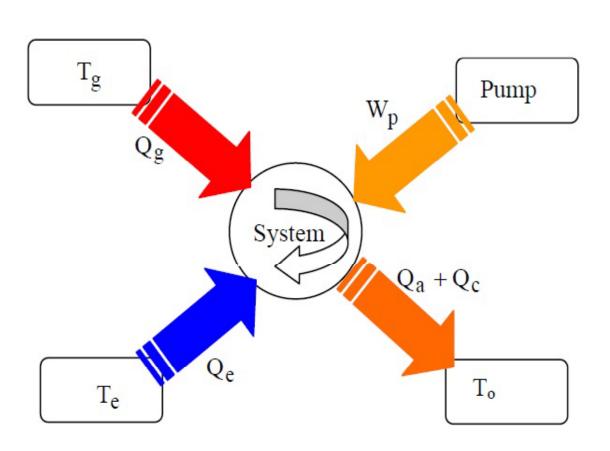
Energy interactions

Maximum COP of An Ideal VARS:



Energy interactions

Maximum COP of An Ideal VARS:



 Neglecting solution pump work W_p

$$COP = \frac{Q_e}{Q_g}$$

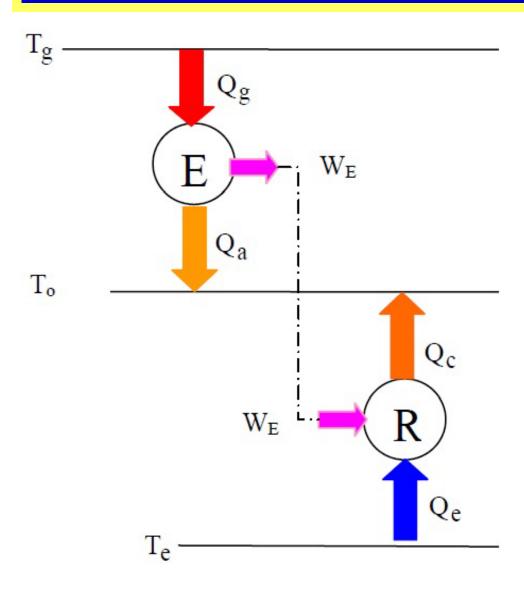
$$COP \le \left(\frac{T_e}{T_o - T_e}\right) \left(\frac{T_g - T_o}{T_g}\right)$$

Ideal or COP of reversible VARS

Energy interactions

$$(COP)_{ideal} = \left(\frac{T_e}{T_o - T_e}\right) \left(\frac{T_g - T_o}{T_g}\right)$$

Maximum COP of An Ideal VARS:



$$(COP)_{ideal} = \left(\frac{T_e}{T_o - T_e}\right) \left(\frac{T_g - T_o}{T_g}\right)$$
 $(COP)_{ideal} = (COP)_c \cdot \eta_c$

That means, ideal
 COP is the product of
 Carnot COP of a
 system operating
 between T_o and T_e
 and Carnot efficiency
 of a system operating
 between T_g and T_o.

Properties of Refrigerant-Absorbent Mixture:

- The mixture is usually considered as a homogeneous binary mixture
- From Gibb's phase rule, for a binary mixture, one more property along with the pressure and temperature is required to fix the thermodynamic state
- The third independent parameter is taken as the mass fraction or the mole fraction
- Another important parameter for the mixture is miscibility

Properties of Ideal, Homogeneous Mixture:

- 1. The total volume of the mixture should be equal to the summation of volumes of its constituents. That means, there should not be any contraction of expansion of the volume
- 2. Neither heat is generated nor destroyed upon mixing, i.e. the heat of solution should be zero.
- 3. The refrigerant should have high affinity for the absorber at low temperature and less affinity at high temperature.
- 4. The mixture should have low freezing point

Properties of Ideal, Homogeneous Mixture:

- 5. The mixture should be non-corrosive
- 6. There should be a large difference in normal boiling points of the refrigerant and the absorbent.
- 7. The mixture should have high degree of negative deviation from Raoult's law

Thermodynamic Requirements of Mixture:

The two main thermodynamic requirements of the refrigerant-absorbent mixture are as follows:

- 1. Solubility Requirement: The refrigerant should have more than Raoult's law solubility in the absorbent or adsorbent so that a strong solution, highly rich in the refrigerant, is formed.
- 2. Boiling Point Requirement: There should be a large difference in the normal boiling points of the two substances, at least 200oC, so that the absorbent exerts negligible vapour pressure at the generator

Thermodynamic Requirements of Mixture:

Along with the above two requirements, the mixture should possess the following desirable characteristics:

- It should have low viscosity to minimise the pump work
- It should have low freezing point
- It should have good chemical and thermal stability
- The irreversible chemical reactions are to be avoided