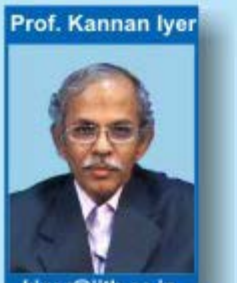
IIT Bombay

Overview

Tthis course aims at presenting the basic physical principles that govern the operation of a nuclear reactor. The treatment is broadly analytical so that a student can get a good feel of the quantitative aspects of the subject. Several problems have been solved for illustrations and additional problems are included as exercise problems. The target audience is third/fourth year engineering students and first year graduate students. It is assumed that the students have had two courses in engineering mathematics up to the level of ordinary differential equations and an exposure to fluid mechanics and heat transfer principles. The latter is required only to appreciate the pressure drop and heat removal aspects. Practicing engineers may also find this course useful owing to its analytical treatment.

Prof. Kannan Iyer



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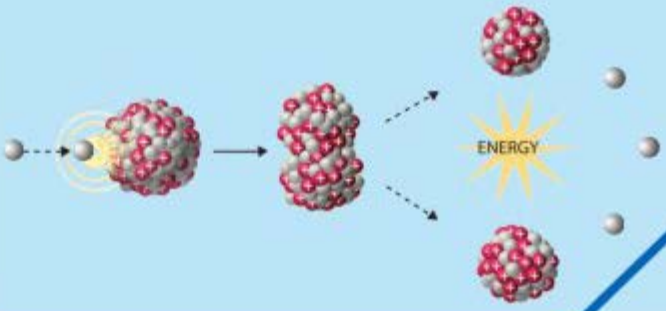


NPTEL

Objectives

objectives

In this lecture you will learn the following

- In this lecture the population and energy scenario in India are reviewed.
- The imminent rapid growth of nuclear power is brought out.
- Subsequently a bird's eye view of the various principles involved are presented.

Role of Nuclear Engineering

- Before we can start understanding about nuclear power, it is essential to look at the motivation.
- We shall realise that if India has to meet its aspirations of providing decent living to its people, it is necessary that nuclear has to grow fast.
- If this is planned properly and executed, there are tremendous opportunities for nuclear engineers.
- Let us show this in a brief manner.

Population Growth

- The source for the information on facts and figures are taken from a recent publication by Grover and Chandra *(R.B. Grover and S. Chandra, "Scenario for growth of energy in India", Energy Policy, Vol. 34, 2006, pp 2834-2847)*.
- Energy is generated for people to use it and hence we need to have a good picture of the population dynamics before we can project the energy requirements.
- The population statistics past, present and projected are given in the following Table:

Year	Population (Billions)	Growth Rate (per year)
2001	1.027	1.50
2011	1.19	1.02
2021	1.32	0.7
2031	1.41	0.4
2041	1.47	0.2
2051	1.50	0.0

- We expect to stabilise around 2050.
- We need to provide electricity for 1.5 billion people.

Electricity Production

Year	Power Production (Mwe)
1947	1363
1981	30214
1991	66086
2003	138730

- From the above table, we see that the average growth rate is 8.6% and is impressive.
- However, per capita power production is still small compared to global average.

Total Energy Scenario

- In 2001, total energy consumed by the world was 382 EJ (1 Exa Joule = 10^{18} J).
- Indians consumed 3.4% with a population share of 16.6%.
- USA consumed 24.5% with 4.6% population.
- Indian consumption was $1/5^{th}$ of the world average.
- Thus to improve the quality of life there has to be a growth in energy production and consumption.

Energy Split

- In 2003, Indians consumed 18.96 EJ.
- Domestic resources accounted for 15 EJ and imported 3.96 EJ.
- 71% of the above was for commercial purposes and 29% was for domestic purposes.
- Of the commercial, 92% was from fossil, 6% hydro, 1.7% nuclear and 0.2% wind.
- The import content was 8% for coal, 71% for oil and 13% for nuclear.

Energy Growth Rate Projection

Year	Total Energy (%)	Electrical Power (%)
2002-22	4.6	6.3
2022-32	4.5	4.9
2032-42	4.5	4.5
2042-52	3.9	3.9

- For a sustainable development a model has been evolved for electricity production. This is summarised in the table shown above.
- The contributions from coal, Hydrocarbon, Hydel, Renewable and Nuclear are 46%, 16%, 8%, 4% and 26% respectively. Thus nuclear has to grow fast if we have to meet our energy demands.

Nuclear Perspective

- For the projections given, if we have to succeed, several new technologies have to be demonstrated.
- The foremost among them is the Fast Breeder.
- Demonstration of Accelerated Driven Systems with thorium as fuel.
- It is important to note that India has to take the leadership role as the rest of the world does not have much interest in thorium based cycle.
- Having understood the need for accelerated growth of nuclear power, let us now learn the fundamentals of the subject.
- Before going into the details, a small overview is given to appreciate the details of the subject that will be taught.

Discovery of Fission

- Rutherford described as the father of nuclear physics, bombarded alpha particles on Nitrogen to eject a proton and thus started the business of splitting atom in 1917.
- James Chadwick is credited with discovering neutron in 1932. Being neutral, it can easily penetrate the Columb barrier and interact directly with the nucleus.
- Enrico Fermi, considered to be one of the most brilliant physicist, postulated and succeeded in producing transuranic elements by bombarding neutrons on Uranium. However, he did not realise that he had fissioned Uranium in 1934.
- Otto Hahn, Fritz Strassman and Lise Meitner in 1938 conducted experiments similar to Fermi.
- They noticed that instead of a heavy element formation they found lighter elements in the product.
- They were the first ones to postulate fission and pointed the large energy that will be released.
- We shall see later how this energy release is made possible.

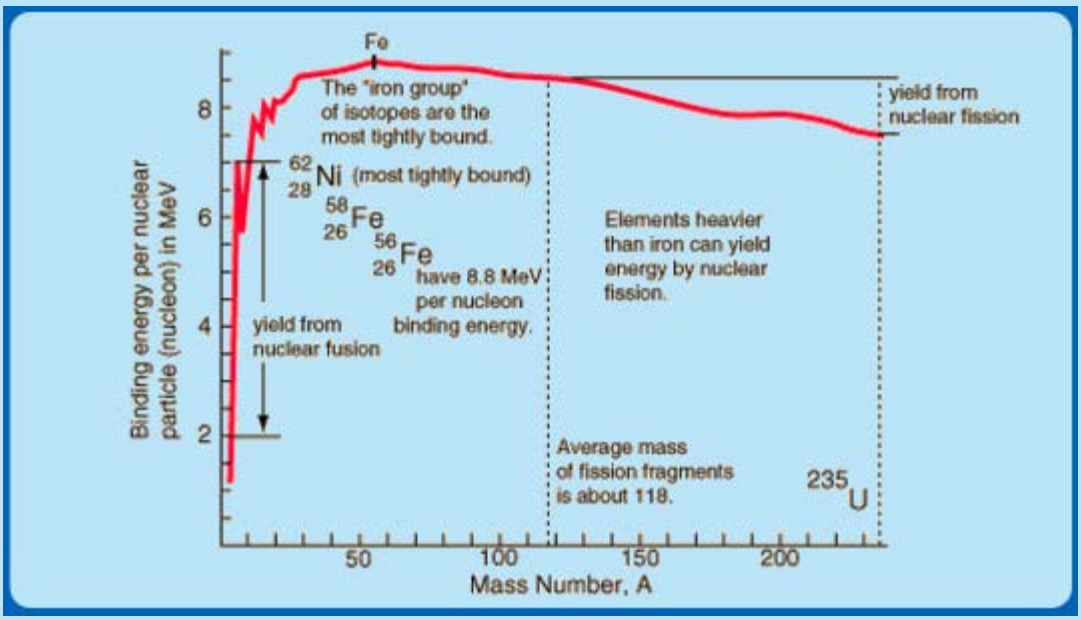
Rutherford's Model

- Rutherford, based on experimentation, had proposed a model for the atom.
- It has a central positively charged nucleus consisting of protons and neutrons.
- It has electrons revolving around to make the atom neutral.
- The charge and mass are summarised in the following table.

	Mass	Charge
Proton	1.67261×10^{-24} g	$+1.60219 \times 10^{-19}$ Columbs
Electron	9.10956×10^{-28} g	-1.60219×10^{-19} Columbs
Neutron	1.67492×10^{-24} g	0

Binding Energy

- Binding energy of a nucleus is defined as the energy required to break a nucleus into its constituents.
- It increases with the number of nucleons in a nucleus.
- It is often plotted in per nucleon basis as shown below



- We shall understand this figure in more detail in a later lecture.
- Higher the binding energy per nucleon, more stable is the nuclei.
- Iron is the most stable nuclei.
- We also note that as light elements fuse or heavy elements undergo fission, the products are more stable.

Fissile Elements

- Consider the following equation

$$^{233}_{92}U + ^{1}_{0}n \rightarrow ^{234}_{92}U + \gamma(6.6MeV)$$

- 6.6 MeV is released on account of nuetron being absorbed or bound to U^{-234} nucleus.
- Thus, Binding Energy of Last Neutron is 6.6 MeV.
- Energy necessary to be supplied to induce fission is called Critical Energy for Fission.
- The binding energy of last neutron and critical energy for fission for some isotopes are listed below:

Isotope	Critical Energy for Fission (MeV)	Binding Energy of Last Neutron (MeV)
Th^{232}	6.5	5.1
U^{234}	4.6	6.6
U^{236}	5.3	6.4
U^{239}	5.5	4.9
Pu^{240}	4.0	6.4

- Thus when neutron is absorbed in U^{233} the neutron deposits its binding energy in the compound nucleus (U^{234}), which is more than what is required to make U^{234} fission. Hence it splits spontaneously.
- Those elements which can be fissioned by zero energy neutrons are called fissile elements.
- Thus U^{233}, U^{235} and Pu^{239} are fissile elements.
- On the other hand, Th^{232} and U^{238} need neutrons with 1.4 MeV and 0.6 MeV respectively for inducing fission. Hence these are called fissionable nuclei.
- Natural Uranium has 99.3% of U^{238} and 0.7% of U^{235}.
- U^{235} is the only naturally occuring fissile element.

Prompt Neutron Spectrum

- Nutrons released from fission have an average Energy = 1.98 MeV.
- A large fraction of neutron is above the threshold energy of 0.6 MeV (refer table in previous page) needed to fission U^{238}.
- Thus some U^{238} is also directly fissioned in reactors.
- It has been noticed that as the energy of the colliding neutron is high, the probability of reaction is poor.
- This is attributed to the reduced time a neutron spends in the zone of interaction.
- Hence the probability of reaction is inversely proportional to the speed of the neutron.
- As the interaction decreases rapidly with increase in energy, and that the energy of the prompt neutrons are high, we need to slow them down.
- This is done by the use of moderator.

Moderators and Coolants

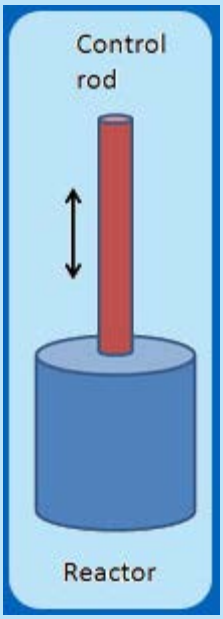
- Hydrogen in the form of water (H_2O) and heavy hydrogen in the form of heavy water (D_2O) are the most common moderators.
- In gas cooled reactors graphite (C) is used as the moderator.
- Coolant is employed to transport the heat generated in the core to the steam generator, where steam is generated.
- In water moderated reactors the same fluid also serves as coolant.
- There are some safety advantages if we use gas as a coolant. In such cases CO_2 or He is used.

Critical Size

- We shall see later in the course that all neutrons produced do not get absorbed in the reactor.
- Many neutrons leak out of the reactor.
- To keep the neutron population steady in a reactor we need to produce more neutrons than what is absorbed in fuel.
- As the number of neutrons produced depends on the volume and number of neutrons leaking is proportional to surface area, there exists a minimum size below which the reactor cannot operate (due to excess leakage).
- This is called the critical size.
- Thus, for a reactor to operate we need a critical size.
- This size depends on the concentration of the fissile elements.
- The size increases with the reduction in concentration.
- Thus conceptually, as fuel burns out we need to increase the size, which is impractical.
- To overcome this, the size of the reactor is increased to begin with and to compensate for the excess production, neutron absorbers are used.
- As fuel burns out the absorbers are gradually removed.

Control of Nuclear Reactors

- The process of power addition or reduction is done by insertion or removal of control rods.
- Originally when the fuel is fresh and the reactor is not operating, these rods are fully in.
- To start the reactor, these are slowly pulled out and the rector begins operation when the rods are out by a small amount.
- As the fissile elements deplete, to compensate for them these are continuously pulled out.
- When they get pulled out completely, then reactor is stopped and fresh fuel added.



Fast Breeders

- We shall show later in the course that we get around 2 effective neutrons from fission induced by fully moderated neutrons.
- However, if we do not fully slow down and allow the neutron to induce fission at higher speeds, we get more effective neutrons per fission.
- While only one neutron is required to get the reactor operating steadily, the excess neutrons can be used to produce fresh fuel.
- This is called breeding.
- Since neutrons are not fully slowed down, it is called fast breeder.
- Such rectors are usually cooled by sodium.

Power Reactors

- Some of the characteristics of nuclear power reactors are summarised in the following Table:

Type	Moderator	Coolant	Pressure (bar)	Maximum Temp (C)	Eff. %
Pressurized Water Reactor	Light Water	Light Water	150	320	33
Boiling Water Reactor	Light Water	Light Water	70	250	33
Pressurised Heavy Water Reactor	Heavy Water	Heavy Water	80	320	32
Gas Cooled Reactor	Graphite	Carbondioxide	15	410	35
High Temperature Gas Reactor	Graphite	Helium	45	800	45
Liquid Metal Fast Reactor	-------	Sodium	1	580	42

Agenda for the Course

- Having realised that nuclear reactors have an important role to play in India's power generation scene, we shall now understand the principles of operation of a nuclear reactor.
- We shall understand that present and the reactors of immediate future shall operate on fission process.
- We shall understand how reactors are sized, moderated, cooled and controlled.
- We shall also understand the safety aspects that have to be kept on mind while designing these reactors.

Objectives

In this lecture you will learn the following

- This lecture brings out the features of nuclear power reactors.
- Description of BWR, PWR, PHWR and FBR's are included.

Fig. 1 Nuclear Reactor with Containment

- This lecture gives some of the basic concepts used in power stations.
- A bird's eye view of a reactor site is shown in fig. 1.
- The two cylindrical buildings are the reactor building.
- The rectangular buildings behind the reactors are the turbine buildings.
- Two large hyperbola shaped towers are the cooling towers.

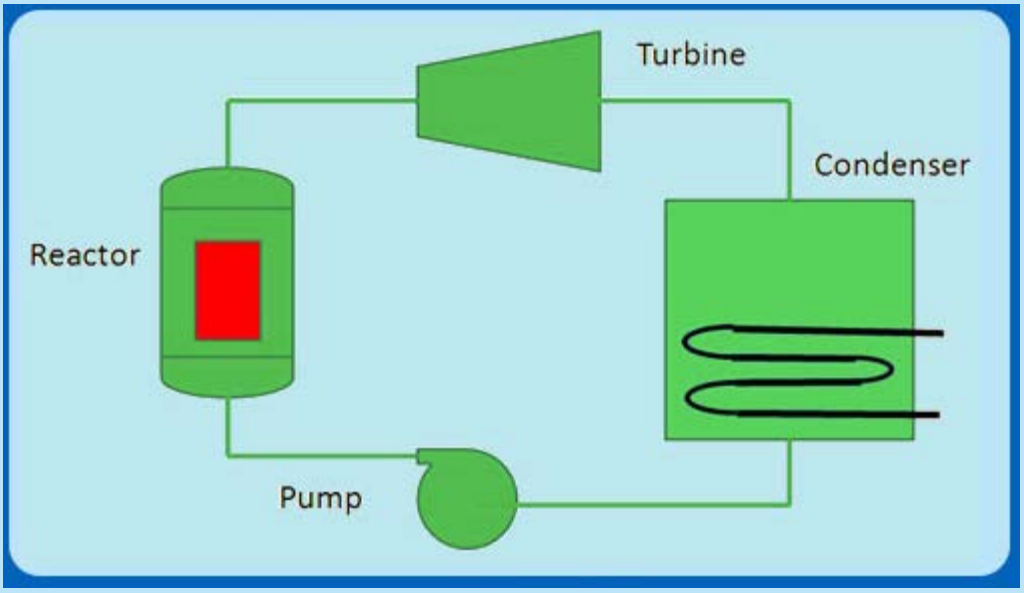


Fig. 2 Boiling Water Reactor (BWR)-I (BWR)

- The schematic view of Boiling Water Reactor (BWR) is shown in fig.2.
- In this reactor water boils in the reactor itself.
- The steam so produced is sent to turbine to produce work.
- The expanded steam is condensed and is then pumped back into the reactor.

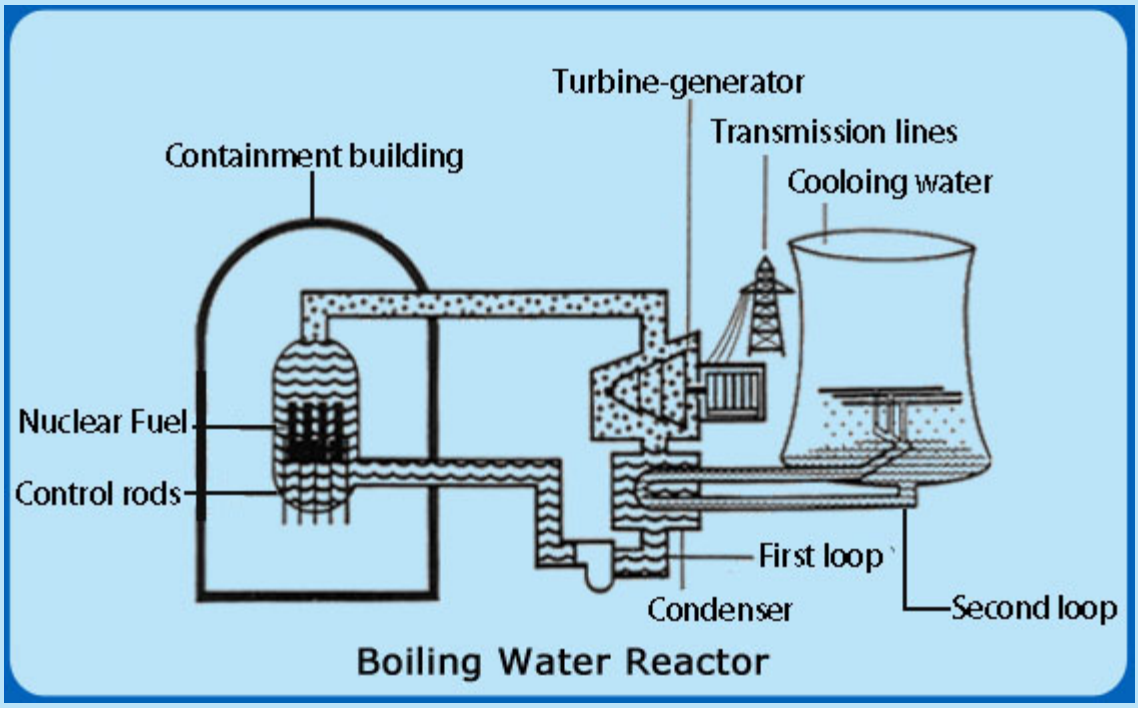


Fig.3 Typical Layout of BWR

- A more detailed schematic layout is shown in fig. 3.
- Note that control rods are inserted from the bottom of the vessel.
- Note that the reactor building is also called containment building.
- This is because in case of any accidents, the radioactive products are trapped (contained) in this building.
- Note that only reactor is in the containment building.
- The turbine and condenser are housed outside.

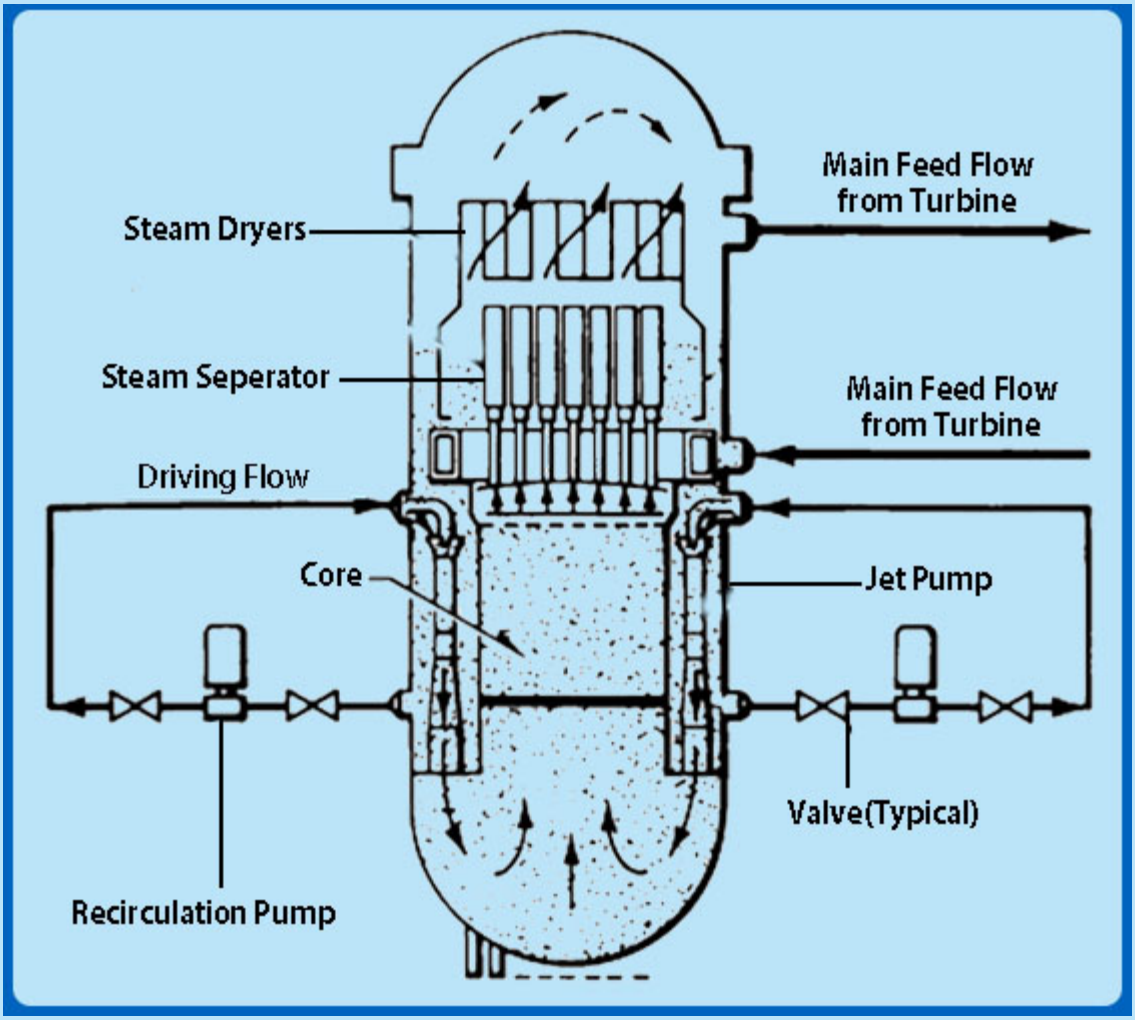


Fig.4 BWR Reactor Vessel

- The details of components housed inside the reactor vessel are shown.
- Jet pumps housed inside the reactor vessel helps in circulating fluid into the core.
- Steam seperators at the top separate steam from water.
- Steam dryers are used to remove water droplets that are carried by steam.

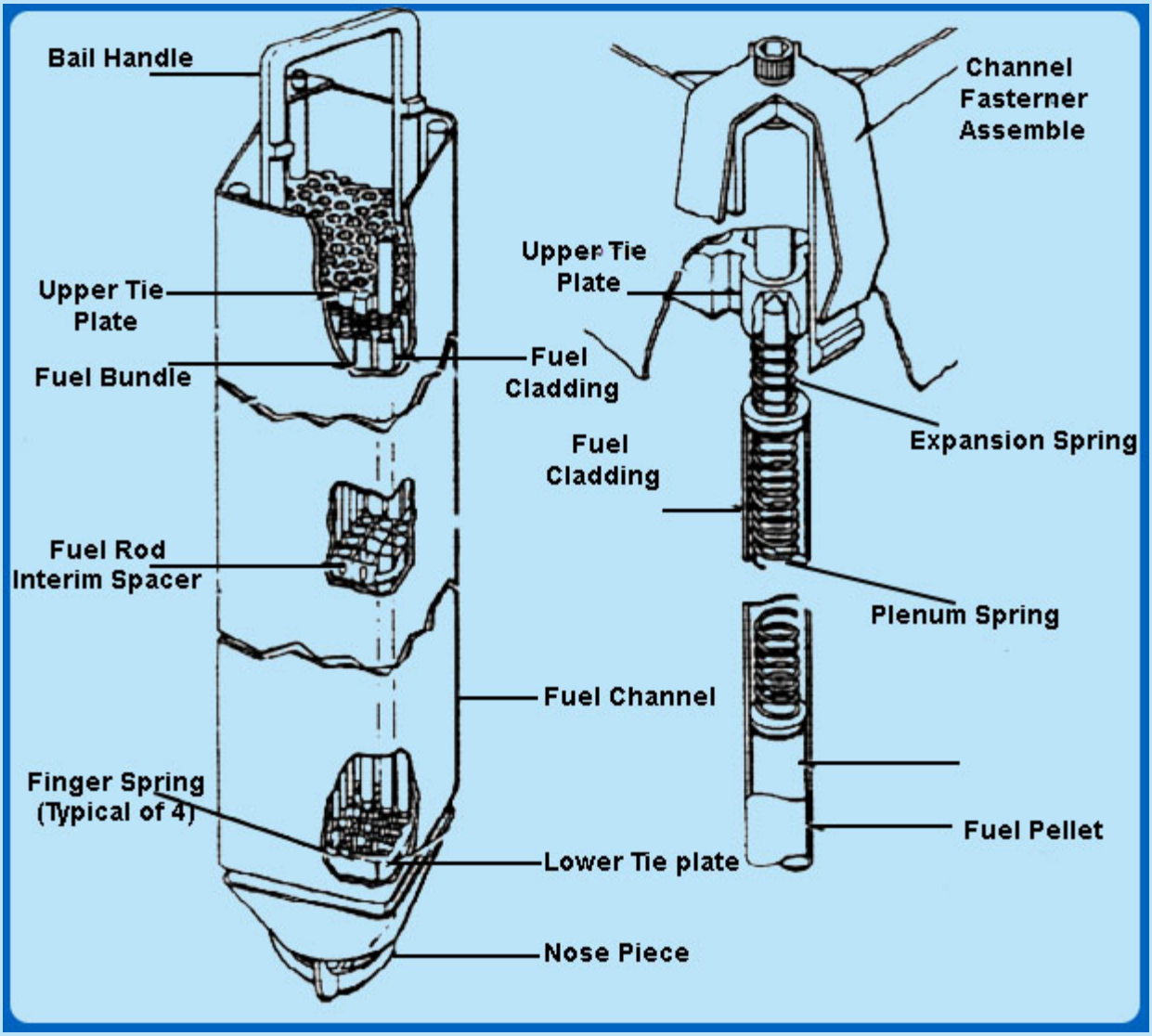


Fig.5 BWR Fuel Bundle

- The fuel bundle of a BWR is shown in fig. 5.
- It is an assembly of fuel rods arranged in a square array.
- Details inside each fuel rod is shown separately.
- The fuel is in the form of cylindrical pellets.
- A gas plenum(space) is provided at the top of fuel rod.
- This is to accommodate fission gases that diffuse out of the fuel.
- The bundle is wrapped with a metal sheet to prevent mixing of steam from adjacent bundles.

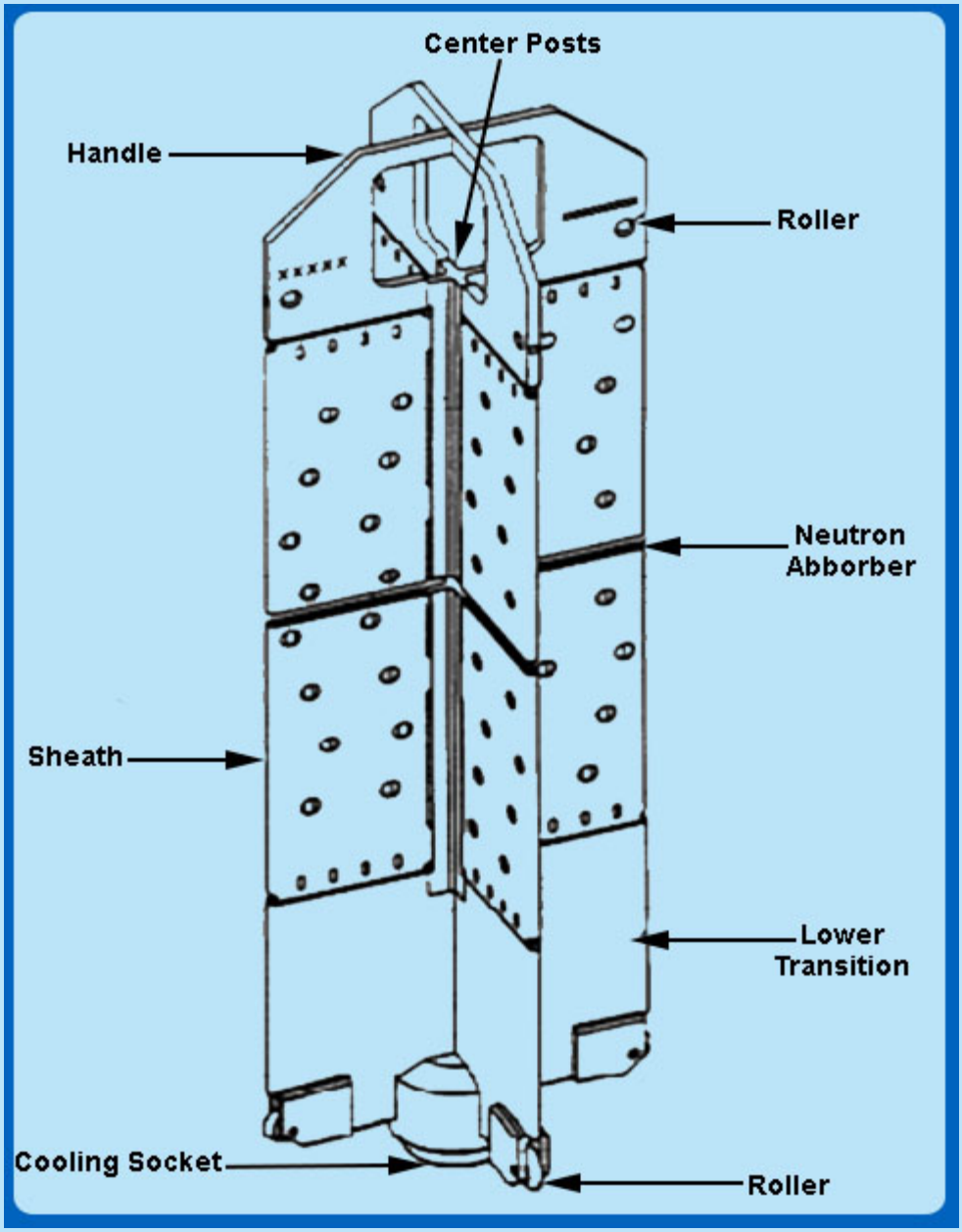


Fig.6 BWR Control Rod

- The schematic view of the control rod is shown in fig.6.
- It has a cross section in the form of a "cross" and is called cruciform control rod.
- Nuetron absorbers are placed inside the cross.

Fig.7 BWR/6 Fuel assemblies & control rod module

- Figure 7 clearly shows the assembly of rod bundles.
- It also shows the position of the cruciform control rods.
- One cruciform control rod is positioned in between four fuel bundles.
- The handles on the rod bundles are used for lifting and positioning the rod bundles.

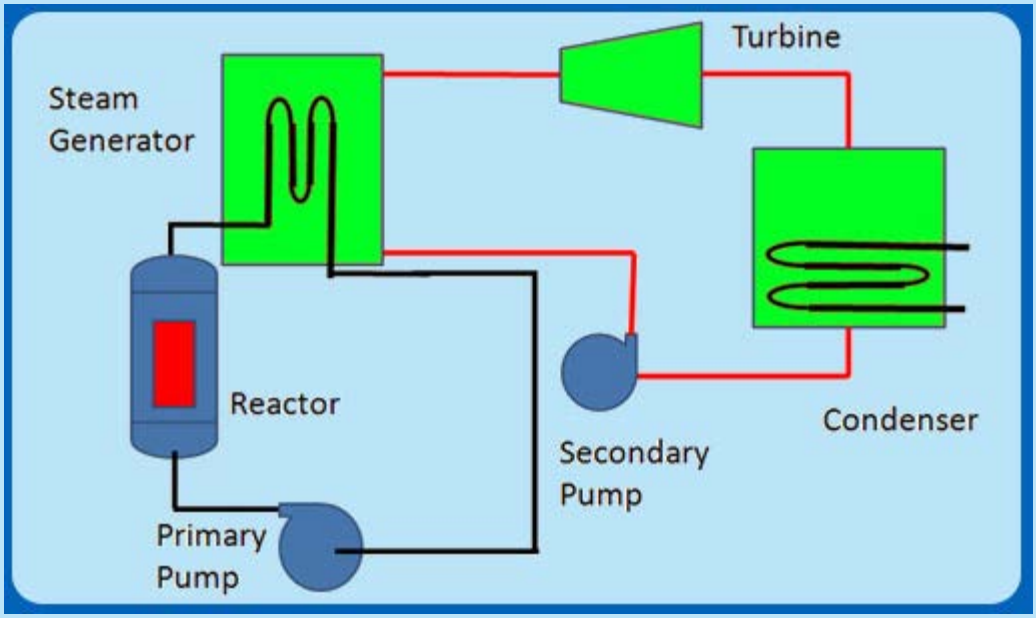


Fig.8 Pressurised Water Reactor

- The schematic view of a Pressurised Water Reactor (PWR) is shown in fig.8.
- In a PWR, the water is not allowed to boil in the reactor.
- Steam is produced in a steam generator.
- Thus there are two circuits, called primary and secondary.
- The primary coolant takes heat from fuel rods in the reactor and transports it to steam generator.
- In the steam generator, the primary coolant transfers heat to the secondary coolant.
- The primary coolant is pumped back using primary pump.
- In the secondary circuit, water is pumped into the steam generator by the secondary pump.
- The steam generated is expanded in the turbine, condensed in the condenser and pumped back.

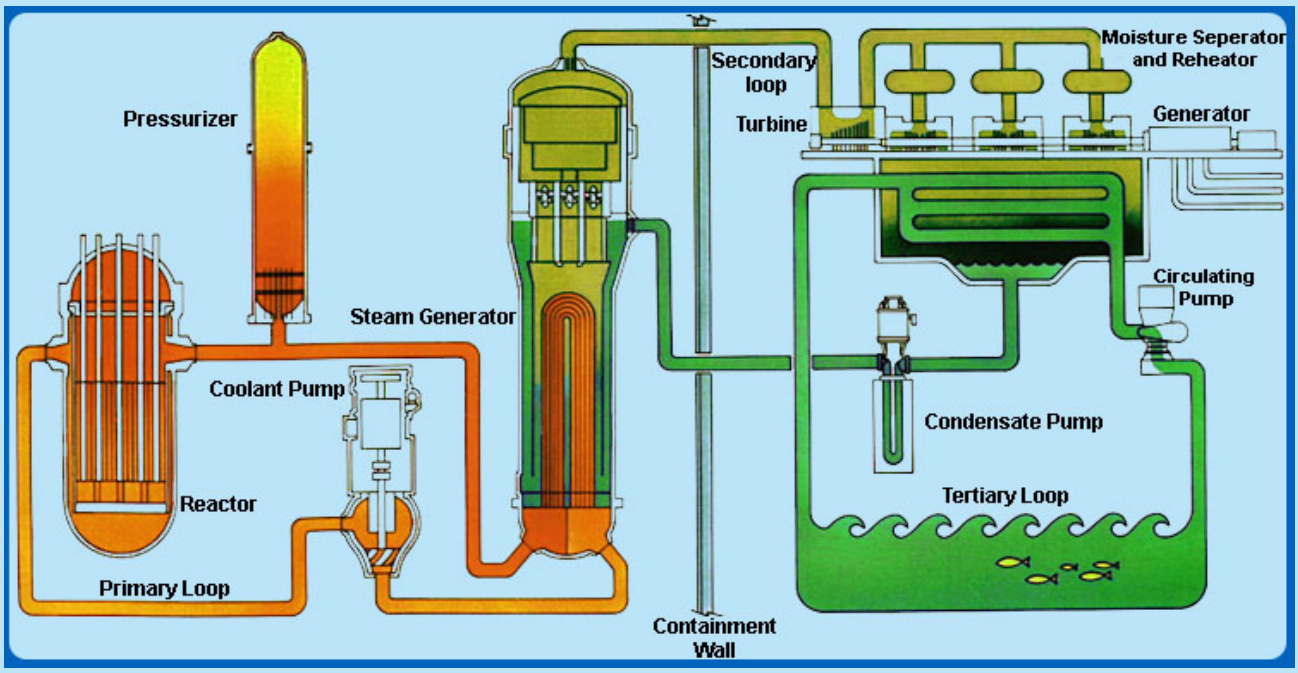


Fig.9 Westinghouse PWR

- The schematic view of the PWR manufactured by Westinghouse is shown in fig.9.
- An additional component called pressuriser is seen.
- It maintains the pressure in the system, should it try to rise or fall due a mismatch between heat generated in fuel and heat transmitted in the steam generator.
- It may be observed that the size of steam generator is larger than the reactor.
- More than one steam generator is used in a PWR.

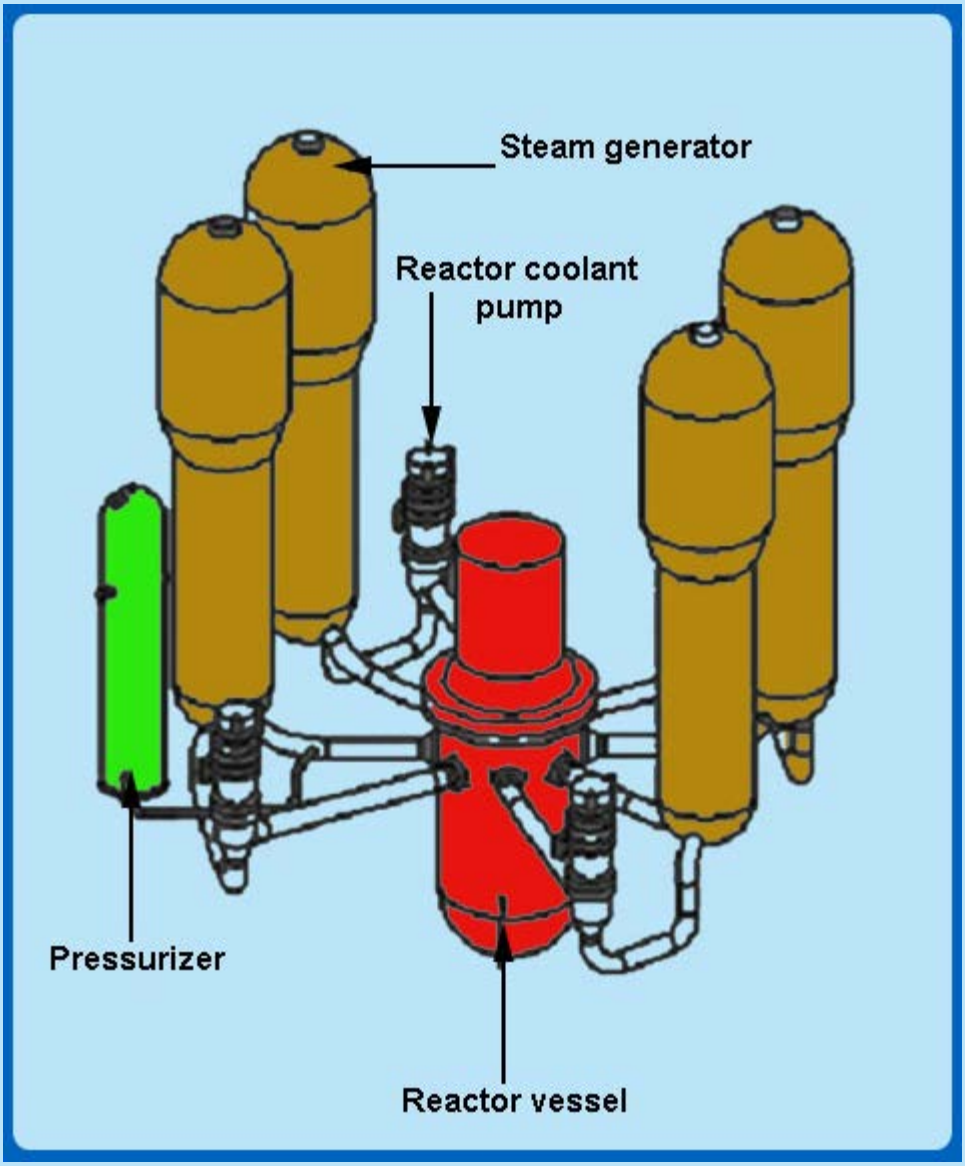


Fig.10 Four-Loop Plant

- A typical four loop plant is shown in fig. 10.
- It may be noted that one of the pump is hidden.

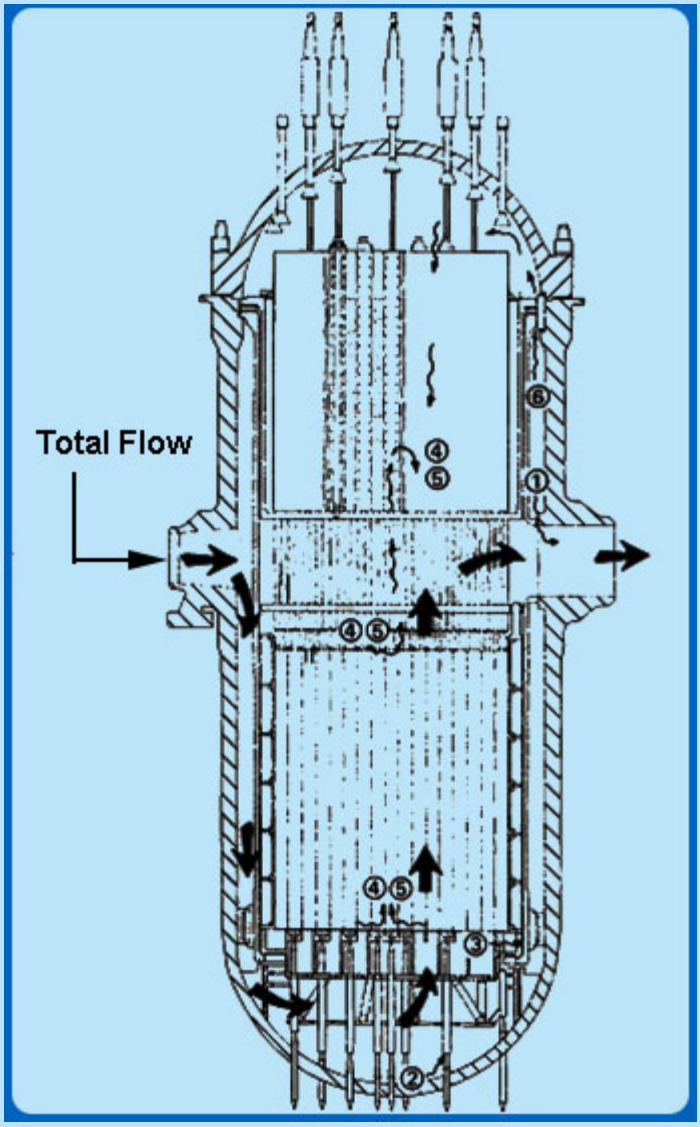


Fig.11 PWR Reactor Vessel

- Figure 11 shows the details of the PWR pressure vessel.
- It may be observed that the top half is used to house the control rods.
- The primary coolant fluid movement path is also shown.
- The fluid is made to move down by the presence of a core baffle.
- The fluid then rises from the bottom of the core.
- The heated water then flows out from the top of the core.

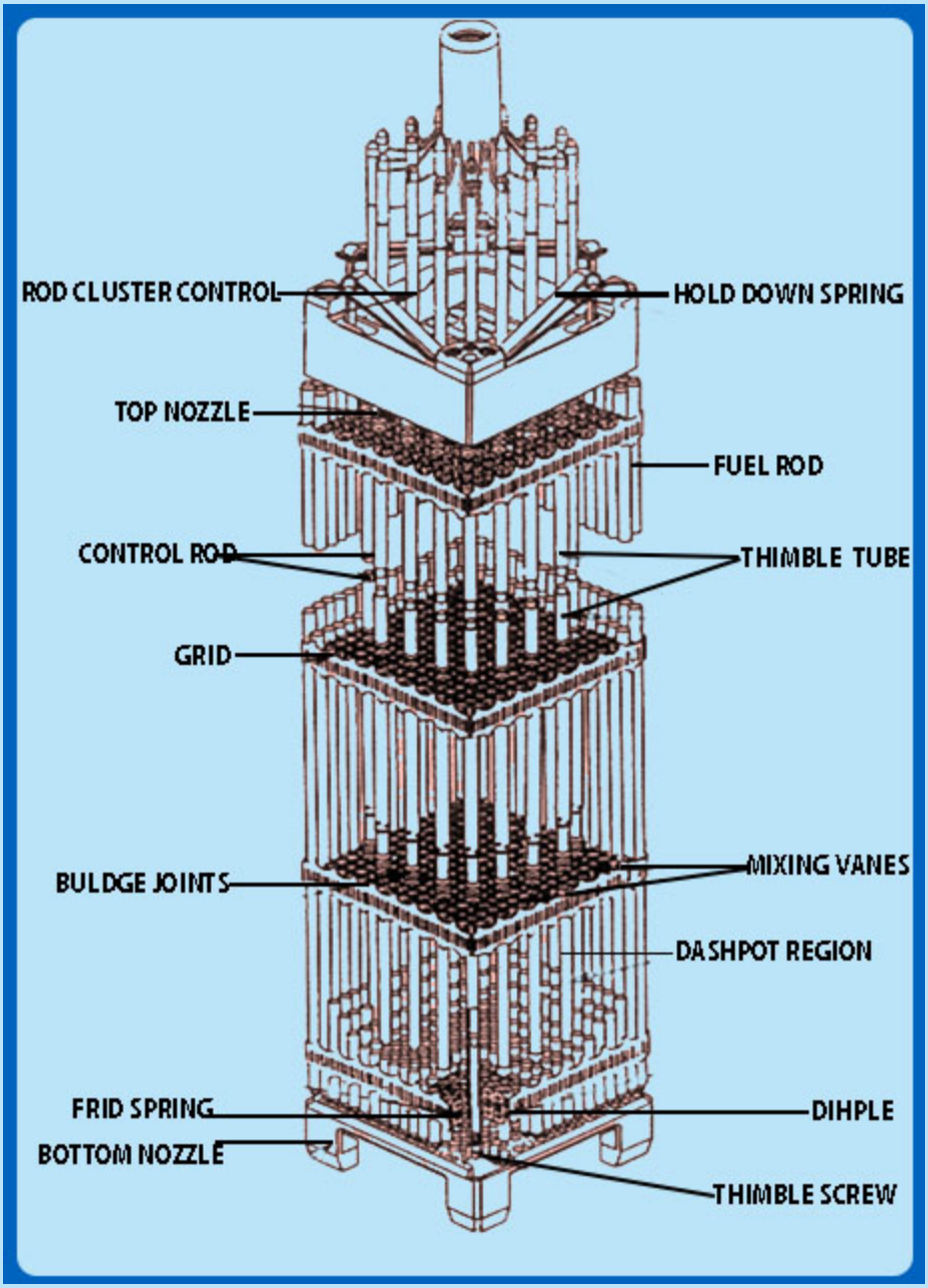


Fig.12 Fuel Assembly

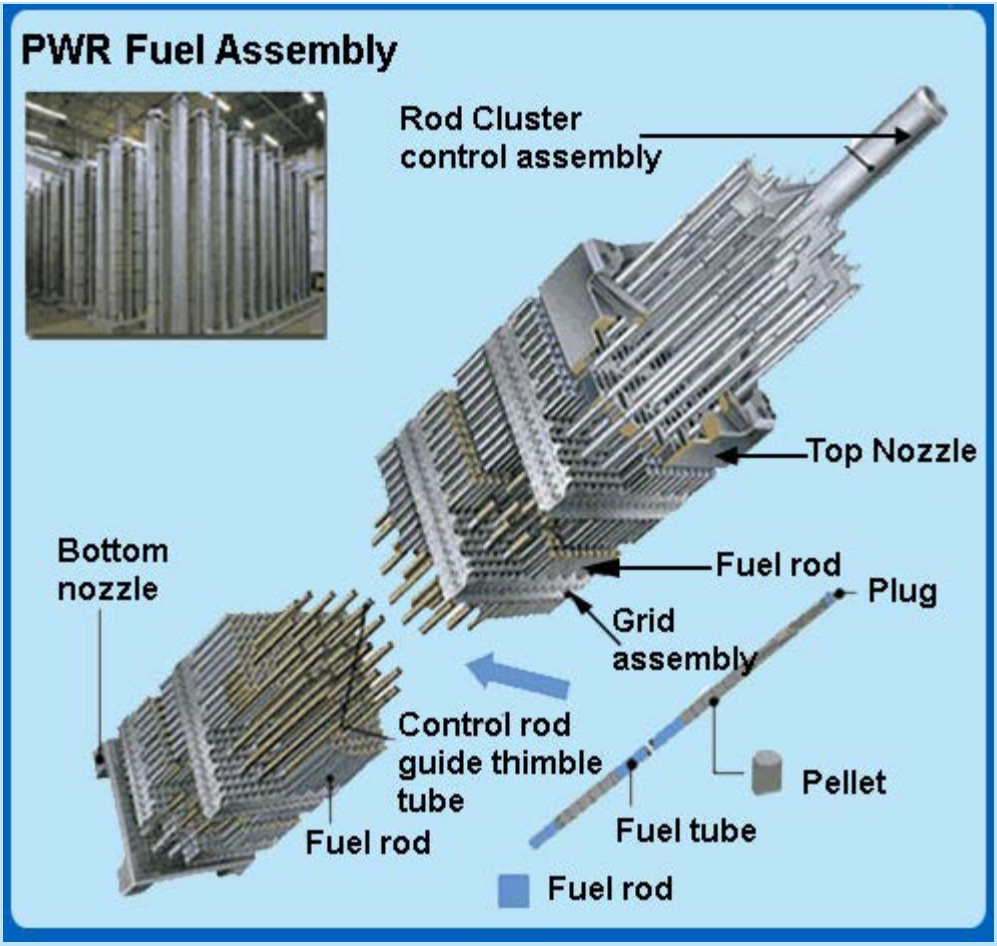


Fig.13 PWR Fuel Assembly

- Figures 12 and 13 shows three dimensional view of the PWR rod bundle.
- The construction of fuel rod is similar to that of a BWR.
- The control rod cluster is also shown in the figure.
- It may be noted that several control rods are moved simultaneously.

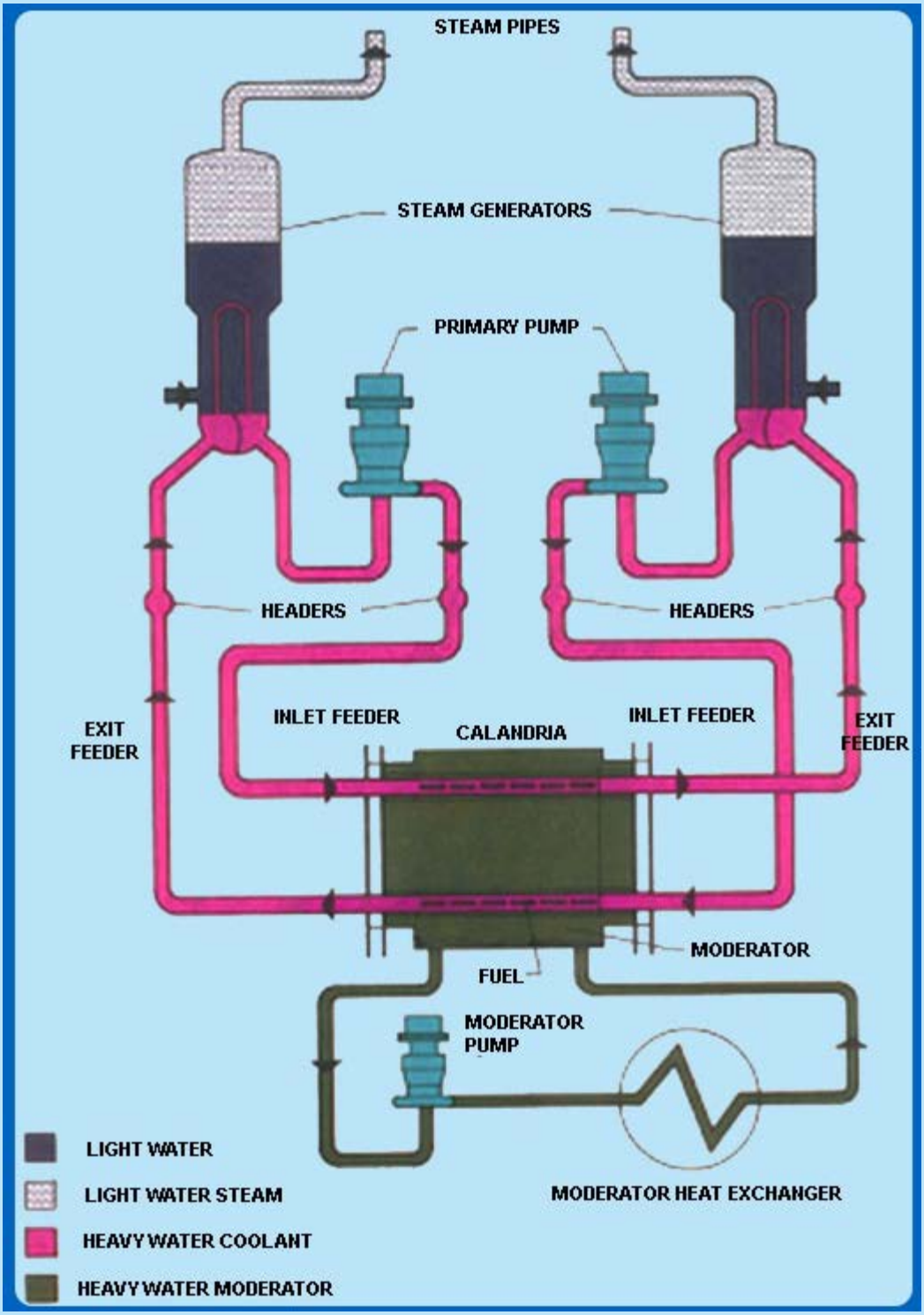


Fig.14 Pressurised Heavy Water Reactor

- A schematic view of a Pressurised Heavy Water Reactor (PHWR) circuit is shown in fig. 14.
- This differes substantially with that of a PWR.
- It employs the pressure tube concept rather than pressure vessel concept employed by PWR and BWR.
- It employs the figure of eight configuration.

- Let us begin tracing the fluid movement from one of the pumps shown in fig.14.
- The fluid is pushed into an inlet header.
- This is also marked as component no. 5 in fig.15.
- It is a long pipe which distributes fluid into many smaller tubes called feeders (component no. 4 in fig. 15).
- Each of this feeder sends the coolant into one of coolant channels in the core. Let us say it goes from right to left in fig. 14.
- A PHWR typically has 300-400 coolant channels.
- Inside the coolant channels, fuel bundles are placed.

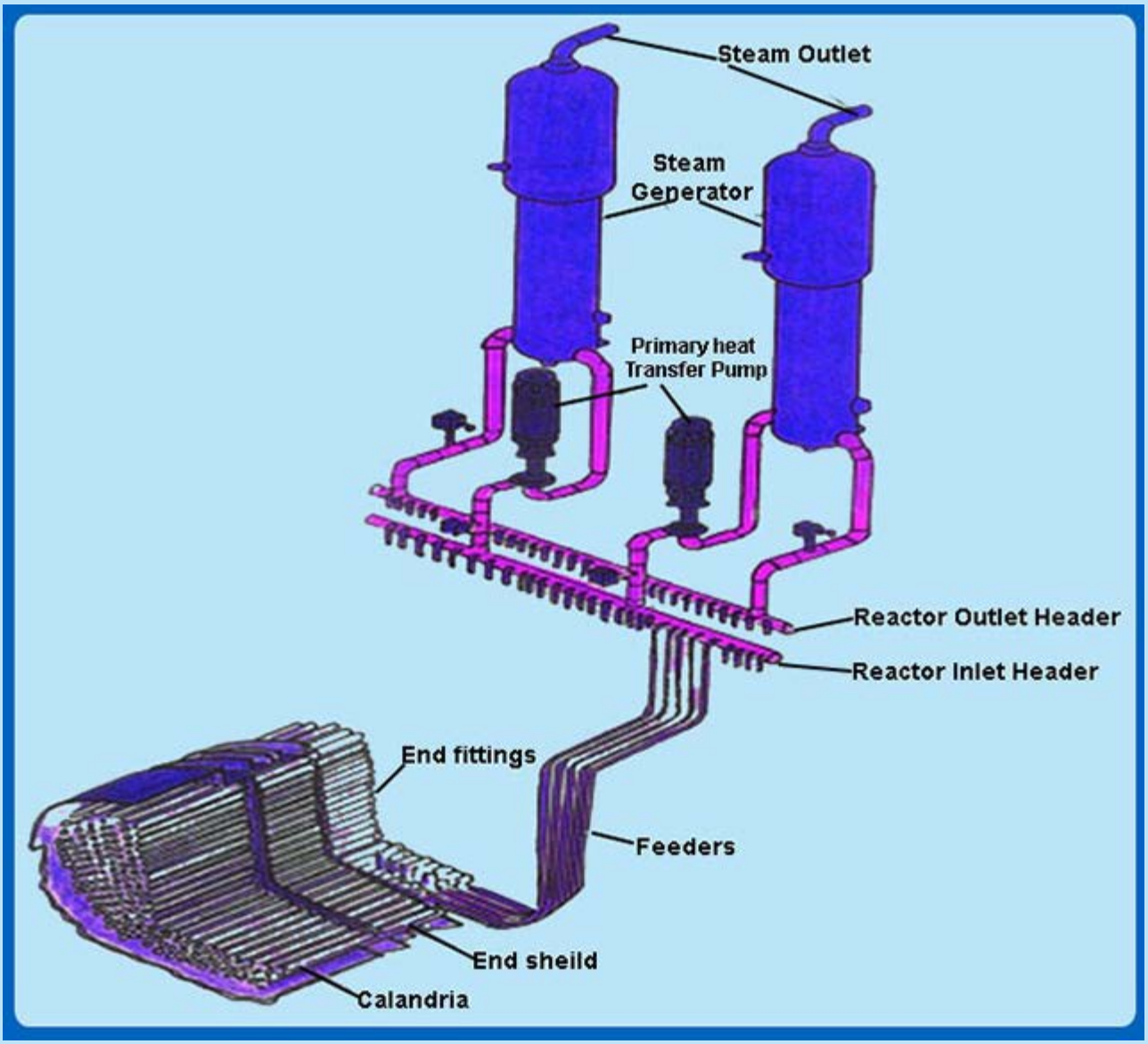


Fig.15 Typical Layout

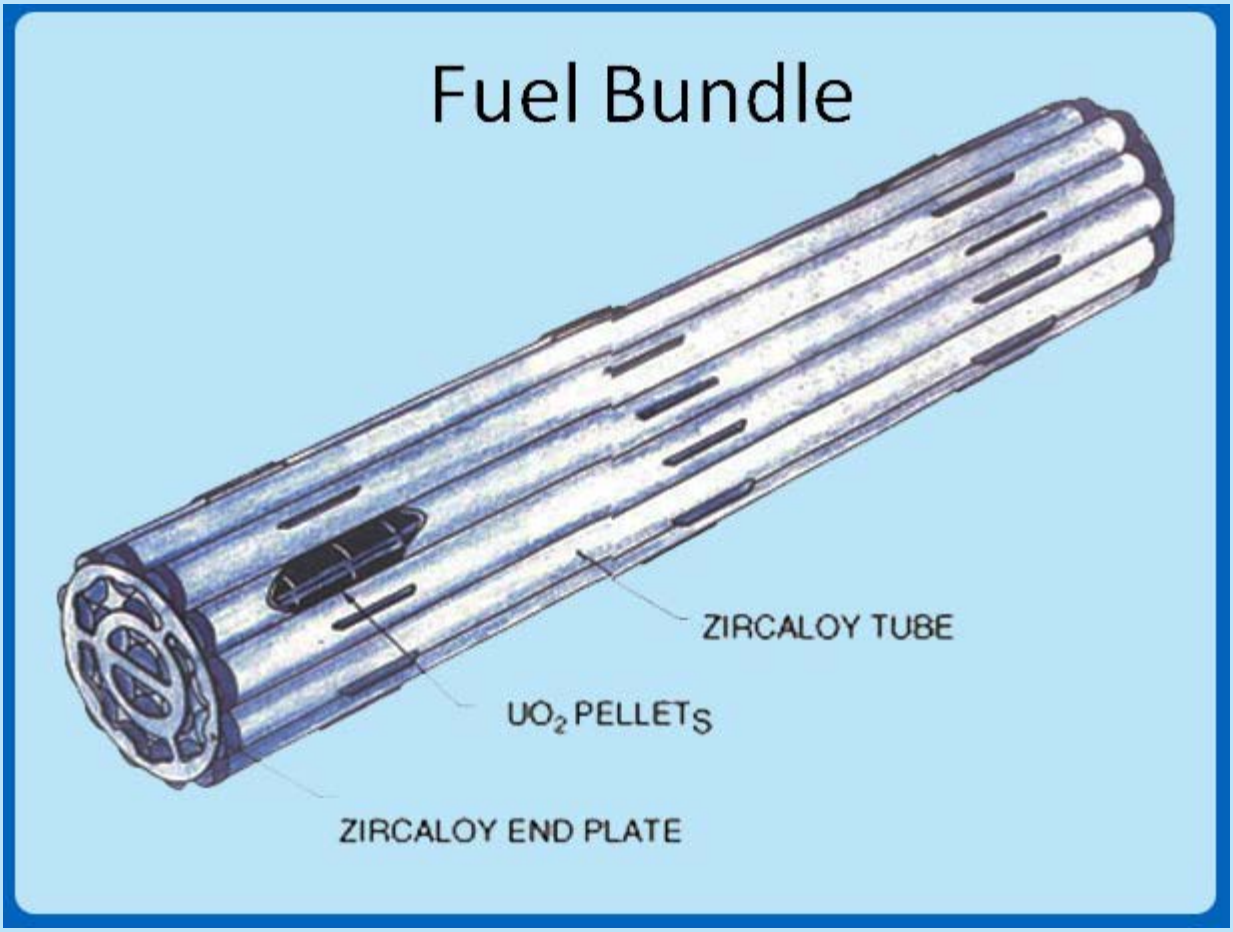


Fig.16 Fuel Bundle

- A schematic view of a fuel bundle is shown in fig. 16.
- It has 19 rods stuck together by end plates.
- Inside each rod, fuel pellet and gas plenum exists as in a PWR.
- Typically there are 10-15 fuel bundles in a coolant channel.
- These are kept short so that they can be removed and inserted easily.
- The heat generated in the fuel bundle increases the temperature of coolant.
- Each coolant channel feeds the hot water into exit feeders which merge into the exit header as shown in fig. 14.
- Note that this feeder is on the other side of the reactor (left in fig. 14).
- This fluid is now sent to the steam generator.
- Here it exchanges heat with secondary coolant (water) and generates steam.
- The coolant after transferring heat in steam generator is directed towards pump.
- The pump increases the pressure of the coolant and sends it to the inlet header.
- From the inlet header, the coolant is sent to coolant channel.
- Now the fluid goes from left to right.
- The coolant is sent to the exit header, then to steam generator and then to the pump, all

located on the right side.

- Thus the circuit is complete. It was noted that the coolant flows first from right to left and then left to right before the circuit is completed.
- This configuration is called figure of eight.
- In such cases, the components are duplicated on either side of the reactor.
- In a PHWR, the coolant channels are surrounded by moderator.
- The moderator is held in a vessel called Calandria.

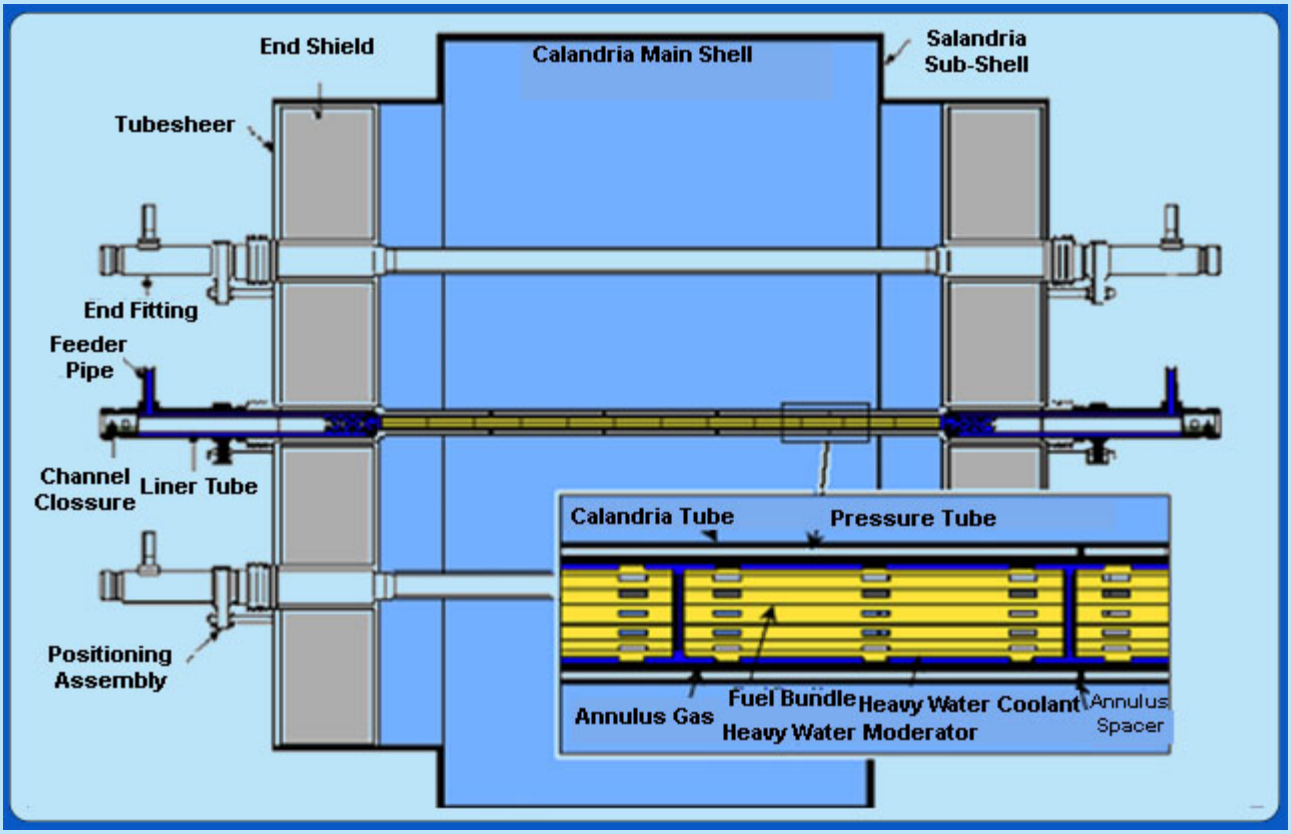


Fig.17 Calandria and Coolant Channel

- Figure 17 shows a schematic of calandria vessel and coolant channel.
- The coolant channel penetrate into the calandria vessel.
- Since the moderator is held around 60°C, while the coolant flows at around 275°C, an insulating air gap is provided.
- This is accomplished by providing a coaxial tube surrounding the coolant channel
- This tube is called the calandria tube.
- The coolant channel tube is often called the pressure tube.
- Thus we have the pressure tube, that is surrounded by calandria tube, with a gap in between the two. This is shown in fig.17.
- CO_2 is circulated in this gap. Outside this calandria tube is the moderator.

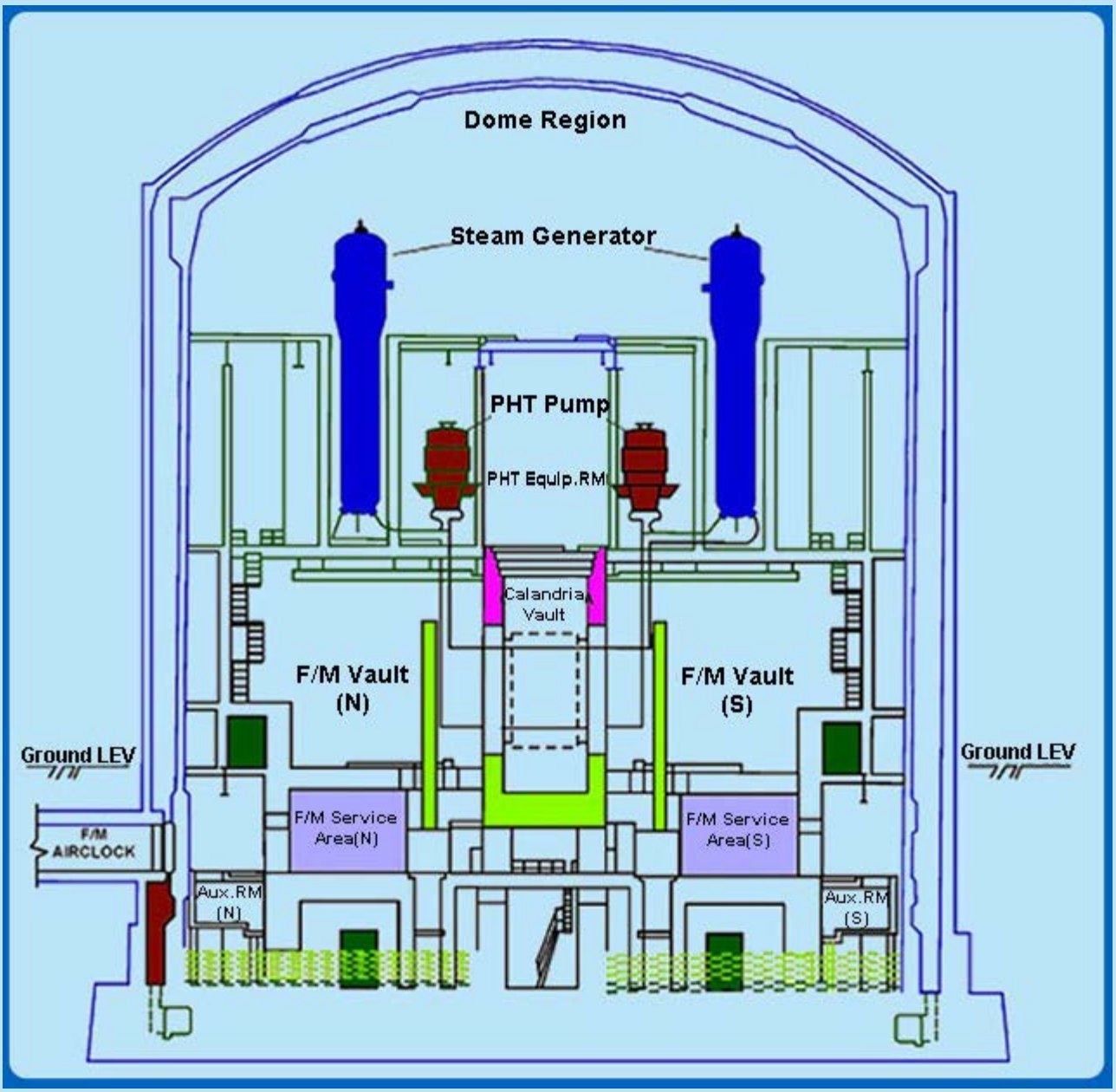


Fig.18 Containment and Equipment Layout

- A typical layout of the components in the containment of a PHWR is shown in fig. 18.

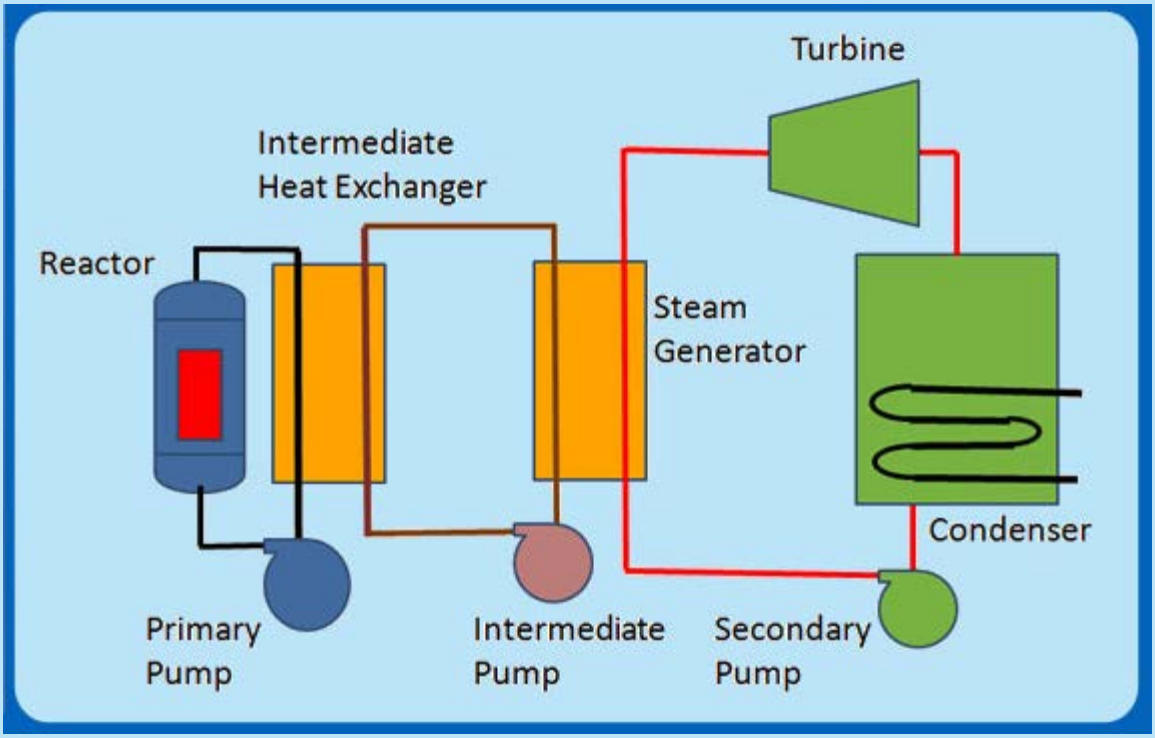


Fig.19 Fast Breeder Reactor

- As discussed earlier, if we do not slow down the neutron fully and allow it to react with fuel, then such a reactor is called fast reactor.
- We shall see later that fast reactors breed fuel and hence they are attractive.
- Since water is not allowed as a coolant, as it will slow down the neutrons, liquid metals are usually used as coolant.
- Sodium is the most commonly used as coolant.
- However as the turbines are run using steam, a sodium water steam generator has to be employed.
- Since sodium violently reacts with water, should there be a leak, fast breeders employ three circuits as shown in fig. 19.
- It consists of a primary loop, an intermediate loop and a secondary loop.
- In primary loop sodium transports the heat from core to the Intermediate Heat Exchanger(IHX).
- The primary pump circulates this fluid.
- The intermediate loop consists of a pump transporting sodium coolant that transports heat from IHX to the steam generator.
- In case of a leak in IHX only primary sodium and secondary sodium mix. Hence there will be no chemical reaction.
- In the secondary circuit, the steam is sent to turbine, expanded, condensed and sent back to the steam generator.
- In case of a leak in steam generator, only non radioactive sodium will react with water.

- This will prevent radioactive sodium leak.
- If IHX were not there, then radioactive sodium can react with water and result in radioactive sodium spill.

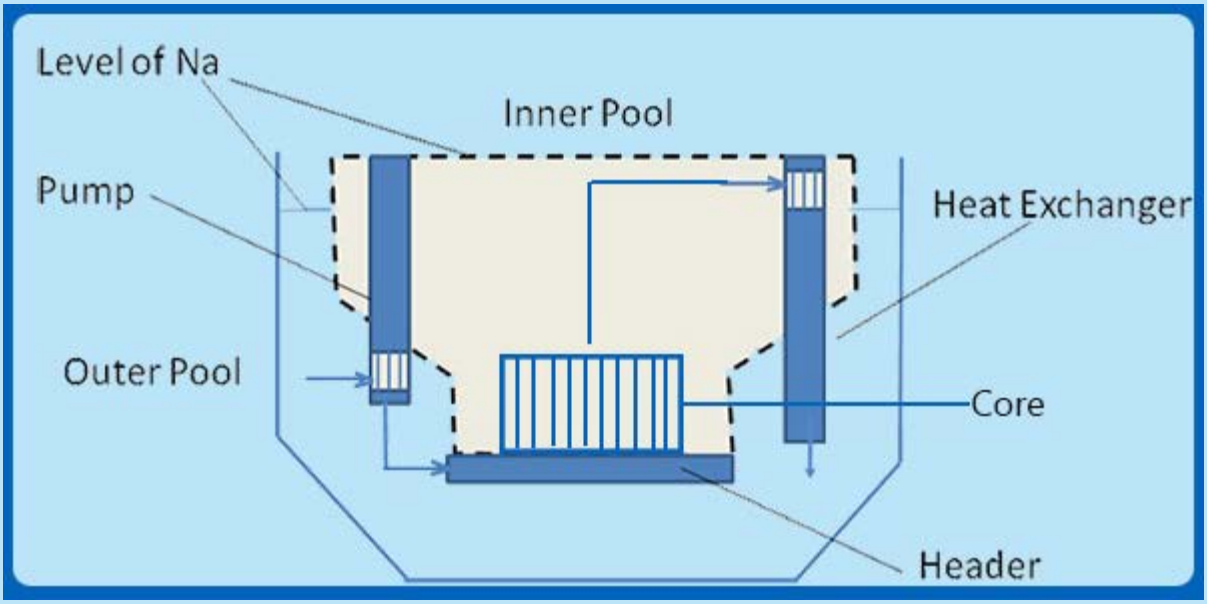


Fig.20 Pool Schematic

- Generally the power reactors use the pool concept. It consists of two pools, an inner pool and an outer pool as shown in fig. 20.
- The primary pump and the IHX are submerged in the pools.
- The pump sucks the fluid from the outer pool into the header and pumps it into the reactor core.
- The heated sodium flows in the inner pool and gets directed towards the IHX.
- It then flows from IHX into the outer pool completing the circuit. This is schematically shown in fig. 20.

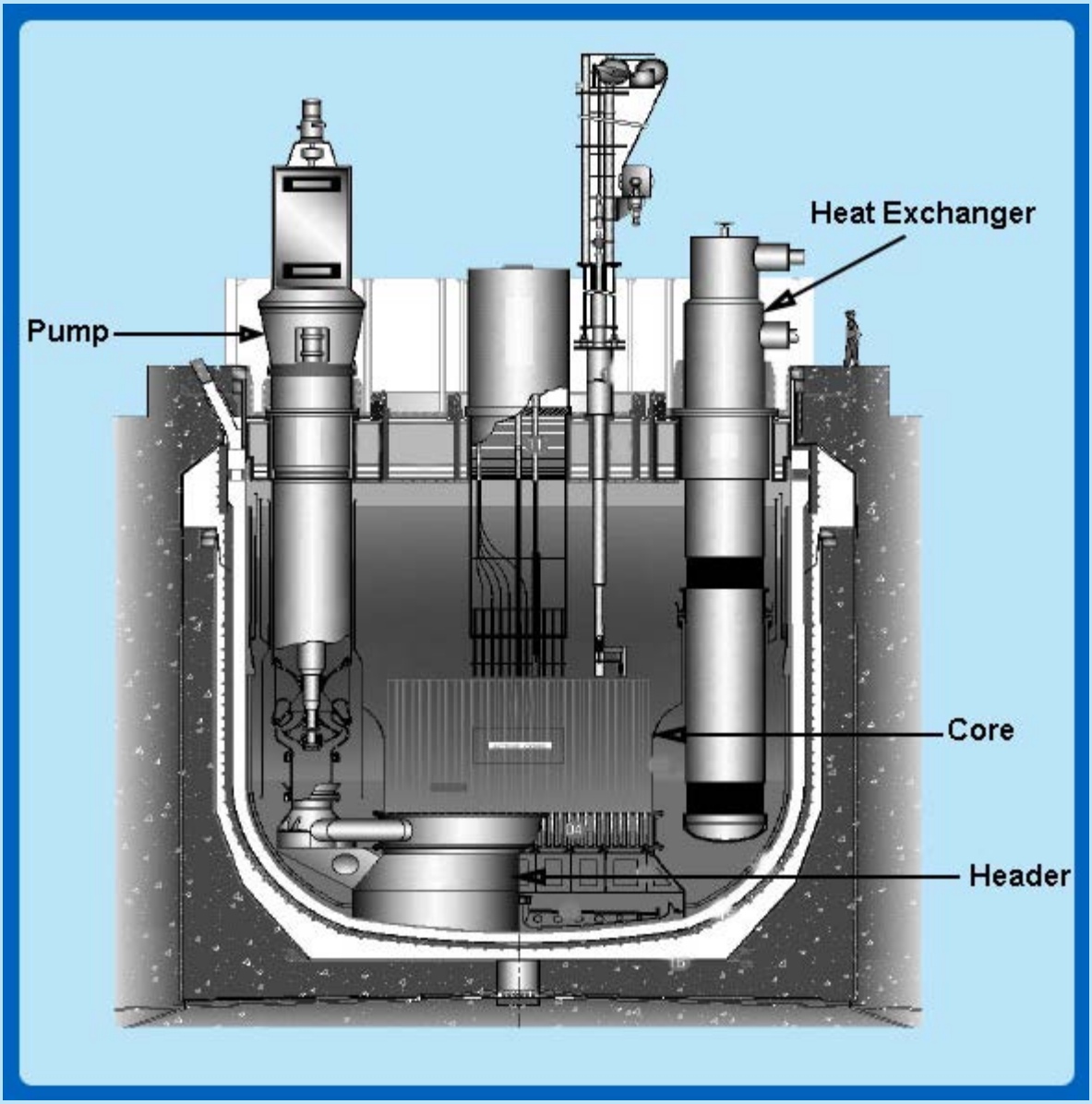


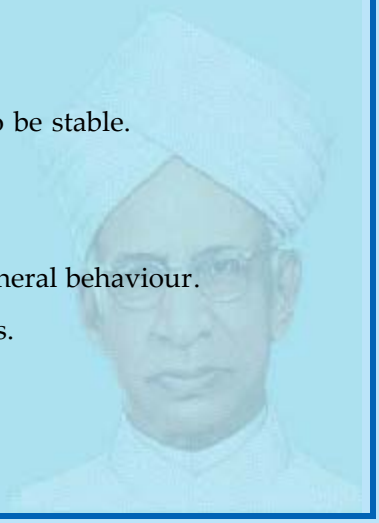
Fig. 21 Pool of Fast Breeder

- A comprehensive schematic view of the fast reactor is shown in fig. 21 with the components approximately to scale.
- Having gotten a feel of the features of the power reactors, we now move on to study the subject in detail.

Objectives

In this lecture you will learn the following

- We shall begin with a general discussion on the nucleus.
- Learn about some characteristics of nucleons.
- Understand some concepts on stability of a nucleus.
- In particular, we shall discuss Neutron/Proton ratio for a nucleus to be stable.
- Move on to quantify radioactivity and associated concepts.
- Understand the radioactive law and learn the concept of half life.
- Finally digest the analysis of radioactive chains and observe the general behaviour.
- Analyse a typical three element chain and identify its characteristics.



Review of Nuclear Physics

- Most of the concepts in nuclear engineering can be understood by considering only protons, neutrons and electrons as described by Rutherford's model.
- Just as chemical energy is released by the rearrangement of electrons, nuclear reactions can be understood by considering rearrangement of protons and neutrons.

Chemical Energy **Rearrangement of Electrons**

Nuclear Energy **Rearrangement of Protons and Nuetrons**

- Mass of Proton and Neutron are approximately equal to $1/N_{Avogadro}$ in grams.
- Every element can be represented by

$${}^{A}_{Z}X$$

In the above

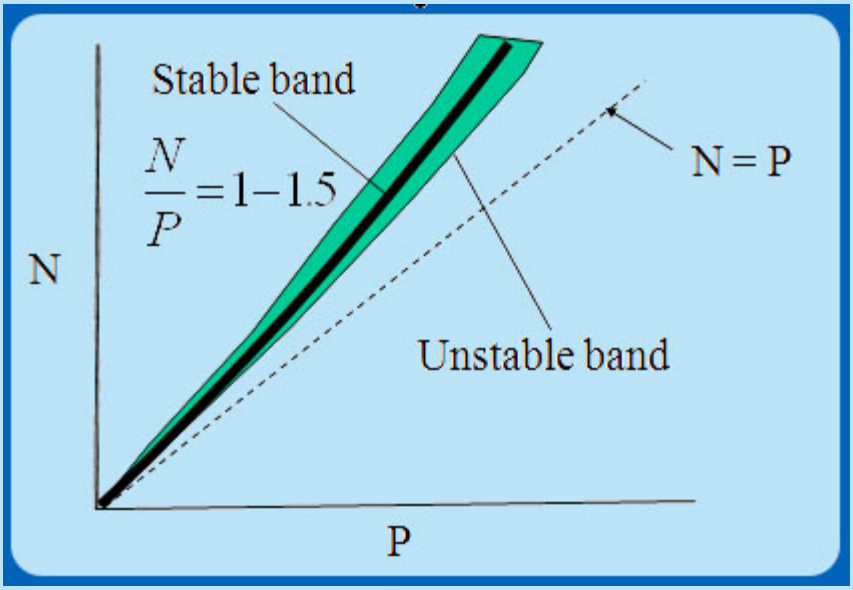
Z - No of Protons or Charge Number.

A - No of Protons + No. of Neutrons or Mass Number.

- Isotope have Same Z, but different A e,g. ${}^{233}_{92}U$, ${}^{235}_{92}U$, ${}^{238}_{92}U$.
- Typically the Radius of Nucleus = 10^{-15}m.
- Radius of Atom = 10^{-10}m.
- Volume ratio = 10^{15}.
- From above we understand that there is enormous hollow space.
- This implies that we need a large number of neutrons for collisions to occur.

Stability Of Nucleus

- In nature all elements are not stable.
- For example, Oxygen is stable, while Uranium is not.
- Interestingly, Neutron/Proton ratio influences stability.
- There is an optimum ratio needed for a nucleus to be stable and this ratio changes with the mass of the nucleus.
- The figure shown below indicates the zone where stable and unstable isotopes lie.
- The ratio of N/P varies from 1-1.5.
- The stable band is shown by a thick line, while the unstable one is shaded.



- The unstable ones emit radiation, transform themselves and move towards stability.
- The process of transformation of unstable nucleus by spontaneous emission of radiation is called radioactivity.

Radioactivity

The fundamental radioactive law is that the probability of nuclear disintegration rate is constant.

Consider the following:

Population	------>	N
Time	------>	dt
Disintegration	------>	dN

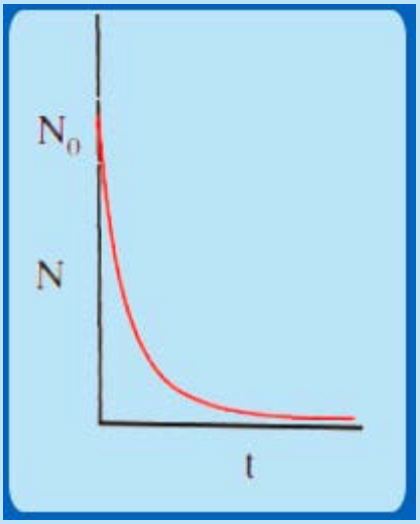
$$\frac{dN}{Ndt} = \text{Constant} = -\lambda \text{ (say)} \qquad \Rightarrow \frac{dN}{dt} = -\lambda N$$

- The above equation is a first order Ordinary Differential Equation (ODE).
  If the initial value is $N = N_o$ at $t = 0$.

Separating the variables and integration leads to

$$\Rightarrow N = N_o e^{-\lambda t}$$

- This implies that the population decays exponentially as shown in the figure.



- Thus, technically speaking a radioactive isotope has infinite life.

Activity

- Every disintergration results in emission of radiation
  dN/dt = Rate of emission of Radiation.
- This is termed as Activity and is denoted by α.
- α is numerically equal to λN (from radioactive law).
- SI unit of α is Becquerel denoted by Bq.
- 1 Bq = 1 disintegration per second (dps) or s^{-1}.
- An older unit is often used and is called Curie.
- 1 – Curie = 3.7 $X10^{10}$ Bq.
- It represents activity of 1g of Radium.

Half Life

- As pointed out earlier, the life of a radioactive isotope is technically infinite. Hence a half life is defined for every radioactive isotope

$$T_{1/2} \Rightarrow \frac{N}{N_0} = 0.5 \quad \Rightarrow e^{-\lambda T_{1/2}} = 0.5 \quad \Rightarrow \lambda T_{1/2} = \ln 2$$

$$or \quad T_{1/2} = \frac{0.693}{\lambda}$$

- We can now rewrite the radioactive decay law as

$$N = N_0 e^{\frac{-0.693t}{T_{1/2}}}$$

$\Rightarrow$ In 5 halflives, $\frac{N}{No} = 0.0313$ $\Rightarrow$ In 10 halflives $\frac{N}{No} = 0.000978$

- Thus in about 10 half lifes the population is reduced by about 1000 times.

Analysis of Decay Chains-I

- Fission Products are Radioactive A→ B → C → D……..
- Practical chains are long.
- A three isotope chain has all the characteristics of any long chain.
- Consider A → B → C (Stable)
- The Population Balance Equation can be written as
  Rate of change = Production Rate – Destruction Rate.
- Let N_A, N_B and N_e be the population of A, B and C respectively.
- The conservation of species A, B & C can be written as

$$\Rightarrow \frac{dN_A}{dt} = 0 - \lambda_A N_A, \quad \frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B, \quad \frac{dN_C}{dt} = \lambda_B N_B$$

The solution of first equation with the initial condition leads to the solution

$$\frac{dN_A}{dt} = 0 - \lambda_A N_A$$ With $N_A = N_{A0}$ at $t = 0$ $$\Rightarrow N_A = N_{A0} e^{-\lambda_A t}$$

Similarly the second equation can be written as

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \quad \Rightarrow \frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_{A0} e^{-\lambda_A t}$$

Using $e^{\lambda_B t}$ as Integral factor, we can write,

$$\frac{dN_B}{dt} e^{\lambda_B t} + \lambda_B N_B e^{\lambda_B t} = \lambda_A N_{Ao} e^{(\lambda_B - \lambda_A)t}$$

$$\Rightarrow d\left(N_B e^{\lambda_B t}\right) = \lambda_A N_{Ao} e^{(\lambda_B - \lambda_A)t} dt$$

Integration of the above with the initial condition, $N_B = N_{B0}$ leads to

$$N_B e^{\lambda_B t}\Big|_{N_B}^{N_B} = \lambda_A N_{Ao} \frac{e^{(\lambda_B - \lambda_A)t}}{\lambda_B - \lambda_A}\Big|_0^t$$

$$\Rightarrow N_B e^{\lambda_B t} - N_{B0} = \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A}\left[e^{(\lambda_B - \lambda_A)t} - 1\right]$$

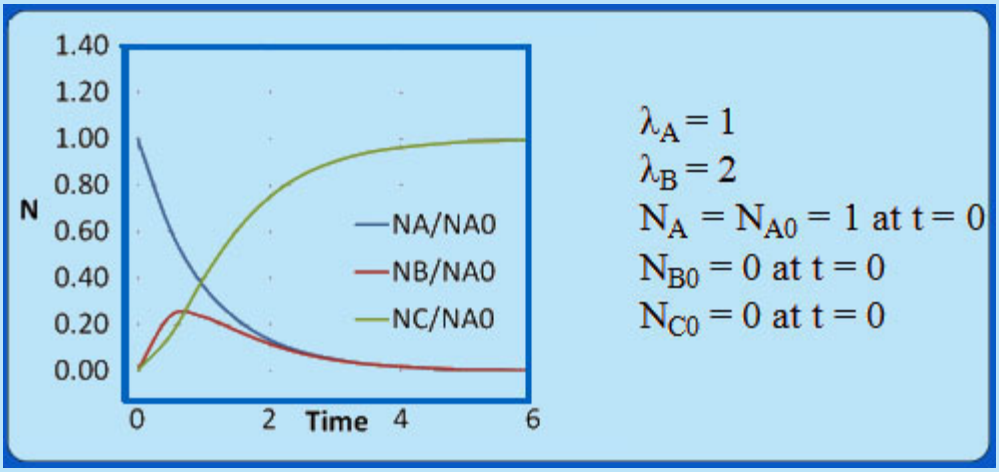
$$\Rightarrow N_B = N_{B0} e^{-\lambda_B t} + \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$$

Since $\frac{d}{dt}(N_A + N_B + N_c) = 0$

$\Rightarrow N_A + N_B + N_C = N_{A0} + N_{B0} + N_{C0}$

$\Rightarrow N_{C0} = N_A + N_B + N_C - N_{A0} - N_{B0}$

- The following figure shows the variation of population of A, B and C for the data shown inside the figure.



Special Cases-I

$$N_B = N_{B0}e^{-\lambda_B t} + \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A}\left[e^{-\lambda_A t} - e^{-\lambda_B t}\right]$$

For $\lambda_B > \lambda_A$ $\quad e^{-\lambda_B t} << e^{-\lambda_A t}$

- After some time, the first term goes to zero and the term 2 in the parenthesis of second term also becomes negligible. This leads to

$$\Rightarrow \frac{N_B}{N_A} = \frac{\lambda_A}{\lambda_B - \lambda_A} \qquad \Rightarrow N_B = \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A} e^{-\lambda_A t}$$

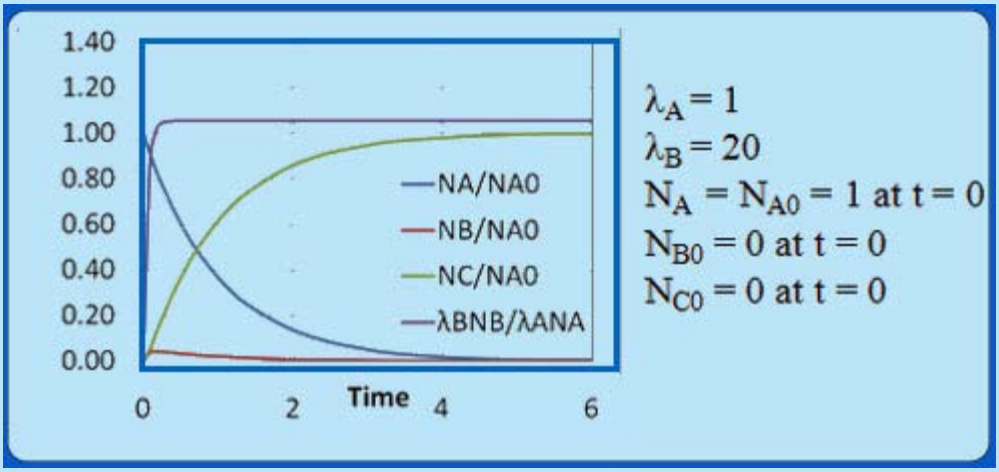
- This is called transient equilibrium.
- Here though both N_A and N_B varies with time, the ratio of them remains a constant.
- It may be observed in the last slide that for $t > 3$, $N_B/N_A = 1$.

Special Cases-II

For $\lambda_B >> \lambda_A$ $\Rightarrow \frac{N_B}{N_A} \approx \frac{\lambda_A}{\lambda_B}$ $\Rightarrow \lambda_B N_B \approx \lambda_A N_A$

$\Rightarrow \alpha_B \approx \alpha_A$

- The above condition is called secular equilibrium.
- It implies that irrespective of their initial activities they quickly attain the same activity.
- This can be appreciated by the figure shown below, where for t > 0.3, the ratio of the activity reaches the value 1.05.



Objectives

objectives

In this lecture you will learn the following

- In this lecture, we shall practice solving problems.We will solve 5 representative problems.

Background Information

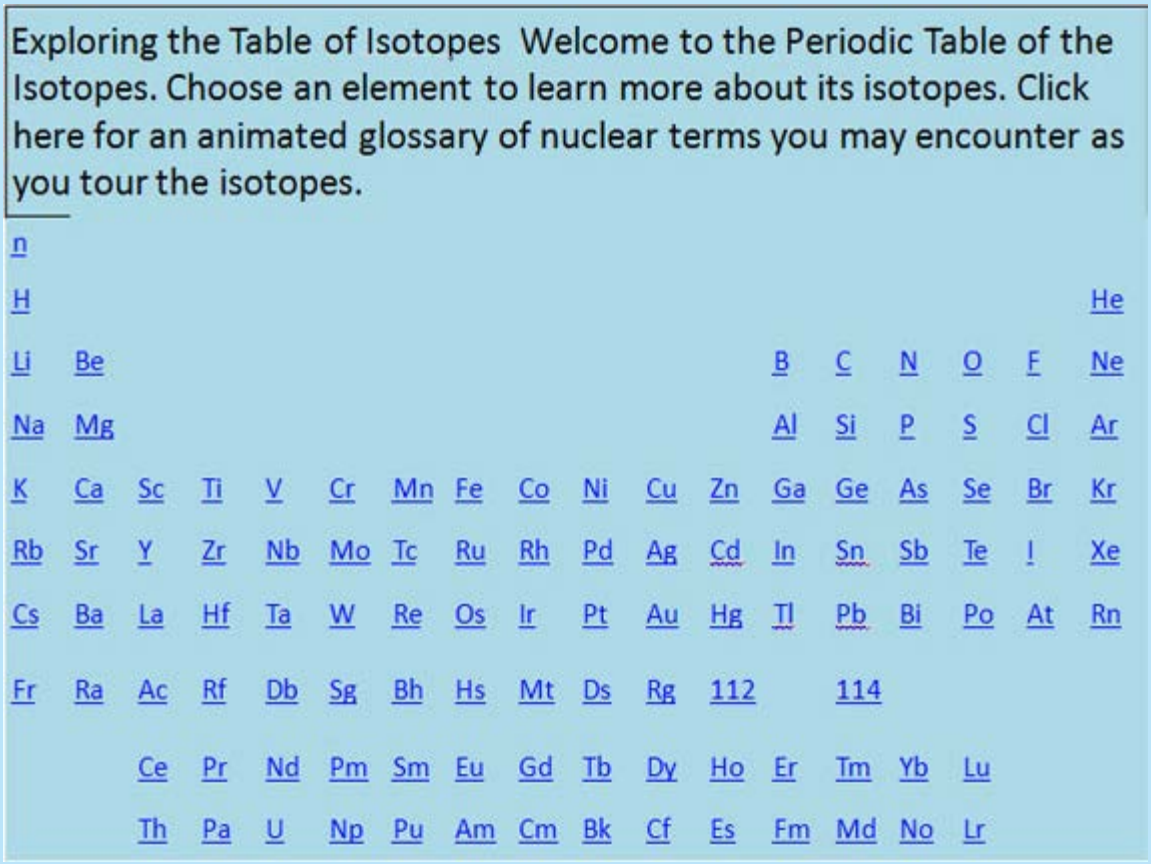
- Mole – Molecular weight expressed in grams.
- 1 mole of any element has Avogadro number of molecules. Every molecule has 1 nucleus.
- 1 MeV = 1.603×10^{-13} Joules.
- Avogadro Number = 6.023×10^{23}.

Question-1

How may neutrons and protons are there in the nuclei of the following atoms;
(a) Li^{7}, (b) Mg^{24},(c) Xe^{135}, (d) Rn^{222}.

- **Sol.** The following website gives properties of isotopes.
  http://ie.lbl.gov/education/isotopes.htm

The Berkeley Laboratory Isotopes Project



Isotopes of Lithium (Z=3)Click on an isotope to get more information about its decay	
Isotope	**Half-life**
6Li	stable
7Li	stable
8Li	838 ms
9Li	178.3 ms

- Now from the above source or from any book we can construct the following table.

	Charge Number	Mass Number	Protons/ Electrons	Neutrons
Li	**3**	**7**	**3**	**4**
Mg	**12**	**24**	**12**	**12**

Xe	54	135	54	81
Rn	86	222	86	136

Question-2

Tritium (H^3) decays by negative beta decay with a half-life of 12.33 years. The atomic weight of H^3 is 3.016. What is the mass in grams of 1 mCi of tritium ?

Solution - 2

Activity = α = 1 mCi = 3.7 10^{10} x 10^{-3} = 3.7 x 10^{7} s^{-1} = λN

$T_{1/2}$ = 12.33 years

λ = $0.693/T_{1/2}$

$=0.693/(12.33 \times 365.25 \times 24 \times 3600)$

= 1.781 x 10^{-9} s^{-1}

N= 3.7 x 10^{7}/1.781 x 10^{-9}

= 2.08 x 10^{16} nuclei

Mass of Tritium Atom = 3.016/6.023 x 10^{23}

= 5.007 x 10^{-24} g

Hence total mass =5.007 x 10^{-24} x 2.08 x 10^{16}

= 1.040 x 10^{-7} g

Question 3

The radioisotope battery is fuelled with 500g of $Pu^{238}C$ (Plutonium-238 carbide), which has a density of 12.5 g/cm^3. The Pu^{238} has a half life of 89 years, and emits 5.6 MeV per disintegration, all of which may be assumed to be absorbed in the generator. The thermal to electrical efficiency of the system is 6 percent. Calculate (a) the specific power in watts (thermal) per gram of fuel; (b) the power density in watts (thermal) per cm^3; (c) the fuel efficiency in curies per watt (thermal); (d) the total electrical power of the generator.

Solution 3

Mol-wt-PuC	**250**	
$T_{1/2}$	**89**	**years**
Energy per disintegration	**5.6**	**MeV**
Density - PuC	**12.5**	**g/cm^3**
Total Mass of PuC	**500**	**g**
Efficiency	**0.06**	

$\lambda = 0.693/T_{1/2}$

$= 0.693/(89 \times 365.25 \times 24 \times 3600)$

$= 2.467 \times 10^{-10} s^{-1}$

Molecules/unit mass = $N = 6.023 \times 10^{23}/250$

$= 2.409 \times 10^{21} g^{-1}$

Specific activity = $\lambda N = 2.467 \times 10^{-10} \times 2.409 \times 10^{21}$

$= 5.944 \times 10^{11} s^{-1}/g$

$= 5.944 \times 10^{11} / 3.7 \times 10^{10}$

$= 16.07$ Ci/g

Specific Power= $\lambda NE = 5.944 \times 10^{11} \times 5.6 \times 1.603 \times 10^{-13}$

$= 0.534$ W/g

Power Density = $\lambda NE\varrho = 0.534$ W/g x 12.5 g/cm^3

$= 6.67$ W/cm^3

Fuel Efficiency = $\lambda N / \lambda NE = 16.07$ Ci/g / 0.534 W/g

= 30.11 Ci/W

Total Power = Specific power x Mass x Conv. Eff.

$=\lambda n E m \eta$ = 0.534 W/g x 500 g x 0.06

= 16.01 W(electric)

Question 4

Three elements A, B and C, not connected by a chain, decay individually. At t=10s, the ratio of their activities are 1:0.5:0.25, and at t=15s, their activities are in the ratio 1:0.25:0.0625. What would be the ratio of their activities at t=0s.

Solution 4

$$N_A = N_{A0}e^{-\lambda_A t} \Rightarrow \lambda_A N_A = \lambda_A N_{A0}e^{-\lambda_A t} \Rightarrow \alpha_A = \alpha_{A0}e^{-\lambda_A t}$$

Similarly

$$\alpha_B = \alpha_{B0}e^{-\lambda_B t}$$

From above, we can write, $\frac{\alpha_A}{\alpha_B} = \frac{\alpha_{A0}}{\alpha_{B0}}e^{(\lambda_B-\lambda_A)t}$

Using the data at 10 s, $2 = \frac{\alpha_{A0}}{\alpha_{B0}}e^{(\lambda_B-\lambda_A)10}$ (1)

Using the data at 15 s, $4 = \frac{\alpha_{A0}}{\alpha_{B0}}e^{(\lambda_B-\lambda_A)15}$

Dividing the latter by former we get, $2 = e^{(\lambda_B-\lambda_A)5}$

$$\Rightarrow \ln 2 = (\lambda_B - \lambda_A)5 \Rightarrow (\lambda_B - \lambda_A) = \frac{\ln 2}{5}$$

Substituting the value in Eq. (1), we get

$$2 = \frac{\alpha_{A0}}{\alpha_{B0}}e^{\frac{(\ln 2)}{5}10} \Rightarrow 2 = \frac{\alpha_{A0}}{\alpha_{B0}}e^{2(\ln 2)} \Rightarrow 2 = \frac{\alpha_{A0}}{\alpha_{B0}}e^{\ln 2^2}$$

$$\Rightarrow 2 = \frac{\alpha_{A0}}{\alpha_{B0}}4 \quad \Rightarrow \frac{\alpha_{A0}}{\alpha_{B0}} = \frac{1}{2}$$

Since the ratio of αB/ αC is similar, we can write,

$$\Rightarrow \frac{\alpha_{B0}}{\alpha_{C0}} = \frac{1}{2} \quad \Rightarrow \alpha_{A0} : \alpha_{B0} : \alpha_{C0} = 1 : 2 : 4$$

Question 5

Tata Memorial Hospital has been procuring a short-lived radio-isotope for medical applications. The doctors, of late, had been complaining that the supplied isotope did not have the necessary strength for which the order was placed. In order to verify this, the inspection department carried out the following test. The sample was counted for 1 minute for the number of disintegration, immediately on its arrival. In the first one minute 1791 disintegrations were counted. After 10 minutes from the arrival time, the sample was again counted for 1 minute. This time the number of counts were 1620. Based on the above facts, compute the strength of the source in Becquerels (dps), when supplied.

Solution 5

Every disintegration results in a radiation and this can be detected in a counter. Thus if a counter detects N counts, it implies N disintegrations have taken place.

Let the Number of nuclei in the sample at t=0 be N(0)

Number of Nuclei after one minute $= N(1) = N(0)e^{-\lambda.1}$

This implies that $N(0) - N(1) = N(0)\left(1 - e^{-\lambda}\right) = 1791$ (1)

Similarly, $N(10) - N(11) = N_0\{e^{-10\lambda} - e^{-11\lambda}\} = 1620$

$\Rightarrow N(10) - N(11) = N_0 e^{-10\lambda}(1 - e^{-\lambda}) = 1620$ (2)

Dividing Eq. (1) by Eq. (2), we get $\frac{1791}{1620} = e^{10\lambda}$

$10\lambda = \ln\frac{1791}{1620}$ $\lambda = \frac{1}{10}\ln\frac{1791}{1620} = 0.01\,min^{-1}$

Substituting this in Eq. (1), we get

$$N(0) = \frac{1791}{(1 - e^{-0.01})} = 1.8 \ x \ 10^5$$

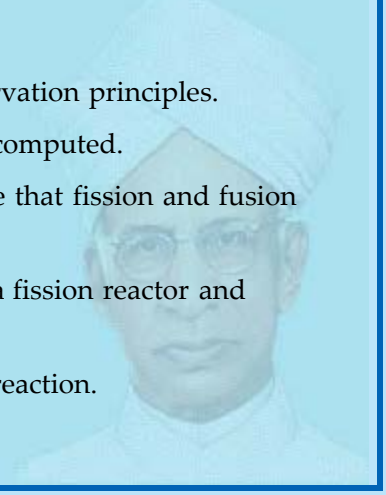
Activity at t = 0 is given by

$\lambda N(0) = 0.01 \ X \ 1.8X10^5 = 1800\,\text{min}^{-1} = 30\,Bq$

Objectives

In this lecture you will learn the following

- We shall understand the concept of kinetic energy from the perspective of particle physics.
- We shall conclude that for all practical purposes, mass can be assumed constant and kinetic energy is same as $0.5\ MV^2$.
- Then we will introduce nuclear reactions and the associated conservation principles.
- Then we shall discuss how the energy released from a reaction is computed.
- The concept of Binding Energy is then introduced and we shall see that fission and fusion reactions release energy.
- Then we shall look into what makes a nucleus suitable for fuel in a fission reactor and understand the terms fissile and fissionable.
- Finally we will understand some of the characteristics of a fission reaction.



Kinetic Energy

- Towards understanding nuclear reactions, first we will define kinetic energy.
- From the theory of relativity, we conclude that mass and energy are interconvertible.
- The total energy of a particle, whose mass is M can be written as

$$E = MC^2 \qquad (1)$$

- In the above equation C is the velocity of light.
- When a particle is at rest, we can write the energy possessed by it as the rest mass energy and can be written as

$$E_0 = M_0 C^2 \qquad (2)$$

In the above equation M_0 is the rest mass.

- From the theory of relativity, we note that the mass, M, changes with velocity, V, and this can be represented by

$$M = \frac{M_0}{\sqrt{1-\frac{V^2}{C^2}}} \qquad (3)$$

- Now if we would like to extract the energy from a moving particle by slowing it down to zero velocity, then the extractable energy shall be

$$E - E_0 = (M - M_0)C^2 \qquad (4)$$

- This we shall define as kinetic energy and denote it as T.
- Thus T is the difference between the total energy and the rest mass energy.
- Now we will try to connect this to the kinetic energy in Newtonian Mechanics.
- From Eq. (4) and Definition of T, we can write

$$T = (M - M_0)\, C^2 \qquad (5)$$

- Substituting for M from Eq. (3), we get,

$$T = M_0 \left(\left(1-\frac{V^2}{C^2} \right)^{-\frac{1}{2}} - 1 \right) C^2 \qquad (6)$$

$$\approx M_0\left(\left(1+\frac{1}{2}\frac{V^2}{C^2}\right)-1\right)C^2 \qquad \text{if} \qquad \frac{V^2}{C^2}<<1$$

Thus, $T \approx \frac{1}{2}M_0V^2$

- Thus for V << C, the kinetic energy reduces it to the classical Newtonian mechanics definition.
- To decide on a criterion for up to what velocity we can use Newtonian mechanics, consider the case of V = 0.1 C.
- From Eq. (6), we get,

$$T = M_0C^2\left((1-0.01)^{-\frac{1}{2}}-1\right)$$

$$= M_0\frac{V^2}{2}\frac{2C^2}{V^2}(0.005038)$$

$$= M_0\frac{V^2}{2}\frac{(0.005038)}{0.005} = M_0\frac{V^2}{2}1.0075$$

- Thus for V<0.1C, the Newtonian Mechanics is adequate as the results are within 0.75%.

Nuclear Reactions

Consider the nuclear reaction

$$ {}^{A_1}_{Z_1}X_1 + {}^{A_2}_{Z_2}X_2 \rightarrow {}^{A_3}_{Z_3}X_3 + {}^{A_4}_{Z_4}X_4 $$

The following conservation equations hold:

- $Z_1 + Z_2 = Z_3 + Z_4$ (Conservation of charge).
- $A_1 + A_2 = A_3 + A_4$ (Conservation of nucleons).
- $P_1 + P_2 = P_3 + P_4$ (Conservation of momentum).
- $E_1 + E_2 = E_3 + E_4$ (Conservation of Energy).
- Note that in most practical cases V < 0.1 C and hence mass of a particle can be assumed as constant.

Energy from a Nuclear Reaction

- In nuclear reactions, the extractable energy possessed by a particle is its kinetic energy.
- Hence net energy of a reaction can be defined as the difference of kinetic energy of products and kinetic energy of reactants.
- As the total energy is equal to rest mass energy + kinetic energy, we can write $E_{01} + T_1 + E_{02} + T_2 = E_{03} + T_3 + E_{04} + T_4$.
- Thus energy from a reaction, also called its Q value, can be written as $Q = T_3 + T_4 - T_1 - T_2 = E_{01} + E_{02} - E_{03} - E_{04}$.

$$\Rightarrow Q = \Delta T = -\Delta E_0 = -\Delta M_0 C^2$$

- The masses expressed in the previous equation are the rest masses of the participating nuclei.
- However, since $Z_1 + Z_2 = Z_3 + Z_4$, We can add the respective masses of electrons to products and reactants and can conclude that

$$\Rightarrow Q = -\Delta M_x C^2$$

Where, Mx, are the masses of neutral atoms that can be measured by a mass spectrometer.

- A website where all the masses are tabulated is given below. The tabluated masses can be used to calculate the Q-value.

http://physics.nist.gov/cgi-bin/Compositions/stand_alone.pl

Atomic Weights and Isotopic Compositions for All Elements

Z	Isotope	A	Mass (amu)	Isotope Composition	Atomic Weight
1	H	1	1.007 825 032 1(4)	99.9885(70)	1.007 94(7)
	D	2	2.014 101 778 0(4)	0.0115(70)	
	T	3	3.016 049 2675(11)		
2	He	3	3.016 029 309 7(9)	0.000 137(3)	4.002 602(2)
		4	4.002 603 2497(10)	99.999 863(3)	

Concept of Binding Energy

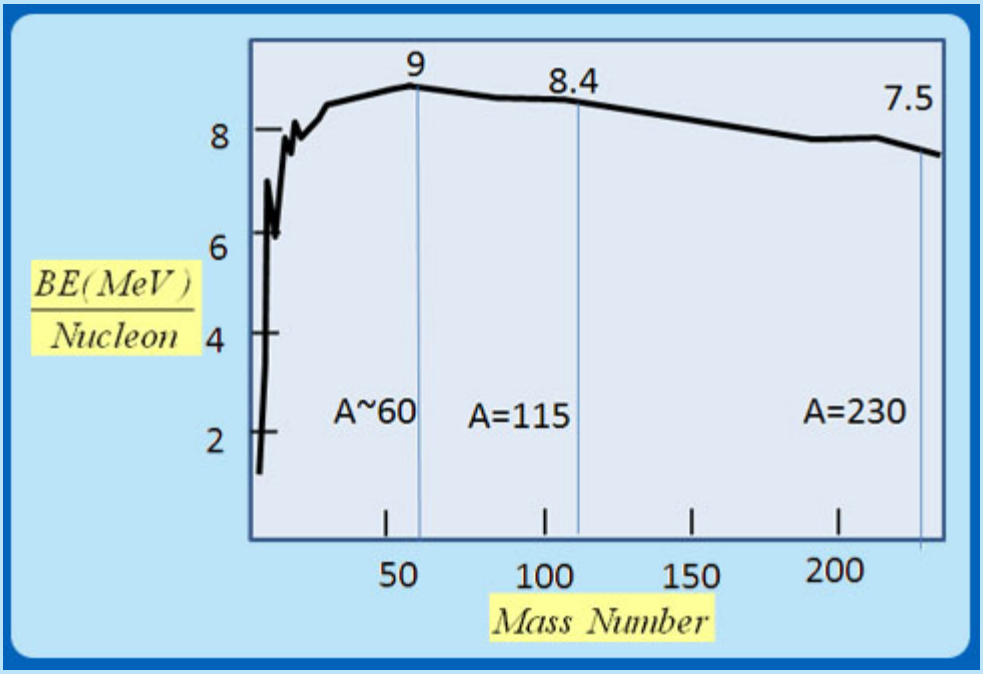
Consider the following reaction

$$ {}_1^1H + {}_0^1n \rightarrow {}_1^2H + \gamma(2.23MeV) $$

- Mass of the product nucleus is less than the sum of the masses of reactant nuclei by an equivalent of 2.23 MeV.
- Another way of looking at this is that a proton and neutron, the constituents of deuterium nuclei, bind themselves by releasing binding energy of 2.23 MeV.
- This implies that we need to supply 2.23 MeV to break deuterium nuclei into its constituents.
- In general, when particles bind themselves they release energy.

Binding Energy Curve

- The binding energy per nucleon curve is shown in the figure below.
- The highest binding energy is for A ~ 60.



Fission and Fusion Energetics

- If we split a nucleus of mass A = 230 into two masses of A = 115. From the figure in last frame, BE/Nucleon for A=230 is 7.5MeV and the same for A=115 is 8.4 MeV. Hence the reaction will release energy equal to 0.9 x 230 = 217 MeV.
- Thus fission process releases a large amount of energy.
- Similarly if two light elements fuse to become a heavy element, energy would be released as the product nuclei has a higher binding energy than the reactants.
- Though fusion energy release per reaction is less, but specific energy (energy release per unit mass) is high.
- While the curve indicates possibility of energy release, ways and means have to be found to enable the reactions to take place.

Fission Energetics

- Consider the following equation

$$^{233}_{92}U + ^{1}_{0}n \rightarrow ^{234}_{92}U + \gamma(6.6MeV)$$

- Binding Energy of Last Neutron is (BELN) 6.6 MeV.
- Energy necessary to be supplied to induce fission is called Critical Energy for Fission.
- The critical energy for fission and BELN for some isotopes are summarised in the following table.

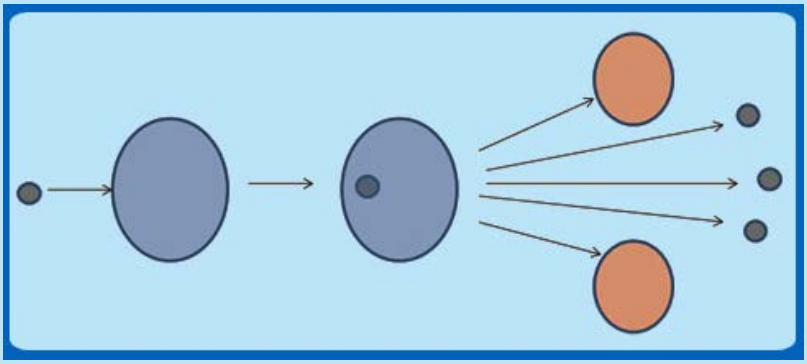
Isotope	Critical Energy for Fission (MeV)	Binding Energy of Last Neutron (MeV)
Th233	6.5	5.1
U234	4.6	6.6
U236	5.3	6.4
U239	5.5	4.9
Pu240	4.0	6.4

Fissile and Fissionable Fuel

- Thus when a neutron is absorbed in U^{233}, the neutron deposits its binding energy in the compound nucleus, which is more than what is required to make U^{234} to fission (refer to table shown on the previous page).
- Those elements which can be fissioned by zero energy neutrons are called fissile.
- Thus U^{233}, U^{235} and Pu^{239} are fissile.
- On the other hand, Th^{232} and U^{238} need neutrons with 1.4 MeV and 0.6 MeV respectively for inducing fission. Hence these are called fissionable nuclei.
- Natural Uranium has 99.3% of U^{238} and 0.7% of U^{235}.

Fission Reaction

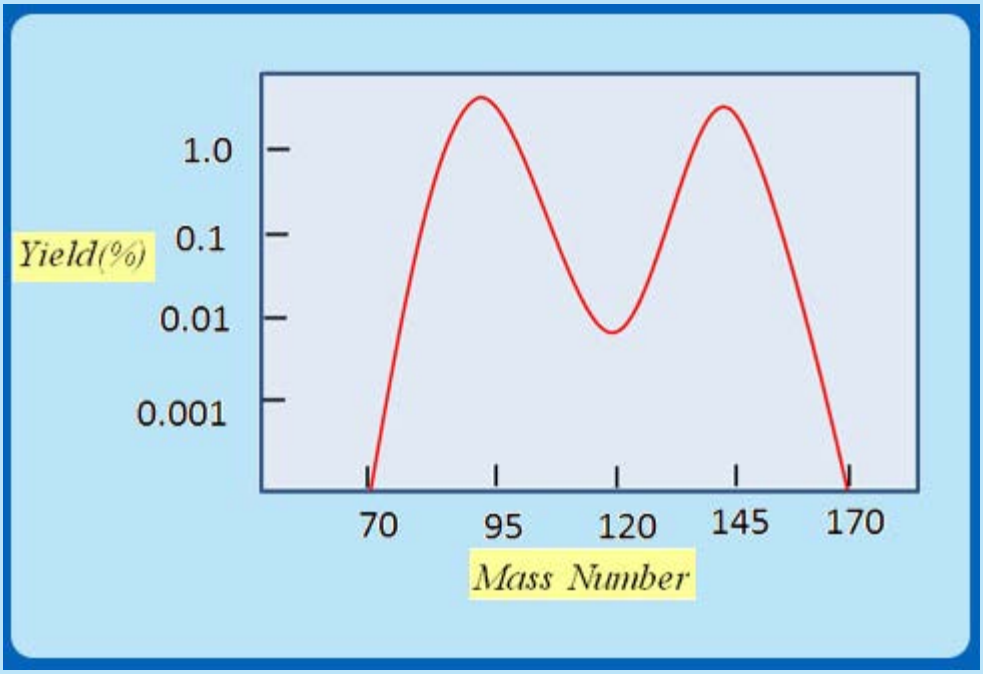
- The process of fission is depicted in the following figure.
- The compound nucleus is first formed, which splits into two fission products along with some neutrons, and releases energy.
- The number of neutrons released vary statistically.



$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{236}_{92}U \rightarrow ^{144}_{56}Ba + ^{89}_{36}Kr + 3^{1}_{0}n + 177MeV$$

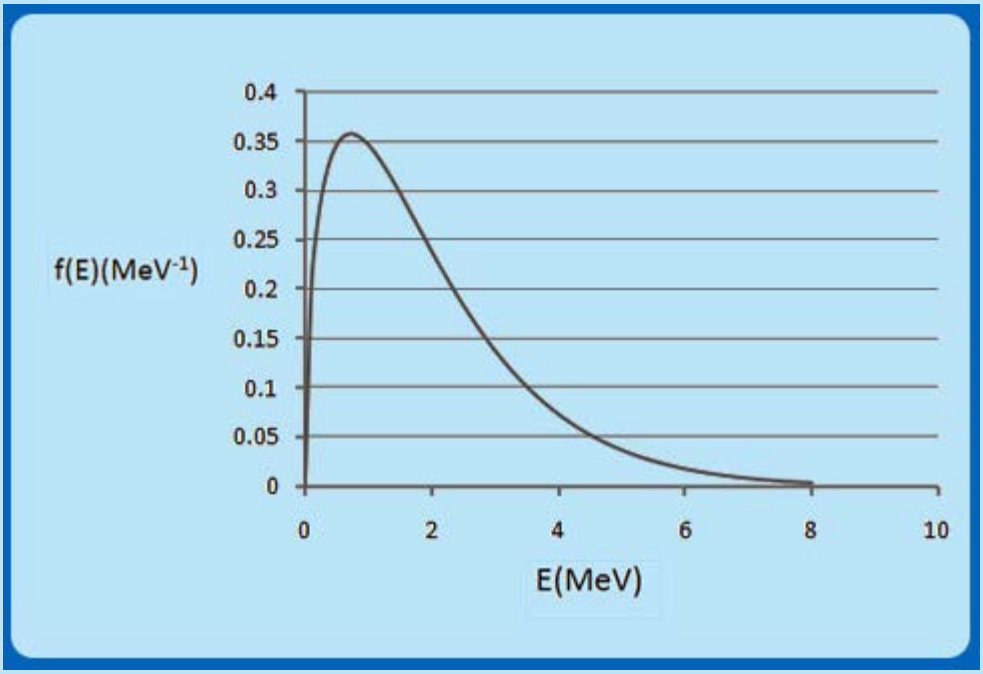
Fission Product Distribution

- The fission product yield curve is shown below.
  - Variety of fission products are released.
  - Symmetric fission is less likely.



Prompt Neutron Spectrum

- Neutrons that are released instantly are called prompt neutrons.
- The energy spectra of the prompt neutrons is shown below.
- An average of ~2.5 neutrons is released when U^{235} fissions. This is denoted by the symbol $\bar{\nu}$.
- The most probable energy = 0.73 MeV.
- The average Energy = 1.98 MeV.
- A large fraction of neutron are above the threshold energy to fission U^{238}.
- Hence some U^{238} fissions also occur in nuclear reactors.



Fission and Capture

- A neutron colliding with U^{235} does not guarantee a fission reaction.
- There is a finite probability that the neutron will be captured without resulting in fission.
- If the probability fission and capture are F and C, then we can define the multiplication of neutron in the fuel, η, as

$$\eta = \frac{F\bar{\nu}}{F+C} = \frac{\bar{\nu}}{1+\frac{C}{F}} = \frac{\bar{\nu}}{1+\alpha}$$

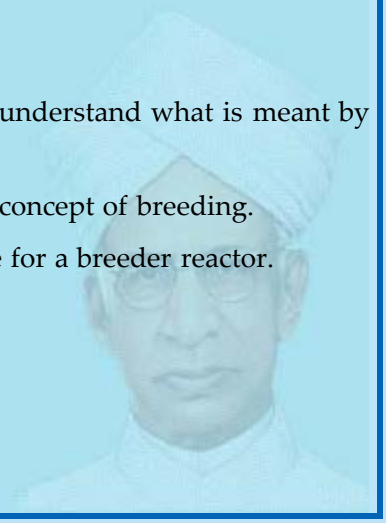
- In the above expression F $\bar{\nu}$ is number of new neutrons released, while F + C is number of neutrons absorbed.
- Since $\alpha > 0$, $\eta < \bar{\nu}$.
- Typical value of $\bar{\nu}$ and η are listed in the adjacent table.

Isotope	$\bar{\nu}$	η
U^{233}	2.49	2.29
U^{235}	2.42	2.09
Pu^{239}	2.87	2.11

Objectives

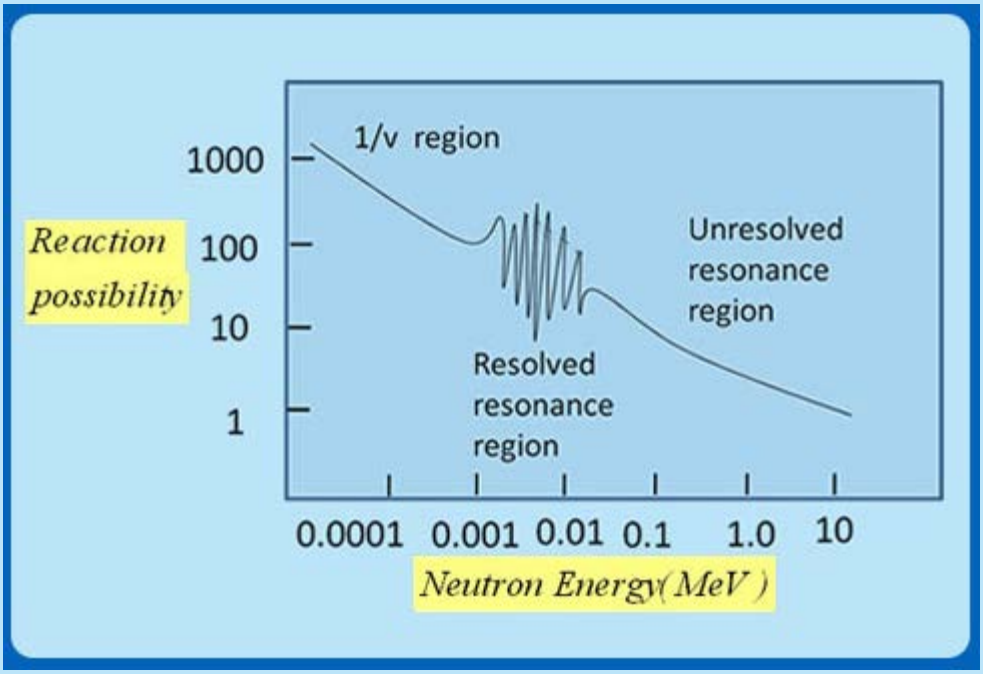
In this lecture you will learn the following

- We shall first digest that the probability of occurrence of fission decreases with the energy of the attacking neutron.
- We will study the process of moderation.
- We shall then study about the infinite multiplication constant and understand what is meant by critical reactor.
- We will then move on to conversion reactions and understand the concept of breeding.
- Finally we shall see some simple models to estimate doubling time for a breeder reactor.



Probability of Fission Reaction

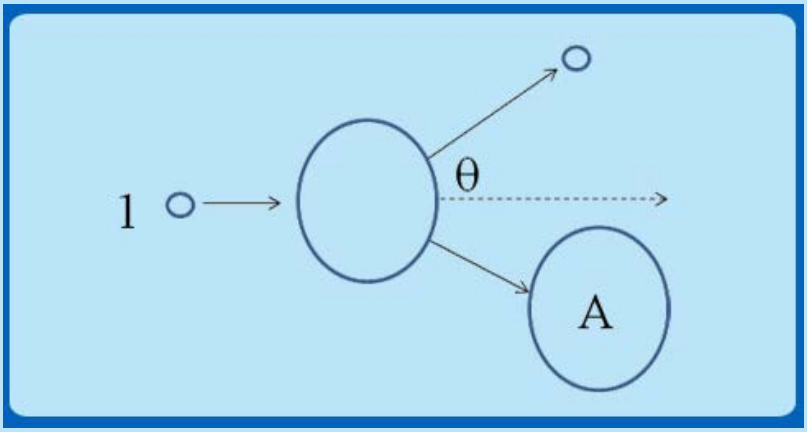
- Towards understanding nuclear reactors, first we will look at the probability of fission reaction as shown below.



- It is observed that as the energy of the colliding neutron is high, the probability of reaction is poor.
- This is attributed to the reduced time a neutron spends in the zone of interaction.
- It is inversely proportional to the speed of the neutron.
- In addition to these, there are resonances that are attributed to quantum mechanical interactions.
- As the interaction decreases rapidly with energy, and that the energy of the prompt neutrons are high, we need to slow them down.
- This is done by the use of moderator.

Moderation

- The energy is reduced by collision process and bulk of the slowing is through elastic collision process.
- The typical scattering process is shown below.
- In the figure a neutron of mass 1 (amu) is shown colliding with a nucleus of mass A (amu) which is originally at rest. After the collision process they move in different directions as shown.



- When angle of scattering is 0 ($\theta = 0$), there is no energy loss.
- When $\theta = 180$, there is maximum energy loss.
- We shall discuss about these later in the course.
- For a neutron with energy E_0, colliding with a nucleus of mass A, the minimum energy a neutron can attain can be shown to be αE_0, where

$$\alpha = \left(\frac{A-1}{A+1}\right)^2$$

The value of mean energy loss per collision is summarised in the following table.

Element	A	α	$\Delta\bar{E}/E$
H	1	0	0.500
D	2	0.111	0.444
Be	9	0.640	0.180
C	12	0.716	0.142
U	238	0.983	0.008

- H as H_2O and D as D_2O are the most commonly used moderators.
- Gas cooled reactors use C as the moderator.
- Be is used as moderator in some research reactors.

- It may be observed that heavy element such as U is a very poor moderator.

Overall Multiplication

- We had seen that the fuel multiplies neutron by a factor η.
- However, as many other materials are used, such as moderator, structure and control materials, neutron is also absorbed by them.
- If the probability of a neutron being absorbed in fuel is *f*, then the overall multiplication denoted by k_∞ can be written as (∞ denotes that reactor is very large and hence there is no leakage)

$$k_\infty = \eta f$$

- $k_\infty = 1$ implies that the population of neutrons remains steady and the system is called critical.
- For an infinite system, if $k_\infty < 1$, the system is called Sub-Critical and if $k_\infty > 1$, the system is called Super-Critical.
- In a subcritical system the neutron population will decrease with time and in supercritical system, it will increase with time.

Conversion Reactions

- When a neutron is captured in U^{238}, it results in U^{239}, which undergoes two β decays as follows

$$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{92}U \xrightarrow{\beta^-,\ 23.5\,min} ^{239}_{93}Np \xrightarrow{\beta^-,\ 2.35\,day} ^{239}_{94}Pu$$

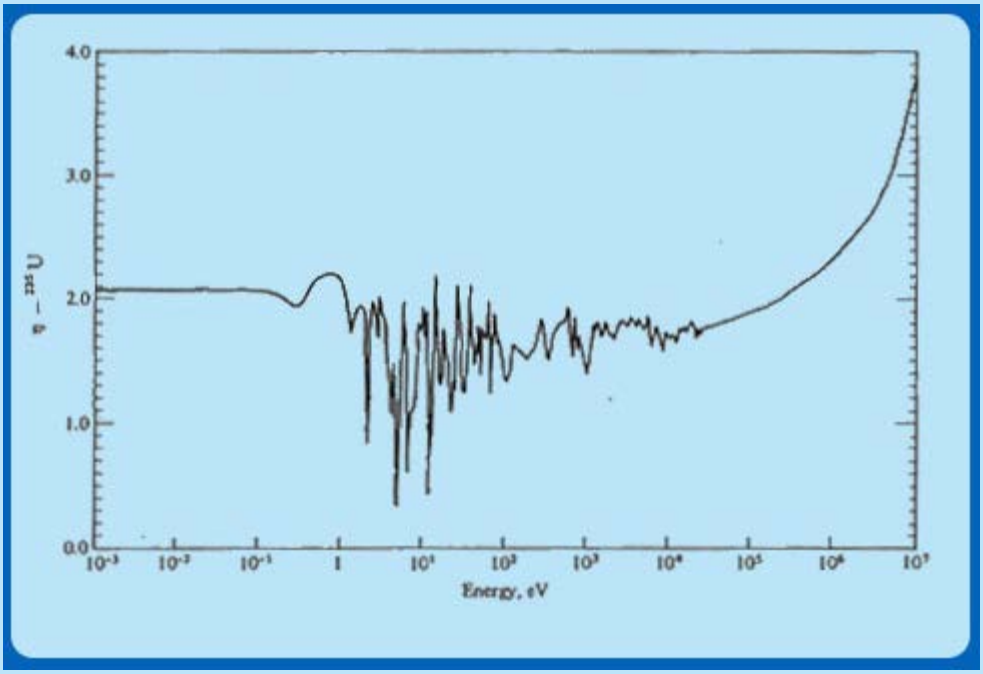
- One can view Beta Decay as a process in which a neutron converts into a proton and electron and the electron is ejected.
- Thus, the charge number increases by one in beta decays.
- However since total nucleons are still same the mass number does not change.
- Thus we see that the non-fissile U^{238} has been converted into a fissile Pu^{239}.
- Similar conversion is also possible to get Th^{232} converted into U^{233} with the following reaction

$$^{232}_{90}Th + ^{1}_{0}n \rightarrow ^{233}_{90}Th \xrightarrow{\beta^-,\ 22.2\,min} ^{233}_{91}Pa \xrightarrow{\beta^-,\ 27.4\,day} ^{233}_{92}U$$

- For this reason, Th^{232} and U^{238} are also called fertile elements.

Conversion and Breeding

- We noted that for every neutron absorbed in fuel, we get η number of second generation neutrons.
- Thus η-1 have to be absorbed elsewhere other than fuel to keep the reactor steady.
- While we can use control material to absorb the excess neutrons, it will be useful if the excess is diverted into fertile elements, so that fuel can be generated.
- It has been seen that if a high energy neutron is allowed to cause fission, η increases and allows the possibility of breeding fuel.
- The term breeding used above implies that more fuel can be generated than the amount of fuel consumed.
- The variation of η with energy of fissioning neutron is shown in the figure below.



Fast Reactor

- In a fast reactor, neutrons are not allowed to slow down by moderators.
- The excess neutrons are directed at the fertile blanket.
- As the probability of neutron reaction with the fuel is low at high energies, the fuel has to be highly enriched.
- Usually, Sodium is used as the coolant.
- Hence these are called Liquid Metal Fast Breeder Reactors.

Breeder Reactors

- Since all of η-1 are not available for conversion, let C be the number of neutrons absorbed in fertile material.
- That would mean for every fissile nucleus consumed, C fertile will be converted.
- If C = 1, then the reactor will produce enough fuel for itself perpetually, so long as enough fertile elements are available.
- If C >1, then the reactor will not only produce enough fuel for itself perpetually, but also produce fuel for new reactors. Such reactors are called breeder reactors.

Elementary Models for Breeding

- In order to get an idea as to how fast one can breed, let us look at some simple models.
- Let the power of the reactor be P and let β be the rate of the mass of fuel consumed per unit power produced (kg/s-W).
- Now we shall derive a model for estimating the inventory of fissile fuel, M, in a reactor.

$$\frac{dM}{dt} = -P\beta + P\beta C = P\beta(C-1)$$

- The first term on RHS represents fuel consumed and the second term, the fuel produced.
- Introducing C-1 = G, the gain constant, we can write,

$$\frac{dM}{dt} = P\beta G$$

- If the reactor operates at constant power, we can write,

$$\frac{dM}{dt} = P\beta G = constant$$

- This implies that the inventory will increase linearly with time. If M_o is the fissile inventory at time = 0, then

$$M = M_0 + P\beta Gt$$

- The time taken for the inventory to double is called linear doubling time. This can be obtained as

$$\Rightarrow 2M_0 = M_0 + P\beta G T_{d-l} \qquad \Rightarrow T_{d-l} = \frac{M_0}{P\beta G}$$

- We shall see later that the power extracted will be proportional to inventory. Thus ideally, we can increase power proportional to inventory and breed more fuel. Thus, the theoretical doubling time for maximum breeding can be obtained as follows

$$P \propto M \quad \Rightarrow P = KM$$

$$\frac{dM}{dt} = P\beta G = KM\beta G \qquad \frac{dM}{M} = K\beta G$$

$$M = M_0 e^{K\beta Gt}$$

- The inventory increases exponentially in this case.
- The doubling time obtained by this model is called exponential doubling time and can be

evaluated as follows

$$2M_0 = M_0 e^{K\beta G T_{d-\exp}} \qquad 2 = e^{K\beta G T_{d-\exp}}$$

$$\ln 2 = K\beta G T_{d-\exp} \qquad T_{d-\exp} = \frac{\ln 2}{K\beta G}$$

$$As\ K = \frac{P}{M} = \frac{P_0}{M_0} \qquad T_{d-\exp} = \ln 2 \frac{M_o}{P_o \beta G}$$

- By comparison, we can say that

$$T_{d-\exp} = \ln 2 T_{d-l}$$

- While the models described are over simplified, these are illustrative. We can gain some insights during our problem solving session.

Objectives

In this lecture you will learn the following

- In this lecture we shall practice solving problems. We will solve 5 out of 10 problems in Assignment-2.

Background Information

- Mole – Molecular weight expressed in grams.
- 1 mole of any element has Avogadro number of molecules.
- 1 MeV = 1.603×10^{-13} Joules.
- Avogadro Number = 6.023×10^{23}.

Question-1

1. (a) Show that the energy released in the nth generation of a fission chain reaction, initiated by one fission is given by $E_n = k^n E_R$: where k is the multiplication factor and E_R is the recoverable energy per fission. (The first neutron is considered as generation - 0)

   (b) Show that the total energy released up to and including the nth generation is given by

$$E_n = \frac{k^{n+1} - 1}{k - 1} E_R$$

Solution-1

Generation	No of Fissions	Energy released
0	1	E_R
1	K	KE_R
2	K^2	K^2E_R
-	-	-
N	K^n	K^nE_R

Total Energy

$$E_{0\to n} = E_R\,[1 + k + k^2 \ldots . k^n]$$

$$\Rightarrow E_{(0\to n)} = \frac{E_R 1[k^{(n+1)} - 1]}{(k - 1)}$$

$$= E_R \frac{k^{n+1} - 1}{k - 1}$$

Question-2

2(a) Show that the fraction F, of the energy released in a super critical chain reaction that originates in the final m generation of the chain is given approximately by $F = 1 - k^{-m}$, provided the total number of generations is large.

(b) Most of the energy from a nuclear explosion is released during the final moments of the detonation. Using the result of the previous part, compute the number of fission generations required to release 99 percent of the total explosive yield. Use the nominal value k = 2.

(c) If the mean time between generation is in the order of 10^{-8} sec, over what period of time is energy released during a nuclear explosion?

Solution-2

Consider the n generations

$$0,\ 1,\ 2,\ n-(m+1),\ n-m,\ n-(m-1),.....n-1\ ,n$$

Energy released in the final m generation can be expressed as

$$E_{n-(m-1)\to n} = \left(E_{0\to n} - E_{0\to(n-m)}\right)$$

$$= E_R\left(\frac{k^{n+1}-1}{k-1} - \frac{k^{n-m+1}-1}{k-1}\right)$$

$$= \frac{(k^{n+1} - k^{n-m+1})}{k-1}\ E_R$$

$$\text{Fraction F} = \frac{\frac{(k^{n+1} - k^{n-m+1})}{k-1}E_R}{\frac{(k^{n+1}-1)}{k-1}E_R}$$

$$= \frac{(k^{n+1} - k^{n-m+1})}{k^{n+1}-1}$$

Assuming n is large & k ~ 2

$$K^{n+1} >>> 1$$

$$\therefore F = \frac{k^{n+1} - k^{n-m+1}}{k^{n+1}} = 1 - k^{-m}$$

If $F = 0.99 \Rightarrow 0.99 = 1 - 2^{-m}$

$\Rightarrow 2^{-m} = 0.01$ or $2^{m} = 100$

$$\Rightarrow m = \frac{\ln(100)}{\ln 2} = 6.644 \approx 7 \text{ generations}$$

$$\therefore \text{Time} = 6.664 \times 10^{-8} s$$

Question-3

The fission product I^{131} has a half life of 8.05 days and is produced with a yield of 2.9% (0.029 atoms per fission). Calculate the equilibrium activity of this isotope in a reactor operating at a thermal power of 3300 MW.

Solution-3

Mass balance of I^{131} in reactor

$$\frac{dI^{131}}{dt} = -\lambda^{131}N^{131} + \dot{N}_{fission}Y_{I^{131}}$$

At equilibrium

$$\frac{dI^{131}}{dt} = 0 \Rightarrow \lambda^{131}N^{131} = \dot{N}_{fission}Y_{I^{131}}$$

Assuming 200MeV / fission

$$N_{fission} = \frac{3300x10^{6} J/S}{200x1.602x10^{-13} J/fission}$$

$$= 1.030x10^{20} fission/s$$

$$\therefore \alpha^{131} = \lambda^{131}N^{131} = \dot{N}_{fission}Y_{I^{131}}$$

$$= 1.030x10^{20} x0.029$$

$$= 2.987x10^{18}/s$$

$$= 8.073x10^{7} Ci$$

Question 4

(a)Define of the 'Q' value of a nuclear reaction:

$$X_1 + X_2 \longrightarrow X_3 + X_4$$

(b) Some tables tabulate the mass excess 'Δ ', defined as the M-A, where M and A are the rest mass of the neutral atom and the mass number of a given element expressed in energy units respectively. Derive a relation for the Q value in terms of

$$\Delta_{X_1}, \Delta_{X_2}, \Delta_{X_3} \text{ and } \Delta_{X_4}$$

(c) Given the values of 'Δ ' s of $^{3}H, ^{2}H, ^{2}D, ^{4}He$ and ^{1}n are 14.950, 13.136, 2.425 and 8.071 MeV respectively, compute the Q value for the reaction,

$$^{3}_{1}H + ^{2}_{1}D \longrightarrow ^{4}_{2}He + ^{1}_{0}n$$

(d) Compute the binding energy of the last neutron for $^{236}_{92}U$, given that the Δ values in MeV for $^{235}_{92}U, ^{236}_{92}U$ and $^{1}_{0}n$ are 40.93, 42.46 and 8.071 respectively.

Solution-4

(a) $Q = (M_{Reactancts} - M_{Products})\, C^2$

$= \Delta_i = M_i - A_i \quad M_i = A_i + \Delta_i$

(b) $Q = (M_1 + M_2 - M_3 - M_4)\, C^2$

$= [(A_1 + \Delta_1) + (A_2 + \Delta_2) - (A_3 + \Delta_3) - (A_4 - \Delta_4)]$

$Q = (\Delta_1 + \Delta_2 - \Delta_3 - \Delta_1)\, C^2$

(c) $Q = 14.950 + 13.136 - 2.425 - 8.071$

$= 17.59$ MeV

(d) $^{235}_{92}U + ^{1}_{0}n = ^{236}_{92}U$

$$Q = \Delta_{U^{235}} + \Delta_n - \Delta_{U^{236}}$$

$$= 40.93 + 8.071 + 42.46$$

$$= 6.541 MeV$$

Question 5

Assuming that the fissioning nucleus is U^{235}, compute the value of β, defined as the mass of the fuel consumed per unit energy release. You may assume that 200 MeV is released per fission and the value of capture to fission ratio is 0.17.

Solution-5

$$\frac{C}{F} = 0.17 = \alpha$$

For every fission $1+\alpha$ will be consumed

For every 200 MeV 1.17 nuclei consumed.

$\Rightarrow$ 1.17 nuclei gives 200 x 1.602 10^{-13} J

$$\Rightarrow \frac{235g}{6.022 \times 10^{23}} \times 1.17 \text{ gives } 3.204 \times 10^{-11} \text{J}$$

$$\Rightarrow 4.565 \times 10^{-2} \text{ g gives } 3.204 \times 10^{-11} \text{J}$$

$$\Rightarrow 1g = \frac{3.204 \times 10^{-11}}{4.565 \times 10^{-22}} \text{J}$$

$$= 7.017 \times 10^{10} \text{ J}$$

$$\therefore \beta = \frac{1g}{7.017 \times 10^{10} J}$$

$$= \frac{1}{7.017 \times 10^{10}} \frac{g}{J}$$

$$= 1.425 \times 10^{-11} \frac{g}{J}$$

$$= 1.425 \times 10^{-14} \frac{kg}{J}$$

Question 6

Because of an error in its design, a thermal reactor that was supposed to breed on the Th^{232}-U^{233} cycle unfortunately has a breeding ratio of only 0.96. If the reactor operates at a thermal power level of 500 MW, how much Th^{232} does it convert in one year? Capture to fission ratio for U^{233} = 0.09.

Solution-6

$$C = 0.96, \quad P = 500\,MW, \quad a_{U233} = 0.09, \quad E_R = 200\,MeV/fission$$

$$\dot{N}_{Fission} = \frac{500 \times 106\ J/s}{200\,MeV \times 1.602 \times 10^{-13}\ J/MeV}$$

$$= \frac{500}{3.204} X\ 10^{17}$$

$$= 1.5605 \times 10^{19}\ fissions/s$$

$$\therefore Fissile\ consumed = 1.5605 \times 10^{19} \times 1.09$$

$$= 1.700 \times 10^{19}\ nuclei/s$$

$$\therefore Fissile\ produced = 1.700 \times 10^{19} \times 0.96$$

$$= 1.633 \times 10^{19}\ nuclei/s$$

$$= 1.633 \times 10^{19} \times 365.25 \times 24 \times 3600$$

$$= 5.153 \times 10^{26}\ nuclei/yr$$

$$\therefore Mass\ of\ U^{233} = \frac{5.153X10^{26}}{6.022X10^{23}} \times \frac{233}{1000}$$

$$= 199\ kg/yr$$

Question 7

What value of the breeding gain is necessary for a fast breeder operating on the U^{238}-Pu^{239} cycle to have an exponential doubling time of 10 years, if the specific power for this type of reactor is 0.6 MW/kg of Pu^{239}? Capture/fission ratio for Pu^{239} = 0.42.

Solution-7

$$T_{de} = 10\,\text{yr}, \quad K = \frac{0.6\,\text{MW}}{\text{Kg}}, \quad \alpha_{Pu} = 0.42$$

Assume fission Energy = 200 MeV = $200 \times 1.603 \times 10^{-13}$

$$= 3.204 \times 10^{-11}\ \text{J}$$

For every fission, actual number of nuclei consumed which includes capture = 1.42

$$= 1.42 \times \frac{239}{6.022 \times 10^{23}}$$

$$= 5.635 \times 10^{-22}\ \text{g}$$

$$\therefore \beta = \frac{5.635 \times 10^{-22}}{3.204 \times 10^{-11}} = 1.759 \times 10^{-11}\ \frac{\text{g}}{\text{J}}$$

$$= 1.759 \times 10^{\cdots}\ \text{kg/J}$$

$K = 0.6\ \text{MW/kg} = 0.6 \times 10^{6}\ \text{J/kg-s}$

$T_{de} = 10\ \text{yr} = 10 \times 365.25 \times 24 \times 3600 = 3.156 \times 10^{8}\ \text{s}$

$$T_{de} = \frac{0.693}{K\beta G}$$

$$\Rightarrow G = \frac{0.693}{K\beta T_{de}} = \frac{0.693}{0.6 \times 10^{6} \times 1.759 \times 10^{-14} \times 3.156 \times 10^{8}}$$

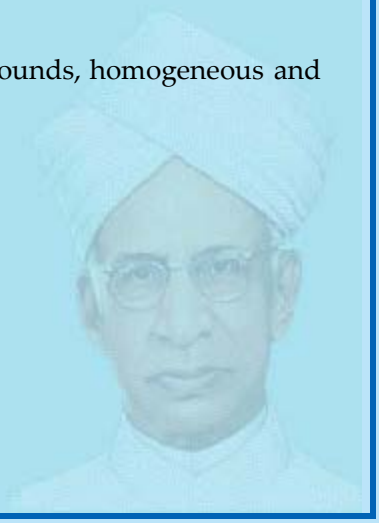
$$\therefore G = 0.208$$

Objectives

objectives

In this lecture you will learn the following

- Now we shall go into details of nuclear reactions in a quantitative manner.
- We will introduce a material property called Cross Section.
- Then we shall understand how to calculate cross sections for compounds, homogeneous and heterogeneous mixtures from the elemental values.

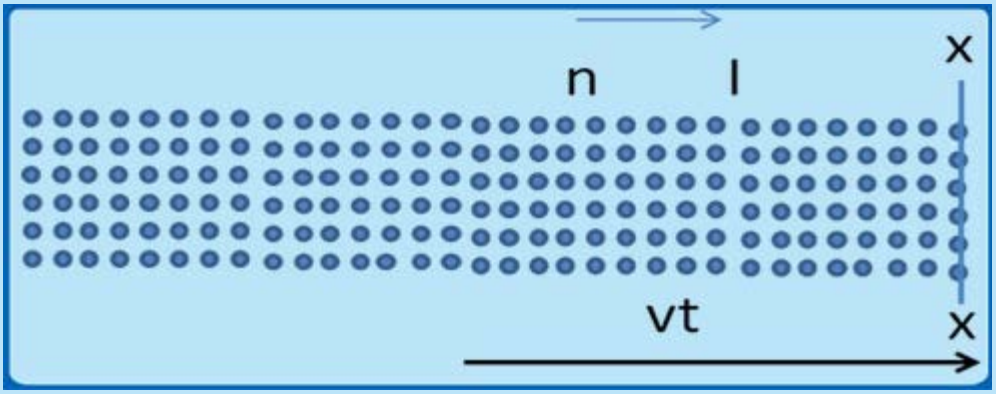


Neutron Reactions

- Neutrons are of interest to nuclear engineers primarily for the following reasons;
  - Neutrons being electrically neutral, directly interact with nucleus.
  - Chain reactions are possible as they are both in the reactant as well as the product side.
- We shall now see some definitions.

Definitions

- Refer to the figure showing a neutron beam in which the neutrons move with a uniform velocity v.
- Similar to the definition in optics, we can define intensity (I) as the number of neutrons that impinge on a unit area per unit time.
- We shall define neutron density (n) at a point as the number of neutrons avaliable per unit volume.
- Now we can relate the parameters that we have defined viz., n, v and I.
- Consider the plane X-X. If the plane was transparent, and we allow a time t to elapse, then the length of travel of neutrons = vt.



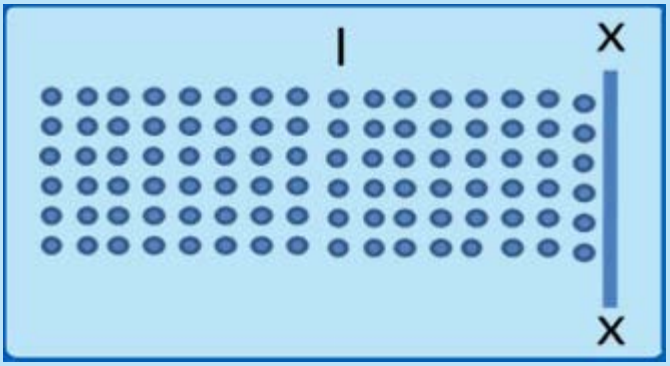
- If A is the area of the beam perpendicular to paper, then volume swept = Avt.
- Total number of neutrons crossed = nAvt.
- Thus, $I = \frac{nAvt}{At} = nv$

Types of Neutron reactions

- Neutron reactions can be mainly grouped into two types, viz., Absorption and Scattering.
- In absorption reaction neutron is absorbed and hence lost.
- In scattering, the neutron collides and changes its direction of motion.
- Scattering can be classified into elastic and inelastic.
- Absorption reaction can be denoted as (n,α), (n,β), (n,γ), (n,2n), etc.
- The first letter in the parenthesis denotes the reactant neutron and the second letter stands for the one of the products of the reaction.
- For e.g. (n,α) indicates that one of the product is an alpha particle.
- (n,2n) is a fission reaction with two product neutrons.

Cross Section for a Reaction

- To quantify reaction rates we shall define some parameters.
- Consider a neutron beam of intensity I striking a target as shown



- From common sense, one can say that the number of interactions between a target nucleus and neutrons will be proportional to I.

$$\therefore \dot{R}/nucleus \propto I \qquad \Rightarrow \dot{R}/nucleus = \sigma I$$

Where σ is the proportionality constant

- If the number of target nucleus per unit target volume, called number density, is denoted by N, then, reaction rate per unit volume of the target can be expressed as

$$R''' = N\sigma I \qquad (1)$$

$$\Rightarrow R''' = \Sigma I \qquad (2)$$

- The parameter σ is called the microscopic cross section and Σ is the corresponding macroscopic cross section. It may be noted that

$$\Sigma = N\sigma \qquad (3)$$

- If we look at the dimensions of Eq. (1), we can conclude the following

$$R''' = N\sigma I \quad \Rightarrow \frac{1}{m^3 s} = \frac{1}{m^3}\sigma\frac{1}{m^2 s} \quad \Rightarrow \sigma \rightarrow m^2$$

- Thus the unit of microscopic cross section is m^2.
- The interpretation is that σ represents the effective cross section every nucleus offers for collision with neutrons.
- Similarly. From Eq. (3), the unit of Σ shall be

$$\Sigma = N\sigma$$

$$\Rightarrow \Sigma = \frac{1}{m^3} m^2 \quad \Sigma \rightarrow \frac{1}{m}$$

- The numerical value of σ is very small. Hence these are usually expressed in barns

$$1\ barn = 10^{-28}\ m^2$$

- To compute the number of neutron reactions per unit volume in the bulk of any material can be written as

$$\frac{\dot{R}}{V} = \Sigma I \qquad \dot{R} = \Sigma I V$$

- In deriving the above equation, it is assumed that the intensity of neutrons is uniform throughout the thickness of the target.

Computations of Cross Section

- The values of microscopic cross section for specific reactions for most of the materials have been experimentally obtained and are tabulated in table of isotopes.
- Usually they are labelled with appropriate subscripts

σ_f	Fission Cross Section
σ_c	Capture Cross Section
σ_α	Absorption Cross Section
σ_s	Scattering Cross Section
σ_t	Total Cross Section

- Since neutron absorbed in a fissile isotope can either induce fission or capture, we can write

$$\sigma_a = \sigma_f + \sigma_c$$

- Similarly, when a neutron-material interaction takes place, it can either be absorbed or scattered. Hence the total cross section will be

$$\sigma_t = \sigma_a + \sigma_s$$

- As scattering can be elastic or inelastic, we can write

$$\sigma_s = \sigma_{elastic} + \sigma_{inelastic}$$

- As already stated, the cross sections for pure elements are listed in handbooks.
- The cross section for a compound can be obtained as shown in the example

$$\sigma_a^{H_2O} = 2\sigma_a^H + \sigma_a^O$$

- The above relation assumes that one reaction is independent of the other. This is generally true as the neutron density is very large.
- The macroscopic cross section can be computed as

$$\Sigma_a^{H_2O} = N_{H_2O}\,\sigma_a^{H_2O}$$

- For homogeneous alloys, say for stainless steel, the macroscopic cross section can be computed from the microscopic cross section of its constituents.
- Suppose the density of an alloy is ϱ, and if we have three constituents, whose mass fractions are ω_1 , ω_2 and ω_3, we proceed as follows.
- Consider unit volume of the alloy (1 m^3).

- Mass of alloy = ϱ (kg/m^3).
- If the mass numbers of the alloying elements are A_1, A_2 and A_3, the number densities of these elements are:

$$N_1 = \frac{\rho\omega_1}{A_1} N_{Avo} \quad N_2 = \frac{\rho\omega_2}{A_2} N_{Avo} \quad N_3 = \frac{\rho\omega_3}{A_3} N_{Avo}$$

- Now the macroscopic cross section can be computed as

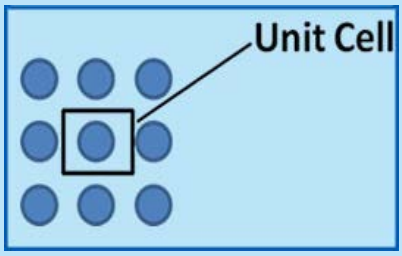
$$\Sigma_{mix} = N_1\sigma_1 + N_2\sigma_2 + N_3\sigma_3$$

- Often for simplified treatment of an assembly of material, homogenised cross sections are computed.

- Let us consider a regular geometry as shown in the figure.

- Let the volume of the unit cell be V.

- Let the volume fractions of fuel and moderator be α_1 and α_2.



- The volumes of fuel and moderator are $\alpha_1 V$, $\alpha_2 V$.

- If ϱ_1 and ϱ_2 are the densities of fuel and moderator, then the mass of fuel and moderator are $\alpha_1 V\ \varrho_1$ and $\alpha_2 V\ \varrho_2$.
- Thus, mass per unit volume shall be, $\alpha_1\varrho_1$ and $\alpha_2\varrho_2$.
- If the mass numbers of the fuel and moderator materials are respectively, A_1 and A_2, then the number densities can be written as

$$N_1 = \frac{\rho_1\alpha_1}{A_1}N_{Avo} \qquad N_2 = \frac{\rho_2\alpha_2}{A_2}N_{Avo}$$

- The macroscopic cross section can now be computed as

$$\Sigma_{mix} = N_1\sigma_1 + N_2\sigma_2 + N_3\sigma_3$$

- If 1 stands for fuel and 2 for moderator, we can write

$$\alpha_1\frac{\rho_1}{A_1}N_{Avo} = \alpha_1 N_{fuel} \qquad \alpha_2\frac{\rho_2}{A_2}N_{Avo} = \alpha_2 N_{Mod}$$

- Thus macroscopic cross section of the mixture can be computed as

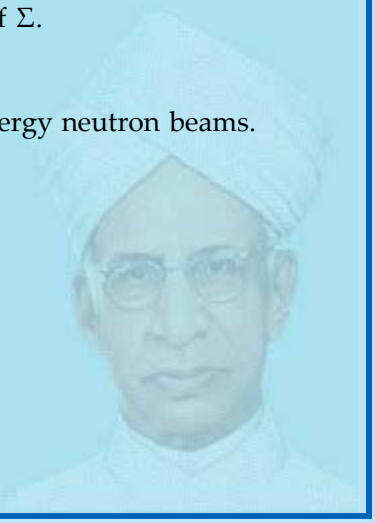
$$\Sigma_{mix} = \alpha_1 N_{fuel}\sigma_{fuel} + \alpha_2 N_{mod}\sigma_{mod}$$
$$\Sigma_{mix} = \alpha_1\Sigma_{fuel} + \alpha_2\Sigma_{mod}$$

- Thus macroscopic cross section of a mixture is volume fraction weighted average of its segregated constituents.

Objectives

In this lecture you will learn the following

- Today we shall understand the difference between thin and thick targets.
- Then we will proceed to define mean free path and measurement of Σ.
- Then we will understand some concepts in activation.
- Finally we will study how to compute reaction rates for variable energy neutron beams.

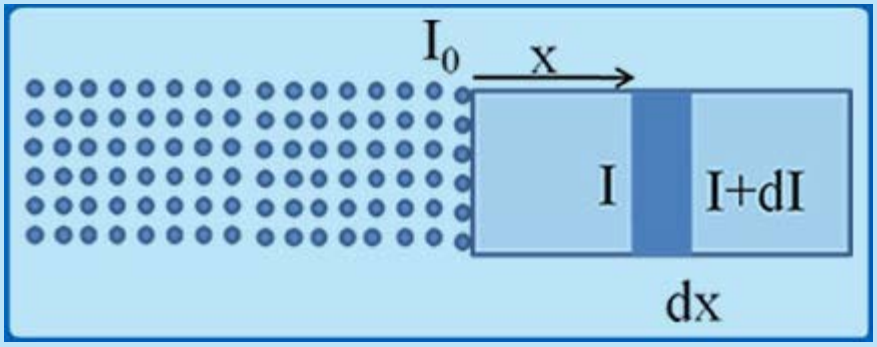


Thin and Thick Targets

- The number of neutron reactions per unit volume in the bulk of any material can be written as

$$\dot{R}''' = \frac{\dot{R}}{V} = \Sigma I \qquad \Rightarrow \dot{R} = \Sigma I V$$

- In deriving the above equation, it is assumed that the intensity of neutrons is uniform throughout the thickness of the target.
- The above equation is strictly never valid. However, from an engineering perspective, we may assume the above assumptions is correct, if the result is within 1% deviation.
- Such an approximation may be termed as **thin target approximation**.
- To arrive at a suitable criterion for the validity of the approximation, we can perform a simple analysis.
- Consider a target that is bombarded by a neutron beam of intensity, I_0.



- Consider an infinitesimal strip of thickness dx at a distance x from the front face.

- If we assume the process to be steady, we can write the neutron population balance for the strip as follows:

Neutron Balance Equation

IA	$-(I+dI)A$	$-\Sigma IAdx \quad = 0$
Rate of neutrons entering the control volume	Rate of neutrons leaving the control volume	Rate of neutrons absorbed in the control volume

$$\Rightarrow \frac{dI}{dx} = -\Sigma I$$

- The boundary condition for the above equation is

$$I = I_0 \; at \;\; x = 0$$

- The Solution for the equation can be written as

$$I = I_0 \, e^{-\Sigma x}$$

- This implies that I at x = L shall be

$$I = I_0 \, e^{-\Sigma L} \qquad \Rightarrow \frac{I_L}{I_0} = e^{-\Sigma L}$$

- If we accept 1% loss of intensity, we get

$$\Rightarrow \frac{I_L}{I_0} = e^{-\Sigma L} = 0.99 \qquad \Sigma L = 0.01$$

- Thus the criterion for a thin target approximation is

$$\Sigma L \leq 0.01$$

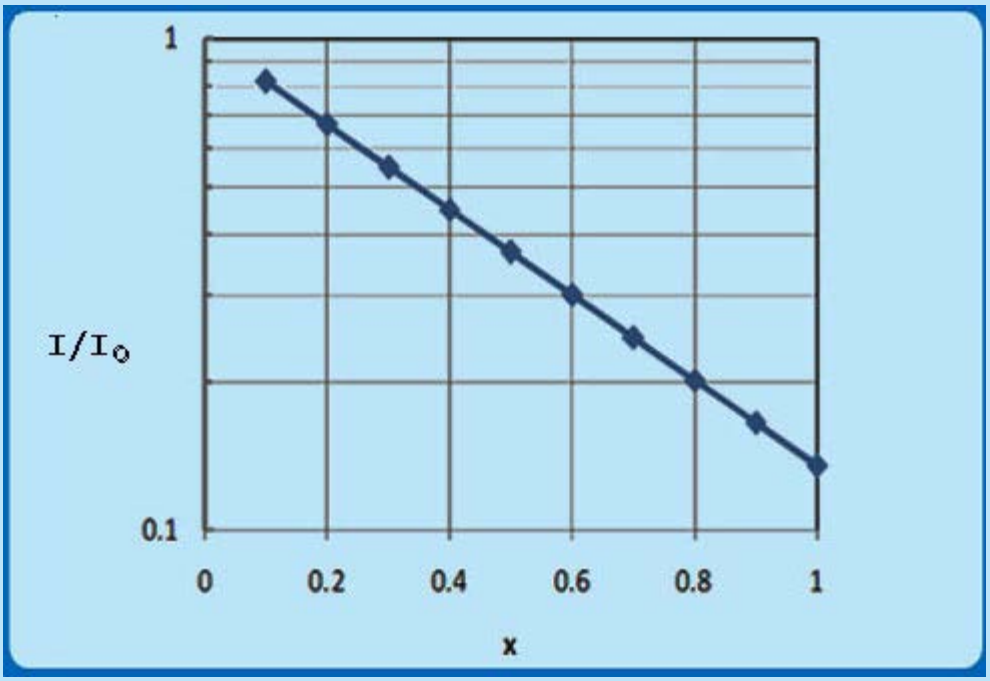
Mean Free Path

- By virtue of exponential decay, the intensity will fall to zero only at infinite distance.
- However, neutrons travel from almost no distance to infinite distance.
- Mean free path can be viewed as the mean distance travelled by the neutron.
- Since the distance travelled by neutrons is a continuous function, we can calculate the mean free path as,

$$\frac{\int_{I_0}^{0} x(-AdI)}{\int_{I_0}^{0} (-AdI)} = \frac{\int_{0}^{\infty} A \text{ x } I_0 \Sigma\, e^{-\Sigma x} dx}{AI_0} = \int_{0}^{\infty} x \Sigma e^{-\Sigma x} dx = \frac{1}{\Sigma}$$

Measurement of Σ

- If we use targets of varying thickness and measure the intensity of a beam as a function of thickness and plot its variation in a semi-log plot, we get



- The slope of the line = -Σ.

Activation Analysis

- A coolant flowing in the neutron field gets activated(becomes radioactive). Similarly, structural materials in neutron field gets activated.
- The amount of activity has to be known to plan for maintenance procedure.
- Hence quantification of activation is an engineering requirement.
- Activation analysis is also of importance in radioisotope production for medical applications.
- Let us look at the analysis to quantity activation.
- Consider that Co^{59} is present in a neutron field.
- As Co^{59} absorbs neutrons it is converted into Co^{60} that is radioactive and it undergoes radioactive decay

$$Co^{59} + n \rightarrow Co^{60*}$$

- If we assume that the target is thin, and its volume is V, we can write the balance equation as

$$\frac{d(N^{59}\,V)}{dt} = -\Sigma^{59} I\,V = -N^{59}\sigma^{59} I\,V$$
$$\Rightarrow \frac{dN^{59}}{dt} = -N^{59}\sigma^{59} I$$

- The solution of the above equation with the initial condition $N^{59} = N^{59}(0)$ at t = 0, is

$$\Rightarrow N^{59} = N^{59}(0)e^{-\sigma^{59} It}$$

- Similar to the earlier argument of thin target approximation, if the total time of interest T is such that $\sigma^{59}IT < 0.01$, then $N^{59} \sim N^{59}(0)$.
- We may call this approximation as **Infinite Mass Approximation**.
- Now if we write the balance equation for N^{60}

$$\Rightarrow \frac{dN^{60}}{dt} = N^{59}\sigma^{59} I - N^{60}\lambda^{60}$$

- If infinite mass approximation is valid, then the first term, which is the production of Co^{60}, is constant. Hence we can write,

$$\dot{P}^{m} = N^{59}\sigma^{59} I = cons\tan t$$
$$\Rightarrow \frac{dN^{60}}{dt} = \dot{P}^{m} - N^{60}\lambda^{60}$$

- We can separate the variables and can get the solution for initial condition $N^{60} = N^{60}(0)$ at t = 0 as follows

$$\Rightarrow \frac{dN^{60}}{\dot{P}^{ss} - N^{60}\lambda^{60}} = dt \Rightarrow \left[\frac{\ln\left(\dot{P}^{ss} - N^{60}\lambda^{60}\right)}{-\lambda^{60}}\right]_{@t=0}^{@t=t} = t\Big|_0^t$$

$$\Rightarrow \ln\frac{\left(\dot{P}^{ss} - N^{60}\lambda^{60}\right)}{\left(\dot{P}^{ss} - N^{60}(0)\lambda^{60}\right)} = -\lambda^{60}t \quad \Rightarrow \frac{\left(\dot{P}^{ss} - N^{60}\lambda^{60}\right)}{\left(\dot{P}^{ss} - N^{60}(0)\lambda^{60}\right)} = e^{-\lambda^{60}t}$$

- If $N^{60}(0) = 0$, we can write

$$\Rightarrow \frac{\left(\dot{P}^{ss} - N^{60}\lambda^{60}\right)}{\dot{P}^{ss}} = e^{-\lambda^{60}t} \quad \Rightarrow N^{60} = \frac{\dot{P}^{ss}}{\lambda}\left(1 - e^{-\lambda^{60}t}\right)$$

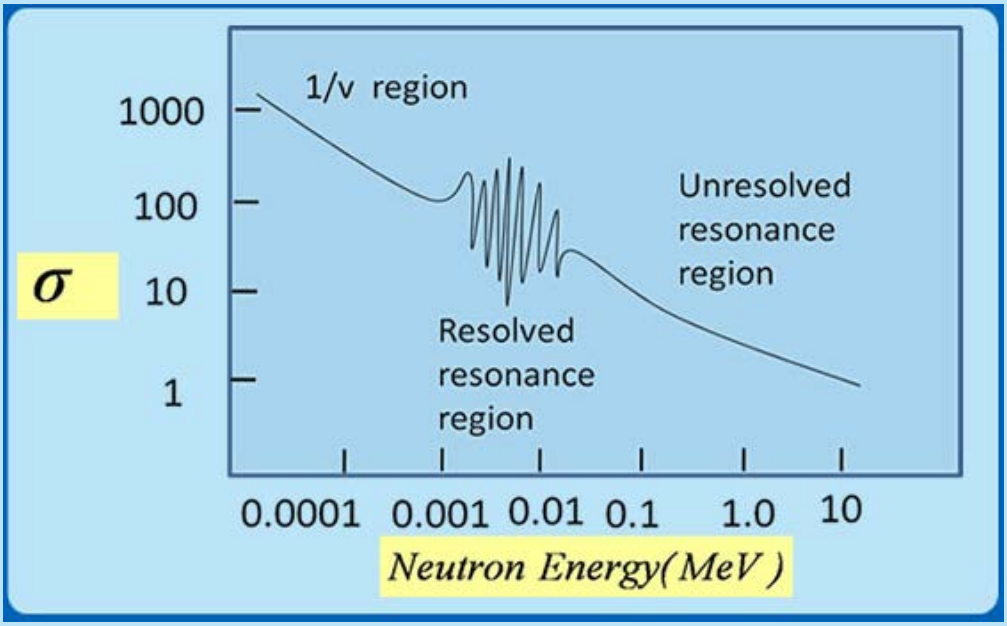
- At large times, we can write

$$\Rightarrow N^{60} = \frac{\dot{P}^{ss}}{\lambda} \qquad \Rightarrow \lambda N^{60} = \dot{P}^{ss}$$

- We note that at large times the production rate is equal to decay rate as expected.

Variation of σ with E

- We have already discussed the graph shown below in an earlier lecture.
- Now the same is quantified.



Poly-Energetic Neutrons

- When neutrons are not mono-energetic, then the distribution of energy is represented by spectrum.
- In such cases, the number density of neutrons n(E) shall be energy dependent. The number density of neutrons lying between E and E+dE is represented by n(E)dE.
- It should be easy to understand that if we integrate over all energy then

$$\int_0^\infty n(E)dE = n$$

- To find the reactions when poly-energetic neutrons are encountered, we proceed as follows:

$$d\dot{R}'''(E) = n(E)dE v(E)\ N\sigma(E)$$

- The total number of reactions over all energy can now be computed as

$$\dot{R}''' = \int_0^\infty n(E)dE v(E)\ N\sigma(E)$$

- If the cross section varies as 1/v, then we can write

$$\sigma(E)v(E) = \sigma(E_{ref})v(E_{ref})$$

- With the above relation in mind, we can now evaluate the integral as

$$\dot{R}''' = \sigma(E_{ref})v(E_{ref})N\int_0^\infty n(E)dE$$

$$\dot{R}''' = N\sigma(E_{ref})nv(E_{ref}) = \Sigma_{ref}I_{ref}$$

- The reference energy is chosen as thermal energy, which corresponds to 0.0253 eV or 2200 m/s neutrons.
- All reference cross sections are usually tabulated at 0.0253 eV.

Objectives

In this lecture you will learn the following

- To illustrate the application of the material studied in last two classes and reinforce them through some interesting applications.
- First we shall illustrate computation of cross sections using realistic problems.
- Then we illustrate application of reaction rates using simple problems.
- Finally. we shall look at an activation problem.

Background Information

- Mole – Molecular/atomic weight expressed in grams.
- 1 mole of any element has Avogadro number of molecules/atoms.
- 1 MeV = 1.603 x 10^{-13} Joule.
- Avogadro Number = 6.023 x 10^{23}.
- Number of density of the target can be computed by

$$N = \frac{6.023x10^{23}}{Atomic\ wt}\ x\ density$$

- 1 Curie = 3.7 x 10^{7} dps (s^{-1}).

Question-1

Stainless steel type 304 having a density 7.86 gm/cm^3 has been used in some reactors. The nominal composition by weight of this material is as follows: carbon 0.08 percent; chromium 19 percent; nickel 10 percent; iron the remainder. Calculate the macroscopic absorption cross section of SS-304 at 0.0253 eV. The microscopic cross section (in barns) of C, Cr, Ni and Fe are respectively, 0.0034, 3.1, 4.42, 2.55. (0.2428 cm^{-1})

Consider volume = 1cm^3

Density = 7.86 g/cm^3

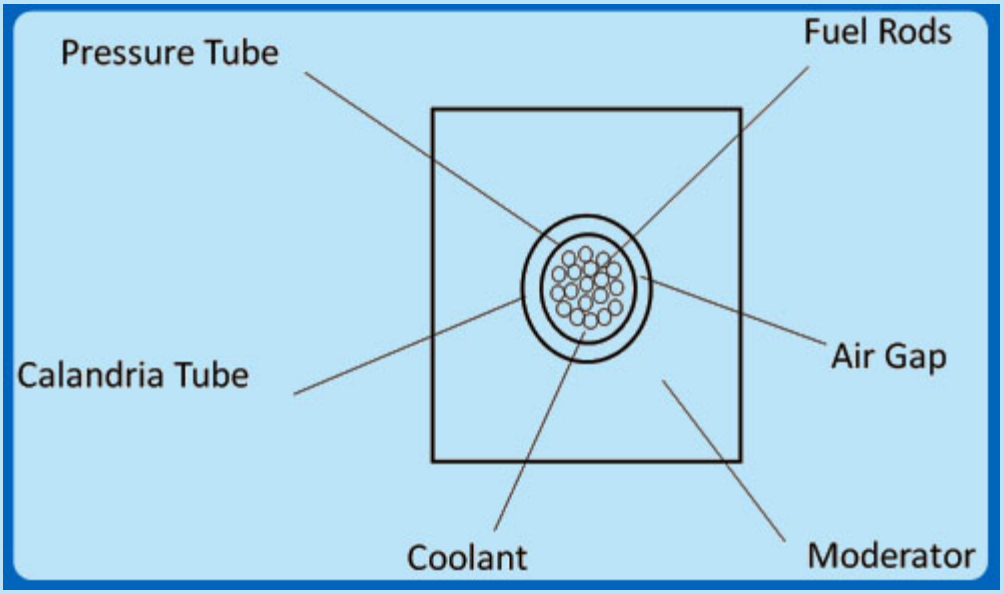
Hence mass = 7.86 g

$$N = \frac{6.023x10^{23}}{Atomic\ wt} x\ mass$$

Material	Mass fraction	Mass in (1cm3)	Atomic wt	Number density (cm-3)	σ(cm2)	Σ(cm-1)
C	0.0008	0.006288	12	3.15605x1020	3.4x10^{-27}	0.000001073
Cr	0.19	1.4934	52	1.72976x1022	3.1x10^{-24}	0.053622
Ni	0.1	0.786	58.7	8.06487x1021	4.42x10^{-24}	0.03564
Fe	0.7092	5.574	55.8	6.01686x1022	2.55x10^{-24}	0.15343
Total	1	7.86				0.242700

Question-2

The typical unit cell of a Rajasthan Atomic Power Station (RAPS) is shown in the following figure.



The computed volumes of the various materials per unit length of the reactor are:

UO_2	29.2 cm^3
Zr (A=91)	20.3 cm^3
D_2O (coolant)	19.1 cm^3
Air gap	27.4 cm^3
D_2O (moderator)	426.6 cm^3

It may be assumed that air may be treated as a non-participating medium (does not react with neutrons) and the Uranium in UO_2 is natural.

(a) Given that the density of UO_2, Zr and D_2O to be 10.5, 6.5 and 1.1 g/cc respectively, calculate the homogenised number density of each material.

(b) Given the volumes as above, calculate the homogenised macroscopic absorption cross section of UO_2, Zr and D_2O and the core, given the following

Material	$\sigma_a (barns)$	$\sigma_f (barns)$
U^{235}	680	580
U^{238}	2.7	0.0
Zr	0.198	---

D_2O	4.6×10^{-4}	---
O	0.0	0.0

For Natural Uranium, assume that $N_{235}/N_{238} = 7/993$.

Solution

Consider unit length of core

Material	Volume	Volume fraction
UO2	29.2	0.0559
ZR	20.3	0.0388
D2O(Coolant)	19.1	0.0365
Air-gap	27.4	0.0524
D2O(Moderator)	426.6	0.8164
Total	522.6	1

- For 1 cc of core, the volumes of the components shall be same as the volume fractions.

Now we can construct the following table.

σ Nat-U = σ U_{235} x 0.007+ σ U_{238} x 0.993

Material	Volume per cc-core	Density (g/cc)	A	Mass (g/cc)	N (cm-1)	σ (barn)	Σ (cm-1)
UO_2	0.0559	10.5	269.98	0.587	1.30884E+21	7.4411	0.009739
Zr	0.0388	6.5	91	0.252	1.67113E+21	0.198	0.000331
D_2O(Cool)	0.0365	1.1	20	0.0402	1.21071E+21	0.00046	5.57E-07
Air-gap	0.0524	----	----	---	---	---	---
D_2O(Mod)	0.8160	1.1	20	0.898	2.70413E+22	0.00046	1.24E-05
Total	1			1.777			0.010083

Question-3

1. A monoenergetic beam of neutrons having an intensity of 4×10^{10} neutrons/cm^2-s impinges on a target 1 cm^2 area and 1 mm thick. There are 0.048×10^{24} atoms per cm^3 in the target and the total cross section at energy of the beam is 0.45 b. (a) what is the macroscopic total cross section? (b) How many neutron interactions per second occur in the target? (c) What is the collision density?

Cross Section = $\sigma = 0.45 \times 10^{-24}$ cm^{-2}

Intensity = I = 4.0 x 1010 n/cm^2-s

Target Number Density = N = 0.048×10^{24} nuclei/cm^3

Σ (cm-1) = N

$\sigma = 0.048 \times 10^{24} \times 0.45 \times 10^{-24}$

$= 0.0216$ cm^{-1}

Volume of Target = V = 1 cm^2 x 0.1 cm =0.1 cm^3

Reaction Rate = $= \Sigma IV = 0.0216 \times 4 \times 10^{10} \times 0.1$ s^{-1}

$= 8.64 \times 10^7$ s^{-1}

Collision density = $\Sigma I = 0.0216 \times 4 \times 10^{10}$

$= 8.64 \times 10^8$ s^{-1} cm^{-3}

Check for thin target approximation

$\Sigma L = 0.0216 \times 0.01 = 0.00216 << 0.01$ **Hence OK**

Question 4

Calculate the mean free path of 1 eV neutrons in graphite (density = 1.6 g/cm^3). The total cross section of carbon at this energy is 4.8 b. (2.59 cm)

$$N = \frac{6.023x10^{23}}{Atomic\ wt}\ x\ density$$

Density(g/cc)	1.6
A	12
N	8.03067E+22
σ(cm^2)	4.8
Σ(cm^-1)	0.385472
MFP(cm)	2.594222148

Question-5

The β^- emitter ^{28}Al (half life 2.30 min) can be produced by the radiative capture of a neutron by ^{27}Al. The 0.0253 eV cross section for this reaction is 0.23 b. Suppose that a small, 0.01 g aluminum target is placed in a beam of 0.0253 eV neutrons having an intensity of 3 x 10^8 neutrons/cm^2 , which strikes the entire target. Calculate (a) the neutron density in the beam; (b) the rate at which ^{28}Al is produced, (c) the maximum activity (in curies) which can be produced in this experiment. (2.17 x 10^{-4} cm^{-3}, 1.5^4X10^4 s-1, 4.16 X 10^{-7} Ci)

- The speed of the neutron is computed by equating energy to kinetic energy

$$V = \sqrt{\frac{2E}{m}}$$

Mass of neutron(kg)	1.675E-27
energy(eV)	0.0253
energy(J)	4.05559E-21
Speed(m/s)	2200.567091
I(n/cm^2-s)	3.000E+08
n(cm^{-3})	1.363E+05

$$Al^{27} + n \rightarrow Al^{28*}$$

- Check if infinite mass approximation is valid

σ59IT < 0.01

$$\Rightarrow T \leq \frac{0.01}{\sigma I} = \frac{0.01}{0.23 \ x \ 10^{-24} x \ 3 \ x \ 10^8} = 1.45 \ x \ 10^{16} s$$

- This is about 4.6 x 10^6 years, which is too large. Hence the approximation is valid.
- Production rate of Al^{28} can be computed as

$$\dot{P} = N_{total}^{27} \sigma^{27} I$$

Mass of Al(g)	0.01
A	27
density of Al(g/cc)	2.7
Number of nuclei	2.23074E+20
σ(cm^2)	2.3E-25
Production rate(s^{-1})	1.54E+04

- During activation the activity of aluminum will build and reach a maximum, when production rate is equal to decay rate

$$\dot{P} = \lambda_i^{28} N_{total}^{28} = activity$$

- Therefore maximum activity = 1.54E+04 s^{-1}

= 4.16×10^{-7} Ci

Objectives

In this lecture you will learn the following

- We shall understand the essence of the slowing down process.
- Then we shall look at the mechanism of elastic collision.
- We shall conclude that lighter materials are better moderators.
- Then we shall identify the common moderators.

Moderation

- The process by which the energy of neutron is decreased is called moderation.
- As discussed in our previous lecture, the cross section for neutron reactions can be assumed to vary approximately as 1/v, where v is the speed of the neutron.
- We also learnt that the average energy of the fission neutron is about 2 MeV, where as a neutron in thermal equilibrium with the 300 K environment is 0.025 eV.
- Thus the ratio of the speed is approximately 10^4 and hence the probability of a average energy neutron is 10^4 times less than the thermal neutron.

The Process of Moderation

- Moderation occurs in the nuclear reactor by collision process with the material present in the system.
- This process is called scattering.
- There are two kinds of scattering process, viz. Elastic scattering and Inelastic Scattering.In inelastic scattering kinetic energy is not conserved as the target nucleus gets into excited state consuming energy.

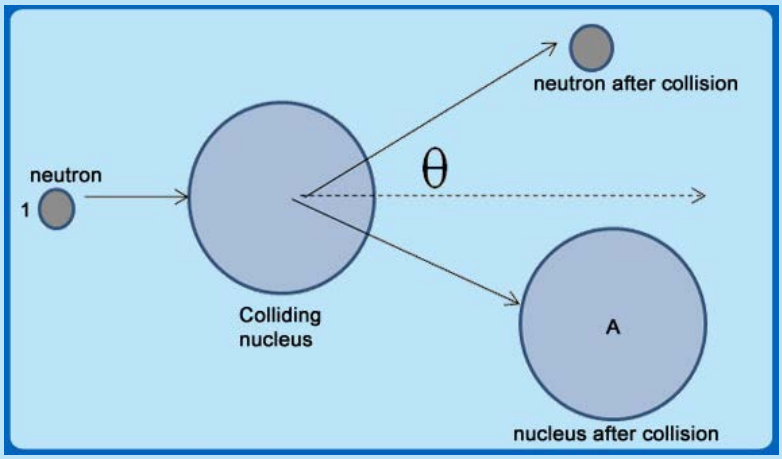
Elastic Collision Process

- We had shown in previously that it is enough to consider Newtonian Mechanics and relativistic effects need not be considered, if the neutron travels at speeds less than 1/10 of the speed of light.
- For all practical purposes with neutron reactions in reactors this is valid.
- From mechanics we can write that momentum, p, and kinetic energy E as

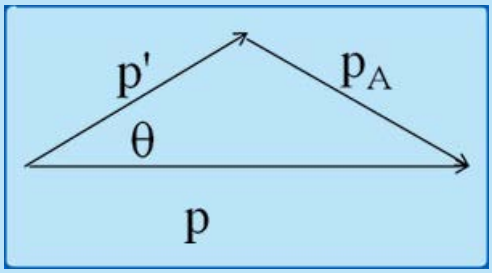
$$p = mv \qquad E = 0.5\ mv^2$$

$$\Rightarrow\ p^2 = 2mE$$

- Consider the elastic collision process



- The momentum conservation can be expressed by the vector diagram shown
- Here p, p' and p_A are the momenta of the incident neutron, scattered neutron and the recoil nucleus respectively.



- From Triangle Law of vector addition, we can write

$$\Rightarrow \; p_A^{\,2} = p^2 + p'^2 - 2pp'\cos\theta$$

- This can be rewritten as

$$2m_A E_A = 2m_n E + 2m_n E' - 4m_n\sqrt{EE'}\cos\theta$$
$$\Rightarrow \; 2AE_A = 2E + 2E' - 4\sqrt{EE'}\cos\theta$$

- In the above equation E, E' and E_A are the kinetic energies of incident neutron, scattered neutron and the recoil nucleus respectively.
- Dividing both sides by 2 and invoking kinetic energy conservation, $E = E' + E_A$, We can write

$$A(E-E') = E + E' - 2\sqrt{EE'}\cos\theta$$
$$\Rightarrow (A+1)E' - 2\sqrt{EE'}\cos\theta - (A-1)E = 0$$

- Dividing by E, we get a quadratic in $\sqrt{\frac{E'}{E}}$

$$\Rightarrow \; (A+1)\frac{E'}{E} - 2\sqrt{\frac{E'}{E}}\cos\theta - (A-1) = 0$$

- The solution of the above quadratic equation is

$$\sqrt{\frac{E'}{E}} = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta + 4(A^2-1)}}{2(A+1)}$$
$$= \frac{\cos\theta \pm \sqrt{\cos^2\theta + (A^2-1)}}{(A+1)}$$

- Only positive sign will be considered for physically realisable solution.

- Till now the treatment is rigorous. From here on we make simple approximations to get some general results. These results can be shown to hold using more rigorous arguments in a next level course.
- If we assume that angle $\theta = 0$, often referred as gracing collision, we can write

$$\sqrt{\frac{E'}{E}} = \frac{1 + \sqrt{1 + (A^2-1)}}{(A+1)}$$
$$= \frac{A+1}{A+1} = 1$$

- Thus, the energy after collision is same as before and there is no energy loss.

General Observations

- If we take θ = 180, which is often referred as head on collision, we can write

$$\sqrt{\frac{E'}{E}} = \frac{-1+\sqrt{1+(A^2-1)}}{(A+1)}$$
$$= \frac{A-1}{A+1}$$
$$\frac{E'}{E} = \left(\frac{A-1}{A+1}\right)^2 = \alpha$$

- Thus, the minimum energy a neutron can have after collision is αE.
- Note that if A = 1, which is true for hydrogen nucleus, $\alpha = 0$ and thus a neutron can be brought to rest in one collision.
- Further, if A >>1, then $\alpha \sim 1$ and thus, there is no energy loss.
- Thus for the moderator to be effective, A should not be large.
- If we assume that every angle is equally likely, then the mean energy of neutron after collision is

$$(E+\alpha E)/2 = E(1+\alpha)/2$$

- Thus the mean energy loss is

$$E-E(1+\alpha)/2 = E(1-(1+\alpha)/2) = \mathrm{E}(1-\alpha)/2$$

- We can construct the following table

Element	A	α	$\Delta\bar{E}/E$
H	1	0	0.500
D	2	0.111	0.444
Be	9	0.640	0.180
C	12	0.716	0.142
U	238	0.983	0.008

- H as H_2O and D as D_2O are used as most common moderators.
- Gas cooled reactors use C as the moderators.

Choice of Moderator

- We can construct the following table illustrating the choice of the elements as moderator.

Element	Atomic No	Comment
H	1	Good moderator, but absorbs neutrons and hence a reactor needs enriched fuel
D	1	Good moderator and can be used with natural U
He	2	Noble Gas and hence not used
Li	3	Not used as it has high absorbtion cross section
Be	4	Difficult to machine and used only in research reactor
B	5	Strong Absorber, hence not used
C	6	Good moderator, can use Natural U

Estimation of Number of Collisions

- To understand the effectiveness of a moderator, we can estimate the average number of collisions to reduce the energy from 2 MeV to 0.025 eV.
- The average energy of neutron after one collision is

$$E_1 = E_0(1+\alpha)/2$$

- The average energy after two collisions shall be

$$E_2 = E_1(1+\alpha)/2) = E_0((1+\alpha)/2)^2$$

- The average energy after n collisions shall be

$$E_n = E_0((1+\alpha)/2)^n$$

- Thus for a given E_n, E_o and α, n can be estimated.

- The number of collisions required to reduce the energy from 2 MeV to 0.025 eV is given in the following Table.

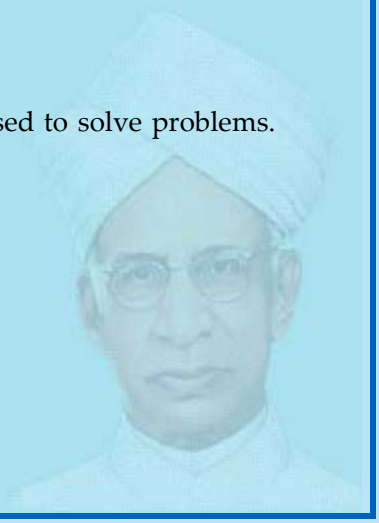
Element	A	α	(1+α)/2	n
H	1	0	0.500	26.2
D	2	0.111	0.555	30.9
Be	9	0.640	0.820	91.7
C	12	0.716	0.858	118
U	238	0.983	0.9915	2132

- As the number of collisions increase, the size of the reactor also has to be increased to ensure less leakage.

Objectives

In this lecture you will learn the following

- We shall understand the essence of the movement of the slowed down neutrons.
- We shall introduce the concept of neutron diffusion.
- We shall then derive the neutron balance equation with diffusion.
- Then we shall discuss the common boundary conditions that are used to solve problems.



Definitions

- We had seen previously in the course that the intensity of neutrons in a beam was defined as the number of neutrons crossing a plane per unit area and unit time and this was shown to be

$$I = nv$$

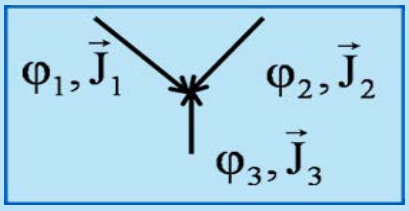
- Since the beam travels in the same direction unless it is absorbed, the problem is a scalar as there is no change of direction is involved.
- However, when we deal with neutrons in a reactor, they are born at different places and travel in different directions.
- Further their directions are modified due to the process of scattering.
- To predict the neutron population density variation in a reactor we need a more complex treatment.
- It will be practically impossible to track every neutron and study its motion as there are very large number of neutrons present in the system.
- To handle this problem in a simple manner, the concept of diffusion is introduced.
- Before we formally look at the concept of diffusion, we shall define a few terms.
- The term Neutron Flux, ϕ, is defined as the product of neutron density and the speed of the neutron and it is a scalar quantity.

$$\phi = nv$$

- Another term Neutron Current, J, is defined as number of neutrons crossing a unit area per unit time in a specific direction.
- J is a vector quantity and shall have components such as J_x, J_y, etc.
- Consider the case of several beams intersecting at a point as shown in the figure below.
- At the point of intersection

$$\phi = \phi_1 + \phi_2 + \phi_3$$

$$\vec{J} = \vec{J}_1 + \vec{J}_2 + \vec{J}_3$$



- When several beams cross, it is easy to visualise that, the former is a scalar sum, while the latter is a vector addition.
- If all the neutrons travel in the same direction as in a single beam, both ϕ and J are identical in a non scattering system.

Diffusion-I

- Consider two nearby points A and B such that the neutron density is high at A and low at B.

- Since the population density is more at A, the number of scattering reactions at A will be larger than the number of scattering at B.
- Assuming that all directions are equally likely as neutrons are travelling in random directions, the number of neutrons that will migrate from A to B will be larger than number of neutrons migrating from B to A.
- Thus the population density at A will decrease and at B will increase.
- This process by which the population migrates from higher density to lower density is called diffusion.
- This can be mathematically treated as follows:
- The current $\vec{J}$ will be proportional to the gradient of n

$$\vec{J} \alpha -\nabla n$$

- The negative sign signifies that the direction of current is opposite to the direction of gradient.
- Assuming that all neutrons are moving at one speed, we can write

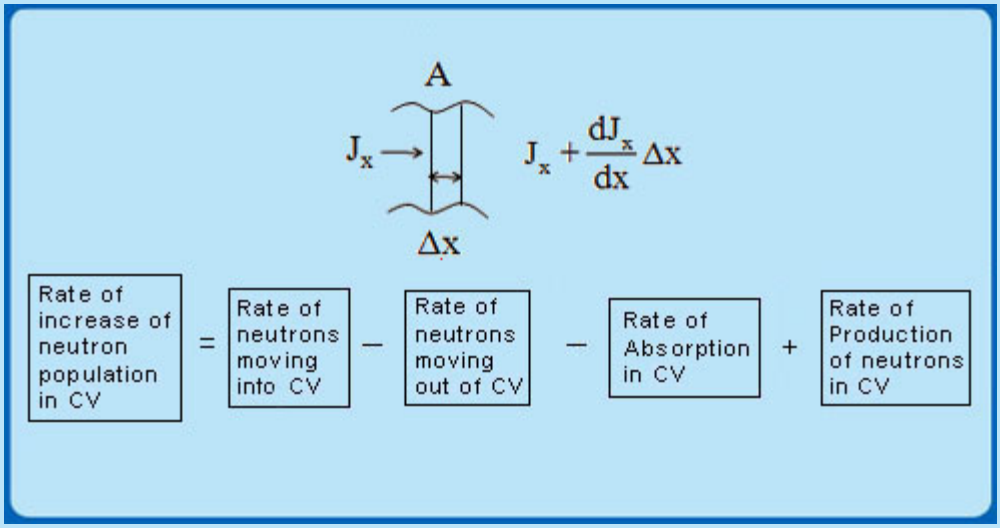
$$\vec{J} \alpha -\nabla nv \qquad \Rightarrow \vec{J} \alpha -\nabla \phi \qquad \Rightarrow \vec{J} = -D\nabla \phi$$

- In the last equation D is the proportionality constant and is called diffusion coefficient and the equation is called Fick's Law of Diffusion.
- It should be clear that the process that influences diffusion is scattering and hence D will depend on Σ_s.
- In this introductory course we shall assume that the value of D is available from text books/hand books.
- D for some common materials are summarised in the table shown below.

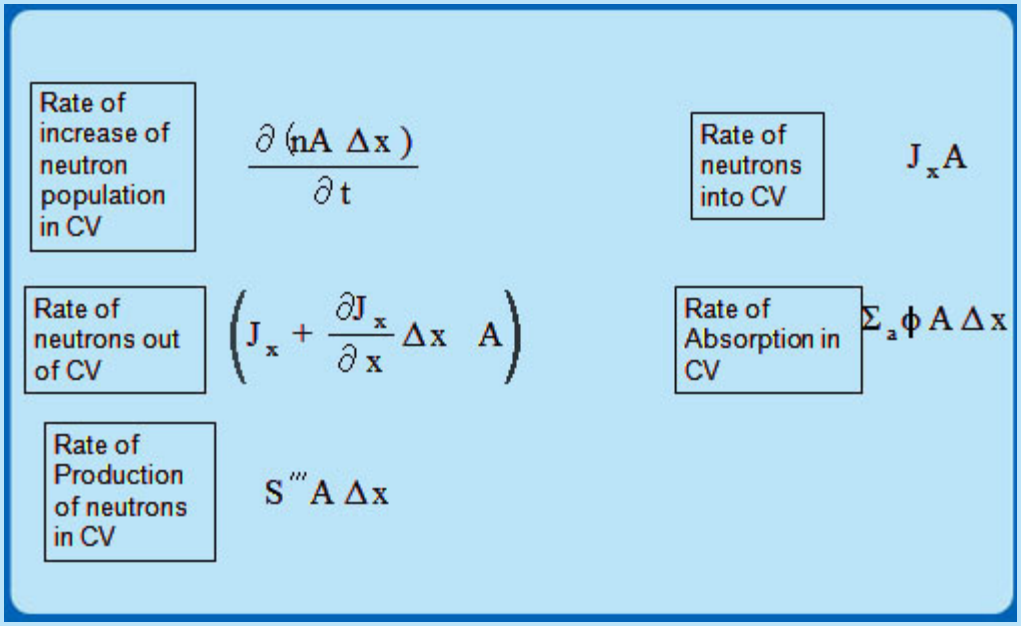
Material	D (cm)
H_2O	0.16
D_2O	0.87
Be	0.50
C	0.84

Neutron Balance Equation

- We shall now derive the neutron balance equation with diffusion also present.
- We will first do this in one dimension Cartesian and then extend to three dimensions.
- Subsequently we will generalise for other coordinates.
- Consider a small strip of area, A, and thickness, Δx, sliced from a media that is infinitely large.
- The rate of increase of neutron population in the control volume (CV) has 4 components as shown in the figure below



- Now each of the componants can be quantified in the figure below



- In the above figure S''' is the source strength of neutrons (number of neutrons emitted per second) per unit volume.

- Thus the neutron population balance can be written as,

$$\frac{\partial(nA\Delta x)}{\partial t} = \left[J_x + \frac{\partial J_x}{\partial x}\Delta x\right]A - \quad J_x A - \quad \Sigma_a \phi A\Delta x + \quad S'''A\Delta x$$

- The above equation can be simplified as

$$\frac{\partial n}{\partial t} = -\frac{\partial J_x}{\partial x} - \Sigma_a \phi + S'''$$

- This can be modified using Fick's law as

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2} \quad - \Sigma_a \phi + S'''$$

- By using the definition of flux we can rewrite the equation as

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\frac{\partial^2 \phi}{\partial x^2} \quad - \Sigma_a \phi + S'''$$

- The only difference when we extend to three dimensions is the diffusion occurs in all three directions and the equation can be modified as

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}\right) - \Sigma_a \phi + S'''$$

- In vectorial form we can write the same as

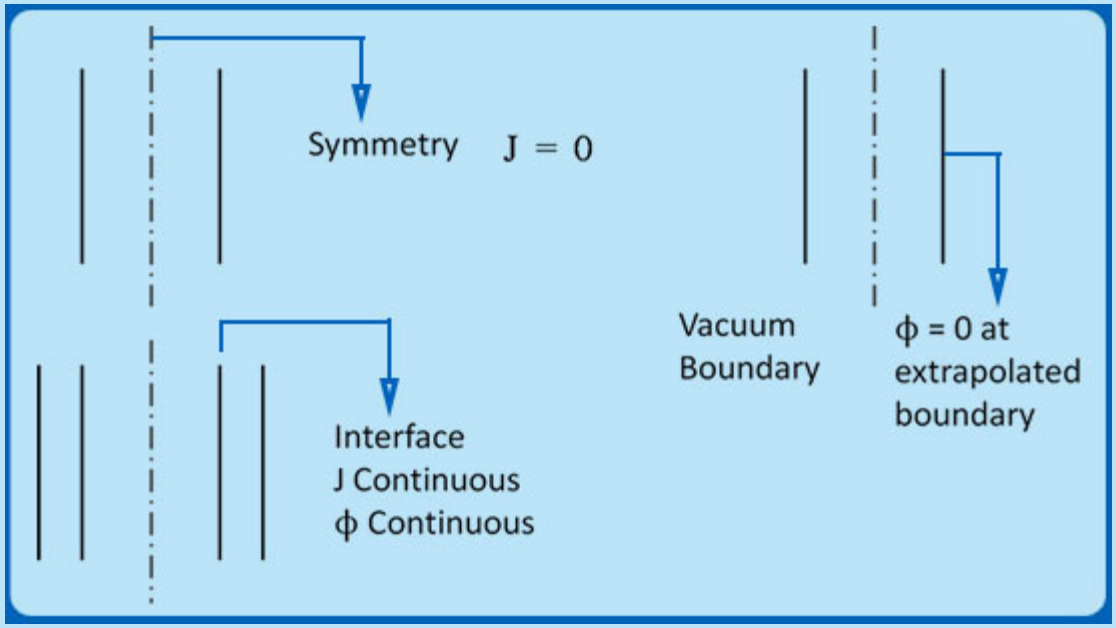
$$\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2 \phi \quad - \Sigma_a \phi + S'''$$

- Where, the Laplacian term can be represented as

$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$	Cartesian
$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	Cylindrical
$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \phi}{\partial r}\right)$	Spherically Symmetric

General Boundary Conditions

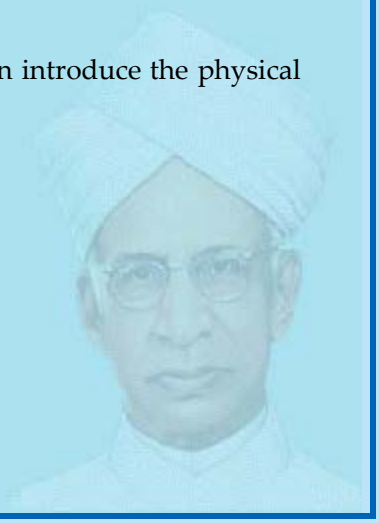
- The following are the general boundary conditions. These are schematically shown in the figure below.
  - Symmetric Boundary Condition
    - This implies that J=0 at the symmetric plane.
  - Vacuum Boundary Condition
    - When the boundary of a system ends in a non-interacting media, it is called vacuum boundary condition.
    - Such boundaries are approximated by using ϕ=0 at a distance approximately 2.1 D away from the boundary.
    - Since the value of D is small in comparison with system dimensions, often flux is assumed to vanish at the system boundary itself.
  - Interface Boundary Condition
    - Wherever the property of a media changes, the plane separating the two media is termed as the interface boundary.
    - Every interface satisfies two continuity conditions viz, both flux and current are continuous across the boundary.



Objectives

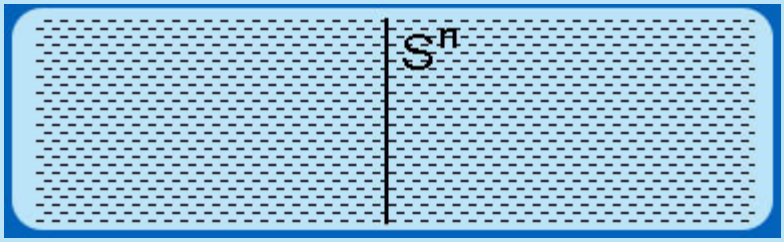
In this lecture you will learn the following

- In this lecture, we shall understand source boundary condition.
- Then we shall look at some simple solutions.
- Finally, we shall define a parameter called diffusion length and then introduce the physical interpretation of this term.



Source Boundary condition

- Consider an infinite planar source immersed in an infinite media as shown

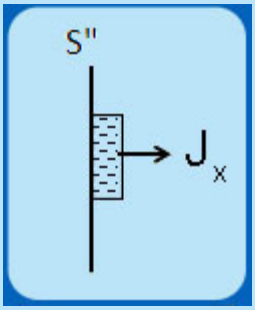


- Let us assume that a planer source emits S'' neutrons per unit area and unit time.
- If we consider the right half of the media away from the source, the media as such does not have a source in it.
- Thus, the planar source is just at a boundary.
- To prescribe a suitable boundary condition, let us look at a small volume of small thickness as

- If the area is A and the Current in x direction is J_x, then we can perform a neutron population balance and write

$$\lim_{x \to 0} J_x A = \frac{S''}{2} A$$

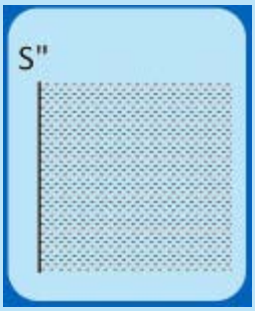
$$\Rightarrow J_x = \frac{S''}{2}$$



- This concept can also be extended to a point source and we shall discuss it later.

Solution for the Infinite Planar Source

- Let us now attempt to solve for the neutron flux distribution for the infinite planar source problem.
- Exploiting the symmetry we shall only solve for the positive half of the coordinate system with the origin at the source.
- Note that the source is only a boundary condition and no source is present inside the media.
- The diffusion equation for steady source free media can be written as



$$\left(\frac{1}{v}\frac{\partial \phi}{\partial t} = D\nabla^2\phi\right) - \Sigma_a\phi + S'''$$

$$\Rightarrow D\frac{d^2\phi}{dx^2} - \Sigma_a\phi = 0$$

- Dividing by D we can write,

$$\Rightarrow \frac{d^2\phi}{dx^2} - \frac{\Sigma_a\phi}{D} = 0$$

- Defining D/Σ_a as L^2, we can write,

$$\Rightarrow \frac{d^2\phi}{dx^2} - \frac{\phi}{L^2} = 0$$

- L in the above equation is called the **diffusion length** and its physical interpretation will be visible shortly.

- The solution for the equation in the last slide is

$$\phi = A_1 e^{-x/L} + A_2 e^{x/L}$$

- This can easily verified by substituting the solution in the differential equation.
- A_1 and A_2 are obtained by satisfying the boundary conditions.
- The applicable boundary conditions are

$$J_x\big|_{x\to 0} = S'''/2 = -D\left.\frac{d\phi}{dx}\right|_{x\to 0}$$

$$\phi\big|_{x\to\infty} = 0$$

The second boundary condition makes $A_2 = 0$.

- Now to satisfy the first boundary condition, we get,

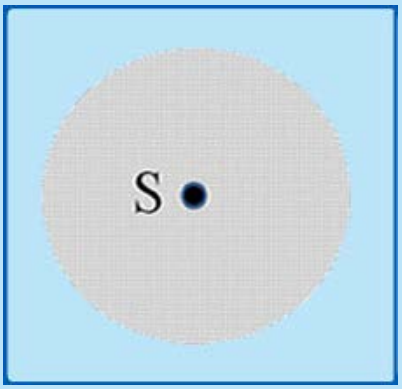
$$-D\frac{-A_1}{L}e^{-x/L}\Big|_{x\to 0} = S''/2$$

$$\Rightarrow A_1 = \frac{S''L}{2D} \qquad \Rightarrow \phi = \frac{S''L}{2D}e^{-x/L}$$

- Flux decays exponentially.

Point Source in an Infinite Media

- Consider a point source in an infinite media as shown with source emitting S neutrons per second.
- We shall assume that the source is isotropic and the media is uniform.
- This would imply that the problem is spherically symmetric.
- To arrive at the boundary condition on the source we can take a small sphere around the source and write



$$J_r 4\pi r^2\Big|_{r\to 0} = S$$

- As done previously, if we remove the source from the media and assuming steady state, we can write

$$D\nabla^2\phi - \Sigma_a\phi = 0$$

$$D\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) - \Sigma_a\phi = 0$$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) - \frac{\phi}{L^2} = 0$$

- Defining a new function ϕ & such that $\phi = \psi / r$ and substituting for ϕ in the above equation we get

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d(\psi/r)}{dr}\right) - \frac{\psi}{rL^2} = 0$$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2\left(\frac{-\psi}{r^2} + \frac{1}{r}\frac{d\psi}{dr}\right)\right) - \frac{\psi}{rL^2} = 0$$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(-\psi + r\frac{d\psi}{dr}\right) - \frac{\psi}{rL^2} = 0$$

$$\Rightarrow \frac{1}{r^2}\left(-\frac{d\psi}{dr} + r\frac{d^2\psi}{dr^2} + \frac{d\psi}{dr}\right) - \frac{\psi}{rL^2} = 0$$

$$\Rightarrow \frac{1}{r}\frac{d^2\psi}{dr^2} - \frac{\psi}{rL^2} = 0$$

$$\Rightarrow \frac{d^2\psi}{dr^2} - \frac{\psi}{L^2} = 0$$

- For r > 0 it has the same form as for the Cartesian system.

- The solution for the equation in the last slide is

$$\psi = A_1 e^{r/L} + A_2 e^{r/L} \qquad \Rightarrow \phi r = A_1 e^{-r/L} + A_2 e^{r/L}$$

$$\Rightarrow \phi = \frac{A_1 e^{-r/L} + A_2 e^{r/L}}{r}$$

- The applicable boundary conditions are

$$4\pi r^2 J_r\Big|_{r\to 0} = 4\pi r^2 \left(-D\frac{d\phi}{dr}\Big|_{r\to 0}\right) = S$$

$$\phi\Big|_{r\to} = 0$$

- The second boundary condition makes $A_2 = 0$
- Now to satisfy the first boundary condition, we get,

$$\left(4\pi r^2 \left(-D\frac{d}{dr}\left(\frac{A_1}{r}e^{-r/L}\right)\right)\right)\Big|_{r\to 0} = S$$

$$\left(4\pi r^2 \left(-D\left(\frac{-A_1}{rL}e^{-r/L}\frac{-A_1}{r^2}e^{-r/L}\right)\right)\right)\Big|_{r\to 0} = S$$

$$\left(4\pi\left(A_1 D\left(\frac{r}{L}e^{r/L} + e^{-r/L}\right)\right)\right)\Big|_{r\to 0} = S$$

- The first term in the inner most bracket goes to 0 and the second term turns to 1 as r tends to 0.
- Hence we can write,

$$A_1 = \frac{S}{4\pi D}$$

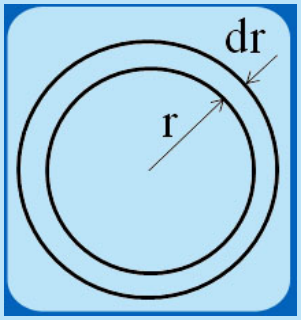
$$\Rightarrow \phi = \frac{S}{4\pi D}\frac{e^{-r/L}}{r}$$

Physical interpretation of L

- Often in more complex calculations one computes the mean square distance travelled before absorption.
- To compare the same with diffusion theory, we can evaluate the same as follows.
- Consider a shell of radius r and thickness dr. The number of neutrons absorbed in the shell can be written as

$$4\pi r^2 dr \Sigma_a \left(\frac{S}{4\pi D}\frac{e^{-r/L}}{r}\right)$$

$$= \frac{S\Sigma_a}{D} r e^{-r/L} dr$$



- The neutrons that were absorbed in the shell have travelled a distance r.
- The mean square of the distance travelled by the neutrons can then be written as

$$\overline{R^2} = \frac{\int_0^\infty \frac{S\Sigma_a}{D} r^3 e^{-r/L} dr}{S}$$

$$\overline{R^2} = \frac{\Sigma_a}{D}\int_0^\infty r^3 e^{-r/L} dr$$

- The above integral is straight forward and this can be shown to be $6L^4$.
- Thus we can write

$$\overline{R^2} = \frac{\Sigma_a}{D} 6L^4 = 6L^2$$

- We can rewrite the above equation as

$$L = \sqrt{\frac{\overline{R^2}}{6}} = 0.41\sqrt{\overline{R^2}} \approx \sqrt{\overline{R^2}}$$

- Thus one can conclude that L is typically the distance travelled by the neutron before it is absorbed.

Objectives

In this lecture you will learn the following

- To illustrate the application of the elastic scattering process and elementary diffusion theory.
- First we shall illustrate several examples on scattering that conveys some physics.
- Then we illustrate application of connection between flux and current density.
- Finally, we shall look at application of superposition for a multiple source problem.

Question-1

1. A 2-MeV neutron travelling in water has a head-on collision with an O^{16} nucleus. (a) What are the energies of the neutron and nucleus after the collision? (b) Would you expect the water molecule involved in the collision to remain intact after the event?

Solution

For head-on collision, the final energy of the neutron after collision is given by αE_0,

where E_0 is the original energy of the neutron and $\alpha = \left(\frac{A-1}{A+1}\right)^2$

For O nucleus, A = 16. Hence $\alpha = \left(\frac{16-1}{16+1}\right)^2 = 0.7785$

Therefore final energy = 0.7785 x 2 = 1.557 MeV.

Hence energy of O nucleus = 2.0-1.557 = 0.443 MeV. Since this shall be much higher than the bond energy, O nucleus will detach and a free radical will be generated.

Question-2

A 1-MeV neutron strikes a C^{12} nucleus that is initially at rest. If the neutron is elastically scattered through an angle of 90^o, (a) what is the energy of the scattered neutron? (b) what is the energy of the recoiling nucleus? (c) at what angle does the recoiling nucleus appear?

Solution

- We had shown that the ratio of the square root of the energy of a neutron after and before collision can be written as

$$\sqrt{\frac{E'}{E}} = \frac{\cos\theta + \sqrt{\cos^2\theta + (A^2 - 1)}}{(A+1)}$$

$$E' = E\left(\frac{\cos\theta + \sqrt{\cos^2\theta + (A^2 - 1)}}{(A+1)}\right)^2$$

- For $\theta = 90^o$, and A = 12, we can write

$$E' = E\left(\frac{0 + \sqrt{0 + (A^2 - 1)}}{(A+1)}\right)^2$$

$$\Rightarrow E' = E\frac{(A^2 - 1)}{(A+1)^2}$$

$$= E\frac{(12^2 - 1)}{(12+1)^2}$$

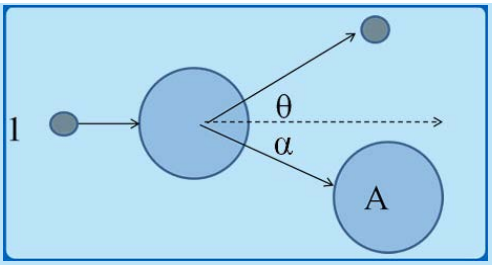
$$= E\frac{143}{169}$$

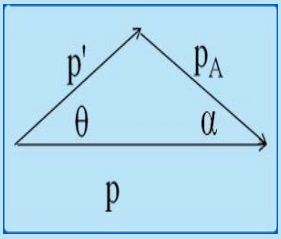
- For E = 1 MeV, we get

$$\Rightarrow E' = \frac{143}{169} = 0.846 \ MeV$$

This implies that energy of C^{12} is = 1 – 0.846 = 0.154 MeV.

The following figures show the collision process and momentum balance





It may be understood from the above figure that

$$\Rightarrow \quad p = p_A \cos\alpha + p' \cos\theta$$

$$\Rightarrow \quad \sqrt{2E} = \sqrt{2AE_A}\cos\alpha + \sqrt{2E'}\cos\theta$$

$$\Rightarrow \quad \sqrt{2} = \sqrt{2*12*0.154}\cos\alpha + \sqrt{2*0.846}*0$$

$$\Rightarrow \cos\alpha = \sqrt{\frac{2}{2*12*0.154}}$$

$$= 0.7356$$

$$\Rightarrow \quad \alpha = 42.6^{o}$$

Question-3

Show that the average fractional energy loss expressed during elastic scattering for large A nucleus is given approximately by,

$$\frac{\overline{\Delta E}}{E} \cong \frac{2}{A}$$

We had shown that the average energy of a neutron after collision as a fraction of initial energy is

$$\frac{\overline{E'}}{E} = (1+\alpha)/2$$

$$\Rightarrow \frac{E-\overline{E'}}{E} = 1-\frac{\overline{E'}}{E} = \left(1-(1+\alpha)/2\right) = (1-\alpha)/2$$

$$\Rightarrow \frac{\overline{\Delta E'}}{E} = \frac{(1-\alpha)}{2} = \frac{\left(1-\left(\frac{A-1}{A+1}\right)^2\right)}{2}$$

$$= \frac{\left((A+1)^2-(A-1)^2\right)}{2(A+1)^2} = \frac{4A}{2(A+1)^2} \approx \frac{4A}{2A^2} = \frac{2}{A}$$

Question 4

Calculate the average number of collisions required to reduce the energy of a neutron from 2 MeV to 0.025 eV in H1 and C^{12} .

Solution

For a neutron originally having energy E after a collision will on the average have energy

$$E_1 = E\frac{(1+\alpha)}{2}$$

After two collisions, the energy shall be

$$E_2 = E\left(\frac{(1+\alpha)}{2}\right)^2$$

Thus, after n collisions, the energy shall be

$$E_n = E\left(\frac{(1+\alpha)}{2}\right)^n$$

For Hydrogen nucleus $\alpha = \left(\frac{1-1}{1+1}\right)^2 = 0$

In this problem, E = 2 MeV, E_n = 0.025 eV

$$\Rightarrow 0.025 = 2x10^6\left(\frac{(1+0)}{2}\right)^n$$

$$\Rightarrow \frac{0.025}{2x10^6} = \left(\frac{1}{2}\right)^n$$

$$\Rightarrow 1.25*10^{-8} = (0.5)^n$$

$$\Rightarrow n = \frac{\ln\left(1.25*10^{-8}\right)}{\ln(0.5)} = 26.2$$

For Carbon nucleus $\alpha = \left(\frac{12-1}{12+1}\right)^2 = 0.716$

$$\Rightarrow 0.025 = 2x10^6\left(\frac{(1+0.716)}{2}\right)^n$$

$$\Rightarrow \frac{0.025}{2x10^{6}} = (0.858)^{n}$$

$$\Rightarrow n = \frac{\ln\left(1.25*10^{-8}\right)}{\ln(0.858)} = 118.8$$

Question-5

The neutron flux in a bare spherical reactor of radius 50 cm is given by $5 \times 10^{13}(\sin(0.0628r)/r)$ neutron/cm^2-s, where 'r' is measured in cm from the centre of the reactor. The diffusion coefficient for the system is 0.80 cm. (a) What is the maximum value of the flux in the reactor? (b) Calculate the neutron current density as a function of position in the reactor. (c) How many neutrons escape from the reactor per second?

Solution

a) The flux is given by

$$\phi = 5*10^{13}\frac{Sin(0.0628r)}{r}$$

$$= 5*10^{13}*0.0628\frac{Sin(0.0628r)}{0.0628*r}$$

- The maximum value of above function shall be at r = 0

$$\Rightarrow \phi_{max} = 5*10^{13}*0.0628 = 3.14*10^{12} \text{ n/cm}^2\text{ - s}$$

b) Current density $\Rightarrow J = -D\frac{d\phi}{dr}$

$$J = -D*5*10^{13}\left(\frac{0.0628Cos(0.0628r)}{r} - \frac{Sin(0.0628r)}{r^2}\right)$$

c) Current density at r = 50 cm is given by

$$J = -0.8*5*10^{13}\left(\frac{0.0628Cos(0.0628*50)}{50} - \frac{Sin(0.0628*50)}{50^2}\right)$$

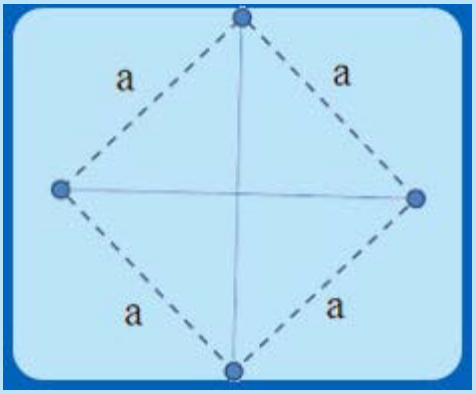
$$= -0.8*5*10^{13}\left(\frac{0.0628}{50} - 0\right) = 5.024*10^{10} \text{ n/cm}^2\text{ - s}$$

Number of neutrons leaking would be given by

$$\Rightarrow 4\pi r^2 J = 4\pi*50^2*5.024*10^{10} = 1.578*10^{15} s^{-1}$$

Question-6

Isotropic point sources each emitting S neutrons/sec are placed in infinite moderator at the four corners of a square of side 'a'. Compute the flux and current at its centre.



Solution

- Distance from centre to any source is $\frac{a}{\sqrt{2}}$
- We have shown that the flux from a point source at a distance r is given by

$$\Rightarrow \phi = \frac{S}{4\pi D}\frac{e^{-r/L}}{r}$$

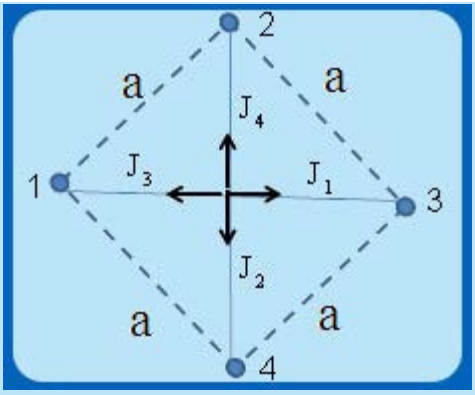
- Thus the flux from a single source shall be

$$\Rightarrow \phi = \frac{S}{4\pi D}\frac{e^{-a/\sqrt{2}L}}{a/\sqrt{2}}$$

- Flux from all the four sources shall be

$$\Rightarrow \phi = \frac{4S}{4\pi D}\frac{\sqrt{2}e^{-a/\sqrt{2}L}}{a} = \frac{\sqrt{2}S}{\pi D}\frac{e^{-a/\sqrt{2}L}}{a}$$

- The current from each source shall be radially outward as shown below



- The vector addition of the current shall be 0. Hence current density in any direction is 0.

Objectives

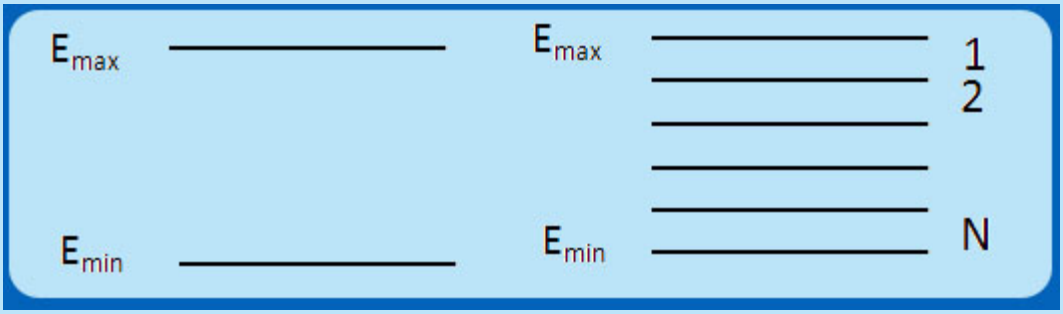
In this lecture you will learn the following

- We shall understand diffusion in systems that have neutrons with varying energies.
- We shall look at a sample application and digest the method of solution.

Introduction

- Our treatment developed in the previous lecture assumes that neutrons are of single energy.
- This is an idealisation as fission neutrons are born with varying energies and they collide with moderator and slow down by the scattering process.
- Thus, as the energy of the neutron changes, the reaction cross section changes and hence constant coefficients for diffusion, absorption, etc. are not valid.
- To solve this problem realistically, neutrons are grouped into energy bands and solution obtained for fluxes in each group.

- Consider a system that has neutrons with energies from E_{min} to E_{max} as shown on the left in the figure.



- The simplest is to lump all the neutrons into one group and calculate the average coefficients for the group.
- This will obviously be very approximate.
- The accuracy can be improved by increasing the number of groups such as the one shown in the figure on the right where N divisions are shown.

- Several questions arise:
  - How many groups should there be?
  - How to get the average cross sections and diffusion coefficients?
- Simple calculations can be done using two groups. However practical reactor calculations would need several tens of groups.
- Fast reactor calculations may involve very large number of groups for accurate predictions.
- We shall limit our discussions to a maximum of two groups in this introductory course.

Definitions

- The following definitions are employed in multi-group analysis.
- The flux for a group is defined as

$$\phi_g = \int_{E_{min}}^{E_{max}} \phi(E)dE = \int_{E_{min}}^{E_{max}} n(E)v(E)dE$$

- n(E) represents the neutron density which have energy between E and E+dE, v(E) is the speed corresponding to energy E. E_{min} and E_{max} represents the minimum and maximum energy for the group 'g'.
- The definitions for average cross sections and diffusion coefficients are given as

$$\Sigma_g = \frac{\int_{E_{min}}^{E_{max}} \Sigma(E)\phi(E)dE}{\int_{E_{min}}^{E_{max}} \phi(E)dE} \qquad D_g = \frac{\int_{E_{min}}^{E_{max}} D(E)\phi(E)dE}{\int_{E_{min}}^{E_{max}} \phi(E)dE}$$

- It may be noted that the coefficients can be determined only if the energy dependence of group flux is known and this has to be postulated.
- As complex concepts will be involved to compute the coefficients for various groups, we will assume in this course as if they are available in the form of a table.

Multi-Group Diffusion Equation

- Now we are ready to lay out the governing equations for multi-group treatment.
- The general equation is

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} = \mathbf{D}_g \nabla^2 \phi_g \; -\Sigma_{ag}\phi_g + S_g''' + \sum_{\substack{i=1 \\ i \# g}}^{N} \Sigma_{s(i\to g)}\phi_i - \sum_{\substack{i=1 \\ i \# g}}^{N} \Sigma_{s(g\to i)}\phi_g$$

- It may be noticed that the equation is very similar to the one speed equation but has two extra terms.
- The last two terms are the transfer terms from one group to another.
- The fourth term on the right hand side represents source of neutrons from other groups through scattering process.
- The scattering cross section for neutrons from any group i to the present group is defined by $\Sigma_{i\to g}$.
- Similarly, the fifth term represents the sink of neutrons from the present group g to other group i.
- Generally, it is assumed that upscatter (neutrons moving from a lower energy group to a higher energy group) is absent.

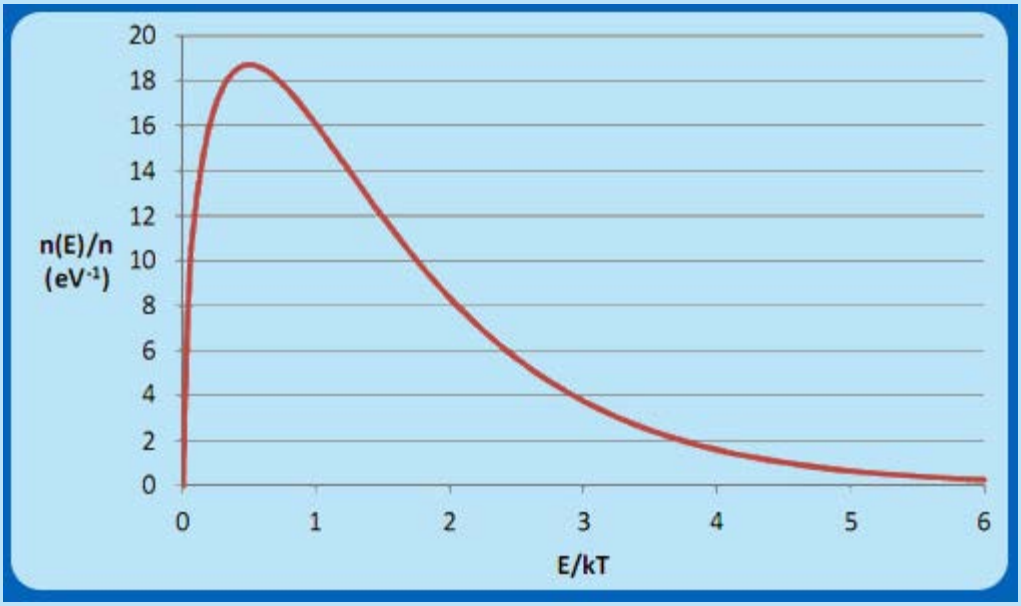
Two Group System

- The simplest treatment is the two group system.
- Usually, the two groups are called thermal and fast.
- The choice of grouping of energy is as follows.
- When neutrons come to equilibrium with moderator at a given temperature, the energy distribution of such neutrons is given by the Maxwellian distribution.

$$n(E) = \frac{2\pi n}{(\pi kT)^{1.5}} E^{0.5} e^{\frac{-E}{kT}}$$

- The above spectra is shown in the figure below.
- The above distribution has a maximum at 0.5 kT and average at 1.5 kT. The population above 5kT is almost zero. Hence thermal group is chosen to have energy from 0 to 5kT.

Maxwellian



Two Group System

- The fast group consists of all the neutrons above 5kT.
- The fast group is denoted as group 1 and the thermal group as 2.
- The two group equation set is written as

$$\frac{1}{v_1}\frac{\partial \phi_1}{\partial t} = D_1 \nabla^2 \phi_1 - \Sigma_{a1}\phi_1 + S_1''' - \Sigma_{12}\phi_1$$
$$\frac{1}{v_2}\frac{\partial \phi_2}{\partial t} = D_2 \nabla^2 \phi_2 - \Sigma_{a2}\phi_2 + S_2''' + \Sigma_{12}\phi_1$$

- It may be noted that there is only one transfer term from 1-2 and no transfer occurs from 2-1.
- Considering only steady state, we can write,

$$D_1 \nabla^2 \phi_1 - \Sigma_{a1}\phi_1 + S_1''' - \Sigma_{12}\phi_1 = 0$$
$$D_2 \nabla^2 \phi_2 - \Sigma_{a2}\phi_2 + S_2''' + \Sigma_{12}\phi_1 = 0$$

- Usually the absorption in group 1 is negligible and hence can be written as

$$D_1 \nabla^2 \phi_1 - \Sigma_{12}\phi_1 + S_1''' = 0$$

- For a source free system, the equation is very similar to what we had already solved for one speed system.

$$\nabla^2 \phi_1 - \frac{\Sigma_{12}}{D_1}\phi_1 = 0$$

- For the fast group, the term $\frac{\Sigma_{12}}{D_1}$ is denoted as τ. Thus,

$$\nabla^2 \phi_1 - \frac{\phi_1}{\tau} = 0$$

- Thus, the solution will be similar to the one speed system with L replaced by $\sqrt{\tau}$.
- For group 2, the equation shall be written as

$$D_2 \nabla^2 \phi_2 - \Sigma_{a2}\phi_2 = -\Sigma_{12}\phi_1$$
$$\Rightarrow \nabla^2 \phi_2 - \frac{\Sigma_{a2}}{D_2}\phi_2 = -\frac{\Sigma_{12}}{D_2}\phi_1$$

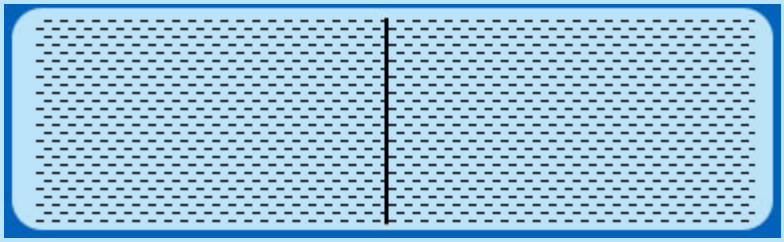
- For the thermal group, the term $\frac{\Sigma_{a2}}{D_2}$ denoted as L^2. Thus,

$$\nabla^2\phi_2 - \frac{\phi_2}{L^2} = -\frac{\Sigma_{12}}{D_2}\phi_1$$

- Note that ϕ_1 has already been solved and thus only ϕ_2 needs to be solved.
- The procedure shall be as follows:
  - Turn the RHS = 0 and get the homogenous solution.
  - Get the particular integral.
  - Substitute the boundary conditions and evaluate the constants.

Example

- The solution procedure is illustrated by the following example.



- Let us solve for the infinite plane source problem. We shall assume that all neutrons are born fast.
- The solution for fast flux is. Note that this is same as one speed solution with $\sqrt{\tau}$ replacing L.

$$\Rightarrow \phi_1 = \frac{S''\sqrt{\tau}}{2D_1} e^{-x/\sqrt{\tau}}$$

- For Group 2, the equation is

$$\nabla^2 \phi_2 - \frac{\phi_2}{L^2} = -\frac{\Sigma_{12}}{D_2} \frac{S''\sqrt{\tau}}{2D_1} e^{-x/\sqrt{\tau}}$$

- For Group 2, the homogeneous solution is

$$\phi_{2H} = A_1 e^{-x/L} + A_2 e^{x/L}$$

- The particular integral is

$$PI = -\frac{\frac{\Sigma_{12}}{D_2} \frac{S''\sqrt{\tau}}{2D_1} e^{-x/\sqrt{\tau}}}{\frac{1}{\sqrt{\tau}} - \frac{1}{L^2}}$$

- The validity of the above expression can be checked by substitution in the governing equation.
- The full solution is

$$\phi_2 = A_1 e^{-x/L} + A_2 e^{x/L} - \frac{\frac{\Sigma_{12}}{D_2} \frac{S''\sqrt{\tau}}{2D_1} e^{-x/\sqrt{\tau}}}{\frac{1}{\sqrt{\tau}} - \frac{1}{L^2}}$$

- The applicable boundary conditions are

$$\left.\frac{d\phi_2}{dx}\right|_{x\to 0} = 0, \quad \left.\phi_2\right|_{x\to\infty} = 0$$

- The final solution is

$$\phi_2 = \frac{S''' L^2}{2D_2(L^2 - \tau)}\left(Le^{-x/L} - \sqrt{\tau}e^{-x/\sqrt{\tau}}\right)$$

Objectives

In this lecture you will learn the following

- In the last few lectures we have understood how to solve diffusion equation and have solved a few cases with sources acting as boundary conditions.
- Reactors have sources present in them.
- Thus we look at problems where multiplying sources are present.

Assumptions

- We make the following assumptions in our treatment
  - First we shall assume that the material in the reactor is homogeneous and of uniform properties.
  - The reactor is in steady state.
  - To begin with we shall assume that all neutrons are of one speed.
- In one of our previous lectures, we had introduced the term infinite multiplication constant, k_∞.
- It represents the number of second generation neutrons born from the absorption of a single neutron in the reactor.
- We shall now see how to utilise it in reactor calculations.

Reactor Theory

- The steady diffusion equation can be written

$$D\nabla^2\phi - \Sigma_a\phi + s''' = 0$$

- Now the source term can be written as

$$s''' = k_\infty \Sigma_a \phi$$

- Thus the diffusion equation becomes

$$D\nabla^2\phi - \Sigma_a\phi + k_\infty\Sigma_a\phi = 0$$

$$\Rightarrow D\nabla^2\phi + \Sigma_a\phi(k_\infty - 1) = 0 \qquad \Rightarrow \nabla^2\phi + \frac{\Sigma_a\phi}{D}(k_\infty - 1) = 0$$

- By introducing the definition of L^2 we can write

$$\Rightarrow \nabla^2\phi + \frac{(k_\infty - 1)}{L^2}\phi = 0$$

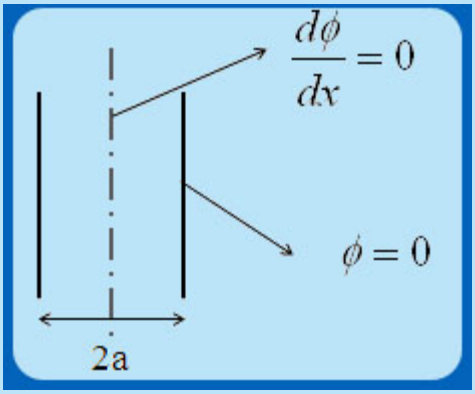
- The above equation is rewritten as

$$\Rightarrow \nabla^2\phi + B_m{}^2\phi = 0 \quad \text{where} \quad B_m{}^2 = \frac{(k_\infty - 1)}{L^2}$$

- The term B_m would represent the curvature of the flux and hence is given the name buckling. Further a subscript m is written to represent that it is determined by the material of the system. Hence B_m is called Material Buckling.

Slab Reactor

- Let us now consider an infinite slab reactor of thickness 2a as shown



- From previous page, we can write the governing equation as

$$\nabla^2\phi + B^2\phi = 0$$

- Note that we have replaced B_m with B.
- For the Cartesian case, the simplified equation shall be

$$\Rightarrow \frac{d^2\phi}{dx^2} + B^2\phi = 0$$

- The solution for the above equation is

$$\phi = A_1 \cos(Bx) + A_2 \sin(Bx)$$

- First we shall apply the symmetry boundary condition.

$$\frac{d\phi}{dx} = A_1(-B\sin(Bx)) + A_2(B\cos(Bx)) = 0 \text{ at } x=0$$

$$\Rightarrow A_2 = 0$$

- Thus the solution reduces to

$$\phi = A_1 \cos(Bx)$$

- Now applying the vanishing flux boundary condition, we get

$$0 = A_1 \cos(Ba)$$

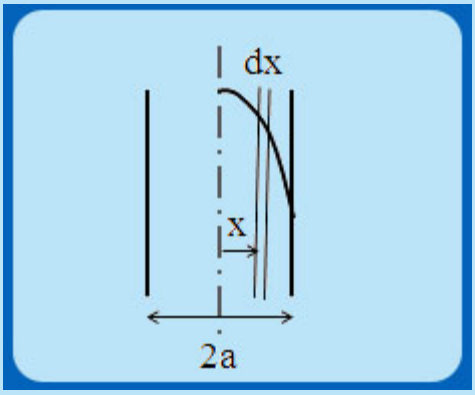
- Thus $A_1 = 0$, which leads to the trivial solution.
- For non-trivial solution,

$$0 = \cos(Ba) \quad \text{or} \quad Ba = (2n+1)\frac{\pi}{2} \qquad n = 0, 1, 2, \ldots$$

- The values of Ba that satisfy the equation are called the Eigen Values of the system.
- The value corresponding to n = 0 is called the fundamental solution. (Ba = $\pi/2$ or B = $\pi/2a$).
- The solution for other values are called harmonics.
- If one solves the transient equation, it can be shown that all the higher harmonics die with time and only fundamental solution survives.
- Hence at steady state $B = \frac{\pi a}{2}$.
- Thus, the steady state solution is

$$\phi = A_1 \cos\left(\frac{\pi}{2a} x\right)$$

- Now we shall see how to determine A_1.
- Consider a reactor that is producing power per unit area perpendicular to the paper equal to P".



- Consider a strip of thickness dx at a distance x from origin as shown.
- The volume of the strip is Adx, where A is the surface area of the reactor.
- Now we shall evaluate A_1 as follows:
- No. of fissions per second = $\sum_f \phi A dx$
- Energy released per second = $\sum_f \phi A dx E_f$
- Total Power, P = $2\int_0^a \sum_f \phi A dx E_f$

$$\Rightarrow P = 2\Sigma_f AE_f \int_0^a A_1 \cos\left(\frac{\pi x}{2a}\right) dx$$

$$\Rightarrow \frac{P}{A} = P'' = 2\Sigma_f E_f A_1 \frac{2a}{\pi}\left[\sin\left(\frac{\pi x}{2a}\right)\right]_0^a$$

$$\Rightarrow P'' = 2\Sigma_f E_f A_1 \frac{2a}{\pi}[1-0] \qquad \Rightarrow A_1 = \frac{P''\pi}{4a\Sigma_f E_f}$$

- Thus, the flux distribution is given by

$$\Rightarrow \phi_1 = \frac{P''\pi}{4a\Sigma_f E_f}\cos\left(\frac{\pi x}{2a}\right)$$

- Power density, defined as power per unit volume in the reactor would be directly proportional to flux.
- Thus it is maximum at the centre and 0 at the surface.
- If the same amount of coolant is passed at the cross section of the reactor, the coolant will get overheated in the centre and will not get heated in the boundary.
- Hence the exit coolant temperature would be unequal. This will lead to a lower mixed mean temperature at the exit and will lead to lower thermodynamic efficiency.

- To get higher exit temperature we need to match the profile of flux and coolant flow in the cross section.
- Exact matching will not be possible, but this is approximately achieved by orificing (putting suitable resistances).
- In practical systems, The flux shape is also modified to keep it as flat as is possible. This is called flux flattening. The degree of flatness is judged by a factor called Peaking Factor that is defined as

$$\Rightarrow Peaking\ \ Factor = \frac{Peak\ \ flux}{Average\ \ flux} = \frac{\phi_{max}}{\bar{\phi}}$$

- We shall define the average flux such that the power is conserved

$$\Rightarrow \Sigma_f \bar{\phi} A a E_f = \int_0^a \Sigma_f \phi A dx E_f$$

$$\Rightarrow \bar{\phi} = \frac{1}{a}\int_0^a \phi dx$$

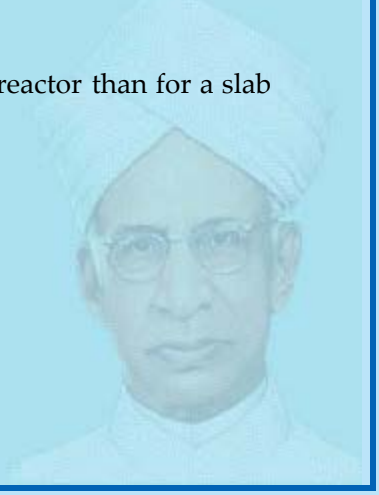
$$\Rightarrow \bar{\phi} = \frac{1}{a}\int_0^a A_1 \cos\frac{\pi x}{2a}\,dx \qquad \Rightarrow \bar{\phi} = \frac{2A_1}{\pi}$$

$$\phi_{max} = A_1 \qquad \Rightarrow \frac{\phi_{max}}{\bar{\phi}} = \frac{\pi}{2}$$

Objectives

In this lecture you will learn the following

- In this lecture we shall look at cylindrical reactor.
- As the solution turns out to be a Bessel function, we shall briefly look at the nature of these functions.
- Finally we will show that the flux is more peaked for a cylindrical reactor than for a slab reactor.



Reactor Theory

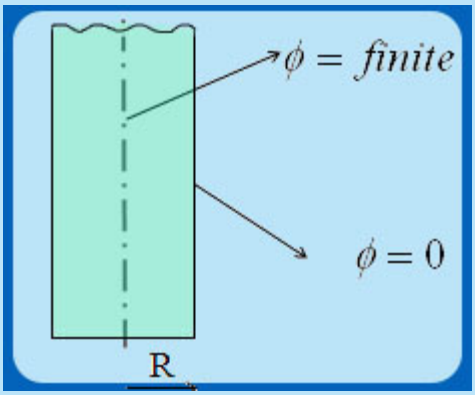
- Now we shall focus our attention towards cylindrical and spherical reactors.
- The governing equation shall continue to be

$$\Rightarrow \nabla^2\phi + B_m^{\ 2}\phi = 0 \qquad \text{where,} \quad B_m^{\ 2} = \frac{(k_\infty - 1)}{L^2}$$

- The term for ^{2} would be different for cylinder and sphere.

Cylindrical Reactor

- Let us now consider an infinite cylindrical reactor as shown



- From last slide, we can write the governing equation as

$$^2\phi + B^2\phi = 0.$$

- Note that we have replaced B_m with B.
- For the Cylindrical case, the simplified equation shall be

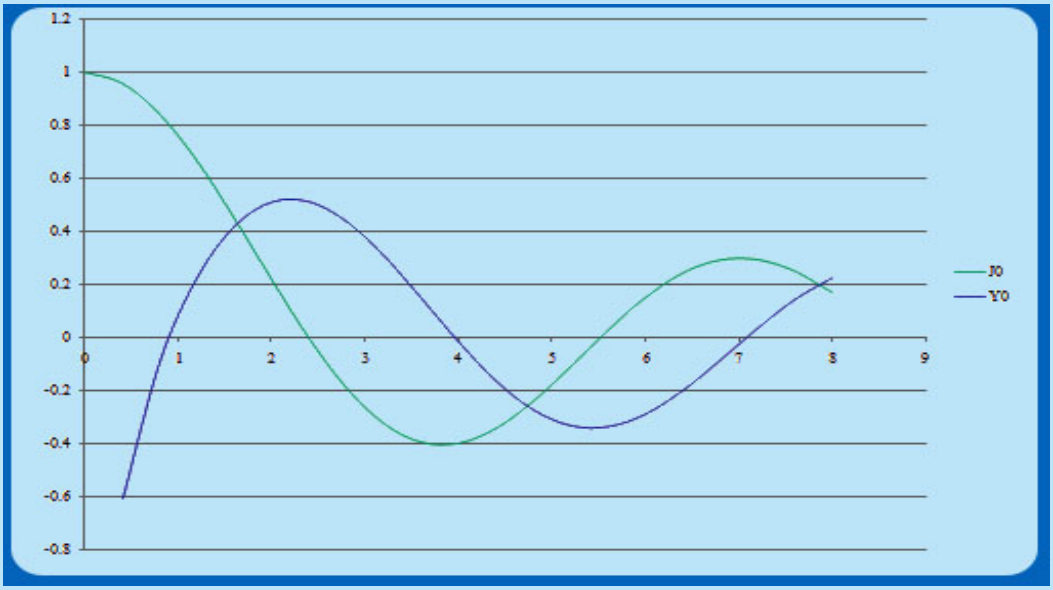
$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) + B^2\phi = 0$$

- The solution for the above equation is Bessel's Equation of order zero

$$\phi = A_1 J_0(Br) + A_2 Y_0(Br)$$

- For those who are not familiar with the Bessel Functions, let us digress a bit and understand them.
- Notice that when a cylinder is large, the curvature effects are small and hence the solution of cylinder must look similar to the Cartesian case. Thus the functions J_0 and Y_0 should be similar to Sine and Cosine.

Bessel Functions



Properties of Bessel Function

- Note that both J_0 and Y_0 are oscillating functions with decaying amplitude.
- The reason for the decay in amplitude can be attributed to the divergence of the coordinate in r direction.
- Similar to Sine and Cosine, they have multiple roots.
- The first root of J_0 is 2.405.
- There are some properties of J_0 and Y_0 which we shall describe below

- $Y_0(0) = -\infty$
- $J_0(0) = 1$
- First root of J_0 is 2.405, That is $J_0(2.405) = 0$.
- Another property of J_0 is that

$$\int J_0(r) r dr = J_1(r) r$$

Cylindrical Reactor

- The boundary conditions are:

$$\phi|_{r\to 0} = finite, \quad \phi|_{r=R} = 0$$

- As Y_0 blows up at r = 0, hence $A_2 = 0$
- Thus the solution reduces to

$$\phi = A_1 J_0 (Br)$$

- Now applying the vanishing flux boundary condition, we get

$$0 = A_1 J_0 (BR)$$

- Thus $A_1 = 0$, which leads to the trivial solution.
- For non-trivial fundamental solution,

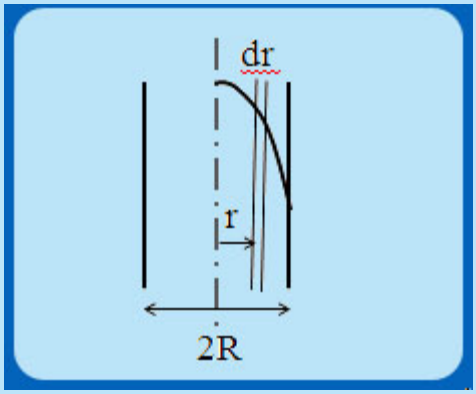
$$0 = J_0(BR) \text{ or } BR = 2.405$$

- Thus the fundamental solution is

$$\phi = A_1 J_0\left(\frac{2.405}{R} r\right)$$

- A_1 shall be similarly derived as done previously by using power.

- Consider a reactor that is producing power per unit length as P'.



- Consider a strip of thickness dr at a distance r from origin as shown.
- The volume of the strip is 2πrdrL, where L is the length of the reactor.
- Now we shall evaluate A_1 as follows:

- No. of fissions per second = $\sum_f \phi 2\pi r dr L$

- Energy released per second = $\sum_f \phi 2\pi r dr L E_f$

- Total Power, P = $\int_0^R \sum_f \phi 2\pi r dr L E_f$

$$\Rightarrow P = 2\pi \sum_f L E_f \int_0^a A_1 J_0\left(\frac{2.405}{R} r\right) r dr$$

$$\Rightarrow \frac{P}{L} = P' = 2\pi \sum_f E_f \frac{R}{2.405} A_1 \left[J_1\left(\frac{2.405 r}{R}\right) r \right]_0^R$$

$$\Rightarrow P' = 2\pi \sum_f E_f \frac{R}{2.405} A_1 [J_1(2.405) R - 0]$$

From tables $J_1(2.405) = 0.518$.

$$\Rightarrow A_1 = \frac{P'}{1.35 \sum_f E_f R^2}$$

- Thus, the flux distribution is given by

$$\Rightarrow \phi_1 = \frac{P'}{1.35\Sigma_f E_f R^2} J_0\left(\frac{2.405r}{R}\right)$$

- As seen previously the power density will be maximum at the centre.
- Hence orificing has to be resorted to.

- We shall define the average flux such that power is conserved.

$$\Rightarrow \Sigma_f \bar{\phi} \pi R^2 L E_f = P$$

$$\Rightarrow \bar{\phi} = \frac{P'}{\pi R^2 \Sigma_f E_f}$$

- Since peak flux = A_1 $\quad \Rightarrow \phi_{max} = \frac{P'}{1.35\Sigma_f E_f R^2}$

$$\Rightarrow \frac{\phi_{max}}{\bar{\phi}} = \frac{\pi}{1.35} = 2.327$$ More peaked than for slab reactor.

Objectives

In this lecture you will learn the following

- We shall first look at the spherical reactor.
- We will then look at rectangular three dimensional reactor.
- We shall show that its governing equation reduces to that of infinite slab reactor in each direction.

Introduction

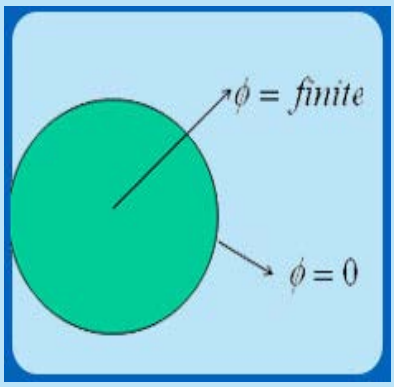
- Now we shall focus our attention on finite systems such as a spherical, rectangular and cylindrical reactors.
- The treatment of spherical reactors is straight forward and similar to what we have already done for a slab reactor.
- The governing equation shall continue to be

$$\Rightarrow \nabla^2\phi + B_m^{\,2}\phi = 0 \quad \text{where,} \quad B_m^{\,2} = \frac{(k_\infty - 1)}{L^2}$$

For a spherically symmetric case $\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right)$

Spherical Reactor

- Let us now consider an infinite spherical reactor as shown



- As done previously, we can write the governing equation as

$$\Rightarrow \nabla^2\phi + B^2\phi = 0$$

Note that we have replaced B_m with B.

- For the spherically symmetric case, the simplified equation shall be

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right) + B^2\phi = 0$$

- As done previously in our lecture on flux distribution from a point source, we can introduce the transformation $\phi = \psi/r$.

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d(\psi/r)}{dr}\right) + B^2\frac{\psi}{r} = 0$$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left[r^2\left(\frac{-\psi}{r^2} + \frac{1}{r}\frac{d\psi}{dr}\right)\right] + B^2\frac{\psi}{r} = 0$$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(-\psi + r\frac{d\psi}{dr}\right) + B^2\frac{\psi}{r} = 0$$

$$\Rightarrow \frac{1}{r^2}\left(-\frac{d\psi}{dr} + r\frac{d^2\psi}{dr^2} + \frac{d\psi}{dr}\right) + B^2\frac{\psi}{r} = 0$$

$$\Rightarrow \frac{1}{r}\frac{d^2\psi}{dr^2} + B^2\frac{\psi}{r} = 0$$

$$\Rightarrow \frac{d^2\psi}{dr^2} - B^2\psi = 0$$

For r > 0 it has the same form as for the Cartesian system.

- The solution for the above equation is

$$\psi = A_1 \cos(Br) + A_2 \sin(Br)$$

- Now replacing $\psi = \phi r$, we get

$$\phi = A_1 \frac{\cos(Br)}{r} + \frac{A_2 \sin(Br)}{r}$$

- Using the boundary condition

$$\phi\big|_{r \to 0} = finite$$

- The above implies that $A_1 = 0$.

- Thus the solution reduces to

$$\phi = A_2 \frac{Sin(Br)}{r}$$

- Now applying the vanishing flux boundary condition at r = R, we get

$$0 = A_2 \frac{Sin(BR)}{R}$$

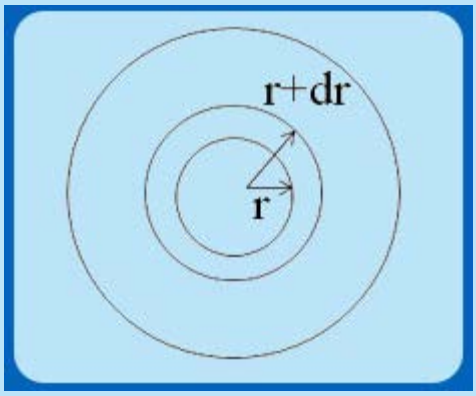
- Thus $A_2 = 0$, which leads to the trivial solution.
- For non-trivial fundamental solution,

$$0 = Sin(BR) \qquad \textbf{or} \qquad BR = \pi$$

- Thus the fundamental solution is,

$$\phi = \frac{A_2}{r} Sin\left(\frac{\pi}{R} r\right)$$

- A_2 shall be similarly derived using power.
- Consider a reactor that is producing power P.
- Consider a strip of thickness dr at a distance r from origin as shown.



- The volume of the strip is $4\pi r^2 dr$.
- Now we shall evaluate A_2 as follows:
- No. of fissions per second $= \sum_f \phi 4\pi r^2 dr$
- Energy released per second $= \sum_f \phi 4\pi r^2 dr E_f$
- Total Power, $P = \int_0^R \sum_f \phi 4\pi r^2 dr E_f$

$$\Rightarrow P = 4\pi \sum_f E_f \int_0^R A_2 Sin\left(\frac{\pi}{R} r\right) r dr$$

- Integrating by parts leads to

$$P = 4 \sum_f E_f A_2 R^2$$

$$\Rightarrow A_2 = \frac{P}{4 \sum_f E_f R^2}$$

- Thus, the flux distribution is given by

$$\Rightarrow \phi_1 = \frac{P}{4\Sigma_f E_f R^2} \frac{Sin\left(\frac{\pi r}{R}\right)}{r}$$

- As seen previously the power density will be maximum at the centre.
- Defining the average flux such that Power is conserved.

$$\Rightarrow \Sigma_f \bar{\phi} \frac{4}{3} \pi R^3 E_f = P$$

$$\Rightarrow \bar{\phi} = \frac{3P}{4\pi R^3 \Sigma_f E_f}$$

- Since peak flux occurs at r=0

$$\Rightarrow \phi_{max} = \frac{P\pi}{4\Sigma_f E_f R^3}$$

$\Rightarrow \frac{\phi_{max}}{\bar{\phi}} = \frac{\pi^2}{3} = 3.29$ More peaked than that for slab and cylindrical reactor.

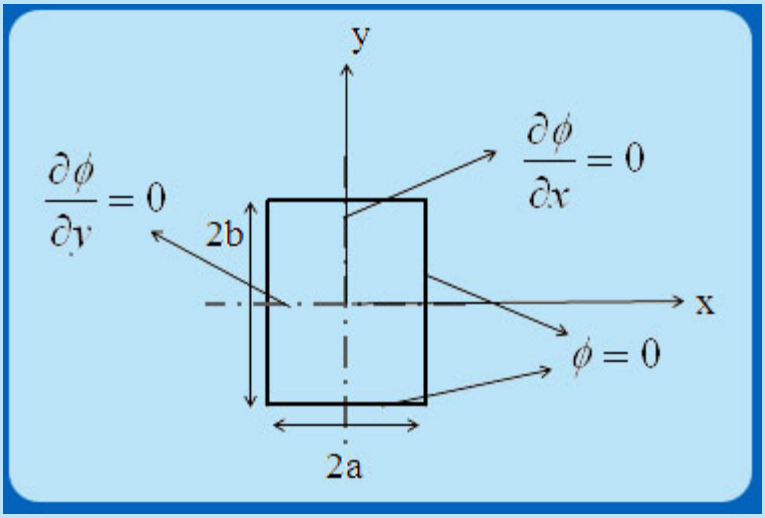
Other Shapes

- Now we shall focus our attention on finite rectangular and cylindrical reactors.
- Since the governing equation is linear, extensions are fairly simple.
- The governing equation shall continue to be

$$\Rightarrow \nabla^2\phi + B_m^{\,2}\phi = 0 \qquad \text{where,} \quad B_m^{\,2} = \frac{(k_\infty - 1)}{L^2}$$

Parallelopiped Reactor

- Let us now consider a three dimensional rectangular slab reactor as shown. Note only two dimensional sketch is shown for simplicity.



- The governing equation is

$$\Rightarrow \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} + B^2 \phi = 0$$

- As the operator is linear and the boundary conditions are zero flux, we can apply separation of variables.
- Let us assume the solution to be

$$\phi = X(x)Y(y)Z(z)$$

- The product solution has X as only a function of x, Y as only a function of y and Z as only a function of z.

$$\Rightarrow \frac{\partial^2 \phi}{\partial x^2} = X''(x)Y(y)Z(z),\ \frac{\partial^2 \phi}{\partial y^2} = X(x)Y''(y)Z(z),\ \frac{\partial^2 \phi}{\partial z^2} = X(x)Y(y)Z''(z)$$

- Substituting the derivatives in the governing equation we get

$$X''(x)Y(y)Z(z) + X(x)Y''(y)Z(z) + X(x)Y(y)Z''(z) + B^2 X(x)Y(y)Z(z) = 0$$

- Assuming flux to be non-trivial, we can divide by XYZ and get

$$\frac{X''(x)}{X(x)} + \frac{Y''(x)}{Y(x)} + \frac{Z''(x)}{Z(x)} + B^2 = 0$$

- Noting that first, second and third terms are independently functions of x, y, and z respectively and yet when summed up with the fourth term should be zero implies that each of them must independently be a constant.

- Let the terms be defined as

$$\frac{X''(x)}{X(x)} = -\alpha^2, \frac{Y''(x)}{Y(x)} = -\beta^2, \frac{Z''(x)}{Z(x)} = -\gamma^2$$

- This implies

$$X''(x) + \alpha^2 X(x) = 0,$$
$$Y''(x) + \beta^2 Y(y) = 0,$$
$$Z''(x) + \gamma^2 Z(z) = 0 \text{ and}$$
$$\alpha^2 + \beta^2 + \gamma^2 = B^2$$

- It may be observed that the first three equations are similar to the governing equation for the infinite slab. The corresponding boundary conditions are also same. Hence the solution for the fluxes can be obtained similarly.

Objectives

In this lecture you will learn the following

- We will obtain the solution for rectangular 3-D reactor using product solution.
- We then will conclude that multi-dimensional reactor solution can simply be obtained from one dimensional solutions.
- Finally we will discuss the criticality condition.

Introduction

- In the last lecture we showed that the three dimensional reactor equation,

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}+B^2\phi=0$$

was reduced to three ODEs given by

$$X''(x)+\alpha^2X(x)=0,$$
$$Y''(x)+\beta^2Y(y)=0,$$
$$Z''(x)+\gamma^2Z(z)=0 \text{ and}$$
$$\alpha^2+\beta^2+\gamma^2=B^2$$

Solution for the Rectangular 3-D Reactor

- We can write the boundary conditions for the first equation as

$$\frac{dX}{dx} = 0 \quad \text{at x = 0, and} \qquad X = 0 \quad \text{at x = a}$$

- The above governing differential equation and boundary conditions are exactly similar to that for the infinite slab reactor.
- The solution for this equation shall be

$$X = A_1 \cos\left(\frac{\pi}{2a} x\right)$$

- Similarly, the solution for Y and Z can be written as

$$Z = A_3 \cos\left(\frac{\pi}{2c} z\right) \qquad Y = A_2 \cos\left(\frac{\pi}{2b} y\right)$$

- Thus, the flux ϕ = XYZ can be written as

$$\phi = A_1 \cos\left(\frac{\pi}{2a} x\right) A_2 \cos\left(\frac{\pi}{2b} y\right) A_3 \cos\left(\frac{\pi}{2c} z\right)$$

- Combining $A_1A_2A_3$ as a single constant, we can write

$$\phi = A \cos\left(\frac{\pi}{2a} x\right) \cos\left(\frac{\pi}{2b} y\right) \cos\left(\frac{\pi}{2c} z\right)$$

- The constant A can be evaluated power as follows.
- Consider a strip of dimensions dx, dy and dz in the x, y and z directions respectively at the location (x,y,z) .
- The power dP in this infinitesimal volume can be written as

$$dP = \Sigma_f \phi E_f dx dy dz$$

- Total Power, P = $\int_{-a}^{a} \int_{-b}^{b} \int_{-c}^{c} \Sigma_f \phi E_f dx dy dz$

$$\Rightarrow P = 8\int_0^a \int_0^b \int_0^c \Sigma_f E_f A \cos\left(\frac{\pi}{2a}x\right)\cos\left(\frac{\pi}{2b}y\right)\cos\left(\frac{\pi}{2c}z\right)dxdydz$$

$$\Rightarrow P = 8\Sigma_f E_f A\int_0^a \cos\left(\frac{\pi}{2a}x\right)dx\int_0^b \cos\left(\frac{\pi}{2b}y\right)dy\int_0^c \cos\left(\frac{\pi}{2c}z\right)dz$$

$$\Rightarrow P = 8\Sigma_f E_f A\frac{2a}{\pi}\frac{2b}{\pi}\frac{2c}{\pi}$$

Hence $A = \dfrac{P\pi^3}{8V\Sigma_f E_f}$ Where V is the volume of the reactor

- As done earlier, we can define the average flux such that

$$P = \Sigma_f \bar{\phi} V E_f$$

$$\Rightarrow \bar{\phi} = \frac{P}{\Sigma_f V E_f}$$

Since peak flux $= \phi_{max} = A = \dfrac{P\pi^3}{8V\Sigma_f E_f}$

$$\Rightarrow \frac{\phi_{max}}{\bar{\phi}} = \frac{\pi^3}{8} = 3.88$$

General Observations

- Now we can make the following observations
  - The functional form of the flux distribution is the product of the solution of the corresponding one dimensional systems.
  - The Buckling B^2 is the sum of corresponding buckling in the one dimensional systems.

$$B^2 = \alpha^2 + \beta^2 + \gamma^2$$

  - The peak to average flux for the 3-D system is the product of the corresponding 1-D systems.

$$\Rightarrow \frac{\phi_{max}}{\bar{\phi}} = \frac{\pi}{2} * \frac{\pi}{2} * \frac{\pi}{2}$$

Parallelopiped Reactor

- Finally as the constant in the flux term is numerically equal to ϕ_{max}, it can be written as,

$$A = \phi_{max} = \frac{\phi_{max}}{\bar{\phi}} \bar{\phi} = \frac{P}{V \Sigma_f E_f} \frac{\phi_{max}}{\bar{\phi}}$$

Cylindrical Reactor

- Now, we can write the solution for cylindrical reactor of Radius R and height H without going into detailed algebra as follows

$$\phi = AJ_0\left(\frac{2.405}{R}r\right)\cos\left(\frac{\pi}{H}z\right)$$

It may be noted that it is the product of radial solution and axial solution, where 2c has been replaced by H, which is the height of the reactor.

- The buckling B^2 for the reactor will be

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$$

- The peak to average flux shall be

$$\frac{\phi_{max}}{\bar{\phi}} = \frac{\pi}{1.35} * \frac{\pi}{2} = 3.65$$

- Finally as the constant A in the flux term is numerically equal to ϕ_{max}, it can be written as,

$$A = \frac{P}{V\Sigma_f E_f}\frac{\phi_{max}}{\bar{\phi}} = \frac{3.65P}{V\Sigma_f E_f}$$

$$A = \frac{P}{V\Sigma_f E_f}\frac{\phi_{max}}{\bar{\phi}}$$

- Now we can construct a generalised Table for all reactors

Geo= metry	Dimen- sions	B2	Flux	Peaking
Infinite Slab	2a	$\left(\frac{\pi}{2a}\right)^2$	$A\cos\left(\frac{\pi}{2a}x\right)$	1.57
Rectan- gular	2ax2bx2c	$\left(\frac{\pi}{2a}\right)^2+\left(\frac{\pi}{2b}\right)^2+\left(\frac{\pi}{2c}\right)^2$	$A\cos\left(\frac{\pi x}{2a}\right)\cos\left(\frac{\pi y}{2b}\right)\cos\left(\frac{\pi z}{2c}\right)$	3.88
Infinite Cylinder	R	$\left(\frac{2.405}{R}\right)^2$	$AJ_0\left(\frac{2.405}{R}r\right)$	2.33
Finite Cylinder	RxH	$\left(\frac{2.405}{R}\right)^2+\left(\frac{\pi}{H}\right)^2$	$AJ_0\left(\frac{2.405}{R}r\right)\cos\left(\frac{\pi}{H}z\right)$	3.65
Sphere	R			3.29

		$\left(\frac{\pi}{R}\right)^2$	$\frac{A}{r} Sin\left(\frac{\pi}{R} r\right)$	

Criticality Condition

- Recall that our governing equation for the reactor was

$$\nabla^2\phi + B_m^{\,2}\phi = 0 \text{ where } B_m^{\,2} = \frac{(k_\infty - 1)}{L^2}$$

- However, when we solved the equation, we modified it as

$$\nabla^2\phi + B^2\phi = 0$$

- We found that B was constrained to take some specific value for the fundamental solution.
- The resulting B^2 is usually termed as Geometric Buckling as it is dictated by the geometry of the reactor.
- Obviously for the reactor to operate steadily, the following condition must be satisfied.

$$B^2 = B_m^{\,2}$$

- This condition is known as criticality condition.
- The physical interpretation of the above condition is as follows.
- If $B_m > B$, then we have more fissile material than the geometrical condition will allow and the reactor will become supercritical.
- Such a reactor will keep increasing power and will destruct itself.
- However $B_m < B$, then there is less fissile material and the reactor will generate no power.
- Thus for a reactor to operate at steady power, we need the delicate balance. This will be achieved by suitable control and we shall discuss it later.

Objectives

objectives

In this lecture you will learn the following

- We will discuss the criticality condition and then introduce the concept of $k_{eff.}$.
- We then will introduce the four factor formula and two group equations.
- Finally we will discuss the criticality condition for two groups.

Introduction

- In the last lecture we showed that the Criticality Condition is $B^2 = B_m^2$.
- From our earlier lecture we know that

$$B_m^{\,2} = \frac{(k_\infty - 1)}{L^2}$$

- Thus, we can write

$$B^2 = \frac{(k_\infty - 1)}{L^2} \qquad \Rightarrow k_\infty = 1 + B^2 L^2$$

$$Or \quad \frac{k_\infty}{1 + B^2 L^2} = 1$$

One group criticality equation .

Interpretation of Criticality Equation

- To appreciate the physical interpretation of the above equation, We shall go back to diffusion theory.

$$\frac{\partial n}{\partial t} = D\nabla^2\phi - \Sigma_a\phi + S'''$$

- In the above the term $D\nabla^2\phi$ represents the net number of neutrons diffusing into the control volume per unit volume.
- This would imply that $-D\nabla^2\phi$ would represent the net number of neutrons leaking out of the control volume by diffusion.
- Since the reactor satisfies the equation $\nabla^2\phi = -B^2\phi$, we can state that $DB^2\phi$ would represent the net number of neutrons leaking out of the control volume by diffusion.
- The neutrons in a finite sized reactor can either get absorbed or can leak out.
- The total number of neutrons that can leak out of the reactor is

$$\int_{Vol} DB^2\phi \, dV$$

- Similarly, the total number of neutrons absorbed in the system is

$$\int_{Vol} \Sigma_a\phi \, dV$$

- Thus, the fraction leaking out is

$$= \frac{\int_{Vol} DB^2\phi \, dV}{\int_{Vol} DB^2\phi \, dV + \int_{Vol} \Sigma_a\phi \, dV}$$

- For a homogeneous system, the leaking fraction will be

$$= \frac{DB^2 \int_{Vol} \phi \, dV}{DB^2 \int_{Vol} \phi \, dV + \Sigma_a \int_{Vol} \phi \, dV} = \frac{DB^2}{DB^2 + \Sigma_a} = \frac{B^2L^2}{B^2L^2 + 1}$$

- Hence the non-leaking fraction called, the non-leakage probability, P_L can be written as

$$P_L = 1 - \frac{B^2L^2}{B^2L^2+1} = \frac{1}{1+B^2L^2}$$

- Thus the one-group criticality equation can be interpreted as follows

$$\frac{k_\infty}{1+B^2L^2} = k_\infty \frac{1}{1+B^2L^2} = k_\infty P_L = k_{eff} = 1$$

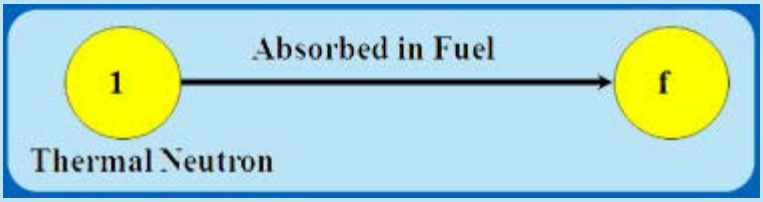
- k_{eff} is the multiplication constant that includes leakage.

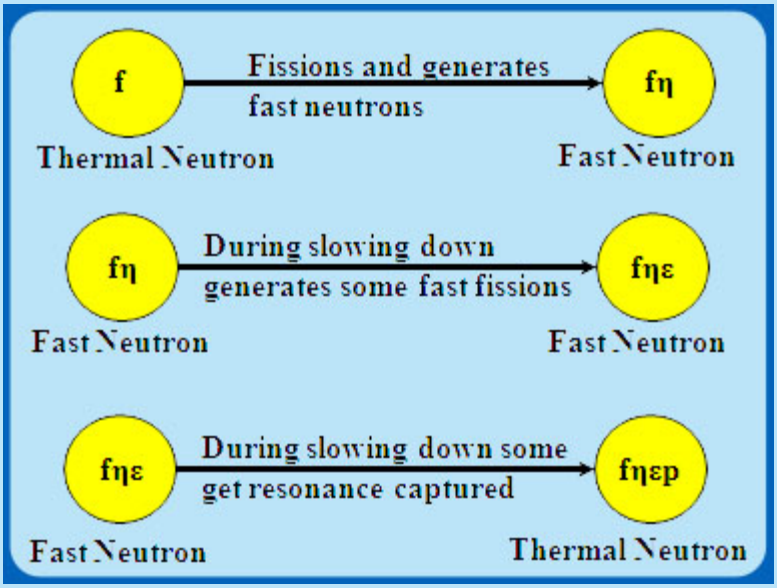
Two-Group Approach

- The calculation performed by one group approach is approximate due to the fact that it ignores the processes that fast neutron undergoes before slowing and eventual absorption.
- It should be understood that most of the absorption takes place in thermal energies.
- As the neutron slows down, some of the fast neutrons can escape.
- During the slowing down process, some neutrons can cause fast fissions.
- Some of the neutrons can be captured in the resonances in the absorption cross section.
- All of this requires some modifications to the theory.

Four Factor Formula

- In two group approach we shall have two groups of neutrons.
- One will be called fast and the other thermal.
- We had discussed in our previous lectures that the thermal group will have energy upto 5kT, while the fast group will have energies above 5kT.
- Such a division will eliminate up-scattering in thermal group.
- Let us now trace the history of a neutron which is thermal.





- Thus the cycle has been completed. By the definition of the infinite multiplication constant, we can write

$$k_\infty = \frac{f\eta\varepsilon p}{1} = \eta\varepsilon pf$$

- The above formula is called four factor formula.
- The definition of each of the term is summarized.
- η is the number of second generation neutrons generated per neutron absorbed in the fuel.
- ε, called the fast fission factor is the amplification in the number of fast neutrons due to fast

fissions.

- p, called the resonance escape probability, is the fraction of fast neutrons that succeed in slowing down.
- Finally, f, the fraction of neutrons that absorb in fuel is called the thermal utilization.
- It may be noted that $\eta \sim 2.5$, $\varepsilon > 1$, $p < 1$, $f < 1$.
- Before we write the governing diffusion equation for two groups, it should be understood that we will write equation for both fast and thermal fluxes.
- The reactor equation for obtaining the fundamental Eigen values are same as before and the boundary conditions for both the fluxes are also same. Hence, the value of B obtained for different geometry continues to be the same as tabulated in the last lecture.

Two-Group Equations

- The neutron balance equation for both the groups are as follows

$$D_1\nabla^2\phi_1 - \Sigma_{1\text{-}2}\,\phi_1 + \Sigma_a\,\phi_2 f\eta\varepsilon = 0$$

- The third term is the source term for fast neutron.

$$D_2\nabla^2\phi_2 - \Sigma_a\,\phi_2 + p\,\Sigma_{1\text{-}2}\,\phi_1 = 0$$

- Note that while slowing down only fraction p succeed in slowing down.
- Since the reactor equation is valid for both the groups, we can rewrite the above equations as

$$-D_1B^2\phi_1 - \Sigma_{1\text{-}2}\,\phi_1 + \Sigma_a\,\phi_2 f\eta\varepsilon = 0$$
$$-D_2B^2\phi_2 - \Sigma_a\,\phi_2 + p\,\Sigma_{1\text{-}2}\,\phi_1 = 0$$

- Note that we have replaced $^2\phi$ by $-B^2\phi$.
- In matrix form the equations can be written as

$$\begin{bmatrix} -D_1B^2 - \Sigma_{1\text{-}2} & \Sigma_a f\eta\varepsilon \\ p\,\Sigma_{1\text{-}2} & -D_2B^2 - \Sigma_a \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \end{Bmatrix} = 0$$

- For non-trivial solution

$$\begin{vmatrix} -D_1B^2 - \Sigma_{1\text{-}2} & \Sigma_a f\eta\varepsilon \\ p\,\Sigma_{1\text{-}2} & -D_2B^2 - \Sigma_a \end{vmatrix} = 0$$
$$\Rightarrow \left(-D_1B^2 - \Sigma_{1\text{-}2}\right)\left(-D_2B^2 - \Sigma_a\right) - p\,\Sigma_{1\text{-}2}\,\Sigma_a f\eta\varepsilon = 0$$

Criticality Condition

$$\Rightarrow \left(-D_1B^2-\Sigma_{1-2}\right)\left(-D_2B^2-\Sigma_a\right)=k_\infty \Sigma_{1-2}\Sigma_a$$

$$\Rightarrow \frac{k_\infty}{\left(\frac{D_1B^2+\Sigma_{1-2}}{\Sigma_{1-2}}\right)\left(\frac{D_2B^2+\Sigma_a}{\Sigma_a}\right)}=1$$

$$\Rightarrow \frac{k_\infty}{\left(B^2\tau+1\right)\left(B^2L^2+1\right)}=1$$

$$\Rightarrow k_\infty P_{LF}P_{LT}=1 \qquad \text{where} \quad P_{LF}=\frac{1}{\left(B^2\tau+1\right)} \qquad P_{LT}=\frac{1}{\left(B^2L^2+1\right)}$$

- In the above equation, P_{LF} and P_{LT} are the fast and thermal non leakage probability.

Objectives

In this lecture you will learn the following

- We will look at couple of design problems for dilute systems.
- Subsequently, we will understand the procedure for computing critical mass.
- Finally we will discuss the effects of heterogeneity and conclude that $k_{\infty\text{-Heterogeneous}} > k_{\infty\text{-Homogeneous}}$.

Introduction

- We had derived the criticality condition for a two-group approach as

$$\Rightarrow k_\infty P_{LF} P_{LT} = 1 \qquad \text{where} \qquad P_{LF} = \frac{1}{(B^2\tau + 1)} \quad P_{LT} = \frac{1}{(B^2 L^2 + 1)}$$

- We had derived the expressions for geometric buckling (B^2) for several common shapes and has been summarized below

Expression for B for Some Simple Geometries

Geometry	Dimensions	B^2
Infinite Slab	2a	$\left(\frac{\pi}{2a}\right)^2$
Rectangular	2ax2bx2c	$\left(\frac{\pi}{2a}\right)^2 + \left(\frac{\pi}{2b}\right)^2 + \left(\frac{\pi}{2c}\right)^2$
Infinite Cylinder	R	$\left(\frac{2.405}{R}\right)^2$
Finite Cylinder	RxH	$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$
Sphere	R	$\left(\frac{\pi}{R}\right)^2$

Design of Reactors

- Now we will study how to use these concepts towards simplified design of the reactors for homogeneous systems.
- As an illustration let us consider a problem where the material to be used is specified and we need to get its size.
- Let us assume that the fuel is a mixture of two or more materials as is often the case.
- We had shown that for a two-group system,

$$k_\infty = \eta \varepsilon p f$$

- To evaluate the value of k_∞ , we have to compute each of these factors.
- First, from the basic data of the fuel composition and the Table of cross sections, we shall compute η and f as follows

$$\eta = \frac{\sum_{i=1}^{N} \nu_i \Sigma_{fi}^{Fuel}}{\sum_{i=1}^{N} \Sigma_{ai}^{Fuel}} \qquad f = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Mod} + \Sigma_a^{Others}}$$

- In the above equation we have assumed a mixture N of fuels.
- In this first level course, the value of ε and p will be assumed to be known as these need more detailed concepts. Often this product is taken as 1.
- Thus k_∞ can be computed.
- For homogeneous dilute system (fuel volume is negligible),

$$L_{mixture}^2 = \frac{D_{mixture}}{\Sigma_a^{mixture}} \approx \frac{D_{Mod}}{\Sigma_a^{Fuel} + \Sigma_a^{Mod} + \Sigma_a^{Others}}$$

$$\cong \frac{D_{Mod}}{\Sigma_a^{Mod}} \frac{\Sigma_a^{Mod}}{\Sigma_a^{Fuel} + \Sigma_a^{Mod} + \Sigma_a^{Others}}$$

$$\cong L_{Mod}^2 (1 - f)$$

- In arriving at the above we have assumed that $D_{mixture} = D_{mod}$ as fuel does not participate in diffusion.
- Thus, by knowing L_{Mod}, which is a property of the moderator, we can compute L_{mix}.
- Similarly, the value of $\tau_{mixture} \approx \tau_{mod}$, which is a property of moderator for a dilute system.
- Thus, for the criticality condition given below, all the terms have been computed except B.

$$\Rightarrow \frac{k_{\infty}}{\left(B^2\tau_{Mixture}+1\right)\left(B^2 L^2_{Mixture}+1\right)} = 1$$

- We can rewrite the above equation as

$$\Rightarrow k_\infty - \left(1 + B^2\left(\tau_{Mod} + L^2_{Mod}(1-f)\right) + B^4 \tau_{Mod} L^2_{Mod}(1-f)\right) = 0$$

- The last term is generally negligible, and hence B can be found explicitly.
- Once B is known, we can use the Table for B and obtain the size.
- In case of spherical reactor, R can be obtained directly. However, if it is a cylindrical reactor, we can obtain the Height or Radius, if the ratio of H/R is assumed.
- Another class of problem is to obtain the material composition for a given geometry.
- For such cases, the procedure followed is outlined.
- The minimum mass of fuel required to make a reactor critical is called Critical Mass.
- For such computations we assume that the reactor is made of fuel and moderator only.

- Let $R = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Mod}}$.

- As the fuel is specified, η, can be computed as described earlier using the fuel property data.
- Similarly L_{mod}, τ_{Mod} can be obtained from the Moderator property data.
- As described earlier, the value of ε and p will be assumed to be known.
- f can be written as

$$f = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Mod}} = \frac{R}{R+1} \quad f = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Mod}} = \frac{R}{R+1}$$

- As described earlier

$$\tau_{Mixture} \approx \tau_{Mod} \qquad L^2_{Mixture} \cong L^2_{Mod}(1-f)$$

- As the geometry is known, the value of B can be estimated using the expressions summarized.
- Thus the critticality equation can be written as

$$\frac{\eta \varepsilon p\left(R/R+1\right)}{\left(B^2\tau_{Mod}+1\right)\left(B^2 L^2_{Mod}\left(1-R/R+1\right)+1\right)} = 1$$

- The only unknown in the above equation is R and it can be estimated.
- As , we can now compute the value of .

$$R = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Mod}} = \frac{N^{Fuel}\sigma_a^{Fuel}}{N^{Mod}\sigma_a^{Mod}} \qquad \frac{N^{Fuel}}{N^{Mod}}$$

Estimation of Critical Mass

- The critical mass can be estimated as follows.
- As our system is assumed dilute, the volume of the system is equal to the volume of the moderator (V).

$$\Rightarrow Mass^{Mod} = V\rho_{Mod}$$

$$\Rightarrow Total\ Moderator\ Nuclei = V\rho_{Mod}\frac{N_{Avo}}{A_{Mod}}$$

$$\Rightarrow Total\ Fuel\ Nuclei = V\rho_{Mod}\frac{N_{Avo}}{A_{Mod}} \qquad R\frac{\sigma_a^{Mod}}{\sigma_a^{Fuel}}$$

$$\Rightarrow Mass\ of\ Fuel = V\rho_{Mod}\frac{N_{Avo}}{A_{Mod}}R\frac{\sigma_a^{Mod}}{\sigma_a^{Fuel}}\frac{A_{Fuel}}{N_{Avo}}$$

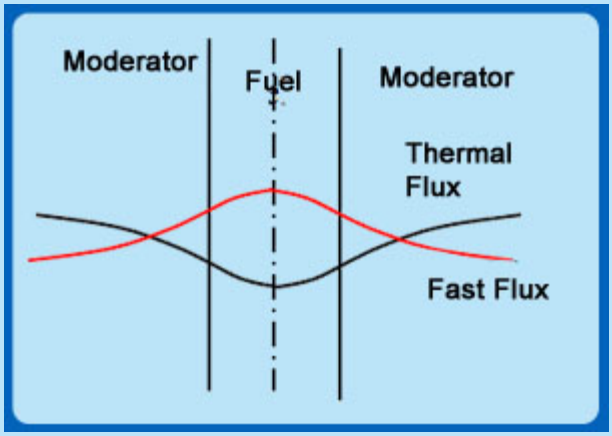
$$= V\rho_{Mod}\frac{A_{Fuel}}{A_{Mod}}R\frac{\sigma_a^{Mod}}{\sigma_a^{Fuel}}$$

A_{fuel}, A_{Mod} are molecular weights of fuel and moderator.

Heterogeneous Systems

- It turns out that the estimation for the reactor size described earlier is a very crude approximation and can be used just to get some idea in a first level course.
- In reality, homogeneous calculations described above, when used for Natural Uranium and Graphite moderated system, predicts that the reactor cannot be made critical.
- However, when the heterogeneity (Lumping of fuel) is considered, the predictions come closer.
- As it involves some more details, we shall skip it and will just give you the qualitative trends.

- The Thermal Flux distribution in a lumped system is as shown



- It may be observed that there is a flux dip in the fuel due to absorption.

- This will reduce *f* for the heterogeneous system.
- The distribution of fast flux shall be just opposite to that of thermal flux and hence will increase ε.
- Also as the fast neutrons do not slow down in fuel, they are not at energies where resonance absorption will take place.
- Hence, p also increases in a heterogenous system.
- As η is a property of the fuel it remains unaffected.
- Overall $k_{\infty\text{-Heterogeneous}} > k_{\infty\text{-Homogeneous}}$.

Objectives

In this lecture you will learn the following

- To illustrate the application of material studied in the last few classes.
- First we shall illustrate application of diffusion theory with source in the system.
- Then we illustrate application of concepts for reactor problems.

Question-1

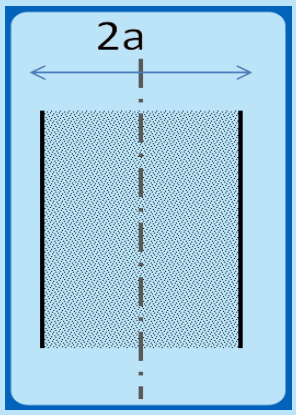
1. An infinite bare slab of moderator of thickness '2a' contains uniformly distributed sources emitting S''' neutrons/cm^3-s. Show that the flux in the slab is given by,

$$\frac{S'''}{\Sigma_a}\left(1-\frac{\cosh(x/L)}{\cosh((a+d)/L)}\right)$$

where 'x' is measured from the centre of the slab and 'd' is the extrapolated length.

- The governing equation for this source problems

$$\frac{d^2\phi}{dx^2}-\frac{\phi}{L^2}=-\frac{S'''}{D}$$



- The solution for the homogeneous part can be written as

$$\phi = A\ Cosh\left(\frac{x}{L}\right)+B\ Sinh\left(\frac{x}{L}\right)$$

- Since the source term is constant, the particular integral shall also be a constant.
- Hence the solution will be of the form

$$\phi = A\ Cosh\left(\frac{x}{L}\right)+B\ Sinh\left(\frac{x}{L}\right)+C$$

- First we shall obtain C.
- Substitution of the solution into the governing equation gives

$$\phi = \frac{A}{L^2}\ Cosh\left(\frac{x}{L}\right)+\frac{B}{L^2}\ Sinh\left(\frac{x}{L}\right)$$

$$-\left(\frac{A}{L^2}\ Cosh\left(\frac{x}{L}\right)+\frac{B}{L^2}\ Sinh\left(\frac{x}{L}\right)+\frac{C}{L^2}\right)=-\frac{S'''}{D}$$

$$\Rightarrow \frac{C}{L^2}=\frac{S'''}{D} \quad Or \quad C=\frac{S'''}{\Sigma_a}$$

- The boundary conditions for the problem are

  At x = 0, J = dΦ/dx = 0,

  x = a+d, Φ = 0

- The symmetric Boundary Condition implies

$$\left.\frac{d\phi}{dx}\right|_{x=0} = 0$$

$$\Rightarrow \left[\frac{A}{L}\ Sinh\left(\frac{x}{L}\right)+\frac{B}{L^2}\ Cosh\left(\frac{x}{L}\right)\right]_{x=0} = 0$$

$$\Rightarrow B = 0$$

- Thus $\phi = A\ Cosh\left(\frac{x}{L}\right)+C$

- Using the flux vanishing at extrapolated boundary, we get

$$0 = A\ Cosh\left(\frac{a+d}{L}\right)+C$$

$$\Rightarrow A = -\frac{C}{Cosh\left(\frac{a+d}{L}\right)}$$

$$\Rightarrow A = -\frac{S'''}{\Sigma_a Cosh\left(\frac{a+d}{L}\right)}$$

- As $\phi = A\ Cosh\left(\frac{x}{L}\right)+C$

$$\phi = -\frac{S'''}{\Sigma_a}\frac{Cosh\left(\frac{x}{L}\right)}{Cosh\left(\frac{a+d}{L}\right)}+\frac{S'''}{\Sigma_a}$$

$$\phi = \frac{S'''}{\Sigma_a}\left[1-\frac{Cosh\left(\frac{x}{L}\right)}{Cosh\left(\frac{a+d}{L}\right)}\right]$$

Question-2

Calculate the fuel utilization factor for a fast reactor consisting of a mixture of liquid sodium and plutonium, in which the plutonium is present to 3.0 w/o. The density of the mixture is approximately 1.0 g/cm^3 . The following properties can be used

$$\sigma_a^{Pu} = 2{,}14b, \quad \eta = 2.61 \qquad \sigma_a^{Na} = 0.0008b$$

- Consider 1 cm^3 of the solution.
- The mass of the solution = 1.0 g.
- Mass of Pu = 0.03 g.
- Mass of Na = 0.97 g.
- Number density of Pu = 0.03 x 6.023 x 10^{23} / 239

  = 7.56 x 10^{19} cm^{-3}

- Number density of Na = 0.97 x 6.023 x 10^{23} / 23

  = 2.54 x 10^{22} cm^{-3}

$$f = \frac{\Sigma_a^{Pu}}{\Sigma_a^{Pu} + \Sigma_a^{Na}} = \frac{1}{1 + \frac{\Sigma_a^{Na}}{\Sigma_a^{Pu}}} = \frac{1}{1 + \frac{2.54 \text{ x } 10^{22} \text{ x } 0.0008}{7.56 \text{ x } 10^{19} \text{ x } 2.14}} = 0.888$$

Question-3

A bare cylindrical reactor of height 200 cm and diameter 100 cm is operating at a steady power of 200 MW. If the origin is taken at the centre of the reactor, what is the power density at the point r = 7 cm, z = - 22.7 cm?

- The flux variation in a cylindrical reactor is given by

$$\phi = \phi_{max} J_0\left(\frac{2.405}{R} r\right) \cos\left(\frac{\pi}{H} z\right)$$

- For the problem, r = 7 cm, R = 50 cm, z = -22.7 cm, H = 200 cm

$$\frac{\phi}{\phi_{max}} = J_0\left(\frac{2.405}{50} 7\right) \cos\left(\frac{\pi}{200}(-22.7)\right)$$

$$= J_0(0.337)\cos(-0.356)$$

$$= 0.97 * 0.937 = 0.909$$

- Since power density is proportional to flux

$$\frac{Q'''(7,-22.6)}{Q'''(0,0)} = 0.909$$

- Further, Peak Power to average power = peaking factor.
- For cylindrical reactor this is equal to 3.65.
- Average power density = 200 MW/ Volume of reactor.
- Volume = π x 50 x 50 x 200 = 1.57 x 10^6 cm^3.
- Hence, Average Power Density = 127.3 W/cm^3.
- Hence Peak Power Density = 464.7 W/cm^3.
- Hence Power density at (7, -22.6) = 464.7x0.909=422W/cm^3.

Question 4

A bare thermal reactor in the shape of a cube consists of a homogeneous mixture of U^{235} and graphite. The ratio of atom densities of N_{Fuel} /$N_{moderator}$ = 1.0 x 10^{-5}. Using the two group theory, calculate the critical dimensions. Use the following properties, Take pε = 1, η = 2.05, L^2 3500 cm^2 , τ = 0 cm^2

Material	σ_a (barns)
U^{235}	640
Graphite	0.0034

$$f = \frac{\Sigma_a^U}{\Sigma_a^U + \Sigma_a^{Graphite}} = \frac{1}{1 + \frac{\Sigma_a^{Graphite}}{\Sigma_a^U}}$$

$$= \frac{1}{1 + \frac{N^{Graphite}\sigma_a^{Graphite}}{N^U \sigma_a^U}}$$

$$= \frac{1}{1 + 10^5 \frac{0.0034}{640}} = 0.653$$

$$\eta = 2.05$$

$$\Rightarrow k_\infty = \eta f = 2.05 * 0.653 = 1.345$$

$$\Rightarrow L^2_{mix} = L^2_{\text{mod}}(1 - f) = 3500 * (1 - 0.653)$$

$$= 1214.5\ cm^2$$

$$\Rightarrow B^2 = \frac{k_\infty - 1}{L^2_{mix}} = \frac{0.345}{3500} = 0.000284\ cm^2$$

$$\Rightarrow 3\left(\frac{\pi}{a}\right)^2 = 0.000284 cm^2$$

$$\Rightarrow a = 323cm$$

Objectives

In this lecture you will learn the following

- We shall solve some more problems to illustrate the applications.

Question 1

Now consider the same problem as problem 4 in the last lecture and calculate (a) the critical mass and (b) the maximum flux, if the reactor operates at 1 kW. Take $\varrho_{graphite}$ = 1.6 g/cm^3, Energy from fission=E_R 200 MeV, σ_f-U^{235}=540 b.

The problem statement is repeated here for convenience.

A bare thermal reactor in the shape of a cube consists of a homogeneous mixture of U^{235} and graphite. The ratio of atom densities of N_{Fuel} /$N_{moderator}$ = 1.0 x 10^{-5}. Using the modified one-group theory, calculate the critical dimensions. Use the following properties, Take $p\varepsilon = 1$, $\eta = 2.05$, L^2 3500 cm^2 , $\tau = 0$ cm^2 .

Material	σ_a (barns)
U^{235}	640
Graphite	0.0034

Question 4 (Previous Solution)

f=0.653

K_∞=1.345

a=323cm

$\eta = 2.05$

Present Solution

From the concepts taught in the lecture 21,

$$\Rightarrow Mass\ of\ Fuel = V\rho_{Mod}\frac{N_{Avo}}{A_{Mod}}R\frac{\sigma_a^{Mod}}{\sigma_a^{Fuel}}\frac{A_{Fuel}}{N_{Avo}}$$

$$= V\rho_{Mod}\frac{A_{Fuel}}{A_{Mod}}\frac{N^{Fuel}}{N^{FMod}}$$

$$= (323)^3 1.6\frac{235}{12}10^{-5}$$

$$= 10558g = 10.56kg$$

$$Power = VN^{Fuel}\sigma_f^{Fuel}\phi_{av}E_R$$

$$N^{Fuel} = \frac{N_{Avo}}{A^{Fuel}}\frac{M^{Fuel}}{V}$$

$$\Rightarrow Power = V\frac{N_{Avo}}{A^{Fuel}}\frac{M^{Fuel}}{V}\sigma_f^{Fuel}\phi_{av}E_R$$

$$\Rightarrow \phi_{av} = \frac{Power\ A^{Fuel}}{N_{Avo}M^{Fuel}\sigma_f^{Fuel}E_R}$$

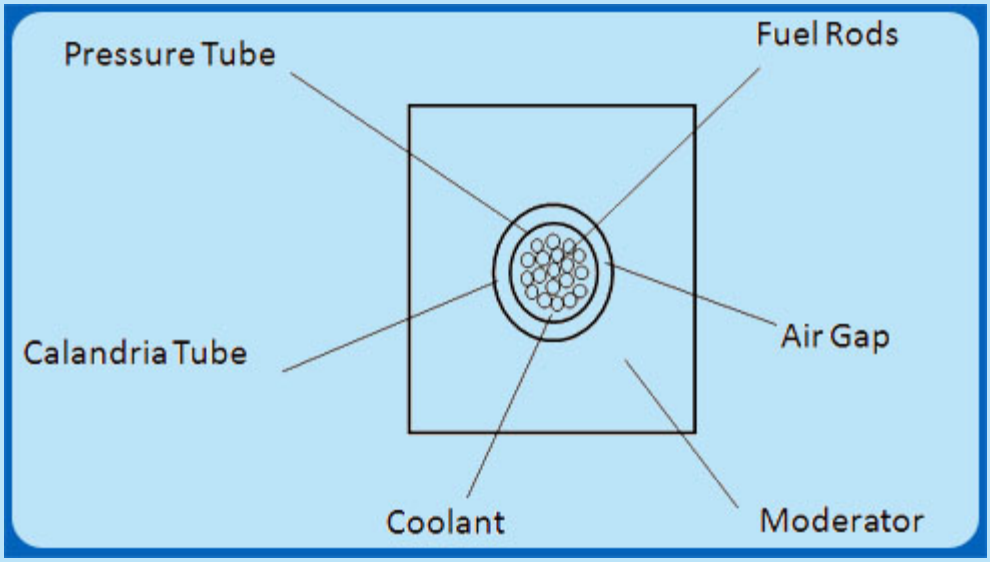
$$\Rightarrow \phi_{av} = \frac{1000x235}{6.023x10^{23}x10558x540x10^{-24}x200x1.602x10^{-13}}$$

$$\Rightarrow \phi_{av} = 2.136 \times 10^{9} n/cm^2 - s$$

$$\Rightarrow \phi_{max} = 2.136 \times 10^{9} x 3.88 = 8.29 x 10^{9}\ n/cm^2 - s$$

Question-2

The typical unit cell of a Rajasthan Atomic Power Station (RAPS) is shown in the following figure



The computed volumes of the various materials per unit length of the reactor are:

UO_2	29.2 cm^3
Zr (A=91)	20.3 cm^3
D_2O (coolant)	19.1 cm^3
Air gap	27.4 cm^3
D_2O (moderator)	426.6 cm^3

It may be assumed that air may be treated as a non-participating medium (does not react with neutrons) and the Uranium in UO_2 is natural.

(a) Given that the density of UO_2, Zr and D_2O to be 10.5, 6.5 and 1.1 g/cm^3 respectively, calculate the homogenised number density of each material.

(b) Given the volumes as above, calculate the homogenised macroscopic absorption cross section of UO_2, Zr and D_2O and the core. For Natural Uranium, assume that N_{235}/N_{238} = 7/993.

c)Compute f.
d)Compute k_∞ .
e)Assuming L/D ratio is 1, compute the diameter.
f)Calculate the critical mass.

The following constants may be used

Material			ν	L (cm)	τ 2

	σ_a (barns)	σ_f (barns)			(cm)
U^{235}	680	580	2.5	---	---
U^{238}	2.7	0.0	---	---	---
Zr	0.198	---	---	---	---
D_2O	4.6 $X10^{-4}$	---	---	100	120
O	0.0	0.0	---	---	---

Consider unit length of core

Material	Volume	Volume fraction
UO_2	29.2	0.0559
ZR	20.3	0.0388
D_2O(Coolant)	19.1	0.0365
Air-gap	27.4	0.0524
D_2O(Moderator)	426.6	0.8164
Total	522.6	1

- For 1 cm^3 of core, the volumes of the components shall be same as the volume fractions.

Now we can construct the following table

- Note that σ Nat-U = σ U_{235} x 0.007+ σ U_{238} x 0.993.

Material	Volume per cm^3-core	Density (g/cm^3)	A	Mass (g/cm^3)	N (cm^{-3})	σ_a (barn)	Σ (cm^{-1})
UO_2	**0.0559**	**10.5**	**269.98**	**0.587**	**1.30884E+21**	**7.4411**	**0.009739**
Zr	**0.0388**	**6.5**	**91**	**0.252**	**1.67113E+21**	**0.198**	**0.000331**
D_2O(Cool)	**0.0365**	**1.1**	**20**	**0.0402**	**1.21071E+21**	**0.00046**	**5.57E-07**
Air-gap	**0.0524**	**----**	**----**	**---**	**---**	**---**	**---**
D_2O(Mod)	**0.8160**	**1.1**	**20**	**0.898**	**2.70413E+22**	**0.00046**	**1.24E-05**
Total	**1**			**1.777**			**0.010083**

$\leftarrow \Sigma_{core}$

$$\Rightarrow f = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Mod} + \Sigma_a^{O}}$$

$$\Rightarrow f = \frac{0.009739}{1.29x10^{-5} + 0.000331} = 0.966 \text{ ----------(c)}$$

$$\eta = \frac{\sum_{i=1}^{N} \nu_i \Sigma_{fi}^{Fuel}}{\sum_{i=1}^{N} \Sigma_{ai}^{Fuel}}$$

$$\eta = \frac{2.5x1.309x10^{21}x0.007x580x10^{-24}}{0.009739} = 1.364$$

$$k_\infty = \eta f p \varepsilon = \eta f = 1.364 x 0.966 = 1.318$$ ----------(d)

$$L^2_{mix} = L^2_{\mathrm{mod}}(1-f) = 10000*(1-0.966) = 340\ cm^2$$

$$\tau_{mix} = \tau_{\mathrm{mod}} = 120\ cm^2$$

$$\frac{k_\infty}{\left(B^2\tau_{Mixture}+1\right)\left(B^2 L^2_{Mixture}+1\right)} = 1$$

$$\Rightarrow \frac{k_\infty}{\left(B^2\left(\tau_{Mix}+L^2_{Mix}\right)+1\right)} \approx 1$$

$$\Rightarrow B^2 \approx \frac{k_\infty - 1}{\tau_{Mix}+L^2_{Mix}} = \frac{0.318}{460} = 6.913 x 10^{-4} cm^2$$

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$$

$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{2R}\right)^2$$

$$B^2 = \frac{1}{R^2}\left[(2.405)^2 + \left(\frac{\pi}{2}\right)^2\right]$$

$$\Rightarrow 6.913 x 10^{-4} = \frac{1}{R^2}\left[(2.405)^2 + \left(\frac{\pi}{2}\right)^2\right]$$

$$\Rightarrow R = 109\ cm$$ D=218cm ----------(e)

Now we can compute critical mass as follows

Volume of UO_2 per cell = 29.2 cm^3.

Mass of UO_2 per cell = 29.2 x 10.6 g.

Mass per unit volume = 29.2 x 10.6/ 522.6 = 0.592 g/cm^3.

Volume of Reactor = π 109^2 x 218 = 8.18 x 10^6 cm^3.

Mass = 0.592 x 8.18 x 10^6 = 4.845 x 10^6 g = 4845 kg. ----------(f)

Question 3

Consider an infinite reactor with the following specifications

Material	$\Sigma_f(cm^{-1})$	$\Sigma_c(cm^{-1})$	ν
Fuel	0.2	0.1	2.5
Moderator	0.0	0.05	---
Others	0.0	0.05	---

Compute k_∞ . How much $\Sigma_A(Absorber)$ should be added to keep the reactor critical at the beginning. As the reactor produces power, the fissile content decreases. Assuming for simplicity that Σ_c does not change, compute the fraction of the fissile nuclei that will be left unconsumed before refuelling.

Solution

$$\eta = \frac{\nu_i \Sigma_f^{Fuel}}{\Sigma_a^{Fuel}} = \frac{2.5 \times 0.2}{0.2+0.1} = 1.667 \qquad f = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Mod} + \Sigma_a^{O}}$$

$$\Rightarrow f = \frac{0.2+0.1}{0.2+0.1+0.05+0.05} = 0.75$$

$$k_\infty = \eta f = 1.667 \times 0.75 = 1.25$$

Since it is an infinite reactor, k_∞ must be 1 for criticality. Hence we need to add absorber equivalent of

$$\Rightarrow 1 = 1.667 \frac{0.3}{0.4 + \Sigma_a^{Abs}} \qquad \Rightarrow \Sigma_a^{Abs} = 0.1$$

At the stage of refuelling, k_∞ must be 1 with $\Sigma_a^{Abs} = 0.0$.

Let the Σ_f of fuel at that time be denoted as Σ_f^{Fuel*}.

$$\Rightarrow k_\infty = \frac{\nu \Sigma_f^{Fuel*}}{\Sigma_f^{Fuel*} + \Sigma_c^{Fuel}} \frac{\Sigma_f^{Fuel*} + \Sigma_c^{Fuel}}{\Sigma_f^{Fuel*} + \Sigma_c^{Fuel} + \Sigma_c^{Mod} + \Sigma_a^{O}}$$

Note Σ_c^{fuel} is assumed to be unchanged.

$$\Rightarrow 1 = \frac{\nu \Sigma_f^{Fuel*}}{\Sigma_f^{Fuel*} + 0.2} = \frac{2.5 \Sigma_f^{Fuel*}}{\Sigma_f^{Fuel*} + 0.2}$$

$$\Rightarrow \Sigma_f^{Fuel*} = 0.133$$

Fraction left

$$\Rightarrow \frac{\Sigma_f^{Fuel*}}{\Sigma_f^{Fuel}} = \frac{0.133}{2} = 0.667$$

Objectives

In this lecture you will learn the following

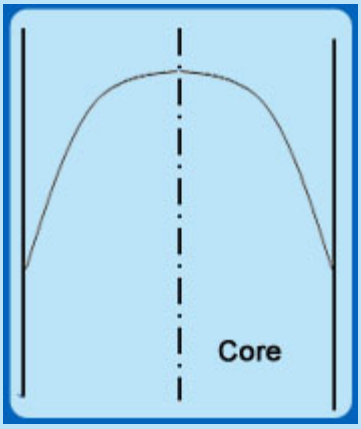
- We will look at the concept of reflected reactors.
- We will demonstrate that when reflectors are employed, the flux flattens and buckling decreases.

Introduction

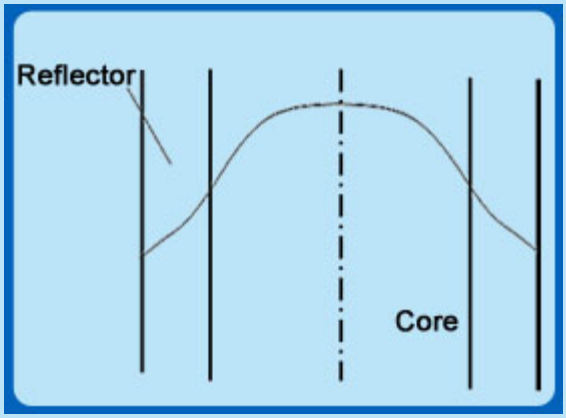
- We have seen that we need to use certain minimum amount of fuel to make a reactor critical.
- Fuel is a very expensive commodity and any attempt to minimise the critical mass will reduce the cost of the reactor.
- One way of reducing the critical size is to provide reflectors.
- Reflectors are made of scattering materials and they throw in the leaking neutrons back into the reactor by back scattering.
- Generally, reflectors in thermal reactors are just moderators surrounding the core.

Reflected Reactors

- The figure shows the flux distribution in an infinite slab reactor.

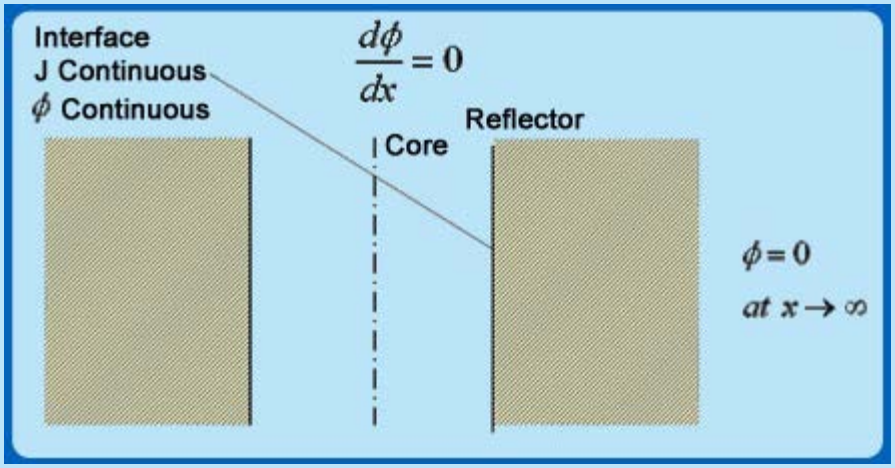


- The flux distribution is also shown in the reflected reactor.



- Notice increase in average flux in the core.

- In this introductory course, we shall demonstrate the associated analysis for a reflected reactor using an infinite slab reactor followed by an infinite reflector.
- The domian of the system and associated boundary conditions are shown below



- The governing equation for the core shall continue to be

$$\Rightarrow \nabla^2 \phi_C + B^2 \phi_C = 0$$

- As far as the core is concerned, the treatment shall be identical, except that the boundary condition has been modified.
- Reflector being a non-source system, the governing equation shall be

$$\Rightarrow \nabla^2 \phi_R - \frac{\phi_R}{L_R^2} = 0$$

- The solution for the core flux is

$$\phi_C = A_1 \cos(Bx) + A_2 \sin(Bx)$$

- The solution for the reflector flux is

$$\phi_R = A_3 e^{-x/L_R} + A_4 e^{x/L_R}$$

- Note that the problem is symmetric and hence applying symmetric boundary condition at x = 0 implies

$$\left.\frac{d\phi_C}{dx}\right|_{x=0} = -A_1 B \sin(0) + A_2 B \cos(0) = 0$$
$$\Rightarrow A_2 = 0$$

- Thus the core flux is of the form

$$\phi_C = A_1 \cos(Bx)$$

- Using the boundary condition that the reflector flux vanishes at x = ∞, we can write,

$$\phi_R = 0 = A_3 e^{-\infty} + A_4 e^{\infty}$$

- Since the positive exponent blows up at infinity, A_4 has to be 0. Hence reflector flux is of the form

$$\phi_R = A_3 e^{-x/L_R}$$

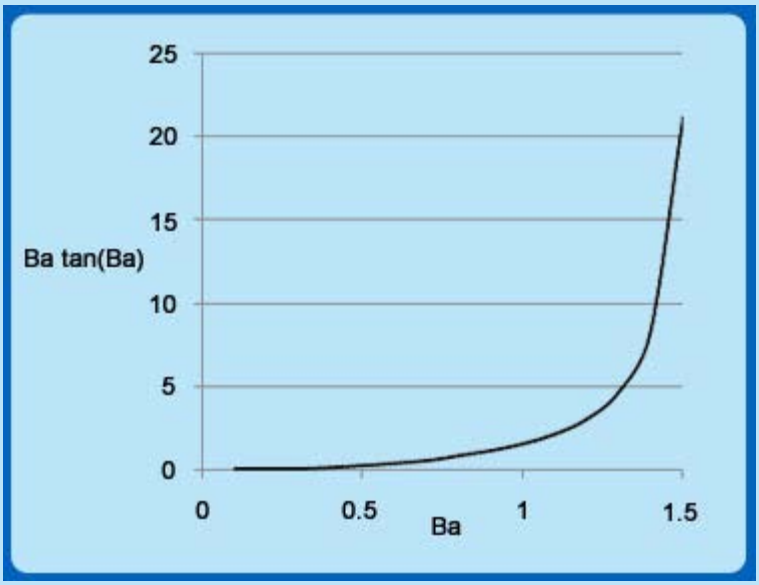
- Applying the interface boundary conditions at x = a

$$A_1 \cos(Ba) = A_3 e^{-a/L_R}$$
$$-D_C A_1 B \sin(Ba) = -D_R \frac{A_3}{L_R} e^{-a/L_R}$$

- From the above two equations, we can write

$$D_C B \tan(Ba) = \frac{D_R}{L_R}$$
$$\Rightarrow Ba \tan(Ba) = \frac{D_R a}{D_C L_R}$$

- The variation of LHS as a function of Ba is plotted in the figure below



- It may be observed that for a given RHS, Ba will be less than π/2. Hence B is less than π/2a.
- Hence buckling is less for a reflected reactor.

Design of Reflected Reactors

- For the estimation of size, the procedure is similar.
- For the composition of the materials specified, the values of η and f are computed as described earlier.
- Thus k_∞ can be estimated.
- For a one group system, we can evaluate L^2 of the mixture as

$$L^2_{Mixture} \cong L^2_{Mod}\,(1-f)$$

- Thus, the material buckling is calculated as

$$B_m{}^2 = \frac{(k_\infty - 1)}{L^2_{mixture}}$$

- At criticality as material buckling is equal to geometric buckling, we can rewrite the condition derive previously as

$$\Rightarrow B_m \tan(B_m a) = \frac{D_R}{D_C L_R}$$
$$\Rightarrow B_m a = \tan^{-1}\frac{D_R}{D_C L_R B_m}$$
$$\Rightarrow a = \frac{1}{B_m}\tan^{-1}\frac{D_R}{D_C L_R B_m}$$

- For a specified dimension, and the moderator and fuel materials specified, the relative concentration can be obtained in a similar manner for non-reflected reactors.
- Again for simplicity, only one group system is considered.
- We begin with the definition of R

$$R = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Mod}}$$

- As the fuel is specified, η, can be computed as described earlier using the fuel property data
- The value of f can be written as

$$f = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Mod}} = \frac{R}{R+1}$$

GO

- As described earlier,

$$L^2_{Mixture} \cong L^2_{Mod}(1-f) = \frac{L^2_{Mod}}{R+1}$$

- From the condition given earlier,

$$\Rightarrow Ba\tan(Ba) = \frac{D_R a}{D_C L_R}$$

- As RHS is known, Ba can be found graphically or numerically by iterative process.
- Thus B can be computed as a is known.
- Now geometric buckling is given by

$$B_m{}^2 = \frac{(k_\infty - 1)}{L^2_{mixture}}$$

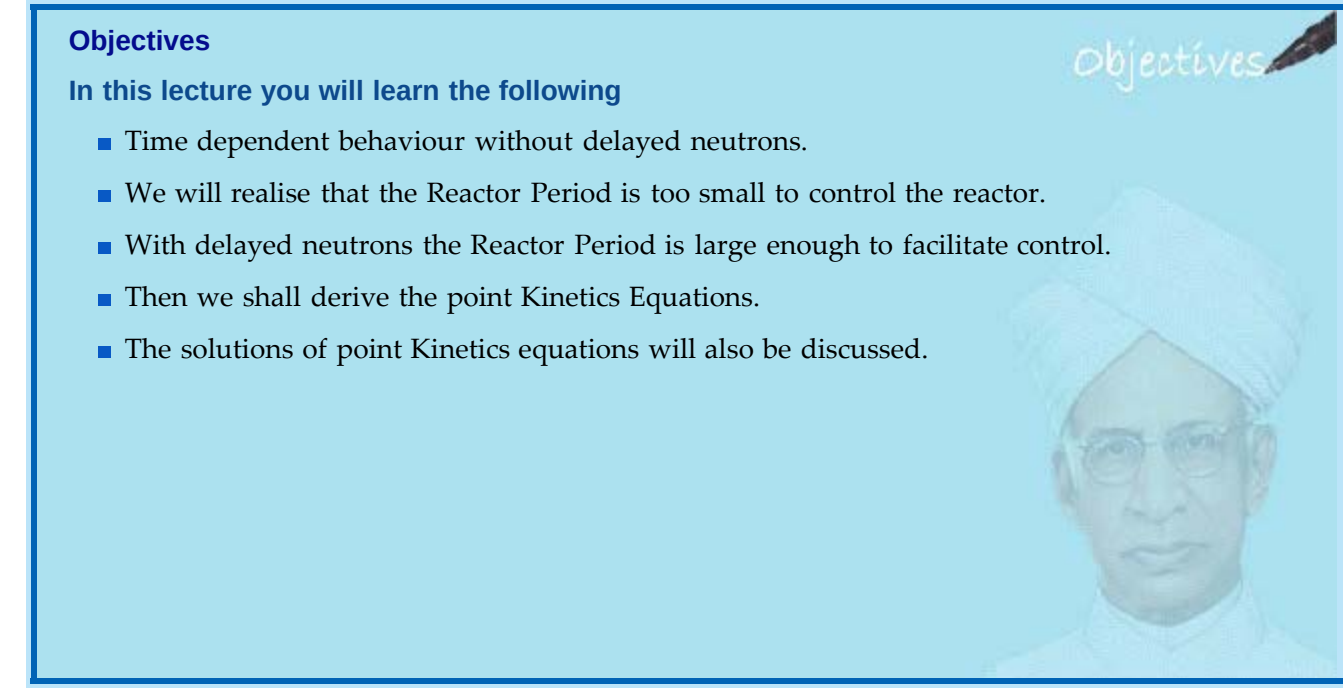
$$B_m{}^2 = \frac{(\eta f - 1)}{L^2_{mixture}} = \frac{\left(\eta \frac{R}{R+1} - 1\right)}{\frac{L^2_M}{R+1}} = \frac{\eta R - R - 1}{L^2_M}$$

- As at criticality $B_m{}^2 = B^2$, we can write

$$B^2 = \frac{\eta R - R - 1}{L^2_M} \qquad \Rightarrow R = \frac{B^2 L^2_M + 1}{\eta - 1}$$

- The computation of critical mass is similar to bare reactors and no new concepts are involved.
- To work out the flux distribution, the constant A_1 has to be determined.
- This is obtained from the power of the operating reactor by methods shown earlier.
- Once the flux shape is obtained, the peaking factor can also be obtained.
- These concepts will be illustrated in our problem solving session.

Objectives



In this lecture you will learn the following

- Time dependent behaviour without delayed neutrons.
- We will realise that the Reactor Period is too small to control the reactor.
- With delayed neutrons the Reactor Period is large enough to facilitate control.
- Then we shall derive the point Kinetics Equations.
- The solutions of point Kinetics equations will also be discussed.

Introduction

- We will now look at the time dependent behaviour of the reactor.
- It should be realised that in strict sense, the reactor is never in steady state and there is a constant change in composition taking place on account of fuel consumption and generation of fission products.
- A well designed control system makes a delicate balance such that the K_{eff} is kept very close to 1 all the time.
- However, when we increase or decrease the power or shut the reactor down there is a transient variation in power and that needs to be understood for safe operation of the reactor.
- When we were studying the steady behaviour of the system, the temporal behaviour was assumed to be absent.
- In a similar manner during transient anaysis, we shall assume that the spatial gradients don't exit.
- This would imply that the spatial term will be neglected and every point in the reactor behaves like every other point and such a representation is called point kinetics treatment.
- The full governing equation for transient variation of heat flux can be given as

$$D\nabla^2\phi - \Sigma_a \phi + s''' = \frac{1}{v}\frac{\partial \phi}{\partial t}$$

- Now making the assumption that spatial variation can be neglected, we can write

$$-\Sigma_a \phi + s''' = \frac{1}{v}\frac{\partial \phi}{\partial t}$$
$$\Rightarrow \frac{1}{v\Sigma_a}\frac{d\phi}{dt} = \frac{s'''}{\Sigma_a} - \phi$$

- Substituting for the source term we can write

$$\Rightarrow \frac{1}{v\Sigma_a}\frac{d\phi}{dt} = \frac{k_\infty \Sigma_a \phi}{\Sigma_a} - \phi \qquad \Rightarrow \frac{1}{v\Sigma_a}\frac{d\phi}{dt} = (k_\infty - 1)\phi$$

- The coefficient in the first term has a physical interpretation and let us look at it first.

- We had shown previously in the course that $1/\Sigma_a$ is the mean distance travelled before a neutron is absorbed.
- The term, $1/(\Sigma_a v)$, then would represent the time taken by neutron to travel the mean distance.
- Thus $1/(\Sigma_a v)$ is the amount of time a fission neutron lives and is called the life of the neutron.
- At this juncture, we assume that all neutrons are prompt and the term $1/(\Sigma_a v)$ is denoted by l_p, the prompt neutron life.
- Thus the neutron balance equation can be written as

$$\Rightarrow \frac{d\phi}{dt} = \frac{(k_\infty - 1)}{l_p}\phi \Rightarrow l_p \frac{d\phi}{dt} = (k_\infty - 1)\phi$$

- Using the initial condition that $\phi = \phi_0$ at $t = 0$, we get the solution as

$$\phi = \phi_0 e^{\frac{(k_\infty - 1)}{l_p}t} \qquad \Rightarrow \frac{\phi}{\phi_0} = e^{\frac{t}{T_P}} \qquad where\ T_P = \frac{l_p}{(k_\infty - 1)}$$

- The term T_p is called the Reactor Period. This is the amount of time taken by the reactor to increase the power by a factor of 'e'.
- The typical values of prompt neutron life in various materials are summarised below

Material	l_p (s)
H_2O	2×10^{-4}
D_2O	4×10^{-2}
Be	4×10^{-3}
Graphite	1.7×10^{-2}

- For an infinite H_2O reactor, if $K_\infty = 1.001$, $T_P = 0.2$ s.
- This implies that in 2 s, the power will increase by e^{10}, or 22000 times! It will be impossible to control such reactors.
- Interestingly, reactors exist and are being controlled. The key for this is the presence of delayed neutrons.
- Some of the fission products like Br^{87}, I^{137} undergo the following radioactive decay chain

$$^{87}_{35}Br \xrightarrow{\beta^-,\ 54s} {}^{87}_{36}Kr \xrightarrow{\sim 0s} {}^{86}_{36}Kr + {}^{1}_{0}n$$

$$^{137}_{53}I \xrightarrow{\beta^-,\ 24s} {}^{137}_{54}Xe \xrightarrow{\sim 0s} {}^{136}_{54}Xe + {}^{1}_{0}n$$

- It may be observed that Br and I after some time delay decay to Kr and Xe respectively. These emit neutrons instantly. Hence Br and I shall be called precursors. Thus the neutrons emitted are delayed in time and hence called delayed neutrons.
- There are several precursors and for simplicity, they can be lumped on to one group with a decay constant of 0.08 s^{-1}.
- The fraction of delayed neutrons are typically 0.0065 and these play a crucial role in controlling the reaction.
- Since we know that the mean life of the radioactive specie is given by $1/\lambda$, we can say that the neutron comes with a delay of 1/0.08 = 12.5 s.
- From a simplistic view point we can define the effective life of neutron to be

  $2 \times 10^{-4} \times (1-0.0065) + 12.5 \times 0.0065 = 0.08$ s

- The Reactor period T_P for the same example of $K_\infty = 1.001$ and water reactor works out to be 80 s, making the reactor amply controllable.
- The above treatment is fairly crude, but illustrative.
- We shall now refine this analysis a bit further to appreciate what are called point kinetics equations.
- There are two point Kinetics equations, one for the neutron population and the other for the precursor concentrations.
- We shall now derive them and appreciate their solutions.

Reflected Reactors-III

- Going back to our point equation again,

$$\Rightarrow \frac{1}{v\Sigma_a}\frac{d\phi}{dt} = \frac{s'''}{\Sigma_a} - \phi$$

- Since both prompt and delayed neutrons control the kinetics, the source term has two components

$$s''' = k_\infty \Sigma_a \phi(1-\beta) + \lambda C$$

- The first term is the prompt neutron fraction and the second one is the rate of production of delayed neutrons.
- Since the concentration of the precursor is involved, we need to write its conservation equation.
- The neutron balance equation after substitution of the source term can be written as,

$$\frac{1}{v\Sigma_a}\frac{d\phi}{dt} = \frac{k_\infty \Sigma_a \phi(1-\beta) + \lambda C}{\Sigma_a} - \phi$$

$$\Rightarrow l_P \frac{d\phi}{dt} = k_\infty \phi(1-\beta) + \frac{\lambda C}{\Sigma_a} - \phi \qquad (1)$$

- The precursor balance equation can be written as

$$\frac{dC}{dt} = \beta k_\infty \Sigma_a \phi - \lambda C \qquad (2)$$

- Note that the first term is the source of the precursor specie. This can be appreciated from the fact that every delayed neutrons is born from one precursor nucleus.
- Eqs. (1) and (2) are the Point Kinetics governing equations.
- To obtain the solution, these are assumed of the form

$$\phi = \phi_0 e^{\omega t}, C = C_0 e^{\omega t}$$

- Substituting the solutions in Eq. (2), we get

$$C_0 \omega e^{\omega t} = \beta k_\infty \Sigma_a \phi_0 e^{\omega t} - \lambda C_0 e^{\omega t}$$

$$\Rightarrow C_0 = \frac{\beta k_\infty \Sigma_a}{\omega + \lambda}\phi_0$$

- Substituting the solutions in Eq. (1), we get

$$\Rightarrow l_P \omega \phi_0 e^{\omega t} = k_\infty \phi_0 e^{\omega t}(1-\beta) + \frac{\lambda C_0 e^{\omega t}}{\Sigma_a} - \phi_0 e^{\omega t}$$

$$\Rightarrow l_P \omega = k_\infty (1-\beta) + \frac{\lambda}{\Sigma_a}\frac{C_0}{\phi_0} - 1$$

- Substituting for C_0 / ϕ_0 , we can write

$$\Rightarrow l_P \omega = k_\infty (1-\beta) + \frac{\lambda \beta k_\infty}{\omega + \lambda} - 1$$

$$\Rightarrow l_P \omega = (k_\infty - 1) - \beta k_\infty \left(1 - \frac{\lambda}{\omega + \lambda}\right)$$

$$\Rightarrow \frac{(k_\infty - 1)}{k_\infty} = \frac{l_P \omega}{k_\infty} + \beta \frac{\omega}{\omega + \lambda}$$

- The excess k_∞ over 1 when normalised with k_∞ is defined as Reactivity and is denoted by ϱ. Thus the above Eq, can be recast as

$$\rho = \frac{l_P \omega}{k_\infty} + \beta \frac{\omega}{\omega + \lambda}$$

- By Substituting for $1/k_\infty = (1-\varrho)$, we get

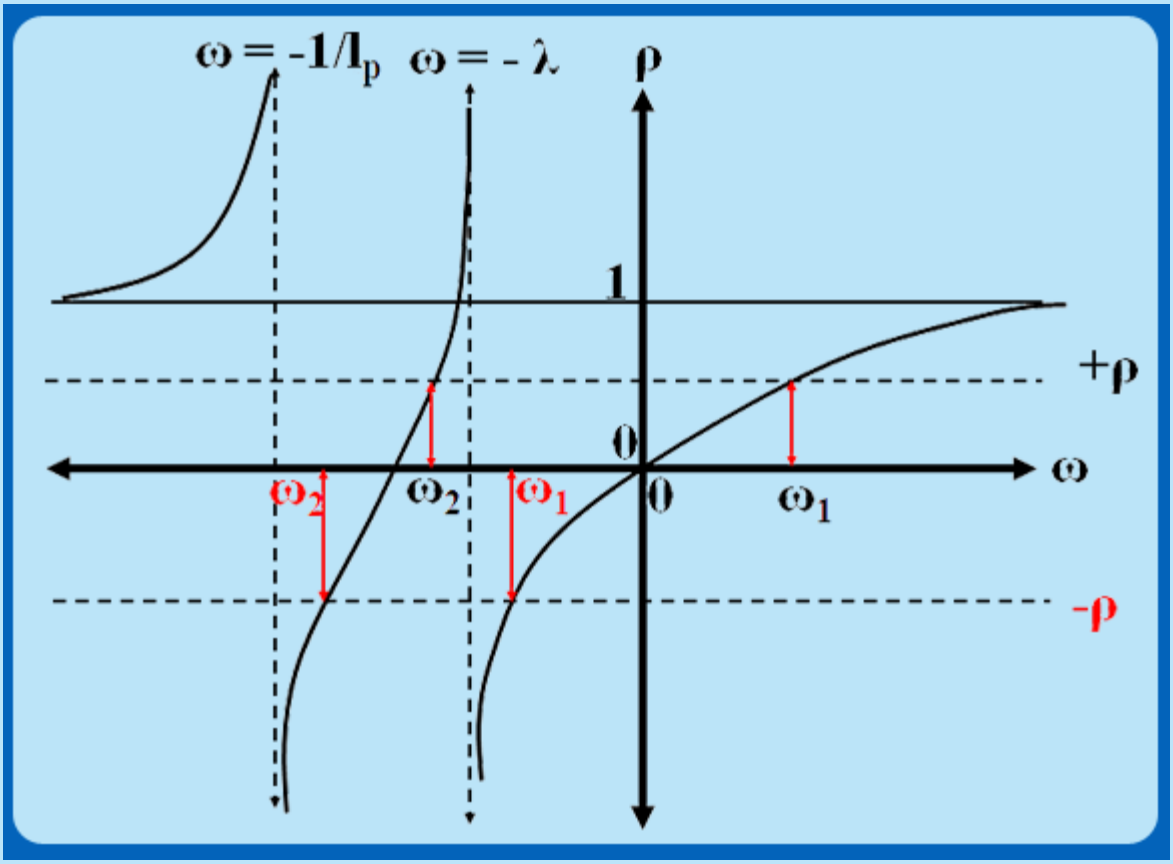
$$\rho = l_P \omega (1-\rho) + \beta \frac{\omega}{\omega + \lambda}$$

- By collecting terms of ϱ and rearranging, we get

$$\rho = \frac{\omega l_P}{1 + \omega l_P} + \frac{\beta}{\omega + \lambda}\frac{\omega}{1 + \omega l_P}$$

- Using the above equation, for a given ϱ, the values of ω can be estimated numerically.

- It is illustrative to look at its graphical solution shown below



- It has two roots ω_1 and ω_2.
- For positive reactivity addition, ω_1 is positive and ω_2 is negative.
- For negative reactivity addition both ω_1 and ω_2 are negative.
- The following points may be noted
  - As $\omega \rightarrow \pm\infty$, $\varrho \rightarrow 1$.
  - At $\omega = -1/l_p$ and $\omega = -\lambda$, ϱ blows up.
  - As ω crosses the singularity points, it changes sign.
  - For positive ϱ, one root is +ve and the other –ve
  - For negative ϱ, both roots are –ve.
  - $\varrho = 0$ when $\omega = 0$.
- We can also get approximate analytical solution as follows
- Starting from,

$$\rho = \frac{\omega l_p}{1+\omega l_p} + \frac{\beta}{\omega+\lambda}\frac{\omega}{1+\omega l_p}$$

- Noting that $\omega l_p << 1$, we can simplify the above equation as

$$\rho = \omega l_p + \frac{\beta\omega}{\omega + \lambda}$$

- Rearranging the above equation, we get

$$\Rightarrow \omega^2 + \omega\left(\frac{\beta - \rho}{l_p} + \lambda\right) - \frac{\lambda\rho}{l_p} = 0$$

$$\approx\Rightarrow \omega^2 + \omega\left(\frac{\beta - \rho}{l_p}\right) - \frac{\lambda\rho}{l_p} = 0$$

- Solving the quadratic equation,

$$\omega = \frac{-\left(\frac{\beta-\rho}{l_p}\right) \pm \left(\left(\frac{\beta-\rho}{l_p}\right)^2 + 4\frac{\lambda\rho}{l_p}\right)^{0.5}}{2}$$

$$= \frac{-\left(\frac{\beta-\rho}{l_p}\right) \pm \left(\frac{\beta-\rho}{l_p}\right)\left(1 + 4\frac{\lambda\rho}{l_p}\left(\frac{l_p}{\beta-\rho}\right)^2\right)^{0.5}}{2}$$

$$\approx \frac{-\left(\frac{\beta-\rho}{l_p}\right) \pm \left(\frac{\beta-\rho}{l_p}\right)\left(1 + 2\frac{\lambda\rho}{l_p}\left(\frac{l_p}{\beta-\rho}\right)^2\right)}{2}$$

- Taking the positive sign first,

$$\omega_1 = \frac{\left(2\frac{\lambda\rho}{l_p}\left(\frac{l_p}{\beta-\rho}\right)\right)}{2} = \frac{\lambda\rho}{\beta-\rho}$$

- Taking the negative sign next,

$$\omega_2 = -\left(\frac{\beta-\rho}{l_p}\right) - \frac{\lambda\rho}{\beta-\rho}$$

$$\omega_2 \approx -\left(\frac{\beta-\rho}{l_p}\right)$$

- For $\varrho = 0.001$, $\lambda = 0.08\ s^{-1}$, $\beta = 0.0065$, $l_p = 2 \times 10^{-4} s$

$$\omega_1 = 0.00145\ s^{-1} \qquad \omega_2 = -27.5\ s^{-1}$$

- The solutions for flux and precursor concentrations are

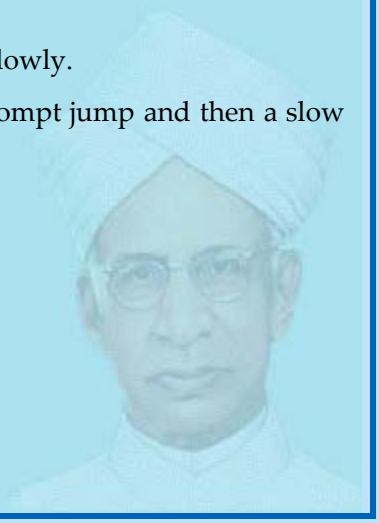
$$\phi = \phi_{10} e^{\omega_1 t} + \phi_{20} e^{\omega_2 t}, C = C_{10} e^{\omega_1 t} + C_{20} e^{\omega_2 t}$$

- The constants are derived in the next lecture.

Objectives

In this lecture you will learn the following

- In this lecture we shall look at the approximate analytical solution for one delayed group point kinetics equations.
- We will observe that the power rises very fast first and then rises slowly.
- The overall behaviour may be modelled adequately by having a prompt jump and then a slow raise.



Introduction

- We showed in the last class that the solutions for flux and precursor concentration are of the form

$$\phi = \phi_{10} e^{\omega_1 t} + \phi_{20} e^{\omega_2 t}, C = C_{10} e^{\omega_1 t} + C_{20} e^{\omega_2 t}$$

Where, $\omega_1 \approx \frac{\lambda \rho}{\beta - \rho}$ and $\omega_2 \approx -\left(\frac{\beta - \rho}{l_p}\right)$

- We also noted that ω_2 is a large negative number and ω_1 is a smaller positive number, if ϱ is a positive number.
- For $\varrho = 0.001$, $\lambda = 0.08\ s^{-1}$, $\beta = 0.0065$, $l_p = 2 \times 10^{-4} s$

$$\omega_1 = 0.0145\ s^{-1} \qquad \omega_2 = -27.5\ s^{-1}$$

Analytical Solution

- We also saw the graphical solution.
- In this lecture we shall get an approximate analytical solution that is very useful to understand the nature of the transient, though it is a bit algebra intensive.
- We shall begin with our governing equation derived last time

$$\Rightarrow l_p \frac{d\phi}{dt} = k_\infty \phi(1-\beta) + \frac{\lambda C}{\Sigma_a} - \phi \qquad (1)$$

$$\frac{dC}{dt} = \beta k_\infty \Sigma_a \phi - \lambda C \qquad (2)$$

- Before the transient was initiated, the reactor was in steady state.
- Hence we can put the time derivatives = 0, k_∞ =1 and C and ϕ shall take the values C_0 and ϕ_0 respectively.
- This would imply

$$0 = \phi_0(1-\beta) + \frac{\lambda C_0}{\Sigma_a} - \phi_0 \qquad \Rightarrow C_0 = \phi_0 \frac{\beta \Sigma_a}{\lambda}$$

- The same thing can also be obtained from the second equation,

$$\Rightarrow C_0 = \phi_0 \frac{\beta \Sigma_a}{\lambda} \qquad 0 = \beta \Sigma_a \phi_0 - \lambda C_0$$

- Our solutions are of the form

$$\phi = \phi_{10} e^{\omega_1 t} + \phi_{20} e^{\omega_2 t}, C = C_{10} e^{\omega_1 t} + C_{20} e^{\omega_2 t}$$

- Substituting the solutions in Eq. (2), we get

$$C_{10}\omega_1 e^{\omega_1 t} + C_{20}\omega_2 e^{\omega_2 t} = \beta k_\infty \Sigma_a \left(\phi_{10} e^{\omega_1 t} + \phi_{20} e^{\omega_2 t}\right) - \lambda\left(C_{10} e^{\omega_1 t} + C_{20} e^{\omega_2 t}\right)$$

- Since the above equation has to hold for any time, the coefficient of the exponentials have to be same

$$\Rightarrow C_{10}\omega_1 = \beta k_\infty \Sigma_a \phi_{10} - \lambda C_{10}$$

$$\Rightarrow C_{10} = \frac{\beta k_\infty \Sigma_a \phi_{10}}{\omega_1 + \lambda}$$

- Similarly, we can write

$$\Rightarrow C_{20} = \frac{\beta k_\infty \Sigma_a \phi_{20}}{\omega_2 + \lambda}$$

- As our solution must also satisfy the initial conditions

$$C_0 = C_{10} + C_{20}$$

- Substituting the values of the above variables, we get

$$\phi_0 \frac{\beta \Sigma_a}{\lambda} = \beta k_\infty \Sigma_a \left(\frac{\phi_{10}}{\omega_1 + \lambda} + \frac{\phi_{20}}{\omega_2 + \lambda} \right)$$

$$\Rightarrow \frac{\phi_0}{\lambda} = k_\infty \left(\frac{\phi_{10}}{\omega_1 + \lambda} + \frac{\phi_{20}}{\omega_2 + \lambda} \right) \qquad \Rightarrow \frac{1}{k_\infty \lambda} = \left(\frac{\phi_{10}}{\phi_0 (\omega_1 + \lambda)} + \frac{\phi_{20}}{\phi_0 (\omega_2 + \lambda)} \right)$$

- The same is rewritten as

$$\frac{(\omega_1 + \lambda)}{k_\infty \lambda} = \left(\frac{\phi_{10}}{\phi_0} + \frac{\phi_{20} (\omega_1 + \lambda)}{\phi_0 (\omega_2 + \lambda)} \right)$$

- Further, as the initial conditions also imply

$$\phi_0 = \phi_{10} + \phi_{20} \qquad \Rightarrow 1 = \frac{\phi_{10}}{\phi_0} + \frac{\phi_{20}}{\phi_0} \qquad \Rightarrow \frac{\phi_{10}}{\phi_0} = 1 - \frac{\phi_{20}}{\phi_0}$$

- Eliminating $\phi_{10}/\ \phi_0$ from the above two equations, we

$$\Rightarrow \frac{(\omega_1 + \lambda)}{k_\infty \lambda} - 1 = \frac{\phi_{20}}{\phi_0} \left(\frac{(\omega_1 + \lambda)}{(\omega_2 + \lambda)} - 1 \right) \qquad \Rightarrow \frac{(\omega_1 + \lambda)}{k_\infty \lambda} - 1 = \frac{\phi_{20}}{\phi_0} \left(\frac{(\omega_1 - \omega_2)}{(\omega_2 + \lambda)} \right)$$

- Substituting $k_\infty = (1/(1-\varrho))$, and rearranging, we get

$$\frac{\phi_{20}}{\phi_0} = \frac{(\omega_1 + \lambda)(1 - \rho) - \lambda}{\lambda} \left(\frac{\omega_2 + \lambda}{\omega_1 - \omega_2} \right)$$

- Neglecting ω_1 when compared with ω_2 , we can write

$$\frac{\phi_{20}}{\phi_0} = \frac{(\omega_1 + \lambda)(1-\rho) - \lambda}{\lambda}\left(-\left(1 - \frac{\lambda}{\omega_2}\right)\right)$$

- Neglecting $\lambda/\ \omega_2$ when compared with 1, we get

$$\frac{\phi_{20}}{\phi_0} = -\frac{(\omega_1 + \lambda)(1-\rho) - \lambda}{\lambda} \qquad \frac{\phi_{20}}{\phi_0} = 1 - \left(\frac{\omega_1}{\lambda} + 1\right)(1-\rho)$$

- Substituting for $\omega_1 = (\lambda\ \varrho\ /(\beta\text{-}\varrho))$, and rearranging, we get

$$\frac{\phi_{20}}{\phi_0} = 1 - \left(\frac{\lambda\rho}{(\beta-\rho)\lambda} + 1\right)(1-\rho) \Rightarrow \frac{\phi_{20}}{\phi_0} = 1 - \left(\frac{\rho}{(\beta-\rho)} + 1\right)(1-\rho)$$

$$\Rightarrow \frac{\phi_{20}}{\phi_0} = 1 - \left(\frac{\beta}{(\beta-\rho)}\right)(1-\rho)$$

- From above, we can imply that

$$\Rightarrow \frac{\phi_{10}}{\phi_0} = 1 - \frac{\phi_{20}}{\phi_0} = \left(\frac{\beta}{(\beta-\rho)}\right)(1-\rho)$$

- The term $\phi_{20}/\ \phi_0$ can be further simplified as

$$\Rightarrow \frac{\phi_{20}}{\phi_0} = 1 - \left(\frac{\beta}{(\beta-\rho)}\right)(1-\rho)$$

$$\Rightarrow \frac{\phi_{20}}{\phi_0} = \frac{\beta - \rho - \beta(1-\rho)}{\beta-\rho} = \frac{-\rho(1-\beta)}{\beta-\rho}$$

- Thus the full approximate solution is

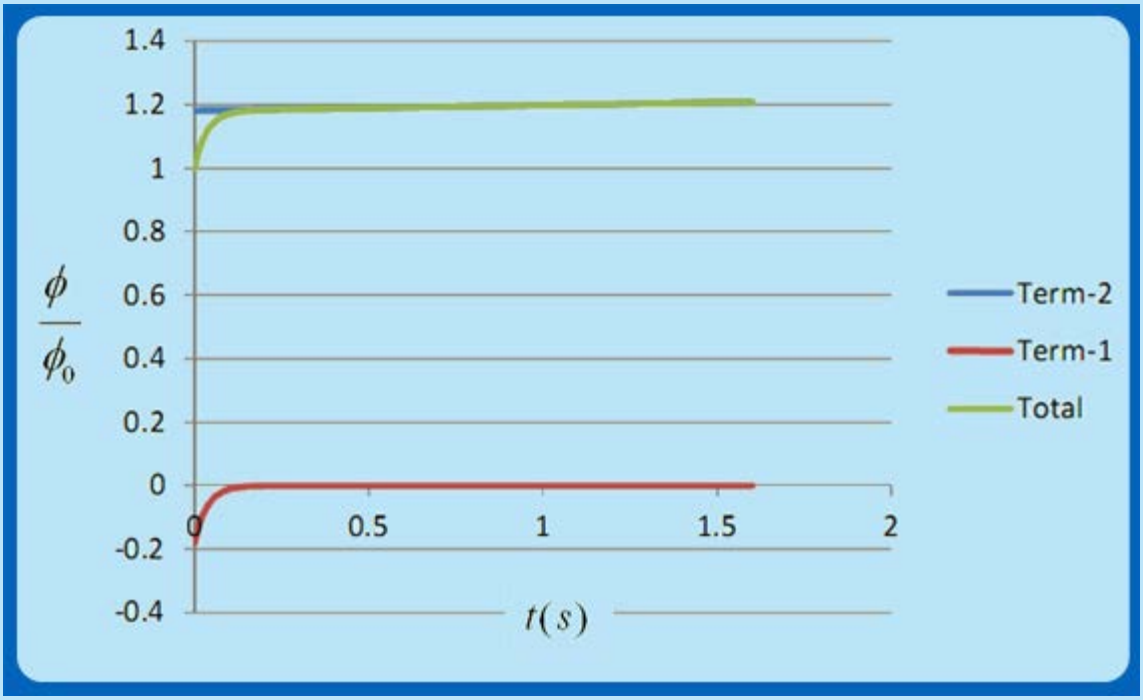
$$\Rightarrow \frac{\phi}{\phi_0} = \left(\frac{\beta(1-\rho)}{(\beta-\rho)}\right)e^{\left(\frac{\lambda\rho}{\beta-\rho}\right)t} - \frac{\rho(1-\beta)}{\beta-\rho}e^{-\left(\frac{\beta-\rho}{l_p}\right)t}$$

- Note that the first term is positive and slowly increasing function and the second term is negative and is a rapidly decaying function.
- It may be noted that as ϱ approaches β, (β-ϱ) approaches 0 and the power will rise very fast and these solution do not apply.
- At β=ϱ, or β-ϱ=0, the reactor is said to be prompt critical as delayed neutrons are no longer required for criticality.

Results for Positive ρ

- For ϱ = 0.001, $\lambda = 0.08\ s^{-1}$, $\beta = 0.0065$, $l_p = 2 \times 10^{-4} s$

$$\omega_1 = 0.0145\ s^{-1} \qquad \omega_2 = -27.5\ s^{-1}$$



- From the solution it may be seen that the second function decays very rapidly and the first term is the overall solution.
- Thus, the increase in flux is very rapid and then it raises slowly.
- Thus the power jumps promptly (almost instantaneously) and then raises very slowly.
- The value of this jump is given by

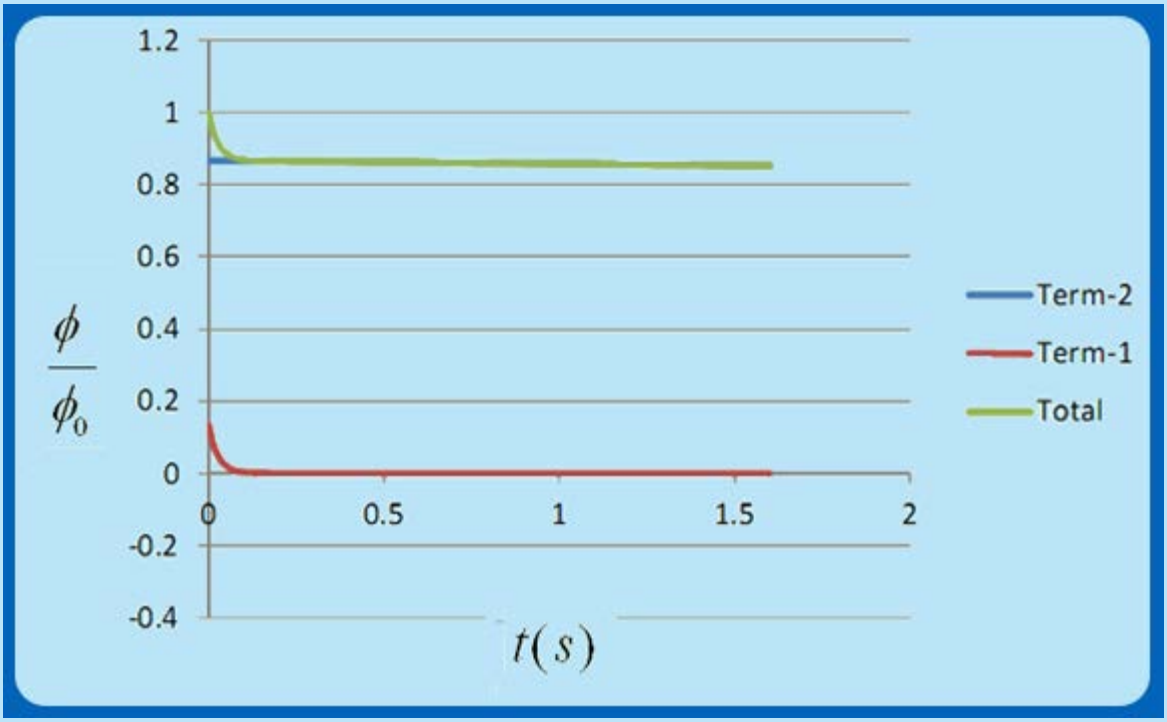
$$\Rightarrow \frac{\phi}{\phi_0} = \left(\frac{\beta}{(\beta - \rho)}\right)(1 - \rho)$$

- In the given example, the jump is 18%.

Results for Negative ρ

- For a similar case when we insert –ve reactivity.
- For $\varrho = 0.001$, $\lambda = 0.08\ s^{-1}$, $\beta = 0.0065$, $l_p = 2 \times 10^{-4} s$

$\omega_1 = -0.01067\ s^{-1}$ $\omega_2 = -37.5\ s^{-1}$



- In this case the prompt fall is to 86.7% of original power.

Prompt Jump Approximation

- The initial part of the transient can be more elegantly derived by the use of the so called prompt jump approximation.
- In this model it is assumed that the precursors do not change their concentrations and stay at their steady state values.
- Thus

$$C = C_0 = \phi_0 \frac{\beta \Sigma_a}{\lambda}$$

- The neutron balance equation can thus be written as

$$\Rightarrow l_p \frac{d\phi}{dt} = k_\infty \phi(1-\beta) + \frac{\lambda}{\Sigma_a}\frac{\phi_0 \beta \Sigma_a}{\lambda} - \phi$$
$$\Rightarrow \frac{d\phi}{dt} = \phi \frac{(k_\infty(1-\beta)-1)}{l_p} + \frac{\phi_0 \beta}{l_p}$$

- Using the integral factor, we get

$$\Rightarrow \frac{d\phi}{dt} e^{\frac{-(k_\infty(1-\beta)-1)t}{l_p}} - \phi \frac{(k_\infty(1-\beta)-1)}{l_p} e^{\frac{-(k_\infty(1-\beta)-1)t}{l_p}} = \frac{\phi_0 \beta}{l_p} e^{\frac{-(k_\infty(1-\beta)-1)t}{l_p}}$$
$$\Rightarrow \frac{d}{dt}\left[\phi e^{-\left(\frac{(k_\infty(1-\beta)-1)t}{l_p}\right)}\right] = \frac{\phi_0 \beta}{l_p} e^{-\left(\frac{(k_\infty(1-\beta)-1)t}{l_p}\right)}$$

- On integration, we get

$$\Rightarrow \frac{d}{dt}\left[\phi e^{-\left(\frac{(k_\infty(1-\beta)-1)t}{l_p}\right)}\right]_{initial}^{final} = -\frac{\phi_0 \beta}{l_p}\frac{l_p}{(k_\infty(1-\beta)-1)} e^{-\left(\frac{(k_\infty(1-\beta)-1)}{l_p}\right)t}\Bigg|_0^t$$

- Using the Initial condition $\phi = \phi_0$ at t =0

$$\Rightarrow \left(\phi e^{-\left(\frac{(k_\infty(1-\beta)-1)t}{l_p}\right)} - \phi_0\right) = -\frac{\phi_0 \beta}{(k_\infty(1-\beta)-1)}\left(e^{-\left(\frac{(k_\infty(1-\beta)-1)t}{l_p}\right)} - 1\right)$$

$$\Rightarrow \phi = \phi_0 e^{\left(\frac{(k_\infty(1-\beta)-1)t}{l_p}\right)} + \frac{\phi_0 \beta}{(1-k_\infty(1-\beta))}\left(1 - e^{\left(\frac{(k_\infty(1-\beta)-1)t}{l_p}\right)}\right)$$

- Thus, it may be seen that for $k_\infty (1-\beta) < 1$, the two exponential terms decay out very quickly

with a period

$$\Rightarrow T = \frac{l_p}{(k_\infty(1-\beta)-1)}$$

- Substituting $k_\infty = (1/(1-\varrho))$, and rearranging, we get

$$\Rightarrow T = -\frac{l_p(1-\rho)}{(\beta-\rho)} \approx -\frac{l_p}{(\beta-\rho)}$$

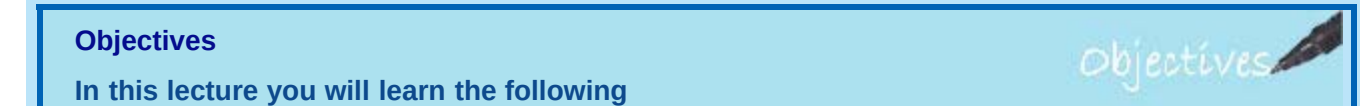
- This is exactly the inverse of ω_2.
- The prompt Jump can also be obtained by noting that the solution obtained quickly reaches the value

$$\Rightarrow \phi = \frac{\phi_0\beta}{(1-k_\infty(1-\beta))}$$

- Substituting $k_\infty = (1/(1-\varrho))$, and rearranging, we get

$$\Rightarrow \phi = \frac{\phi_0\beta(1-\rho)}{(\beta-\rho)}$$

Objectives



In this lecture you will learn the following

- In this lecture we will understand some general concepts on control.
- We will learn about reactivity coefficients and their general nature.
- Finally, we will learn about Xenon poisoning.

Control Rods

- We have understood the response of a reactor for a change in reactivity.
- Such planned reactivity insertions are accomplished by operating control rods (neutron absorbers) provided in the reactor. Typically, compounds of B, Cd, Gd, etc. are used.
- As the rods are discrete entities, their partial insertion or withdrawal changes the homogeneous character of the core composition.
- Consequently, the analysis of their influence is complex and is beyond the scope of an introductory course.
- However, to appreciate the qualitative behaviour, it is enough to consider a reactor as a point system.
- As a control rod is inserted or withdrawn, it affects the fuel utilization factor **f**.

- As $k_{\infty} = \eta f$ and $\rho = \dfrac{k_{\infty} - k_{\infty 0}}{k_{\infty}}$

$$\Rightarrow \rho = \frac{f - f_0}{f} = 1 - \frac{f_0}{f}$$

- Thus an insertion of rod increases neutron absorption in the absorber material and hence the fraction of neutrons absorbed in the fuel, viz., **f**, decreases.
- This implies ϱ is negative as $f_0 > f$.
- In the last lecture we understood that insertion of ϱ = β results in prompt criticality.
- Hence, care has to be taken that an accidental removal of one control rod should not add large enough ϱ.
- At the same time, over the life of the fuel residence time in the core of a reactor, there is a large depletion of fissile material.
- This implies we need to have a large inventory of the control material.
- This leads to a conflict that the reactor may not have space to accommodate all of the control rods.

Burnable Control Materials

- This issue is solved by employing burnable control material, at times called burnable poisons (owing to their very large absorption cross section) or by chemically dissolving control material in moderator and coolant (called chemical shim control)).
- In solid burnable poisons like Gadolinium, once it absorbs a neutron, the cross section of the product nuclei is much smaller and therefore Gadolinium said to have been burnt.
- Similarly the concentration of dissolved Boron in the form of Boric acid can be controlled to control the reactivity of the core.
- Thus by using one of the two or both schemes described above the control rod inventory and its movement can be minimised.

Reactivity Coefficients

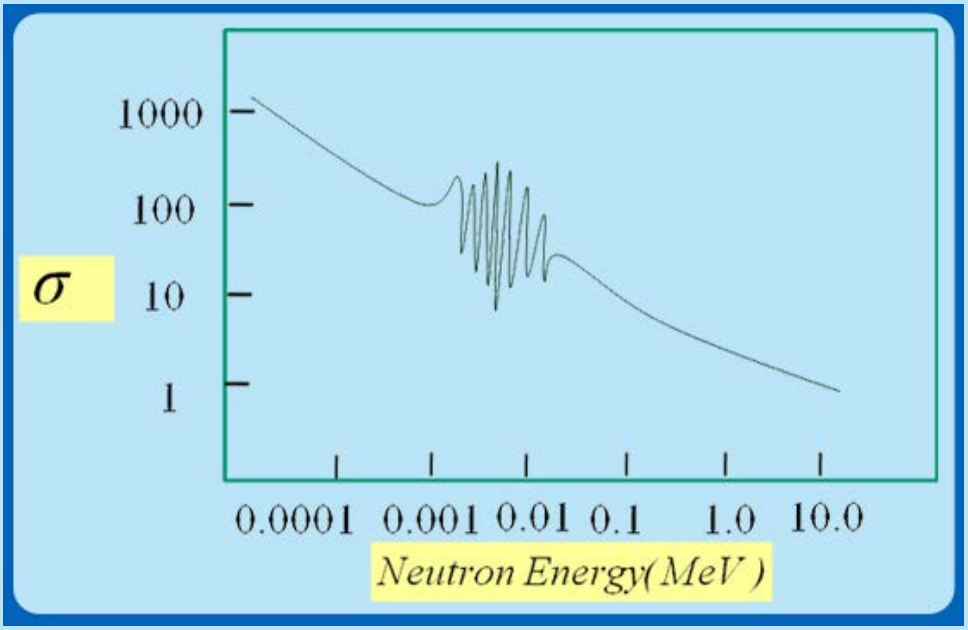
- When a reactor is operated, change of its operating state can induce reactivity changes.
- These can be due to changes in:
  - Temperature of the fuel,
  - Temperature of the moderator,
  - Voids in the coolant.
- Many different reactivity coefficients are used during specific analysis.
- They are expressed as

$$\frac{\partial \rho}{\partial T_f}, \frac{\partial \rho}{\partial T_m}, \frac{\partial \rho}{\partial \alpha}, etc.$$

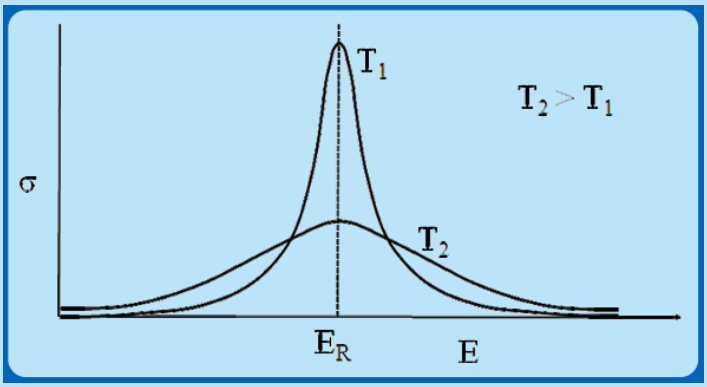
- In the above expression α is the volume fraction of void.
- Larger the reactivity coefficient, larger will be the influence of that parameter on the power.
- A positive value for a reactivity coefficient will increase reactivity change in that parameter and therefore would increase the reactor power.
- A negative value for a reactivity coefficient will decrease reactivity for a positive change in that parameter and therefore would decrease the reactor power.
- As the changes in reactivity induced are often small, the unit of %mk (0.00001) is commonly used.
- Examples of units for reactivity coefficients are, %mk/C (for the temp coefficient),%mk/(g/cm^3) (for the density coefficient), %mk/%FP (for the power coefficient; note FP stands for full power).

The Fuel Temperature Reactivity Coefficient

- This coefficient is determined by the influence of temperature on the neutron absorption cross section.
- Of particular importance is the resonance absorption cross section.
- We had discussed in our lecture on variation of cross section with energy of neutron that if the neutron has a speed which exactly matches with the resonance energy, then there is a high probability of absorption (shown below).
- Ideally, this would be a sharp line, but due to the statistical variation of energy introduced by temperature, neutrons having energy nearby the resonance energy will also be absorbed and hence the spread.



- The net effect of increasing temperature is to broaden the resonance, while at the same time reduce the peak. This is called Doppler Effect or Doppler Broadening as shown below.



- It has been shown using detailed theoretical arguments that area under the curve increases with temperature and hence the absorptions increase.

- The overall effect on reactivity will be dictated by the changes in fission and capture resonance. In general, the increase in capture is more than that of fission and hence this reactivity coefficient is generally negative.
- As fuel responds immediately to power changes, this coefficient plays a crucial role in compensating for accidental insertion of positive reactivity.
- This effect makes the power increase to be self limiting.

The Moderator Temperature Reactivity Coefficient

- The sign of moderator or coolant temperature coefficient depends on many factors.
- For water reactors, increase in temperature results in decreased density.
- This implies less collision leading to less moderation and hence increase in resonance absorption, leading to negative coefficient.
- However, reduction in density leads to reduction in direct absorption of neutrons, leading to positive coefficient.
- Particularly if chemical absorbers are added, the net effect is positive. This is often the case in PHWRs.

The Void Reactivity Coefficient

- This is of particular relevance to the BWRs, where voids are present in the core.
- An increase in void decreases the density of the moderator.
- This results in decrease of moderation leading to large reduction in thermal neutrons as well as increase in resonance absorption leading to negative coefficient.
- Though reduction of moderator density leads to reduction in direct absorption of neutrons, leading to increase in reactivity, this effect is small.
- Hence moderator void coefficient is negative.

The Power Reactivity Coefficient

- As the power of the reactor is increased, all of the effects described comes into play and decides the overall power coefficient of the reactivity.
- This reactivity coefficient is useful in characterising whether the reactor is load following.
- Let the reactor have a positive power coefficient of reactivity.
- Consider a case that due to reduction in grid load, the heat removed from the reactor is decreased.
- This will lead to increase in temperature of the system leading to an overall negative reactivity and reduction in neutron power.
- The opposite will be for an increase in power. Such systems that have positive overall power coefficient are said to be load following.

Xenon Poisoning

- Some of the fission products or daughter products of the fission products are highly neutron absorbing and are denoted as poisons.
- The most important of them are ^{135}Xe and ^{149}Sm.
- In both these cases the nuclides are directly born as fission products as well as produced by the β-decay of other parent fission products.
- The Chain for Xenon is shown below:

$$^{135}Te \xrightarrow{\beta^-,\ 11s} {}^{135}I \xrightarrow{\beta^-,\ 6.7hr} {}^{135}Xe \xrightarrow{\beta^-,\ 9.2hr} {}^{135}Cs$$

- The large absorption cross section of ^{135}Xe plays a crucial role and affects the system reactivity.
- Usually as the half life of Tellurium is very small, its yield is also added to the yield of Iodine.
- We will not worry about the chain after Cesium as it is not of our interest.
- The governing equation for Iodine (I) and Xenon (X) concentration can be represented as

$$\frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I \qquad (1)$$

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_f \phi - \lambda_X X - \sigma_X X \phi \qquad (2)$$

- In the above equations the symbols represent the following:

ϕ - nuetron flux

γ_I - yield of Iodine nuclei

γ_X- yield of Xenon nuclei

λ_I - decay constant of Iodine

λ_X- decay constant of Xenon

σ_X- absorption cross section of Xenon

Σ_f - macroscopic fission cross section

t - time

- At steady state, the time derivatives are 0 and hence we can write,

$$\gamma_I \Sigma_f \phi_{ss} - \lambda_I I_{ss} = 0 \qquad (3)$$

$$\lambda_I I_{ss} + \gamma_X \Sigma_f \phi_{ss} - \lambda_X X_{ss} - \sigma_X X_{ss} \phi_{ss} = 0 \qquad (4)$$

- From above we can write

$$I_{ss} = \frac{\gamma_I \Sigma_f \phi_{ss}}{\lambda_I} \qquad X_{ss} = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_{ss}}{\lambda_X + \sigma_X \phi_{ss}}$$

- It may be noted that While I_{ss} is directly proportional to flux, X_{ss} saturates at higher fluxes.
- The reactivity equivalent of these poisons can be estimated as follows:
- We had derived in the early part of the lecture that

$$\Rightarrow \rho = \frac{f - f_0}{f} = 1 - \frac{f_0}{f}$$

- As $f_0 = \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM}}$ and $f = \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM} + \Sigma_{aX}}$

$$\Rightarrow \rho = 1 - \frac{\Sigma_{aF} + \Sigma_{aM} + \Sigma_{aX}}{\Sigma_{aF} + \Sigma_{aM}} = -\frac{\Sigma_{aX}}{\Sigma_{aF} + \Sigma_{aM}}$$

- For the reactor to be steady before poison build up

$$k_{\infty 0} = \eta f_0 = 1 = \eta \frac{\Sigma_{aF}}{\Sigma_{aF} + \Sigma_{aM}} \Rightarrow \Sigma_{aF} + \Sigma_{aM} = \eta \Sigma_{aF} - = \nu \Sigma_f$$

- Thus we can write

$$\rho = -\frac{\Sigma_{aX}}{\Sigma_{aF} + \Sigma_{aM}} = -\frac{\Sigma_{aX}}{\nu \Sigma_f}$$

- Since $\Sigma_{aX} = \sigma_x X_{ss}$ and $X_{ss-\max} = \frac{(\gamma_I + \gamma_X)\Sigma_f}{\sigma_X}$

$$\rho_{\max} = -\frac{\Sigma_{aX}}{\nu \Sigma_f} = -\frac{(\gamma_I + \gamma_X)}{\nu}$$

- For $\gamma_I = 0.0639$ and $\gamma_x = 0.00237$, $\nu = 2.42$

$$\rho_{max} = -0.0273$$

Objectives

In this lecture you will learn the following

- In this lecture we will understand the transient build up of Xenon. This can lead to dead time in reactors.
- Xenon also induces power oscillations and must be avoided.
- We shall discuss the simplified equations for the fissile inventory depletion.

Introduction

- Towards the end of last lecture we showed that as the ^{135}Xe builds up, it introduces negative reactivity in the reactor.
- Thus, if we need to keep the reactor operating, we need to compensate for this negative reactivity by reducing control materials present in the system.
- Thus, the transient variation of the Xenon build up is of importance.
- Further, another important problem arises during operation of a reactor.
- A reactor is provided with a safety measure of control rods dropping into the reactor, should there be some malfunction and the system parameters go beyond some safe value.
- One example is the pressure of the reactor increasing due to mismatch between the power generated by fission and power removed by coolant.
- The safety measure of the action of inserting of control rods is called **Reactor Scram**.
- Being a very complex system there are several safety parameters, violation of which results in scram.
- Sometimes, the reactor can be scrammed due to some instrumentation faults, or operator can scram it due to the perception that some thing is wrong.
- Once operator realises that the scram was a mistake or due to spurious instrument reading, he or she would like to restart the reactor.
- However, during the time the reactor was shutdown, the Xenon concentration increases and imposes additional negative reactivity.
- Hence the transient build up of the Xenon concentration is of importance.
- We shall now look at this transient variation before more discussion of its consequences.

Xenon Poisoning

- In the last lecture, the governing equations were shown to be

$$\frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I \qquad (1)$$

$$\frac{dX}{dt} = \lambda_I I + \gamma_X \Sigma_f \phi - \lambda_X X - \sigma_X X \phi \qquad (2)$$

- If the reactor has operated for a good amount time, typically more than 60 hours or so, the Iodine and Xenon come to their steady state values.
- Hence, we shall consider the effect of scramming, assuming that the initial concentrations are the steady state values derived in the last class.
- Thus, at t = 0, X = X_{ss} and I = I_{ss} where,

$$I_{ss} = \frac{\gamma_I \Sigma_f \phi_{ss}}{\lambda_I} \quad X_{ss} = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_{ss}}{\lambda_X + \sigma_X \phi_{ss}}$$

- Once the reactor is scrammed, neutron flux is assumed to go to zero immediately, and hence the govering equations reduce to,

$$\frac{dI}{dt} = -\lambda_I I \qquad (3)$$

$$\frac{dX}{dt} = \lambda_I I - \lambda_X X \qquad (4)$$

- We had analysed a similar set of equations in our radioactivity lecture. Hence, we can directly obtain the solutions as,

$$I = I_{ss} e^{-\lambda_I t}$$

$$X = X_{ss} e^{-\lambda_X t} + \frac{\lambda_I I_{ss}}{\lambda_I - \lambda_X}\left[e^{-\lambda_X t} - e^{-\lambda_I t}\right]$$

- We had shown in the last class that the negative reactivity generated on account of Xenon can be written as,

$$\rho = -\frac{\Sigma_X}{\nu \Sigma_f}$$

- Substituting the expressions for X, in the last equation, we get,

$$\rho = -\frac{\sigma_X}{\nu\Sigma_f}\left(X_{ss} e^{-\lambda_X t} + \frac{\lambda_I I_{ss}}{\lambda_I - \lambda_X}\left[e^{-\lambda_X t} - e^{-\lambda_I t} \right] \right)$$

- Now substituting for I_{ss} and X_{ss}, we get

$$\rho = -\frac{\sigma_X}{\nu\Sigma_f}\left(\frac{(\gamma_I+\gamma_X)\Sigma_f\phi_{ss}}{\lambda_X+\sigma_X\phi_{ss}}e^{-\lambda_X t}+\frac{\gamma_I\Sigma_f\phi_{ss}}{\lambda_I-\lambda_X}\left[e^{-\lambda_X t}-e^{-\lambda_I t}\right]\right)$$

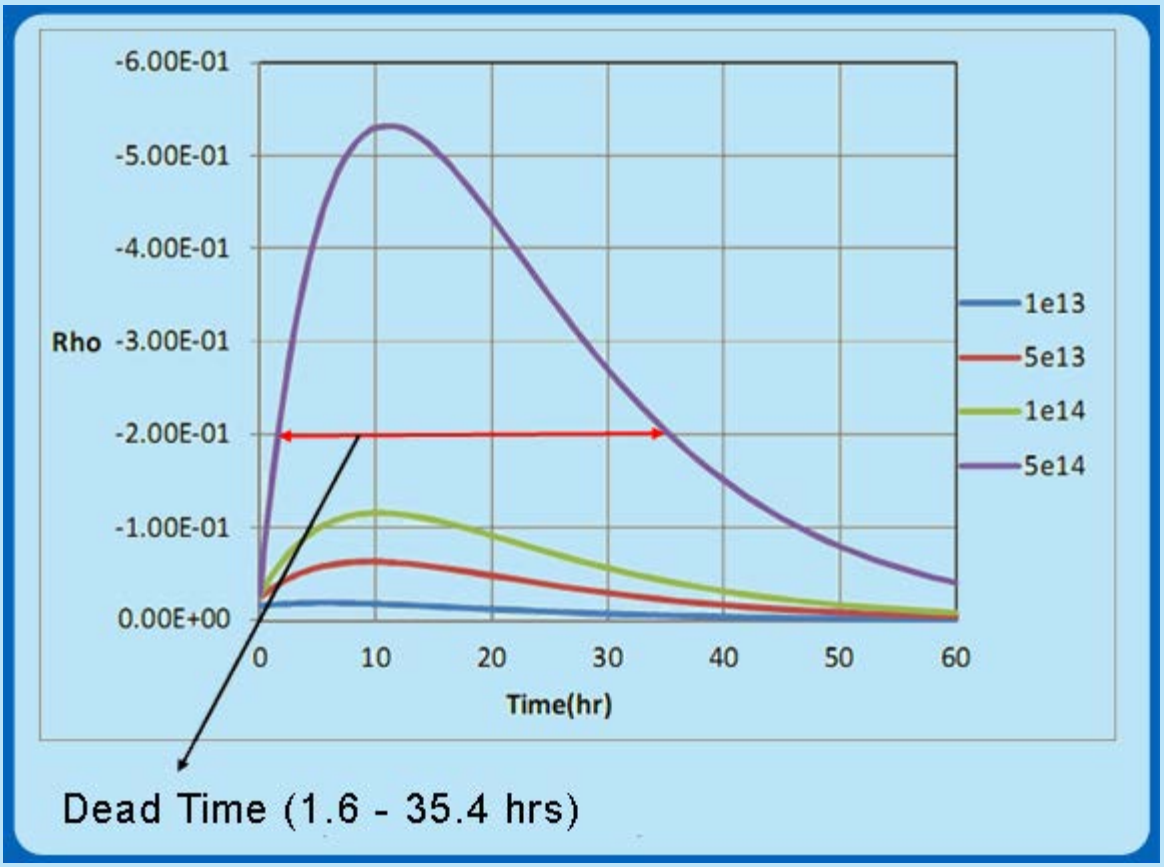
$$\Rightarrow \rho = -\frac{\sigma_X}{\nu}\left(\frac{(\gamma_I+\gamma_X)\phi_{ss}}{\lambda_X+\sigma_X\phi_{ss}}e^{-\lambda_X t}+\frac{\gamma_I\phi_{ss}}{\lambda_I-\lambda_X}\left[e^{-\lambda_X t}-e^{-\lambda_I t}\right]\right)$$

- Often, the above equation is written in the following form

$$\Rightarrow \rho = -\frac{1}{\nu}\left(\frac{(\gamma_I+\gamma_X)\phi_{ss}}{\phi_X+\phi_{ss}}e^{-\lambda_X t}+\frac{\gamma_I\phi_{ss}}{\phi_I-\phi_X}\left[e^{-\lambda_X t}-e^{-\lambda_I t}\right]\right)$$

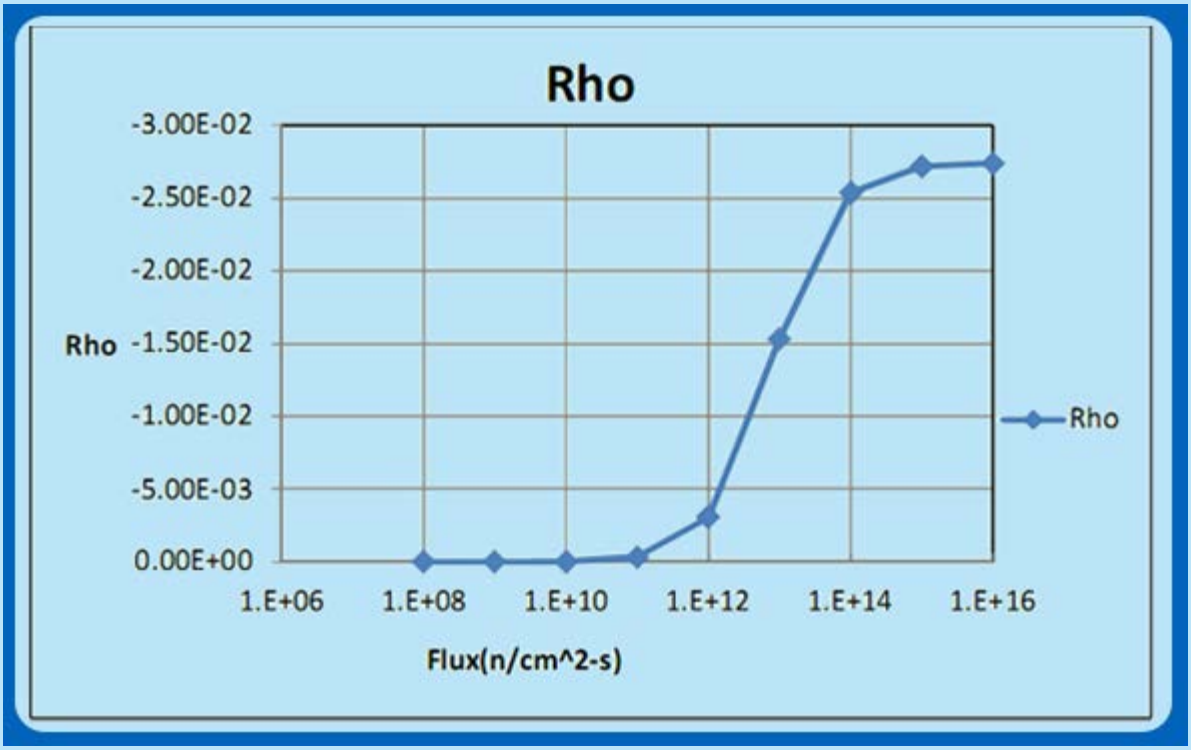
$$\text{where } \phi_I = \frac{\lambda_I}{\sigma_x},\quad \phi_X = \frac{\lambda_X}{\sigma_X}$$

- For $\lambda_I = 2.87 \times 10^{-5}\ s^{-1}$, $\lambda_X = 2.09 \times 10^{-5}\ s^{-1}$, $\phi_I = 1.08 \times 10^{13}\ cm^{-2}\ s^{-1}$, $\phi_X = 7.89 \times 10^{12}\ cm^{-2}\ s^{-1}$, $\gamma_I = 0.0639$, $\gamma_X = 0.00237$, the variation of negative reactivity with time for various steady state fluxes (expressed in nuetrons/$cm^{-2}\ s^{-1}$) are given below.
- Note that it reaches a maximum and then decreases as shown in the figure

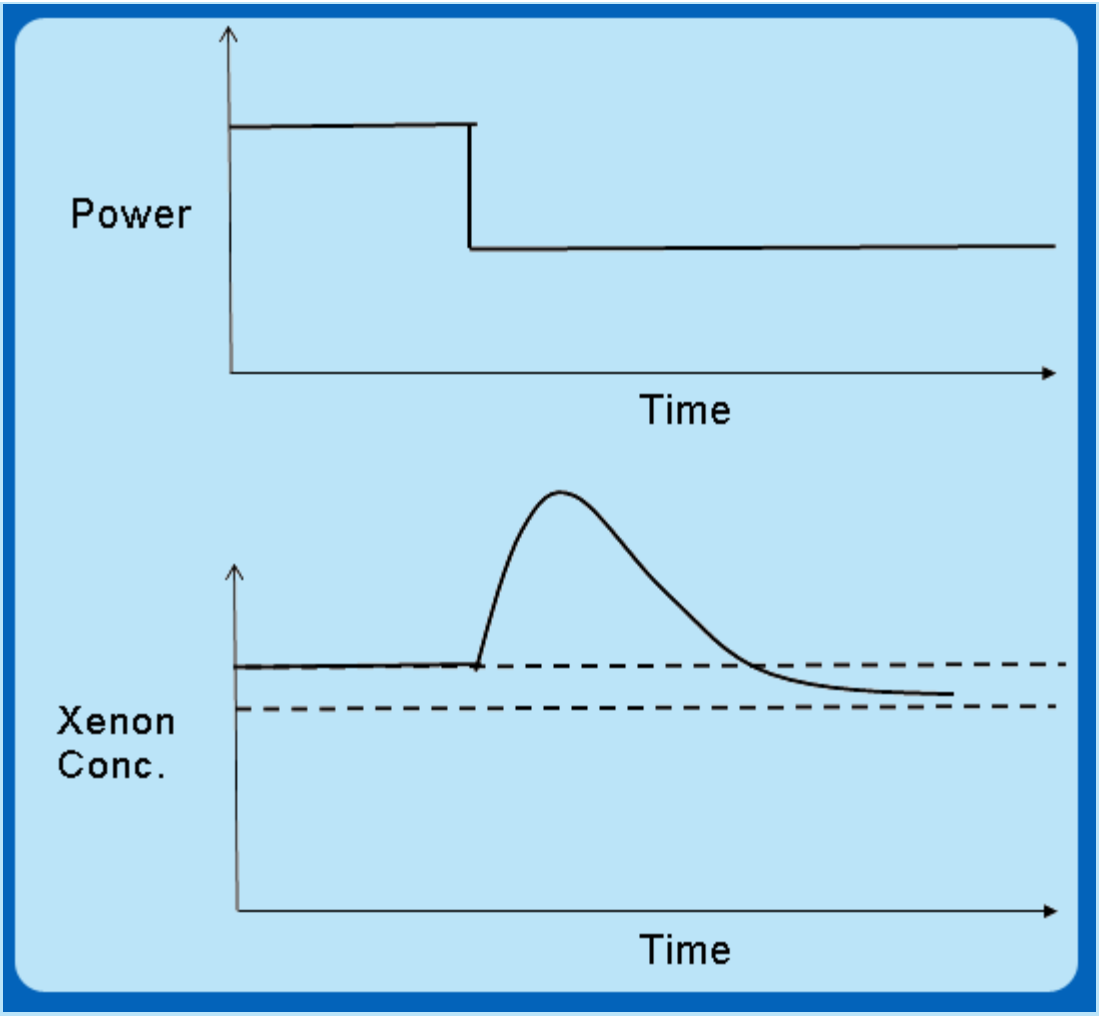


- If the reactor does not have sufficient reserve reativity, then the reactor cannot be restarted.
- Consider a reactor that has a reserve reactivity of 0.2 and operating at a flux of $5x10^{14}$ neutron/ cm^{-2} s^{-1}.
- If after scram, the reactor is not restarted before 1.6 hrs, the reactor cannot be restarted till a time of 35.4 hrs.
- This is because the reactivity penalty due to Xenon build up is more than the reserve reactivity.
- This state of the reactor is called dead state. The dead time for the above example is 35.4 - 1.6 = 33.8 hrs.

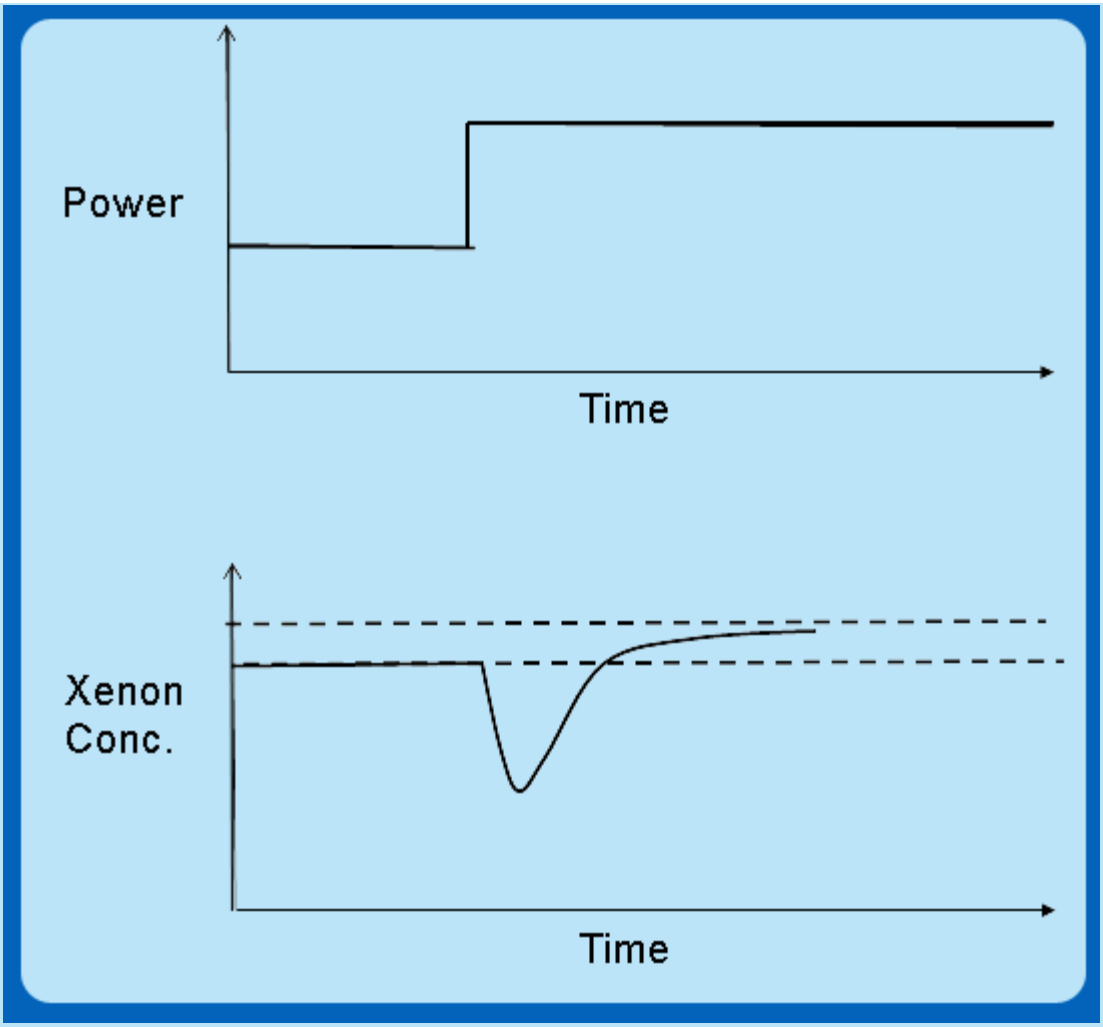
- Since $X_{ss} = \frac{(\gamma_I + \gamma_X)\Sigma_f \phi_{ss}}{\lambda_X + \sigma_X \phi_{ss}}$ and $\rho = -\frac{\Sigma_{aX}}{\nu\Sigma_f}$, we can plot ρ at equilibrium Xenon concentration.
- Note that it depends on steady state flux.
- However, as ϕ_{ss} increases such that $\lambda_X \ll \sigma_X \phi_{ss}$, then dependence on flux vanishes.
- This can be observed in the figure shown below.



- It is interesting to see how the reactor responds for a decrease in power as shown below.



- As the power is decreased, the concentration of Xe first increases due to its depletion on account of increased neutron reactions and then decreases due to the decreased yield of Iodine and Xenon.
- Since the flux will be lower, the settling equilibrium concentration is lower than the original equilibrium value.
- For a step power increase the trends will be opposite to what was described.
- This is shown below


Power
Time
Xenon
Conc.
Time

Xenon Oscillations

- In large power reactors, where the reactor size is also large, various regions are loosely coupled.
- This implies that different parts of the reactor have lesser influence on each other.
- While clear understanding of the mechanism is fairly involved, simple explanation can be offered as follows.
- Let one part of the core has an increase in power due to some perturbation or disturbance.
- As the power increases on one side, the reactor control system would try to maintain the average power and this leads to reduction of power in some other part.
- As we had just seen that increase of power, initially decreases the xenon concentration and vice versa.
- This would imply that the part in which the power was rising will increase further and the opposite will happen on the other side.
- This is self limiting and the trend eventually reverses due to complex coupling and the region that had increasing power now starts decreasing and the opposite on the other side.
- Thus if not properly controlled, this oscillation continues indefinitely, creating hot spots and other undesirable effects.
- Such reactors need what is called zonal control and needs special control philosophy.
- Such measures have been very effective and large reactors operate without the xenon oscillation problem.

Fuel Burn Up

- Fuel Burn Up involves keeping track of the fissile inventory in the reactor.
- The depletion changes fuel utilization (f) and consequently adds to negative reactivity.
- We have discussed two transients earlier. These were due to control rod movements and poison build up.
- These have time scales in several minutes for the former and several hours for the latter.
- The time scales for the fuel depletion is very large typically in months.
- Full treatment is very complex and let us see some very simple analysis using one group.
- The fissile inventory primarily consists of ^{235}U and ^{239}Pu.
- The depletion of ^{235}U can be viewed as follows

$$\frac{d\left(^{235}U\right)}{dt} = -\sigma_{a-235}\,{}^{235}U\phi$$

- In the above equation ^{235}U is the concentration of U-235, $\sigma_{\alpha\text{-}235}$ is the absorption cross section of U-235.
- The solution for this equation can be written as

$$^{235}U(t) = {}^{235}U(0)e^{-\sigma_{a-235}\,\phi\,t}$$

- Similarly, $^{238}U(t) = {}^{238}U(0)e^{-\sigma_{a-238}\,\phi\,t}$.
- Though ^{238}U is not fissile, it contributes to the formation of ^{239}Pu. Hence has been considered.
- Although there is a time delay between ^{238}U neutron absorption and the formation of ^{239}Pu, it is ignored as the time scale of Pu depletion is very large. Hence we can write

$$\frac{d\left(^{239}Pu\right)}{dt} = \left(\sigma_{a-238}\,{}^{238}U - \sigma_{a-239}\,{}^{239}Pu\right)\phi$$

- The solution of above equation is straight forward and left as an exercise.

Objectives

In this lecture you will learn the following

- We shall solve some problems to illustrate the theory discussed in last few lectures.

Problem 1

The core of an infinite planar thermal reactor consists of a solution of Pu^{239} and H_2O (density 1 g/litre) with the Pu concentration at 8.5 g/litre. The core is reflected on both sides by infinitely thick H_2O reflectors. Calculate, (a) the critical thickness of the core, (b) reflector savings, and (c) the critical mass in g/cm^2. Use one speed theory. Use the properties given below

Material	σ_a (barns)	D(cm)	$L^2(cm^2)$	η
Pu^{239}	1011.3			2.108
H2O	0.604	0.16	8.1	

$$N^{H_2O} = \frac{N_{avogadro}\rho_{H_2O}}{M} = \frac{6.023\times10^{23}\times1}{18}$$

$$= 3.346\times10^{22} \quad no/cm^3$$

$$\Sigma_a^{H_2O} = N^{H_2O}\sigma_a^{H_2O} = 3.346\times10^{22} \times 0.604\times10^{-24}$$

$$= 0.0197 \quad cm^{-1}$$

$$N^{Pu} = \frac{N_{avogadro}\rho_{Pu}}{M} = \frac{6.023\times10^{23}\times8.5\times10^{-3}}{239}$$

$$= 2.142\times10^{19} \quad no/cm^3$$

$$\Sigma_a^{Pu} = N^{Pu}\sigma_a^{Pu} = 2.142\times10^{19} \times 1011.3\times10^{-24}$$

$$= 0.02166 \quad cm^{-1}$$

$$f = \frac{\Sigma_a^{Pu}}{\Sigma_a^{Pu}+\Sigma_a^{H_2O}} = \frac{0.02166}{0.02166+0.0197} = 0.523$$

$$\eta = 2.108$$

$$\Rightarrow k_\infty = \eta f = 2.108\times0.523 = 1.1039$$

$$\Rightarrow L^2_{mix} = L^2_{\text{mod}}(1-f) = 8.1\times(1-0.523)$$

$$= 3.864 \quad cm^2$$

$$\Rightarrow {B_m}^2 = \frac{k_\infty - 1}{{L^2}_{mix}} = \frac{0.1039}{3.864} = 0.02689\, cm^2$$

For an unreflected system, we can find the size as follows

$$\Rightarrow \left(\frac{\pi}{a}\right)^2 = 0.02689\ cm^2 \ \Rightarrow a = 19.2\, cm$$

For a reflected system, It was shown that buckling is given by the transcendental equation

$$B\tan(Ba) = \frac{D_R}{L_R D_C}$$

Since in a critical system B = B_m, we can write

$$\Rightarrow B_m a = \tan^{-1}\left(\frac{D_R}{B_m L_R D_C}\right) \tan(B_m a) = \frac{D_R}{B_m L_R D_C}$$

In our system $D_C \approx D_R$, We can write

$$\Rightarrow B_m a = \tan^{-1}\left(\frac{1}{B_m L_R}\right)$$

Substituting the values for the terms, we get

$$\Rightarrow a = \frac{1}{\sqrt{0.02689}} \tan^{-1}\left(\frac{1}{\sqrt{0.02689}\sqrt{8.1}}\right)$$

$$\Rightarrow a = \frac{1}{0.1640} \tan^{-1}\left(\frac{1}{0.4667}\right) = 6.915\, cm$$

$$a = 6.915 cm$$

Reflector savings = 19.2 – 6.91 = 12.3 cm

Critical Mass = Volume/area x density = a x ϱ

= 6.91 cm^3/cm^2 x 8.5 g-Pu/litre x 1 litre/1000 cm^3

= 58.73 mg-Pu/cm^2

Problem-2

Fifty cents of reactivity is suddenly introduced into a reactor fueled with ^{235}U. What is the period of the reactor using one group approach? Use $\lambda = 0.08\ s^{-1}$

$$T_p = \frac{\beta - \rho}{\lambda \rho} = \frac{\beta - 0.5\beta}{\lambda 0.5\beta} = \frac{1}{\lambda}$$

Since $\lambda = 0.08\ s^{-1}$, we can write

$$T_p = \frac{1}{\lambda} = \frac{1}{0.08} = 12.5s$$

Problem-3

A ^{235}U fueled reactor originally operating at a power of 1 MW is scrammed by the instantaneous insertion of 5 dollars in negative reactivity. Approximately how long does it take the power level to drop to 1 milliwatt?

Considering one group approach is adequate, the values of parameters for ^{235}U are as follows:

$\lambda = 0.08\ s^{-1}$, $\beta = 0.0065$

- The prompt jump (PJ) can be estimated as

$$PJ = \frac{\beta(1-\rho)}{\beta-\rho} = \frac{\beta(1-(-5\beta)}{\beta-(-5\beta)} = \frac{\beta(1+5\beta)}{6\beta} = \frac{1+5\beta}{6}$$

$$PJ = \frac{1+5X0.0065}{6} = 0.1721$$

For longer time, the power variation can be expressed as:

$$\Rightarrow \frac{P}{P_0} = \left(\frac{\beta(1-\rho)}{(\beta-\rho)}\right) e^{\left(\frac{\lambda\rho}{\beta-\rho}\right)t}$$

$$\Rightarrow \frac{P}{P_0} = PJ\ \exp\left(\frac{t}{T_P}\right) \Rightarrow \ln\left(\frac{P}{P_0 PJ}\right) = \frac{t}{T_P}$$

T_P can be computed as

$$T_P = \frac{\beta-\rho}{\lambda\rho} = \frac{6\beta}{\lambda(-5\beta)} = -\frac{6}{5\lambda} = -\frac{6}{5x0.08} = -15s$$

$$\Rightarrow \ln\left(\frac{10^{-3}}{10^6 x0.1721}\right) = \frac{t}{-15}$$

$$\Rightarrow t = -15\ln\left(\frac{10^{-3}}{10^6 x0.1721}\right) = -15x-18.96 = 284.4s$$

Problem-4

When a certain research reactor operating at a power of 2.7 megawatts is scrammed, it is observed that the power drops to a level of 1 watt in 15 minutes. How much reactivity was inserted when the reactor was scrammed?

As the power rise with time is a non-linear function of reactivity, as given below, it needs an iterative solution.

$$\Rightarrow \frac{P}{P_0} = \left(\frac{\beta(1-\rho)}{(\beta-\rho)}\right) e^{\left(\frac{\lambda\rho}{\beta-\rho}\right)t}$$

The first guess can be obtained by assuming PJ = 1

$$\ln\left(\frac{P}{P_0 PJ}\right) = -\frac{t}{T_p} \Rightarrow \ln\left(\frac{1}{2.7\times10^6}\right) = -\frac{900}{T_p}$$

$$\Rightarrow T_p = \frac{900}{\ln\left(\frac{1}{2.7\times10^6}\right)} = 60.77s$$

The approximate T_p obtained and be used to get an approximate ϱ as follows

$$T_p = \frac{\beta-\rho}{\lambda\rho}$$

$$\Rightarrow \rho = \frac{\beta}{1+\lambda T_p} = \frac{0.0065}{1-0.08\times60.77}$$

$$= 1.683\times10^{-3}$$

The final results obtained by using Goalseek function of Excel is shown below

Po(W)	2.70E+06
P(W)	1
t(s)	900
Beta	0.0065
lambda(1/s)	0.08
rho(assumed)	-0.00133688
prompt jump	0.080021381
period(s)	-73.2757002
P/Po(assumed)	3.70712E-07
P/P0 (actual)	3.7037E-07
difference/actual	9.21E-04

←Answer

Problem-5

An infinite reactor consists of a homogeneous mixture of ^{235}U and H_2O. The fuel concentration is 5 percent smaller than that required for criticality. What is the reactivity of the system?

For a critical reactor

$$\Rightarrow f = \frac{1}{\eta_{U-235}} = \frac{1}{2.068} = 0.4836 \quad k_\infty = \eta f = 1$$

$$f_0 = \frac{\Sigma_{a0}^{Fuel}}{\Sigma_{a0}^{Fuel} + \Sigma_{a0}^{Mod}} = \frac{1}{1 + \frac{\Sigma_{a0}^{Mod}}{\Sigma_{a0}^{Fuel}}} = 0.4836$$

$$\Rightarrow \frac{\Sigma_{a0}^{Mod}}{\Sigma_{a0}^{Fuel}} = 1.068$$

For the system in question

$$\Sigma_a^{Fuel} = 0.95 \; \Sigma_{a0}^{Fuel} \quad \Sigma_a^{Mod} = \Sigma_{a0}^{Mod}$$

$$\Rightarrow f = \frac{1}{1 + \frac{\Sigma_a^{Mod}}{\Sigma_a^{Fuel}}} = \frac{1}{1 + \frac{\Sigma_{a0}^{Mod}}{0.95\Sigma_a^{Fuel}}} = \frac{1}{1 + \frac{1.068}{0.95}} = 0.4708 \qquad \rho = 1 - \frac{f_0}{f} = 1 - \frac{0.4836}{0.4708} = -0.02719$$

Problem-6

A ^{235}U fueled reactor operating at a thermal flux of 5 X 10^{13} neutrons/cm^2-sec is scrammed at a time when the reactor has 5 percent in reserve reactivity. Compute the time to the onset of the dead time and its duration.

In the lecture we had shown that

$$\Rightarrow \rho = -\frac{1}{\nu}\left(\frac{(\gamma_I + \gamma_X)\phi_{ss}}{\phi_X + \phi_{ss}} e^{-\lambda_X t} + \frac{\gamma_I \phi_{ss}}{\phi_I - \phi_X}\left[e^{-\lambda_X t} - e^{-\lambda_I t} \right] \right)$$

$$\text{Where } \phi_I = \frac{\lambda_I}{\sigma_x}, \quad \phi_X = \frac{\lambda_X}{\sigma_X}$$

For λ_I = 2.87 x 10^{-5} s^{-1}, λ_X = 2.09 x 10^{-5} s^{-5}, φ_I = 1.08 x 10^{13} cm^{-2} s^{-1}, φ_X = 7.89 x 10^{12} cm^{-2} s^{-1}, γ_I = 0.0639, γ_X = 0.00237,

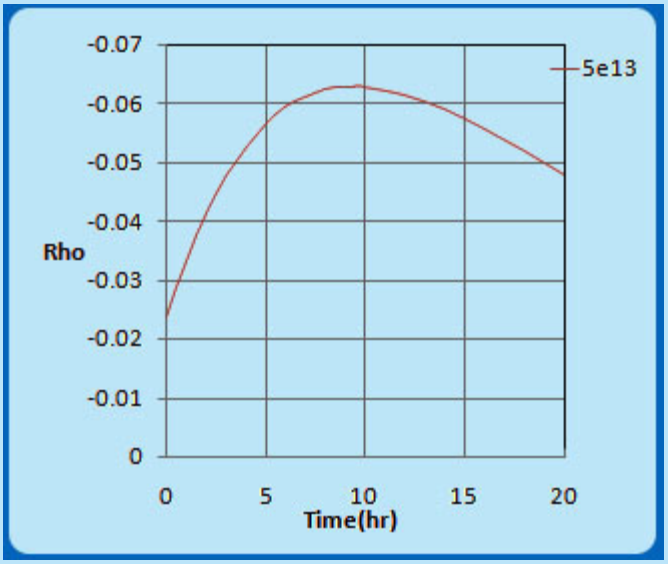
Since the expression is complex, the solution has to be obtained numerically. This was done in Excel.

The onset time = 3.41 hr

The end time = 19.03 hr

Dead time = 19.03-3.41 = 15.6 hrs.

The variation of reactivity with time is shown below.



Problem-7

Gadolinium -157 is a stable nuclide having an absorbtion cross section at 0.0253 eV of 240,000b. It is formed from the decay of the fisson product ^{157}Sm according to the following chain.

$$\underset{\text{Fission}}{\overset{\uparrow}{^{157}Sm}} \xrightarrow[0.5m]{\beta^-} {}^{157}Eu \xrightarrow[15.2h]{\beta^-} {}^{157}Gd.$$

Neither ^{157}Sm nor ^{157}Eu absorbs neutrons to a significant extent.

The ^{235}U fission yeild of ^{157}Sm is $7X10^{-5}$ atoms per fission.

(a) What is the equilibrium reactivity tied up in ^{157}Gd in a reactor having an average thermal flux of 2.5×10^{13} neutrons/ cm^2-sec?

(b) What is the maximum reactivity due to this nuclide after the shutdown of the reactor in part (a)?

Solution

Since half life of Sm is too small compared to that of Eu, It is adequate to assume that Eu is directly born

$$\frac{dEu}{dt} = \gamma_{Sm}\Sigma_f\phi - \lambda_{Eu}Eu$$

$$\frac{dGd}{dt} = \lambda_{Eu}Eu - \sigma_{a-Gd}Gd\ \phi$$

At steady state $Eu_\infty = \dfrac{\gamma_{Sm}\Sigma_f\phi}{\lambda_{Eu}}$

$$Gd_\infty = \frac{\lambda_{Eu}Eu_\infty}{\sigma_{a-Gd}\phi}$$

From the equations derived given above, we can write

$$\Sigma_{a-Gd_\infty} = \gamma_{Sm}\Sigma_f$$

From the class discussion, we note that

$$\rho = -\frac{\Sigma_{poison}}{\nu\Sigma_f} \qquad \Rightarrow \rho_{Gd-\infty} = -\frac{\gamma_{Sm}}{\nu} = -\frac{7x10^{-5}}{2.42} = -2.893x10^{-5}$$

Maximum reactivity after shutdown shall be due the conversion of Eu into Gd. Thus total Gd shall be

$$Gd_{\max} = Gd_\infty + Eu_\infty = \frac{\gamma_{Sm}\Sigma_f}{\sigma_{a-Gd}} + \frac{\gamma_{Sm}\Sigma_f\phi}{\lambda_{Eu}}$$

$$\Rightarrow Gd_{\max} = \frac{\gamma_{Sm}\Sigma_f}{\sigma_{a-Gd}}\left(1+\frac{\phi}{\lambda_{Eu}/\sigma_{a-Gd}}\right)$$

$$\Rightarrow \rho_{\max} = -\frac{\gamma_{Sm}}{\nu}\left(1+\frac{\phi}{\lambda_{Eu}/\sigma_{a-Gd}}\right)$$

$$\Rightarrow \rho_{\max} = -2.895x10^{-5}\left(1+\frac{2.5x10^{13}}{0.693/15.2x3600x2.4x10^{-19}}\right)$$

$$\Rightarrow \rho_{\max} = -2.895x10^{-5}(1+0.4737)$$

$$\Rightarrow \rho_{\max} = -4.266x10^{-5}$$

Objectives

In this lecture you will learn the following

- We shall summarise the principles used in fluid mechanics and heat transfer.
- It is assumed that the student has already been exposed to courses in heat transfer and fluid flow.

Fluid Flow in Circular ducts

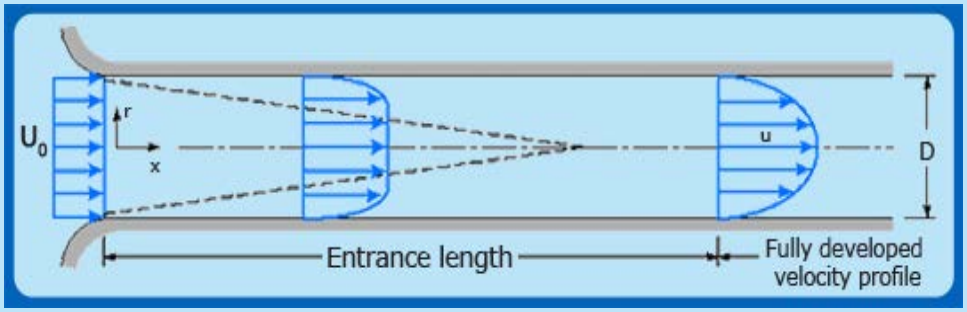
- Flow and heat transfer inside circular passages are the most common applications of Fluid Mechanics and Heat transfer.
- Flow and heat transfer in between rod bundles are the most common application in Nuclear Engineering.
- The Objective of this lecture is to review relevant portions of fluid flow and heat transfer so that we can solve some simple problems in nuclear reactors.
- Before reading the chapter, the students are advised to refresh their concepts in fluid mechanics and heat transfer studied prior to this course.
- When velocity of fluid increases beyond a critical value, several whirls called vortices are formed.
- This is called Turbulent Flow. In this case the velocity and temperature continuously fluctuate with time.
- The transition to turbulent is governed by a non-dimensional quantity called the Reynolds number.

$$\mathrm{Re} = \frac{\rho \bar{V} d}{\mu}$$

$\bar{V}$ – Velocity (m/s)
ϱ – Density (kg/m^3)
d – Diameter (m)
μ –Dynamic Viscosity (Ns/m^2)

- Its value in circular ducts is typically 2300.
- Now we shall review analysis of fully developed laminar flows in circular ducts.
- The approach is to integrate the Navier-Stokes equations. Prior to the analysis let us understand the concept of fully developed flows.

Fully Developed Flow



- In the above figure the fluid velocity profile is shown at different sections in a pipe.
- It has a flat profile at the entrance and develops into a fully developed profile after some distance, called entrance length.
- Fully developed flow implies that the velocity profile does not change along length.
- Detailed analysis indicates that the non-dimensional entrance length (L_h /D) is ~ 0.06 Re.
- This is small in turbulent flow (L_h /D ~ 6-10). The subscript h is used to denote hydrodynamic entrance length.
- Since velocity profile is same, it implies that wall shear stress is same.

Laminar Flow in Pipes

- For axi-symmetric, incompressible, steady flow V = V(r,z).
- If the flow is also fully developed, we can write the following:

Continuity Equation

$$\frac{1}{r}\frac{\partial(rV_r)}{\partial r}+\frac{\partial V_z}{\partial z}=0$$

- V_r and V_z represent the velocities in r and z directiions respectively.
- Since flow is fully developed, $\frac{\partial V_z}{\partial z}=0$

$$\Rightarrow \frac{1}{r}\frac{\partial(rV_r)}{\partial r}$$

- We can calculate the following:
  - rV_r is independent of r.
  - Since V_r at r = R is 0, V_r is 0 everywhere.
  - Hence there is only V_z , and V_z is only a function of r .

r - Momentum Equation

$$\rho\left(V_r\frac{\partial V_r}{\partial r}+V_z\frac{\partial V_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left(\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(rV_r)}{\partial r}\right)+\frac{\partial^2 V_r}{\partial z^2}\right)$$

- Since V_r is zero everywhere, we can write $\frac{\partial p}{\partial r}=0$.
- This implies that p is only a function of z

$$\Rightarrow\frac{\partial p}{\partial z}=\frac{dp}{dz}$$

z - Momentum Equation

$$\rho\left(V_r\frac{\partial V_z}{\partial r}+V_z\frac{\partial V_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)+\frac{\partial^2 V_z}{\partial z^2}\right)$$

- Since V_r, $\frac{\partial V_Z}{\partial z}$ are zero,

$$\Rightarrow\frac{\partial p}{\partial z}=\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)$$

- The left hand side (LHS) of the above equation is only a function of z and the right hand side (RHS) is only a function of r.
- This demands that both LHS and RHS be each equal to a constant.

$$\Rightarrow\frac{dp}{dz}=\mu\frac{1}{r}\frac{d}{dr}\left(r\frac{dV_Z}{dr}\right)=Constant$$

Transposing r, we can write

$$\frac{d}{dr}\left(r\frac{dV_z}{dr}\right)=r\frac{1}{\mu}\frac{dp}{dz}$$

Integrating once with r

$$r\frac{dV_z}{dr}=\left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r^2}{2}+C_1$$

Using the boundary condition that flow is symmetric as r→0

$$\Rightarrow\left.\frac{\partial V_Z}{\partial r}\right|_{r=0}=0$$

Hence $C_1=0$

$$\therefore r\frac{dV_z}{dr}=\left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r^2}{2}$$

Transposing r, we can write

$$\therefore \frac{dV_z}{dr} = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r}{2}$$

Integrating with r

$$V_z = \left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{r^2}{4} + C_2$$

Using the boundary condition $V_z = 0$ at r = R

$$C_2 = -\left(\frac{1}{\mu}\frac{dp}{dz}\right)\frac{R^2}{4}$$

$$\therefore V_z = \frac{1}{4\mu}\left(-\frac{dp}{dz}\right)\left(R^2 - r^2\right)$$

$$= \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Hence $V_z = \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(1 - \frac{r^2}{R^2}\right)$

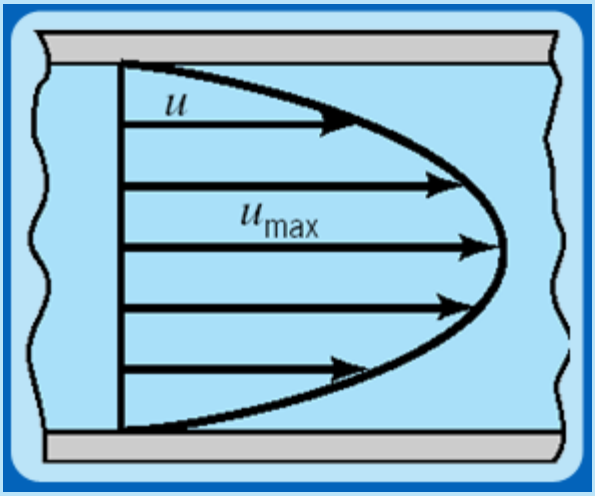
- Velocity distribution is parabolic

$V_z = V_z$ (max) at r = 0

$$\therefore V_z(\max) = \frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)$$

$$\Rightarrow V_z = V_z(\max)\left(1-\frac{r^2}{R^2}\right)$$

- The velocity distribution is sketched below.



Average Velocity

- The average velocity can be obtained as follows.

$$\bar{V}_z = \frac{1}{\pi R^2}\int_0^R v_z\, 2\pi r\, dr \quad \frac{2\pi}{\pi R^2}\int_0^R v_z\, r\, dr = \frac{2}{R^2}\int_0^R v_z\, r\, dr$$

$$\bar{V}_z = \frac{2V_z(\max)}{R^2}\int_0^R \left[1 - \frac{r^2}{R^2}\right] r\, dr = \frac{2V_z(\max)}{R^2}\int_0^R \left[r - \frac{r^3}{R^2}\right] dr$$

$$\bar{V}_z = \frac{2V_z(\max)}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$

$$\bar{V}_z = \frac{2V_z(\max)}{R^2}\frac{R^2}{4} = \frac{V_z(\max)}{2}$$

$$= \frac{-R^2}{8\mu}\frac{dp}{dz}$$

- From above we see that average velocity is equal to one half of the maximum velocity.

Shear Stress

- The shear stress in the z direction can be obtained as follows.

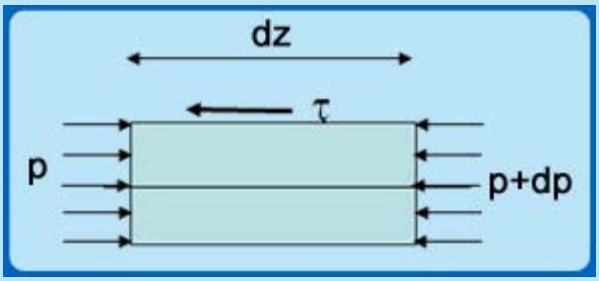
$$\tau_{rz} = \mu\left(\frac{\partial V_z}{\partial r} + \frac{\partial V_r}{\partial z}\right)$$
$$= \mu\frac{\partial V_z}{\partial r} \quad \text{(Since } V_r = 0 \text{ every where)}$$
$$= \mu V_z(\max)\left(\frac{-2r}{R^2}\right)$$
$$= \mu\frac{R^2}{4\mu}\left(-\frac{dp}{dz}\right)\left(\frac{-2r}{R^2}\right)$$
$$= -\frac{r}{2}\left(-\frac{dp}{dz}\right)$$

- Thus shear stress varies linearly with radius.

Force Balance

- Referring to the figure shown below,

$$2\pi r dz\tau + \pi r^2 dp = 0$$
$$\Rightarrow \tau = \frac{r}{2}\left(-\frac{dp}{dz}\right)$$



- Note the answer is same, but the sign is opposite as we have already assigned its direction.

Fanning Friction Factor

- We can introduce Non-dimensional shear stress in the form of Fanning Friction factor. It is defined as

$$f_F = \frac{|\tau_w|}{0.5\rho \bar{V}_z^2}$$

Further $\bar{V}_z = \frac{R^2}{8\mu}\left(-\frac{dp}{dz}\right)$

and $\tau_w = \tau\big|_{r=R} = -\frac{R}{2}\left(-\frac{dp}{dz}\right)$

$$\Rightarrow f_F = \frac{(R/2)/(dp/dz)}{0.5\rho \bar{V}_z (R^2/8\mu)(dp/dz)} = \frac{8\mu}{\rho \bar{V}_z R} = \frac{16\mu}{\rho \bar{V}_z D} = \frac{16}{\mathrm{Re}}$$

Pressure Drop

From force balance $\tau_w = \frac{R}{2}\left(-\frac{dp}{dz}\right)$

$$\Rightarrow \left(-\frac{dp}{dz}\right) = \frac{2\tau_w}{R} = \frac{4\tau_w}{D} = \frac{4\left(0.5\rho\bar{V}_z^2 f_F\right)}{D} = \frac{4\left(\rho\bar{V}_z^2 f_F\right)}{2D}$$

$$= \frac{4\rho\bar{V}_z^2}{2D}\left(\frac{16\mu}{\rho\bar{V}_z D}\right) = \frac{32\mu\bar{V}_z}{D^2}$$

$$\therefore -\Delta p = \frac{32\mu\bar{V}_z L}{D^2}$$

Darcy Friction Factor

- Many books and papers use alternate form of friction factor called Darcy friction factor.
- In this definition, the pressure gradient is computed using the relation

$$\Rightarrow \left(-\frac{dp}{dz}\right) = \frac{\rho \bar{V}_z^2 f_D}{2D}$$

- By comparison with the expression given earlier, we get

$$\Rightarrow f_D = 4 f_F$$

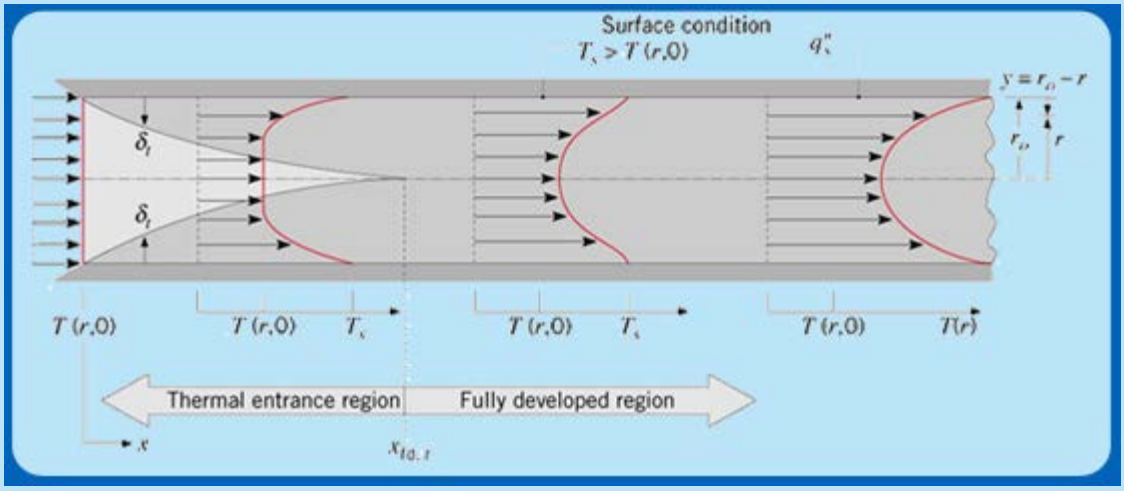
Fluid Flow in Complex Passages

- Often in engineering applications such as nuclear reactors the passage is not circular.
- We could use circular tube correlation by introducing the concept of hydraulic diameter,

$$D_h = \frac{4\ Area}{Wetted\ Perimeter}$$

- In turbulent flow, the method works reasonably well.
- However, specialized correlations exist for several shapes in literature and they may be used for better prediction.

Fully Developed Heat Transfer



- The figure above shows the development of temperature profile in a pipe subjected to constant heat flux.
- It should be noted here that, unlike in velocity development, where the velocities do not change is the fully developed region, the temperature will continue to change.
- However the shape will be similar.
- The entrance length (L_t /D) is ~ 0.05 Re Pr. The subscript 't' denotes thermal entry length.
- Pr = Prandtl number and is computed using $\frac{C_p \mu}{K}$ where C_p is the specific heat at constant pressure, μ is the dynamic viscosity and K is the thermal conductivity of the fluid.
- The entrance length is large for oils (Pr>>1).
- For turbulent flow L_t /D ~ 10.

Convective Heat Transfer in Pipes

- In convective heat transfer, the fluid carries away heat from a wall subjected to constant heat flux by virtue of its motion.
- In such systems, it is conventional to define a term called heat transfer coefficient, “h”.
- It can be defined as the rate of heat carried out per unit surface area and unit temperature difference.
- The purpose of defining it that way is to estimate the wall temperature, if the heat flux and the fluid temperature are known.
- As shown above, the temperature of the fluid continuously varies all along the radius.
- In such cases, we need a reference temperature to be defined for the sake of defining the heat transfer coefficient.
- The most common definition used is the thermodynamic bulk coolant temperature.
- This temperature is also called the mixing cup temperature or mixed mean temperature.

Thermodynamic Mean Temperature

- Mathematically, the total energy carried by a flowing fluid in a pipe can be written as

$$\int_A \rho V C T dA$$

- In the above expression C is the specific heat at constant pressure.
- For the sake of argument, let us assume that the specific heat, C, is constant.
- Now we want to hypothetically define a temperature T_B that will be homogeneous at a cross section such that the energy carried is same as above.
- In such a case the above integral will become

$$T_B C \int_A \rho V dA = \dot{m} C T_B$$

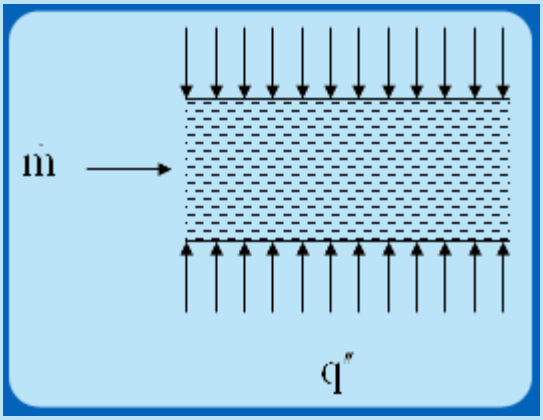
- Thus T_B reduces to

$$T_B = \frac{\int_A T d\dot{m}}{\dot{m}}$$

- For varying specific heat (if C is a strong function of temperature) this would be modified as

$$T_B = \frac{\int_A T \rho V C dA}{\int_A \rho V C dA}$$

Energy Balance



- For the constant heat flux case, the energy balance over a length Δx of pipe gives

$$\int_{outlet} \rho c_p V T dA = \int_{inlet} \rho c_p V T dA + q'' P \Delta x$$

- To make the understanding simple, we shall assume a constant c_p.
- The equation can now be written as

$$\Rightarrow \dot{m} c_p T_B\big|_{inlet} + \left.\frac{d(\dot{m} c_p T_B)}{dx}\right|_{inlet} \Delta x + HOT = \dot{m} c_p T_B\big|_{inlet} + q' \Delta x$$

where HOT stands for higher order terms of Taylor series and q' represents heat per unit length on the tube.

- For steady flow and constant c_p, when we shrink the length to 0, we get

$$\frac{dT_B}{dx} = \frac{q'}{\dot{m} c_p}$$

- The above stated equation is the simplified energy equation that we shall use subsequently.

Heat Transfer Coefficient

- Now we can define heat transfer coefficient as

$$h = \frac{q''}{(T_W - T_B)}$$

- In heat transfer analysis, a non-dimensional heat transfer coefficient, called Nusselt number, denoted by "Nu" is defined

$$Nu = \frac{hd}{k}$$

Where,

h - Heat Transfer Coeff. (W/m^2-K)

d – Diameter (m)

k -Thermal Conductivity (W/m-K)

Heat Transfer in Internal Passages

- In internal passages, boundary layers develop and merge.
- This leads to fully developed regions.
- In the developed region, both friction factor and Nussselt number are constant.
- In laminar flow, the values of Nu = 4.36 for constant wall heat flux case and is 3.66 for constant wall temperature case.
- Skipping details of analysis, we shall only state some of useful correlations.
- If we employ modified Reynolds analogy, we can show that

$$\frac{Nu}{\mathrm{Re}\,\mathrm{Pr}^{1/3}} = \frac{f_D}{8} = \frac{0.184\,\mathrm{Re}^{-0.2}}{8} = 0.023\,\mathrm{Re}^{-0.2}$$

- In the above equation, Pr is the Prandtl number,

$$\Rightarrow Nu = 0.023\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}$$

- The above equation is modified to give the Dittus-Boelter Equation and is the most common correlation used in turbulent flows

$$\Rightarrow Nu = 0.023\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n}$$

n = 0.4 for heating ($T_W > T_B$),
n = 0.3 for cooling ($T_W < T_B$)

- The properties are calculated at mean bulk coolant temperature.
- The validity of the above has been checked for

$0.7 < \mathrm{Pr} < 160$
$\mathrm{Re}_D > 10{,}000$ It is about +/- 15%
$L/D > 10$

- For high temperature difference between the wall and the bulk fluid, Sieder-Tate equation is mostly used

$$Nu = 0.027\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_W}\right)^{0.14}$$

- The properties are calculated at mean bulk coolant temperature, except for μ_w that is taken at wall Temperature.
- The validity of the above has been checked for

$0.7 < \mathrm{Pr} < 16{,}700$

Re_D > 10,000 It is about +/- 15%

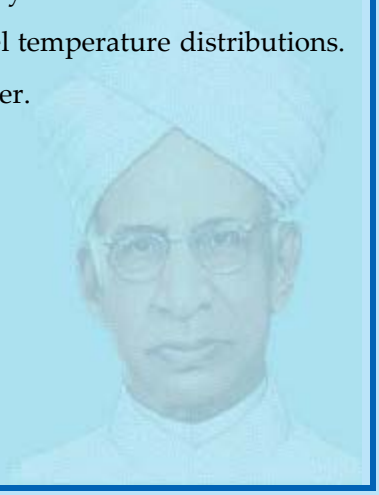
L/D > 10

For more accurate equations, one can refer text books and reference books in heat transfer.

Objectives

In this lecture you will learn the following

- We shall discuss the various coolants used in reactors.
- Then we shall look into the definitions associated with thermal analysis of reactors.
- Then we shall present the analysis to map the coolant, clad and fuel temperature distributions.
- Finally we shall understand a procedure to estimate the decay power.



Reactor Coolants

- Most commercial nuclear power plants use turbines for producing electricity.
- Thermodynamically, the working fluid has to be at a higher temperature to get good efficiency.
- Thus, the coolant used should operate at these temperatures.
- The desirable characteristics are:
  - High specific heat. This would lead to less amount of fluid to be handled for a given temperature difference.
  - Should be easily available and economical.
  - Should have less neutron absorption and should not produce highly active products (due to neutron activation).
  - It will be ideal, if it can also be used as a moderator.
  - Should be compatible with structural material and hence, should not be corrosive.
- Seldom all properties will be satisfied. Usually, it is a compromise. Some of the competitive coolants are:
  - Light Water
  - Heavy Water
  - Carbon-dioxide
  - Helium
  - Liquid Metals
  - Organic Coolants
  - Molten Salts

Light Water

- Used as coolant and moderator.
- It is cheap.
- Has low boiling point and therefore, requires high pressure.
- Decomposes into H_2 and O_2 by the action of radiation.
- Corrosive.
- Strong absorber of neutrons.

Heavy Water

- Expensive.
- Lower thermal neutron absorption cross section than light water.
- Otherwise the same advantages and disadvantages as light water.
- Tritium production can occur.

Liquid Metals

- High boiling point, hence lower operating pressures.
- High melting point requires heating of pipes when not in operation.
- Coolant becomes radioactive.
- Corrosive.
- Excellent heat transfer properties.
- Water must be kept out of reactors to avoid violent reactions.

Figure of Merit

- A figure of merit has been evolved to compare their utility as a coolant.
- Using basic principles, it can be shown that parameter represents the inverse of pumping power for the same amount of heat extracted with same temperature rise in the core, when flow is turbulent.
- Thus, lower this figure, smaller is pumping power.
- The value of this parameter relative to sodium is summarised in the following table.
- It may be observed that water has the highest value, indicating that it is the best coolant.
- Gas coolants are very poor as they need a lot of pumping power.

Material	Figure of merit (Relative to Na)
Liquids	
Water, H_2O or D_2O (300 °C)	60
Sodium hydroxide, NaOH (350 °C)	13
Dowtherm oil (300 °C)	7
Potassium and lithium chlorides, KCl, LiCl (400 °C)	2.5
Sodium, Na (300 °C)	1
Tin, Sn (300 °C)	0.45
Mercury, Hg (300 °C)	0.31
Lead, Pb (500 °C)	0.27
Bismuth, Bi (300 °C)	0.21
Potassium, K (300 °C)	0.21
Gases	
Carbon Dioxide, CO_2 (300 °C, 5 MPa)†	2.8×10^{-3}
Helium, He (300 °C, 5 MPa)†	1.9×10^{-3}
Hydrogen, H_2 (300 °C, 1 MPa)†	4.0×10^{-4}
Carbon dioxide, CO_2 (300 °C, 1 MPa)†	1.1×10^{-4}
Helium, He (300 °C, 1 MPa)†	7.6×10^{-5}
Air (300 °C, 1 MPa)†	4.5×10^{-5}

† 1 MPa is approximately equal to 10 times atmospheric pressure.

Heat Generation in Nuclear Reactor

- Heat generation in nuclear reactor is not spatially uniform.
- Heat generated per unit volume depends on the number of fissions occurring per unit volume.
- This can be shown to be proportional to the variation of neutron density in the reactor.
- This information is provided to a thermal hydraulic engineer by the reactor physicist.
- While the exact evaluation is fairly complex, some simplifying approximations shall be made in our further discussion.

Heat Distribution

- Heat generation in nuclear reactor is distributed amongst
  - Kinetic energy of fission fragments (KE-FF).
  - Kinetic Energy of neutrons (KE-N).
  - Betas, Gammas and Neutrinos.
- Further, not all energy is liberated instantly.
- We can typically approximate them as follows

Receiver	Instantaneous fraction (~90%)	Delayed fraction (~10%)
Fuel Element	KE- FF - 83%	Betas – 4%
Dispersed in Fuel, Mod., structure	KE-N 2.5%, Prompt Gamma 4%	Capture Gamma 3% Fission Product Gamma 3.5%

- Approximately 90% is deposited in fuel and 10% at other places.

Thermal Constraints

- The objective of a thermal hydraulic analysis is to satisfy the following Constraints:
  - The maximum fuel temperature has to be within the allowable limit set by the materials engineer.
  - Typically this is the metal softening temperature.
  - Similarly, the clad temperature shall be within its applicable safety limit.
  - This is dictated by the metal water reaction temperature.
  - In addition, the heat flux should be below the Critical Heat Flux (CHF limit).
  - We shall discuss this during boiling heat transfer.

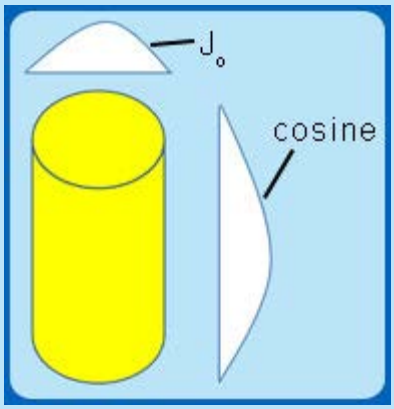
Heat Distribution

- The spatial variation for bare cylindrical reactors has been shown to be Cosine axially and Bessel function of zeroth order in radial direction as shown.

- For complex systems, these are complicated.

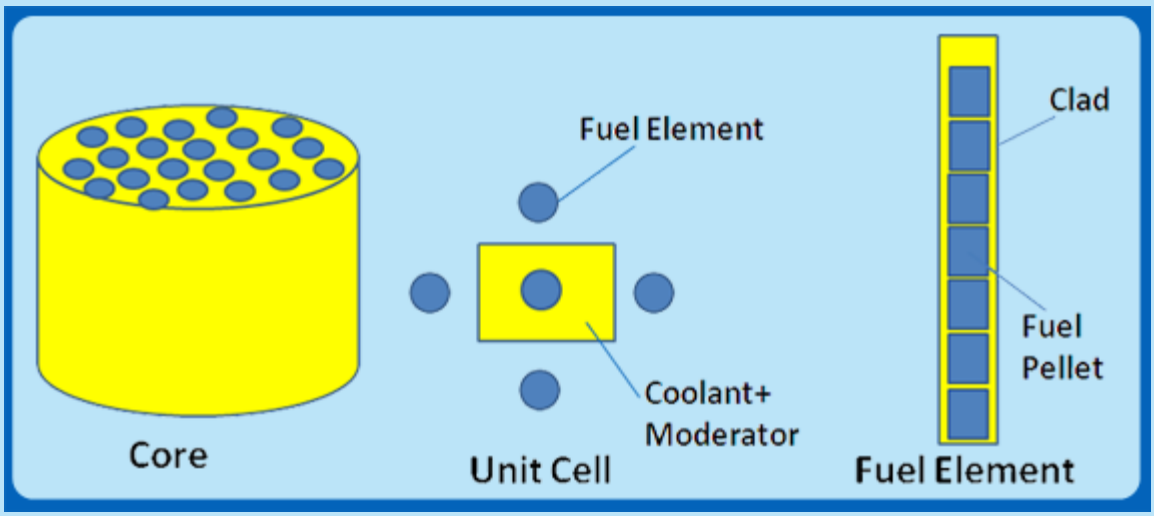
- Shape supplied by physicist.

- We shall keep the discussion general so as to grasp the fundamentals.

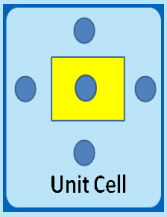


Definitions

- The reactor core by definition is the assembly of the fuel material along with moderator and coolant material.
- It can be assumed to be an array of rods distributed in a uniform manner.
- As discussed, each rod has an outer clad and inner fuel pellets.



- The discussion is presently illustrated using a square pitch (p). It should be fairly straight forward to extend it to any type of unit cell.
- For a unit cell, the volume of the core can be viewed as the total volume of the unit cell.
- Power density (Q''') is defined as the thermal power generated per unit core volume.
- Linear heat rate (q') is defined as the thermal power generated per unit length of the fuel rod.
- Heat flux (q'')is defined as the thermal power generated per unit external surface area of the fuel rod.



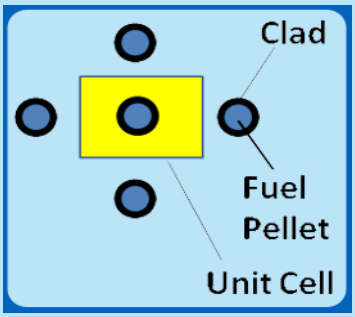
- Volumetric heat generation (q''') rate is defined as the thermal power generated in the core per unit fuel pellet volume.

- Consider an infinitesimal length (dl) of unit cell and let the thermal power generated be (dQ). The power density can be written as

$$Q''' = \frac{dQ}{p^2 dl}$$

- Similarly, the linear heat rate can be written for the control volume of length dl as

$$q' = \frac{dQ}{dl}$$



- Similarly, the heat flux for the control volume can be written as

$$q'' = \frac{dQ}{\pi d_{rod} dl}$$

- Finally, the volumetric heat generation rate can be written as

$$q''' = \frac{dQ}{\frac{\pi}{4} d_{pellet}^2 dl}$$

- Since dQ is same in all the above equations, we note that

$$Q''' p^2 = q' = q'' \pi d_{rod} = q''' \frac{\pi}{4} d_{pellet}^2$$

- Thus, the parameters defined differ only by a geometric factor. Hence if any one of them is known, the others may be computed from the geometry.
- Further, since dQ varies as the neutron density, all the parameters would vary accordingly.

Thermal Analysis

- To keep the analysis general for any shape distribution, we shall introduce the following analysis.
- For the purpose of analysis, the case of the cylindrical reactor is analysed. This is so as most power reactors are cylindrical. The argument can be extended for any other case similarly.
- For a cylindrical bare core, the variation of neutron density varies as

$$n = n_{max} J_0\left(2.405\frac{r}{R}\right) Cos\left(\pi\frac{z}{H}\right)$$

- This would imply that

$$Q''' = Q'''_{max} J_0\left(2.405\frac{r}{R}\right) Cos\left(\pi\frac{z}{H}\right)$$

$$\Rightarrow Q = \int_{Vo} Q''' dV$$

$$= \int_0^R \int_{-H/2}^{H/2} Q'''_{max} J_0\left(2.405\frac{r}{R}\right) Cos\left(\pi\frac{z}{H}\right) 2\pi r \, dr dz$$

$$\Rightarrow \bar{Q}''' = \frac{Q'''_{max}}{\pi R^2} \int_0^R J_0\left(2.405\frac{r}{R}\right) 2\pi r \, dr \frac{1}{H} \int_{-H/2}^{H/2} Cos\left(\pi\frac{z}{H}\right) dz$$

$$\Rightarrow \bar{Q}''' = Q'''_{max} \; P(r) \; P(z)$$

- where

$$P(r) = \frac{1}{\pi R^2} \int_0^R f(r) 2\pi r \, dr \quad \text{and} \quad P(z) = \frac{1}{H} \int_{-H/2}^{H/2} f(z) dz$$

- P(r) and P(z) are called radial and axial power factors.
- Thus the maximum value can be found from the average value if P(r) and P(z) are known.
- The values of maximum heat flux, linear heat rate and volumetric heat generation rates can be computed using the geometric factors.
- Single channel Hot Channel Analysis is generally carried out to assess the overall safety of a nuclear reactor.

The steps are:

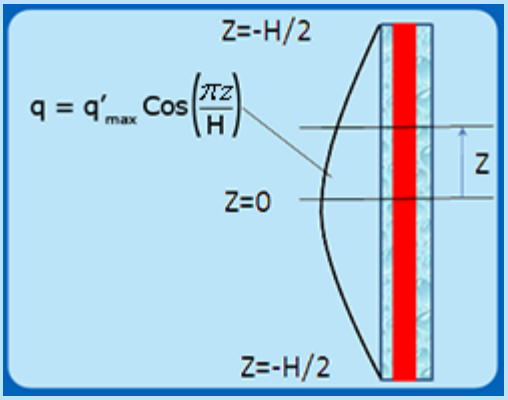
- Identify the hot channel, which is usually the central fuel channel.
- Compute the maximum linear heat rate in central channel by dividing the global average linear heat rate by the product of radial and axial power factor.

- Compute the mass flow rate associated with the channel by dividing the average mass flow rate per channel by any radial factor (introduced by orifices).
- The analysis illustrated for a single phase constant property flow to begin with.

Fluid Temperature

- Energy Balance for an infinitesimal slice at a distance z can be written as

$$\dot{m}c_p dT_B = q'\,dz = q'_{max}\,Cos\left(\frac{\pi z}{H}\right)dz$$



- On integration from the entrance to any general point z, we get

$$\int_{T_{B-in}}^{T_B} dT_B = \frac{q'_{max}}{\dot{m}c_p}\int_{-H/2}^{z} Cos\left(\frac{\pi z}{H}\right)dz$$

$$T_B = T_{B-in} + \frac{q'_{max}}{\dot{m}c_p}\frac{H}{\pi}\left[Sin\left(\frac{\pi z}{H}\right)\right]_{-H/2}^{z} = T_{B-in} + \frac{q'_{max}}{\dot{m}c_p}\frac{H}{\pi}\left[Sin\left(\frac{\pi z}{H}\right)+1\right]$$

- Having found the temperature distribution for the fluid, we turn our attention to the outer clad temperature.
- From the definition of convective heat transfer coefficient, we can write

$$q'' = h\left(T_{CO} - T_B\right)$$

$$\Rightarrow T_{CO}(z) = \frac{q''(z)}{h} + T_B(z)$$

- To find the clad temperature distribution we need to perform conduction analysis in clad.

Clad Temperature

- The governing equation for conduction is

$$\rho_c c_c \frac{\partial T_c}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk_c \frac{\partial T_c}{\partial r}\right) + q'''$$

- For steady and non-heat generating system, we can write

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(rk_c \frac{dT_c}{dr}\right) = 0 \qquad \Rightarrow rk_c \frac{dT_c}{dr} = C = R_{co}k_c \left.\frac{dT_c}{dr}\right|_{R_{co}}$$

- In the above expression R_{co} stands for clad outer radius.
- By definition

$$\Rightarrow -k_c \left.\frac{dT_c}{dr}\right|_{R_{co}} = q'' \qquad \Rightarrow C = -q'' R_{co}$$

$$\Rightarrow -k_c \frac{dT_c}{dr} = q'' \frac{R_{co}}{r} \qquad \Rightarrow -\int_{T_c}^{T_{co}} dT_C = \int_{r}^{R_{co}} \frac{q''}{k_C} R_{co} \frac{dr}{r}$$

- Integration with k_c assumed to be constant gives

$$\Rightarrow T_c - T_{co} = \frac{q''}{k_C} R_{co} \ln \frac{R_{co}}{r}$$

- Taking the other limit to the clad inner radius, R_{ci},

$$\Rightarrow T_{ci} - T_{co} = \frac{q''}{k_C} R_{co} \ln \frac{R_{co}}{R_{ci}}$$

- Inner clad temperature would be the highest clad temperature and if this is safe, then the design is OK.
- Now we shall perform conduction analysis in fuel pellet.

Fuel Temperature

- The governing equation for conduction in fuel is

$$\rho_c c_c \frac{\partial T_f}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rk_f \frac{\partial T_f}{\partial r}\right) + q'''$$

- In steady state $\frac{\partial T_f}{\partial t} = 0$

$$\Rightarrow \frac{1}{r}\frac{d}{dr}\left(rk_f \frac{dT_f}{dr}\right) = -q'''$$

- Transposing and performing the integration we get,

$$\Rightarrow \int_{\left(rk_f \frac{dT_f}{dr}\right)_0}^{\left(rk_f \frac{dT_f}{dr}\right)_r} d\left(rk_f \frac{dT_f}{dr}\right) = -\int_0^r q'''rdr$$

- Using the boundary condition that $\frac{\partial T_f}{\partial r}$ on dTf/dr = 0 at centre

$$\Rightarrow rk_f \frac{dT_f}{dr} = -q''' \frac{r^2}{2}$$

- Transposing and performing integration once again

$$\Rightarrow -\int_{T_f(R_{CI})}^{T_f(r)} k_f dT_f = \int_{R_{CI}}^{r} q''' \frac{r}{2} dr$$

- If k_f is assumed to be constant, then

$$\Rightarrow k_f\left(T_f(r) - T_f(R_{CI})\right) = \frac{q'''}{4}\left(R_{CI}^2 - r^2\right)$$

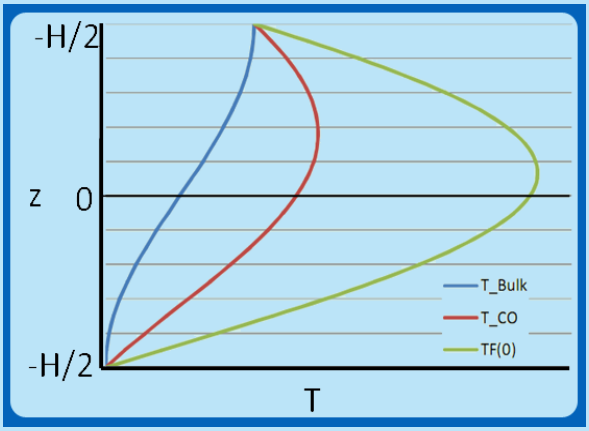
- Taking the other limit of r = 0, we get

$$\Rightarrow T_f(0) = T_f(R_{CI}) + \frac{q'''R_{CI}^2}{4k_f}$$

Summary

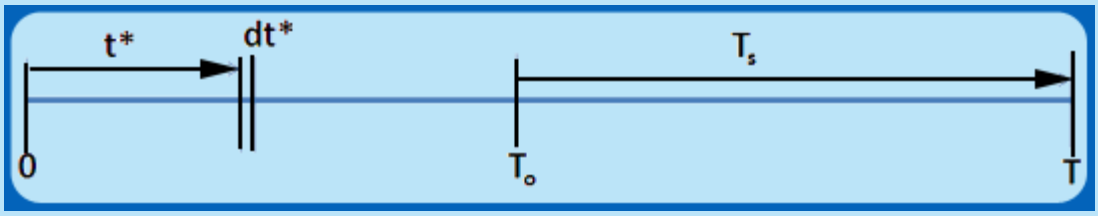
- The axial variation of temperatures are summarised below and are shown in the adjoining figure.

- T_{Bulk} increases monotonously.
- T_{clad} has a maxima after the centreline.
- $T_F(0)$ also has a maxima after the centreline.



Decay Heat

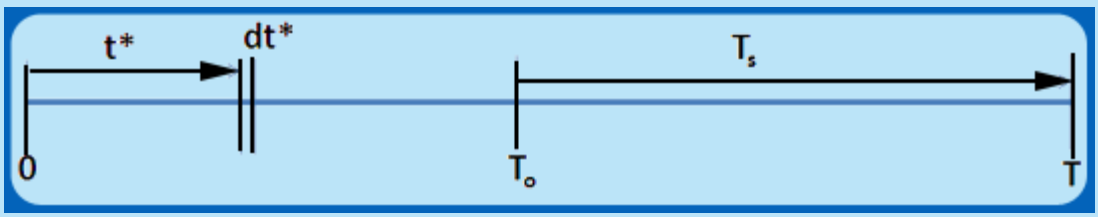
- We have understood the behaviour of temperature in steadily operating reactors.
- Unlike thermal power plants where the reaction ceases when fuel supply is cut off, nuclear reactor continues to produce heat even after shut down due to radioactive decay of fission products.
- Estimation of the power produced after shut down is very complex. However, the simple model of Way and Wigner is useful in understanding the trends.



- Referring to the figure shown above. The coordinate t^* starts when the reactor started producing power, whereas the coordinate T_s starts from the time the reactor is shut down.
- During operation, it is assumed that the power of the reactor is constant.
- Since there are several radioactive species present, there is no specific half time that can be assigned.
- Way and Wigner fitted a curve for the decrease in energy release with time as a power law.
- The energy release rate in the form of β and γ at time t after occurrence of a fission was expressed as

$$\dot{E}_\beta = 1.4t^{-1.2}\ MeV/(fission-s)$$
$$\dot{E}_\gamma = 1.26t^{-1.2}\ MeV/(fission-s)$$

- In the above expressions, t is time expressed in seconds.
- Our aim is to express the decay power as a fraction of the original full power at any given time T_s after the reactor is shut down. It is assumed that the reactor was operating for a time T_o at a uniform power.



- Let us consider the fissions that occurred between t^* and $t^* + dt^*$. Also, let q''' be the volumetric heat generation rate. The number of fissions during this time shall be,

$$No.\ of\ \ fissions = q'''(W/cc)\ 3.16x10^{10}(fissions/J)\ dt^*(s)$$

$$dP_\beta''' = q'''\ 3.16x10^{10}\ 1.4(T-t^*)^{-1.2}dt^*(MeV/cc-s)$$

$$= q'''\ 4.34x10^{10}(T-t^*)^{-1.2}dt^* x1.602x10^{-13}(J/cc-s)$$

$$= q'''\ 6.95x10^{-3}(T-t^*)^{-1.2}dt^*(J/cc-s)$$

$$\Rightarrow P_\beta''' = \int_0^{T_s} q'''\ 6.95x10^{-3}(T-t^*)^{-1.2}dt^*(J/cc-s)$$

- Since q''' is same as P_α''',

$$\Rightarrow \frac{P_\beta'''}{P_0'''} = 6.95x10^{-3}\frac{-(T-t^*)^{-0.2}}{-0.2}\Bigg|_0^{T_s}$$

$$\Rightarrow \frac{P_\beta'''}{P_0'''} = 3.48x10^{-2}[(T-T_o)^{-0.2} - T^{-0.2})]$$

$$\Rightarrow \frac{P_\beta'''}{P_0'''} = 3.48x10^{-2}[T_s^{-0.2} - T^{-0.2}]$$

- Similarly $\frac{P_\gamma'''}{P_0'''} = 3.12x10^{-2}[T_s^{-0.2} - T^{-0.2}]$
- Hence total decay power can be written as,

$$\frac{P'''}{P_0'''} = 0.066[T_s^{-0.2} - T^{-0.2}]$$

- American Nuclear Society (ANS-1971) had suggested a curve that was fitted to the following expression

$$\frac{P}{P_0} = 0.005\ a[T_s^{-b} - T^{-b}]$$

where, a and b are given in the following table

T_S(s)	a	b
0.1-1	12.05	0.0639
10-150	15.31	0.1807

150-8x10	27.43	0.2962

Objectives

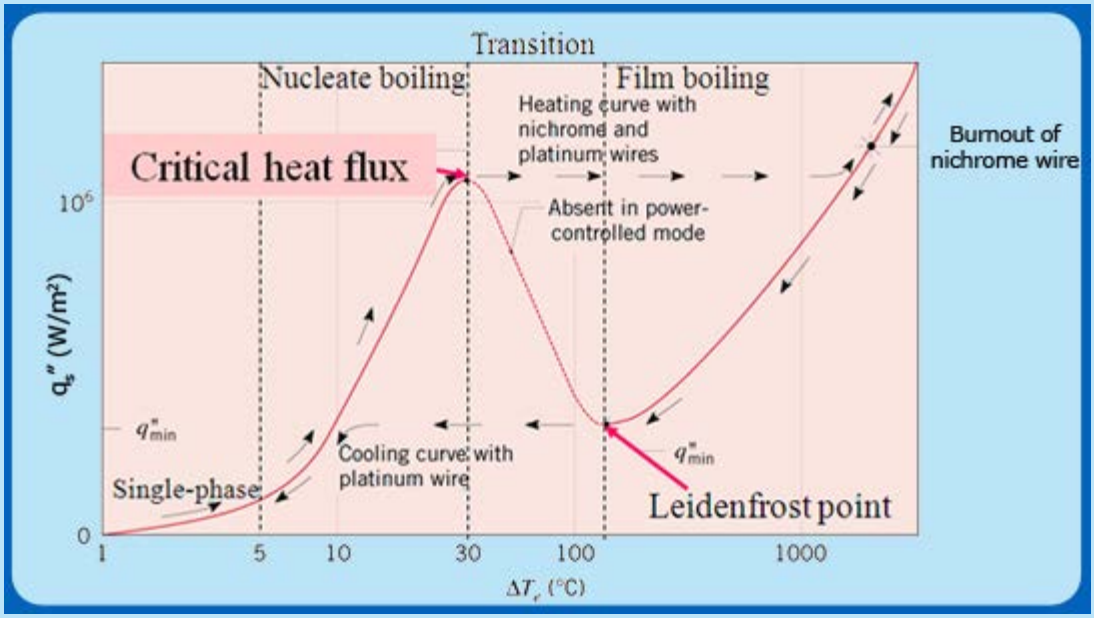
In this lecture you will learn the following

- We will see an overview of boiling heat transfer.
- First we will see the characteristics of Pool Boiling.
- Then we shall discuss salient features of flow boiling.

Boiling Heat Transfer

- Boiling is associated with transformation of liquid to vapor by heating.
- It differs from vaporization in the sense that it is associated with the formation of bubbles.
- The formation of bubbles stir the fluid and breaks the boundary layers thereby increasing the heat transfer coefficient.
- The bubbles are normally formed on the surface scratches. The bubbles do not appear till the wall is heated in excess of the saturation temperature, called wall superheat.
- The excess temperature required for the onset of formation of bubbles decreases with increase in the size of surface scratches.
- Models exist for the prediction of this, but will not be discussed in this first level course.
- One of the main interest is the prediction of heat transfer coefficient.
- The general features can be understood for the experiment conducted by Nukiyama. The results are summarized in what is known as Boiling Curve.

Pool Boiling Heat Transfer



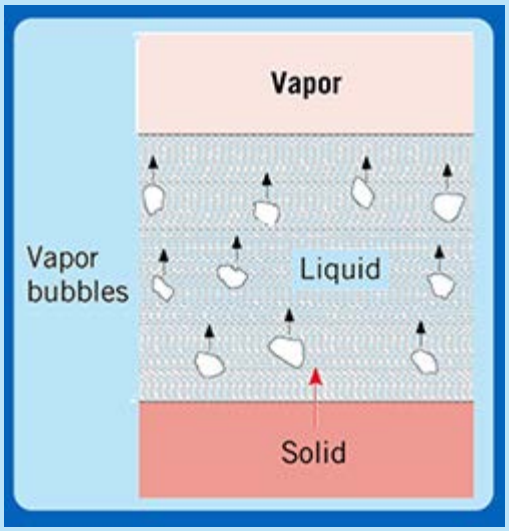
- The above figure shows the boiling curve for water at 1 atm.
- The x axis is $\Delta T_{sat} = T_{wall} - T_{sat}$.
- Single phase free convection region occurs for $\Delta T < 5\ ^oC$.
- Onset of nucleation occurs $\Delta T_{sat} \sim 5\ ^oC$.
- Bubbles nucleate, grow and detach from the surface.
- Increase of wall superheat (ΔT_{sat}) leads to more vigorous nucleation and rapid increase in heat transfer.
- As the superheat is increased, the vapor formation becomes vigorous, it blankets the surface and the slope of the boiling curve decreases.
- As the superheat is increased, more blanketing causes the heat transfer to drop, till the entire heated surface is blanketed and the slope reduces to zero.
- This turn around point is called the Critical Heat Flux or Boiling crisis.
- Now the radiation heat transfer also starts playing a role and eventually, the slope of the boiling curve starts increasing due to increase convection and radiation heat transfer.
- The second turn around point is called Leidenfrost point or rewetting point.
- The heat transfer beyond this point is called film boiling.
- Instead of controlling ΔT_{sat}, if we control surface heat flux as it happens in a nuclear reactor, the behaviour slightly differs.
- The path traced is marked by arrows.
- It may be seen that as heat flux is increased beyond CHF, the wall temperature abruptly increases.
-

Similarly, when heat flux is brought back, there is an abrupt jump at the Leidenfost point.

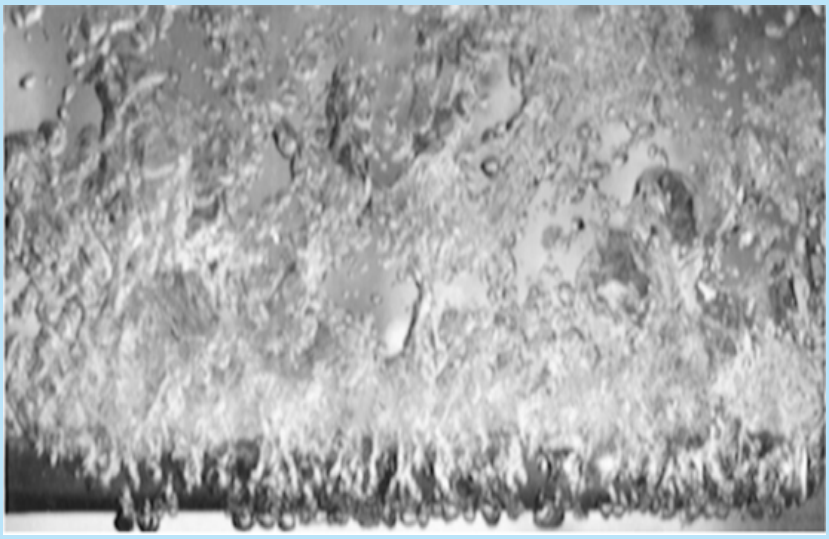
- Thus, there are discontinuities in the boiling curve.

- Use of platinum wire enables one to see the entire path described.
- We shall briefly look at some details.
- Nucleate Boiling (Wall super heat < 30 ^{o}C at 1 atm).

- Isolated bubbles Region $5 < \Delta T_{Sat} < 10\ ^{o}C$
  - Liquid motion is strongly influenced by nucleation of bubbles at the surface.
  - 'h' rapidly increases with wall superheat.
  - Heat transfer is principally due to contact of liquid with the surface (single-phase convection) and not due to vaporization.



- Jets and Columns Region ($10 < \Delta T_{Sat} < 30\ ^{o}C$ at 1 atm)
  - Increasing nucleation density causes bubbles to coalesce to form jets and slugs
  - Liquid wetting impaired.
  - 'h' starts decreasing with increase in superheat.



- Critical Heat Flux (Wall super heat ~30 ^{o}C at 1 atm)
  - Typically its value is 1 MW/m^2 at 1 atm and increases with pressure.
  - If the wall pumps heat flux, there is a potential for the wall to melt as the heat transfer coefficient is very low here due to vapor blanketing.
- Film Boiling (Wall superheat >120 ^{o}C at 1 atm)

- Heat transfer by conduction and radiation across vapor blanket.
- Usually not a preferred mode of cooling but can occur during the the accident situations in a nuclear reactor.

- Transition Boiling (30 ^{o}C < ΔT_{sat} < 120 ^{o}C at 1 atm)
  - Called Unstable film boiling.
  - Surface conditions oscillate between nucleate and film boiling.

- Boiling Heat Transfer Correlations
  - Different models exist and there is no single view on this aspect.
  - We shall just list the correlations for application purposes
- Nucleate Boiling
  - Rohsenow's correlation is most popular

$$q'' = \mu_l h_{fg}\left(\frac{g(\rho_l-\rho_v)}{\sigma}\right)^{0.5}\left(\frac{c_{p,l}\Delta T_{sat}}{C_{s,f}h_{fg}\,\mathrm{Pr}_l^{n}}\right)^{3}$$

    - $C_{s,f}$, n depends on Surface/fluid combination (Given in Table 10.1 of Incropera and Dewitt). Typical values are $C_{s,f} = 0.013$) n=1.
- Critical Heat Flux
  - Lienhard's correlation

$$q'' = 0.149 h_{fg}\rho_v\left(\frac{\sigma g(\rho_l-\rho_v)}{\rho_v^{2}}\right)^{0.25}$$

- Minimum Heat Flux (Leidenfrost's point)
  - Adapted from Zuber's correlation

$$q'' = 0.09 h_{fg}\rho_v\left(\frac{\sigma g(\rho_l-\rho_v)}{(\rho_l+\rho_v)^{2}}\right)^{0.25}$$

Not very reliable

- Film Boiling
  - Both Convection and radiation effects are important.
  - Bromley's correlation is most quoted

$$\overline{h_{overall}}^{4/3} = \overline{h_{conv}}^{4/3} + \overline{h_{rad}}\,\overline{h_{overall}}^{1/3}$$

$$\overline{Nu_L} = \frac{\overline{h_{conv}}L}{k_v} = 0.943\left(\frac{h'_{fg}g(\rho_l-\rho_v)L^3}{\nu_v k_v(T_s-T_{sat})}\right)^{0.25}$$

$$h'_{fg} = h_{fg} + 0.8c_{p,v}(T_s-T_{sat})$$

$$\overline{h_{rad}} = \frac{\varepsilon\sigma\left(T_s^{4}-T_{sat}^{4}\right)}{(T_s-T_{sat})}$$

Vapour properties are evaluated at average film temperature, liquid property at saturated temperature

For cylinder and sphere, the convective heat transfer coefficient is modified as

$$\overline{Nu_D} = \frac{\overline{h_{conv}} D}{k_v} = C\left(\frac{h'_{fg} g (\rho_l - \rho_v) D^3}{\nu_v k_v (T_s - T_{sat})}\right)^{0.25}$$

where D is the diameter

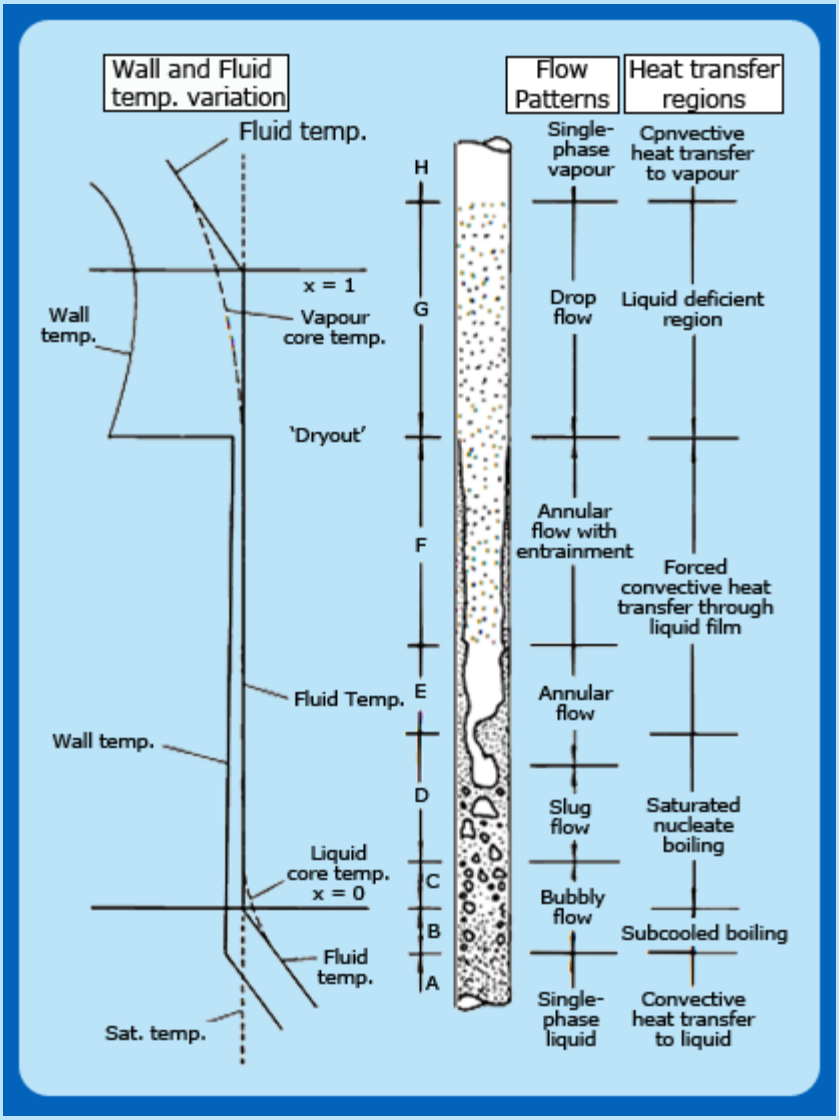
The value of C for Horizontal cylinder = 0.62, for sphere, C = 0.67.

As an approximation, when $\overline{h_{conv}} > \overline{h_{rad}}$

$$\overline{h_{overall}} = \overline{h_{conv}} + \frac{3}{4}\overline{h_{rad}}$$

Flow Boiling Heat Transfer

- Boiling observed during flow of water is heated tube is called Flow Boiling.
- Typical Flow patterns observed when boiling occurs inside tubes subjected to constant heat flux are shown below.



- At the begining single-phase flow occurs.
- As bubble tries to nucleate, it condenses when released.
- This is called subcooled boiling.
- As the liquid temperature increases to the saturated value, bubbles move in the bulk liquid.
- This region is called saturated boiling region.
- The flow pattern changes from bubbly to slug flow.
- Slug flow implies that the bubbles are of the size of the radius of the tube.
- As the bubble become large, the liquid film becomes thin and nucleation is no longer seen.

- As the vapour core region becomes large and the film small, the flow pattern is called annular flow.
- As the vapour velocity increases, it creates waves on the liquid film and tears away drops. This is called entrainment.
- Gradually the film dries off and only entrained droplets are present.
- Finally, the drops also evaporate leading to single-phase vapour flow.
- At the dryout point, the heat transfer coefficient suddenly deteiorates and a large increase in wall temperature is seen.
- The methods used to compute the heat transfer coefficient are complex and hence not treated in this first level course.

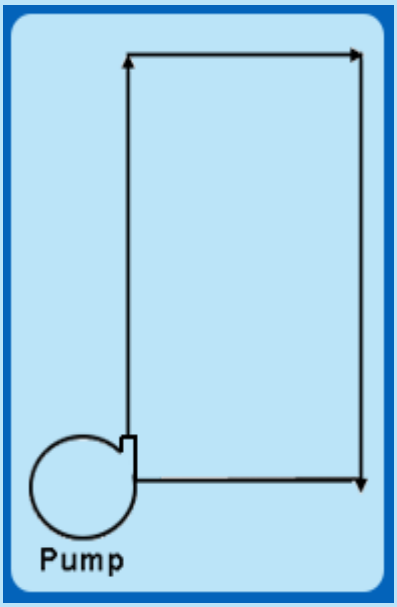
Objectives

In this lecture you will learn the following

- We shall solve some illustrative problems in fluid flow and in nuclear systems heat transfer.

Problem-1

Solving real life reactor problems are tedious and time consuming. However, to illustrate the principles involved, let us look at a closed loop constructed of a pipe with diameter d = 2.5 cm and a total length of 20 m driven by a centrifugal pump as shown in the figure. The fluid circulated is water of density 1000 kg/m^3 and viscosity 10^{-3} Pa.s at a rate of 1.5 kg/s



a. Calculate the frictional pressure drop in the total circuit.
b. The next task is to choose a pump. Calculate the head required to be developed by the pump.
c. To be conservative, add an additional 30% on the estimated head. If the market offers two pumps of the following given characteristics, which one would you choose? The characteristics are:

Pump-A H (m) = 10 – 700,000 Q^2 where Q is flow rate in m^3/s

Pump-B H (m) = 12 – 700,000 Q^2 where Q is flow rate in m^3/s

d. Assuming that the actual circuit behaves ideally with no error (30% accounted in part c is not there), estimate the actual mass flow rate the pump will circulate.
e. Assuming that the desired flow rate is 1.5 kg/s, estimate the friction loss coefficient K for the valve that has to be installed in the circuit.

Problem 1-Solution

a. Given ϱ=1000 kg/m^3 , d = 0.025 m , $\dot{m}$ = = 1.5 kg/s

Average Velocity $\bar{v} = \dot{m}/(\rho A)$

$$= \frac{1.5}{1000(\pi/4)\,0.025^2} = 3.06\ m/s$$

Frictional $\Delta p = = \frac{\rho \bar{v}^2}{2}\frac{fl}{d}$

Assuming smooth pipe, $f = 0.316\,Re^{-0.25}$

Reynolds Number, $Re = \frac{\rho \bar{v} d}{\mu} = \frac{1000x3.06x0.025}{10^{-3}}$

$\Rightarrow Re = 76{,}500$

$\Rightarrow f = 0.316x(76{,}500)^{-0.25} = 0.019$

Frictional $\Delta p = \frac{1000x3.06^2}{2} \frac{0.029x20}{0.025} = 71163Pa$

b. Head $= \frac{\Delta p}{\rho g} = \frac{71163}{1000x9.8} = 7.26m$

c. Conservative Head $= 1.3x7.26 = 9.44m$

Head for Pump A at rated flow $= 10 - 700{,}000x0.0015$

$= 8.425m$

Head for Pump B at rated flow $= 12 - 700{,}000x0.0015$

$= 10.425m$

Hence Pump A would not be adequate.

However, Pump B is adequate and hence chosen.

d. This part of the problem becomes iterative because friction factor is related to flow and the flow is unknown

- Numerical methods may be used to solve this.
- However, a simple procedure for illustration is given.
- Since friction factor is insensitive to Re, we can first assume this to be the old value.

Head for Pump B $= 12 - 700{,}000(\bar{v}A)^2$

$A = \pi/4\, 0.025^2 = 4.91x10^{-4} m^2$

Hence, Head for Pump B $= 12 - 0.169\bar{v}^2$

Frictional Head $= \frac{\bar{v}^2}{2g} \frac{fl}{d} = \frac{\bar{v}^2}{2x9.8} \frac{0.019x20}{0.025}$

Equating the two heads, we get

$$12 - 0.169\bar{v}^2 = 0.776\bar{v}^2$$

$$\Rightarrow \bar{v}^2 = 12.70$$

$$\Rightarrow \bar{v} = \sqrt{12.70} = 3.56m/s$$

- Computing f with 3.56 m/s gives f = 0.0182
- A new iteration can be done with f = 0.018 and obtain more accurate solutions.

Mass flow rate $= \rho A\bar{v} = 1000x4.91x10^{-4}x3.56$

$= 1.748kg/s$

e. Desired Mass flow rate $= 1.5kg/s$

Head required$= 7.26m$

Pump Head at rated flow$= 10.425m$

Head to be dissipated$= 3.165m$

Assuming the valve has the same reference size as the pipe, we can compute K_v as follows:

$$K_v = \frac{H_{dissipated}}{\left(\bar{v}^2 / 2g\right)} = \frac{3.165x2x9.8}{3.06^2} = 6.62$$

Problem-2

A small PWR plant operates at a power of 485 MW(th). The core is approximately 1.9 m in diameter and 2.3 m tall. It consists of 23,142 tubes of thickness 0.5 mm and inner diameter 7.5 mm on a square pitch of 10.5 mm. The tubes are filled with 3.4 w/o enriched UO_2. The core is cooled with water entering at 260°C (c_p = 5.4 kJ/kg-K) flowing at a rate of 15.4 X 10^6 kg/hr at 150 bar. Compute ,

a. average power density in kW/litre.
b. average temperature of water leaving the core, and
c. maximum linear heat rate, surface heat flux and volumetric heat generation rate.

Problem 2-Solution

a. Given Power = 485 MW, D = 1.9 m, H = 2.3 m

Volume of Core $= \frac{\pi}{4} D^2 H = \frac{\pi}{4} x1.9^2 x2.3$

$= 6.521m = 6521\ litres$

Power Density $= \frac{485x10^3}{6521} = 74.37$ KW/litre

b. Energy Balance implies $\dot{Q} = \dot{m} c_p \Delta T$

$$\Rightarrow \Delta T = \frac{\dot{Q}}{\dot{m} c_p} = \frac{485x10^3}{(15.4x10^6 / 3600)x5.4}$$
$$= 21K$$

Therefore $T_{exit} = T_{in} + \Delta T = 260 + 21 = 281^\circ C$

c. Number of fuel tubes = 23142

Hence $\overline{q'} = \frac{4.85x10^3}{23142x2.3} = 9.11 \frac{kW}{m}$

$q'_{max} = \overline{q'} x\ Peaking\ factor = 9.11x3.64$

$= 33.16 \frac{kW}{m}$

$q''_{max} = \frac{q'_{max}}{\pi d_o} = \frac{33.16}{\pi(7.5 + 2x0.5)/1000} = 1241 \frac{kW}{m^2}$

$q'''_{max} = \frac{q''_{max}}{(\pi/4) d_{fuel}^2} = \frac{33.16}{(\pi/4)(7.5/1000)^2}$

$= 750.6x10^3 \frac{kW}{m^3}$

Problem-3

Design a 3000 MW(th) bare cylindrical PWR, whose height to diameter ratio is 1. The fuel to coolant volume ratio is to be maintained equal to 1 in a square lattice. The system pressure and core inlet temperature shall be 175 bars and 285^{o}C respectively. If the reactor has to work under the following three thermal constraints, viz., the

i. maximum power density = 250 W/cc,
ii. maximum clad surface heat flux = 125 W/cm^2 and,
iii. maximum core outlet temperature = 325^{o}C,

Compute, the

a. reactor dimensions and volume ,
b. fuel element diameter and lattice pitch,
c. number of fuel elements, and
d. mass flow rate and average coolant velocity.

You may assume: c_p of coolant = 6 kJ/kg-K , radial peaking factor = 2.32 and overall peaking factor = 3.64

Note: The N square lattices, when properly arranged forms a circular core of diameter D

Problem 3-Solution

a. Given Power = 3000 MW, L/D = 1, Peaking factor = 3.64

$$\overline{Q'''} = \frac{Q}{(\pi/4)D^2L} = \frac{3000x10^3}{(\pi/4)D^2L} = \frac{250x10^6}{3.64x10^3}\frac{kW}{m^3}$$

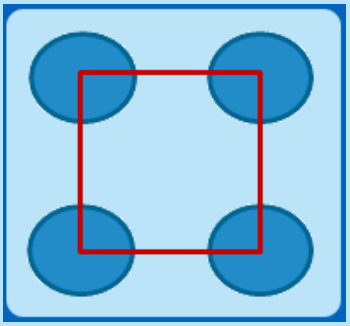
$$\Rightarrow D^2L = \frac{3000x3.64}{(\pi/4)250} = 55.61m^3$$

Since L/D =1$\Rightarrow D^3 = 55.61$

$$\Rightarrow D = 3.82m = L$$

Hence Volume $= \frac{\pi}{4}D^2L = 43.78m^3$

b. For a Unit Cell with diameter of rod, d, and pitch p as shown, we can write



Fuel Volume $= \frac{\pi}{4}d^2$ Core Volume $= p^2$ Coolant Volume $= p^2 - \frac{\pi}{4}d^2$

Given that fuel to coolant volume = 1 $\Rightarrow \dfrac{p^2 - \frac{\pi}{4}d^2}{\frac{\pi}{4}d^2} = 1$

$$\Rightarrow \frac{\frac{p^2}{d^2} - \frac{\pi}{4}}{\frac{\pi}{4}} = 1$$

$$\Rightarrow \frac{p^2}{d^2} = \frac{\pi}{2}$$

$$\Rightarrow \frac{p}{d} = 1.253$$

To solve for the pitch and diameter of the rods, we rewrite the reactor power as

$$Q = np^2 L\overline{Q'''} = n\pi dL\overline{q''}$$

$$\Rightarrow \frac{p^2}{d} = \frac{\pi\overline{q''}}{\overline{Q'''}} = \frac{\pi\overline{q''_{max}}}{\overline{Q'''_{max}}} = \frac{\pi 125x10^4}{250x10^6} = 0.0157m$$

Since p/d =1.253 $\Rightarrow 1.253^2 d = 0.0157$

$$\Rightarrow d = 0.01m$$

$$\Rightarrow p = 0.01253m$$

c. $n = \dfrac{Q}{p^2 L\overline{Q'''}} = \dfrac{3000x10^6}{0.01253^2 x3.82x250x10^6 / 3.63}$

$$= 72{,}631$$

d. The radial peaking factor = 2.32. This implies that power in the hot channel is,

$$Q_{hot} = \frac{Qx2.32}{n} = \frac{3000x10^3 x2.32}{72631} = 95.8kW$$

$$\dot{m}_{hot} = \frac{Q_{hot}}{c_p \Delta T_{max}} = \frac{95.8}{6(325-285)} = 0.399kg/s$$

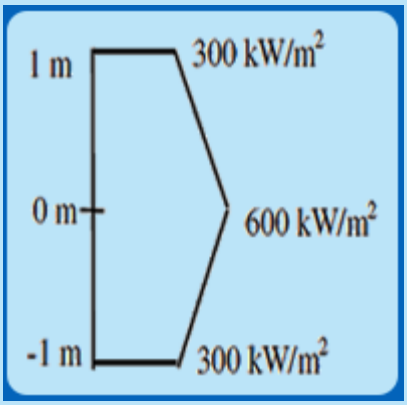
Assuming that flow is uniform in the core, the total mass flow rate for the reactor shall be

$$\dot{m} = n\dot{m}_{hot} = 0.399x72631 = 28979kg/s$$

$$= 8.05kg/hr$$

Problem-4

The heat flux at the surface of fuel rod in an experimental reactor varies as shown in the figure. The fuel rods are 1 cm in diameter and 2 m long. Water (C = 4.5 kJ/kg^{o}C) enters at 65^{o}C and flows at the rate of 160 kg/hr per rod. The heat transfer coefficient is 6000 W/m^{2}-C. Calculate the temperature of coolant and fuel surface temperature at the middle and the exit of the channel.



Problem 4-Solution

The integral for of the energy balance can be written as

$$\int_{T_{in}}^{T_z} \dot{m} c_p dT = \int_{-1}^{z} q'' \pi d dz$$

Integrating from inlet to the centre, we get

$$\Rightarrow \dot{m} c_p \left(T_z(0) - T_{in}\right) = \pi d \left(\frac{300+600}{2}\right) 1$$

$$\Rightarrow T_z(0) = T_{in} + \frac{\pi d 450}{\dot{m} c_p}$$

$$\Rightarrow T_z(0) = 65 + \frac{\pi x 0.01 x 450}{(150/3600)4.5} = 65 + 70.6 = 135.6^{\circ}C$$

Similarly integrating from centre to exit, we shall get

$$\Rightarrow T_z(1) = 135.6 + 70.6 = 206.2^{\circ}C$$

Using the definition of heat transfer coefficient, we can write

$$h\left(T_w(z) - T_z(z)\right) = q''$$

$$\Rightarrow T_w(0) = T_z(0) + \frac{q''}{h}$$

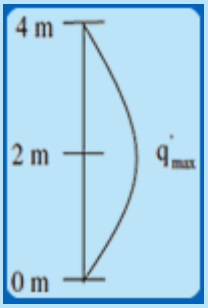
$$\Rightarrow T_w(0) = 135.6 + \frac{600}{6} = 235.6^{\circ}C$$

Similarly at exit

$$\Rightarrow T_w(1) = 206.2 + \frac{300}{6} = 256.2^{\circ}C$$

Problem-5

Length of the coolant channel	4 m
Power profile	Cosine, refer figure
Mass flow rate of coolant	0.5 kg/s
Temperature of the coolant inlet	270°C
cp of the coolant	5.444 kJ/kg-K
Specific heat of the fuel pellet	750 J/kg-K
Specific heat of the clad	1000 J/kg-K
Radius of the fuel pellet	5 mm
Thickness of the clad	0.5 mm
Thermal Conductivity of the fuel	6 W/m-K
Thermal Conductivity of the clad	18 W/m-K
Convective heat transfer coefficient	20 kW/m^2-K



a. If the temperature of the clad surface at 3 m from the base was measured to be 600°C, compute the value of q'_{max} at which the reactor was operating.
b. At what temperature, the coolant will exit the hot channel ?

Problem 5-Solution

Writing the energy balance for a slice

$$\dot{m}c_p dT_B = q'\,dz = q'_{max}\sin\left(\frac{\pi z}{4}\right)dz$$

Integrating from inlet to any arbitrary point, we get

$$\dot{m}c_p\left(T_B(z)-T_{B-in}\right)=\int_0^z q'_{max}\sin\left(\frac{\pi z}{4}\right)dz$$

$$T_B(z)=T_{B-in}+\frac{q'_{max}}{\dot{m}c_p}\frac{4}{\pi}\left[-\cos\left(\frac{\pi z}{4}\right)\right]_0^z=T_{B-in}+\frac{q'_{max}}{\dot{m}c_p}\frac{4}{\pi}\left[1-\cos\left(\frac{\pi z}{4}\right)\right]$$

$$\Rightarrow T_B(3)=T_{B-in}+\frac{q'_{max}}{\dot{m}c_p}\frac{4}{\pi}\left[1-\cos\left(\frac{\pi 3}{4}\right)\right]$$

Further, $T_w(3)=T_B(3)+\frac{q''}{h}$

$$\Rightarrow T_w(3) = T_B(3) + \frac{q''_{max}}{h} \sin\left(\frac{\pi 3}{4}\right)$$

$$\Rightarrow T_w(3) = T_{B-in} + \frac{q'_{max}}{\dot{m} c_p} \frac{4}{\pi}\left[1 - \cos\left(\frac{\pi 3}{4}\right)\right] + \frac{q'_{max}}{\pi dh} \sin\left(\frac{\pi 3}{4}\right)$$

$$\Rightarrow (T_w(3) - T_{B-in}) = \frac{q'_{max}}{\dot{m} c_p} \frac{4}{\pi}\left[1 - \cos\left(\frac{\pi 3}{4}\right)\right] + \frac{q'_{max}}{\pi dh} \sin\left(\frac{\pi 3}{4}\right)$$

$$\Rightarrow (T_w(3) - T_{B-in}) = q'_{max}\left\{\frac{4}{\pi \dot{m} c_p}[1 + 0.707] + \frac{0.707}{\pi dh}\right\}$$

$$\Rightarrow (600 - 270) = q'_{max}\left\{\frac{4x1.707}{\pi x0.5x5.444} + \frac{0.707}{\pi(0.01 + 0.001)20}\right\}$$

$$\Rightarrow 330 = q'_{max}\{0.7985 + 1.023\}$$

$$\Rightarrow q'_{max} = \frac{330}{1.8215} = 181.2 \frac{kW}{m}$$

Total power in the channel can be computed as

$$Q = \int_0^4 q'_{max} \sin\left(\frac{\pi z}{4}\right) dz = \frac{4q'_{max}}{\pi}\left[-\cos\left(\frac{\pi z}{4}\right)\right]_0^4$$

$$Q = \frac{4q'_{max}}{\pi}[1 + 1] = \frac{8q'_{max}}{\pi} = \frac{8x181.2}{\pi} = 461.4kW$$

Hence ΔT_B can be computed as $\Delta T_B = \dfrac{Q}{\dot{m} c_p} = \dfrac{181.2}{0.5x5.444}$

$$\Rightarrow \Delta T_B = 66.6°C \Rightarrow T_{B-out} = 270 + 66.6 = 336.6°C.$$

Objectives

In this lecture you will learn the following

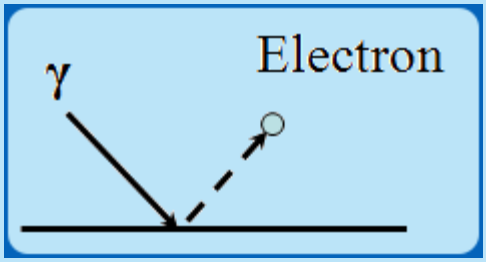
- The objective of this lecture is to introduce the interactions radiation has with matter.
- This is particularly important as radiation affects health of living beings.
- First the types of interactions are introduced.
- Then the units to measure exposure and energy absorbed are introduced.

General-I

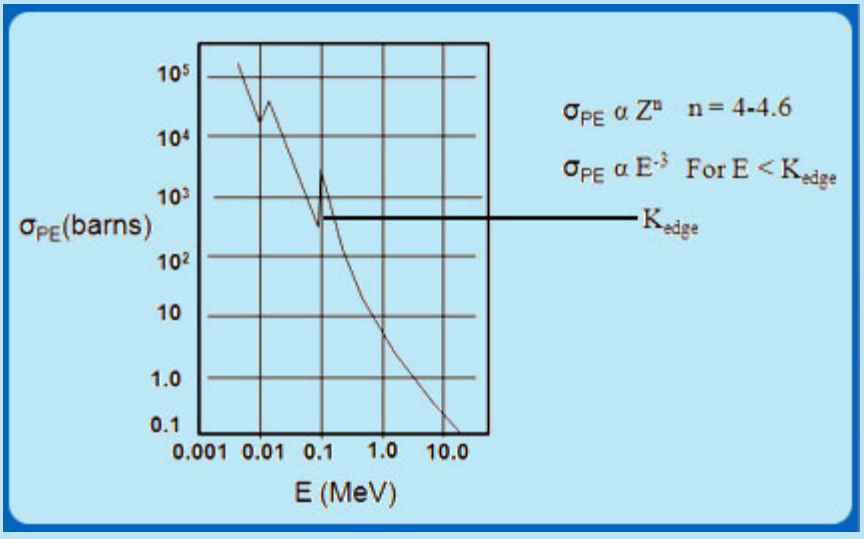
- The Radiation encountered with nuclear reactors can be classified, as
  - Charge particles (α, β, fission fragments).
  - Electromagnetic Radiation (X-ray, γ-ray).
  - Neutrons.
- As the charge particles are mostly absorbed in reactor, they do not pose a direct health threat.
- However, when fission products are discharged due to any accidental release, they will have an influence.
- As we have studied neutron interactions in detail, it will be skipped.
- X-rays and γ-rays have long range and hence can have an influence on the health of operating personnel.
- Hence shielding is required to protect the plant personnel.
- Similarly, neutrons can leak out and hence have to be taken care of.
- Standards have been evolved for the protection of plant personnel and general public and these will be discussed subsequently.
- Prior to understanding the health effects, let us look at how radiation interacts with matter.
- First we shall look at interaction of electromagnetic radiation.
- γ-rays are emitted when nucleus de-excites from an excited state.
- On the other hand, X-rays are emitted when electrons move from one shell to another.

Photoelectric Effect

- In this interaction the γ-ray is completely absorbed and an electron is ejected, as shown in figure below.
- Generally, the recoil nucleus acquires negligible energy.
- The energy of ejected electron is equal to the γ energy minus the binding energy of the electron.



- The binding energy shall depend on which electron is knocked out.
- If the innermost shell in knocked out (K-shell), then, subsequent electron rearrangement (Internal transition) can lead to emission of X-rays or even ejection of one more electron of lesser binding energy (called Auger electron).
- The cross section for photoelectric interaction, σ_{PE} varies with the energy of the γ. Its variation for lead (Pb) is shown in the following figure.
- σ_{PE} varies as Z^n, where Z is the change number and n varies from 4 - 4.6.



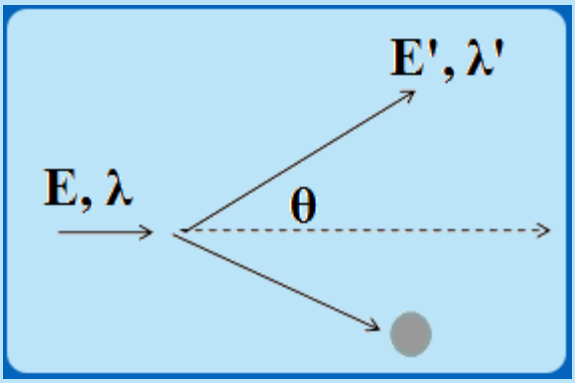
Compton Scattering-I

- In this interaction photon is scattered while the electron is ejected.
- Thus, photon is not fully absorbed.
- We can write momentum and energy equations similar to what we did for neutron elastic scattering and show that

$$E' = \frac{EE_0}{E(1-Cos\theta)+E_0}$$

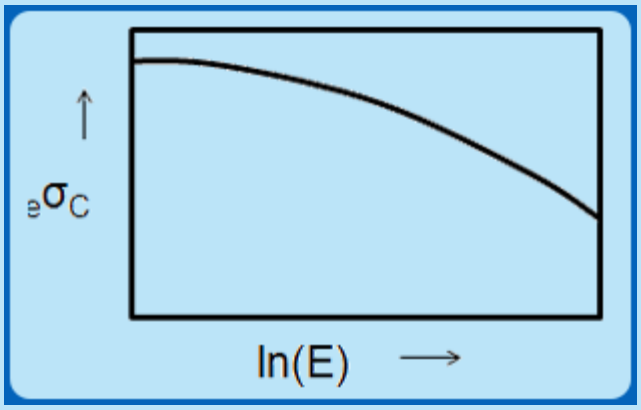
$$\lambda' - \lambda = \frac{h}{m_0 c}(1-Cos\theta)$$

$$where \frac{h}{m_0 c} = \lambda_{Compton} = 2.426x10^{-10} cm$$



- In the above expressions the energy and wavelength of photon after scattering, respectively E' and λ'.
- The Compton cross section per electron $e^{\sigma}C$ decreases with energy as shown in the adjoining figure.
- The total cross section per atom can be written as

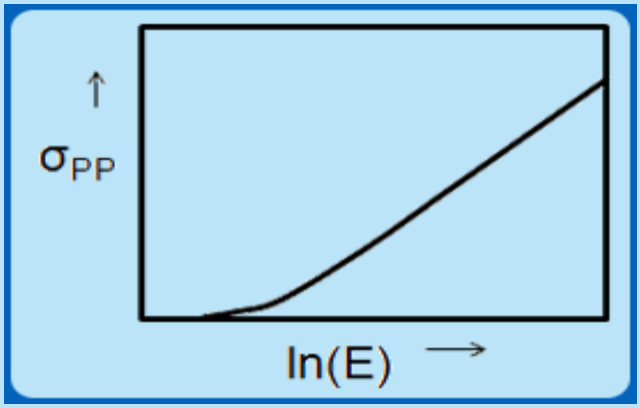
$$\sigma_C = Z \; {}_e\sigma_C$$



- As there are multiple scattering in materials, Compton Scattering poses most difficulty from prediction point of view.

Pair Production

- In Pair Production, the γ-ray completely disappears and a pair of an electron and a positron are produced.
- Since the rest mass energy of electron and positron are each equal to 511 keV, the energy of γ-ray has to be at least 1.02 MeV for pair production to initiate.
- The cross section for pair production, σ_{pp}, varies as Z^2.



$\sigma_{PP} \alpha Z^2$

Combined Effects

- Having gotten a gist of all three effects, now we shall discuss the combine effects of these.
- Similar to the macroscopic cross section for neutrons, we shall introduce the macroscopic attenuation coefficient, μ, as

$$\mu = N\sigma$$

- Similar to neutron interactions, the number of interactions per unit volume can be written as

$$\frac{\dot{R}}{V} = \mu I$$

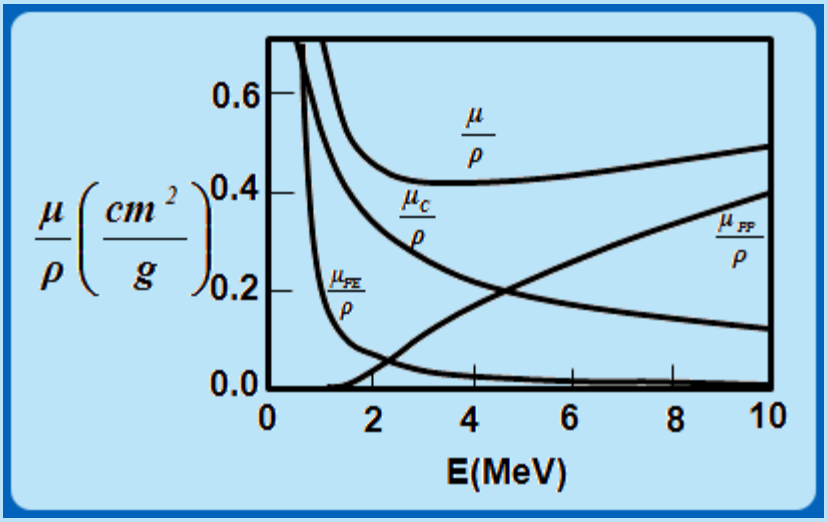
- The total macroscopic attenuation coefficient can be written as,

$$\mu = \mu_{PE} + \mu_C + \mu_{PP}$$

- Usually, one defines a mass attenuation coefficient as,

$$\frac{\mu}{\rho} = \frac{\mu_{PE}}{\rho} + \frac{\mu_C}{\rho} + \frac{\mu_{PP}}{\rho}$$

- The mass attenuation coefficient for lead is shown below



- The uppermost curve represents the summation all the effects.
- It may be noted that for 0.5 < E < 4 MeV, Compton Scattering is dominant.
- Further, the exponent of Z for photo electric field and pair production are higher than that for compton scattering. Hence, for lower Z elements, Compton effect is most dominant.

- Further, by using the fact that

$$\left.\frac{\mu}{\rho}\right|_C = \frac{N\sigma_C}{\rho} = \frac{N_{Avo}\rho}{M}\frac{\sigma_C}{\rho} = \frac{N_{Avo}\sigma_C}{M} = \frac{N_{Avo}Z_e\sigma_C}{M}$$

- Except for Hydrogen isotopes and heavy elements, Z/M ~ 0.5.
- This implies μ/ρ is approximately constant for most low Z materials.

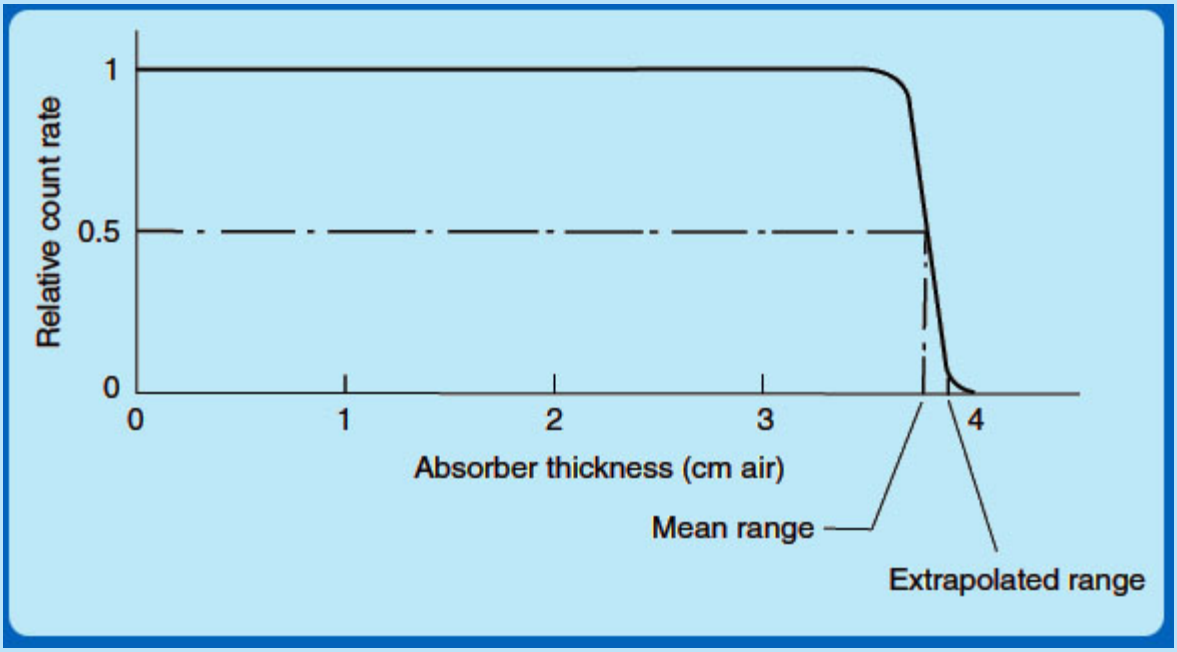
- Extending the concepts from neutrons, we can write,

$$I(x) = I_0 e^{-\mu x}$$

- However, since in Compton Scattering, the photon is not fully attenuated, some correction will be required.

Behaviour of Alphas-I

- Charge particles interact with nucleus and electrons through Coulombic interaction and it ionizes the medium.
- It is a general practice to refer to terms like Specific Ionization, Stopping Power and Range. These are defined as follows.
- Specific Ionization is expressed as the number of ion-pairs produced per unit distance travelled.
- Stopping Power is expressed in energy lost per unit distance travelled. This is also called LET (Linear Energy Transfer).
- Range is the total distance travelled before the radiation is absorbed.
- The Count rate of alphas as a function of absorber thickness in air is shown in the following figure.



$$R_{air}(cm) = 0.322\ (E(MeV))^{1.5} \quad for\ 8MeV < E < 2MeV$$

- The almost abrupt fall in intensity can be attributed to the almost mono-energetic spectra of alphas.

- The range of alphas in other materials relative to that in air is found by using the Bragg-Kleeman Rule

$$\frac{R}{R_{air}} = \frac{\rho_{air}}{\rho}\sqrt{\frac{M}{M_\alpha}} \Rightarrow R \propto \frac{\sqrt{M}}{\rho}$$

- Alphas do not pose any shielding problem, though they affect tissues. More discussion on this

will follow later.

Behaviour of Betas-I

- Unlike alpha sources that emit mono-energetic particles, Betas are emitted with a continuous spectrum.
- However the interaction of Betas are similar.
- The specific ionization of betas are less than that of alphas.
- It has been found that the specific ionization varies exponentially with distance

$$i(x) = i_0 e^{-\mu x} = i_0 e^{-\frac{\mu}{\rho}\rho x}$$

- μ/ϱ is almost a constant for most materials.
- Based on Al measurements, an empirical formula given by Lamarsh is

$$\frac{\mu}{\rho} = \frac{17}{E_{max}^{1.14}}$$

- Similarly Range R_{max} is correlated as

$$R_{max}\rho = 0.042E_{max}^{(1.305-0.0954\ln E_{max})} \qquad E_{max} < 2.5 \text{ MeV}$$
$$R_{max}\rho = 0.530E_{max} - 0.106 \qquad E_{max} > 2.5 \text{ MeV}$$

$R_{max}\,\varrho$ in g/cm^2, E_{max} in MeV.

Exposure and Absorption

- There is a subtle difference between exposure to radiation and absorbed dose.
- Whenever there is a radioactive source emitting radiation, it will irradiate all the material that is exposed to it.
- However, if the material is transparent to it, then, no energy would be deposited on the material and hence there will not be any radiation damage.
- As damage due to radiation will be proportional to the energy deposited on the material, it has more importance.

Unit of Exposure

Roentgen

- Roentgen (denoted by R) is used to quantify the X-ray or γ-ray radiation exposure.
- It is detected by ionization produced in a gas filled detector.
- 1 Roentgen is equivalent to generation of 1 ESU of charge in 1 cm^3 of air at 0 ^oC.
- This is equivalent to 2.58 x 10^{-4} Coulombs/kg-air or 5.47 x 10^{10} MeV/kg-air or 87.7 ergs/g-air (Note that 1 electron charge = 1.602 x 10^{-19} Coulomb and 34 eV is required to create an electron-ion pair).

- The SI Unit of Exposure is X-unit.
- 1 X-unit = 1C/kg-air.
- 1 X-unit = 3881 R.
- 1 R = 2.58 x 10-4 X-unit.
- It should be noted that 1R is equivalent to the 87.7 erg/g of absorbed energy in air.

Absorbed Dose

- The damage caused is a function of specific energy absorption.
- The CGS unit of absorbed dose is called rad.
- 1 rad = 100 ergs/g-media.
- The SI unit is Gray; 1 Gy = 1 J/kg-media = 100 rad.

Dose-Exposure Relationship-I

- The motivation of this relationship is to directly convert the exposure that can be measured by instruments to dose absorbed by material.
- The energy absorbed in a media is directly proportional to the value of μ/ϱ, which is directly proportional to the electronic density.
- The electronic density in air is 3.01 x 10^{23} electrons/g.
- The same in a tissue is typically 3.28 x 10^{23} electrons/g.
- As Dose is directly proportional to μ/ϱ, we can write

$$D_{tis}(rad) = D_{air}(rad)\ \frac{\mu/\rho|_{tis}}{\mu/\rho|_{air}}$$

- In the above expression D refers to dose.

$$\Rightarrow D_{Tis}(rad) = \frac{328}{301} 87.7 = 95\ erg/g = 0.95\ rad$$

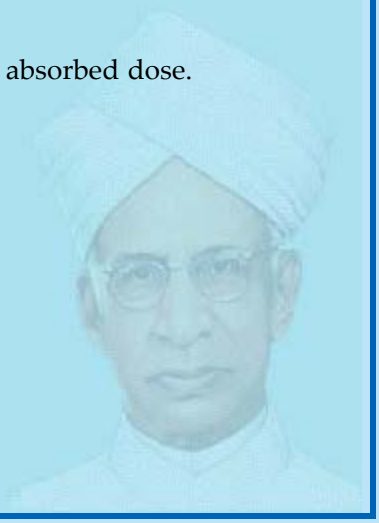
- Thus an exposure of 1 R, which produces 87.7 ergs/g in air, shall produce 0.95 rad in the tissue,

- Similar computations can be done for any other material by employing its μ/ϱ value, which is generally tabulated.

Objectives

In this lecture you will learn the following

- We shall introduce the concept of Relative Biological Effectiveness (RBE).
- Then , we shall derive the radiation balance equation.
- Finally, we shall study methods of estimation of radiation flux and absorbed dose.



Relative Biological Effectiveness

- Biological damage is just not decided only by dose.
- It also depends on the LET (dE/dx).
- Ionizing radiation like α and β have large LET than γ.
- Relative Biological Effectiveness (RBE) is defined as

$$RBE = \frac{\text{Biological damage due to a given radiation}}{\text{Same damage due to 200 keV } \gamma_{ray} \text{ of same dose}}$$

- This depends on the tissue, dose rate, etc., and hence is too complex.

Quality Factor

- International Commission on Radiation Units and measurements (ICRU) has recommended Quality Factors after duly considering the RBE.

Type of Radiation	Quality Factor Q
X-ray or γ-ray	1
β with E < 0.03 MeV	1
β with E > 0.03 MeV	1.7
Alphas	10
Heavy Recoil Nuclei	20
Neutrons 0-1 keV	2
Neutrons 1-10 keV	2.5
Neutrons 10-1 00 keV	7.5

- Q is used to estimate the biological dose.

Dose Equivalent

- The Dose Equivalent, also referred to as biological dose (H) can be computed from absorbed dose denoted (D) by multiplying it with the Quality factor (Q)

$$H = D\ Q$$

- The unit of Dose Equivalent is called rem (Roentgen Equivalent Man)

$$H(rem) = D(rad)\ Q(rem/rad)$$

- The SI equivalent of rem is Sievert

$$1\ Sv = 100\ rem \qquad Also\ 1Sv = 1Gy\ x\ Q$$

- It should, however, be noted that the damage to different organs with 1 rem dose is different as the tolerance for each organ is different.
- Hence permissible dose for each organ is different.
- For converting non-uniformly irradiated organs into an equivalent uniformly irradiated whole body, an effective dose was defined.

Effective Dose

- The effective dose is computed using the relation.

$$\Rightarrow H_E = \sum_{All\ T} w_T H_T$$

- In the above equation w is the weighting factor for a given tissue, H_T is the dose for that particular tissue and H_E is the effective dose.
- The values of W_T for various organs are tabulated below

Dose Equivalence

Tissue/Organ	W_T, ICRP 60
Gonads	0.20
Red bone marrow	0.12
Colon	0.12
Lung	0.12
Stomach	0.12
Bladder	0.05
Breast	0.05
Liver	0.05
Thyroid	0.05
Esophagus	0.05
Skin	0.01
Bone Surface	0.01
Remainder	0.05

Radiation Balance Equation

- From the definition of μ, we can state that the rate of radiation absorbed in an infinitesimal control volume of area A and thickness dx can be written as

$$-AdI = \mu IAdx$$

- The rate of energy absorbed, if the radiation was mono energetic, shall be

$$\Delta E = \mu IEAdx$$

- The rate of specific energy absorbed or the dose can then be calculated as:

$$D = \frac{\mu IEAdx}{Adx\rho} = \frac{\mu IE}{\rho}$$

- Thus, we also note that the dose is directly proportional to μ/ϱ.
- Thus dose at a local point can be computed using the local intensity, energy of the radiation and μ/ϱ of the media.
- To determine the local intensity, one has to write the flux balance for the radiation, often called the Radiation Balance Equation.

Generalised Treatment

- Most of the cases can be handled by deriving the radiation balance from a point source.
- To keep the physical understanding easier for the beginners, the governing equation is derived for a spherically symmetric system.
- The conservation of radiation for the shell shown in the figure would imply

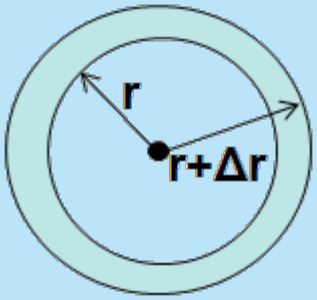
$$4\pi r^2\phi - \left(4\pi r^2\phi + \frac{d}{dr}(4\pi r^2\phi)\Delta r\right) - 4\pi r^2\Delta r\mu\phi = 0$$

Rate of radiation entering the control volume

Rate of radiation leaving the control volume

Rate of radiation absorbed in the control volume

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}(r^2\phi) = -\mu\phi$$



- The equation can be manipulated as follows:

Defining, $r^2\phi = \psi$, we can write,

$$\frac{d\Psi}{dr} = -\mu\Psi \qquad \Rightarrow \Psi = \Psi_o e^{-\mu(r-r_o)} \qquad \Rightarrow r^2\phi = r_o^2\phi_o e^{-\mu(r-r_o)}$$

- As r tends to 0, we can state that

$$4\pi r_o^2\phi_o = S \qquad \Rightarrow \phi_o = \frac{S}{4\pi r_o^2}$$

- Substituting the value of ϕ_0, and putting $r_o = 0$, we can write

$$\Rightarrow \phi = \frac{S}{4\pi r^2}e^{-\mu r}$$

- Few corollaries can be derived from the above.
- If the media does not absorb, the flux of radiation varies as

$$\Rightarrow \phi = \frac{S}{4\pi r^2}$$

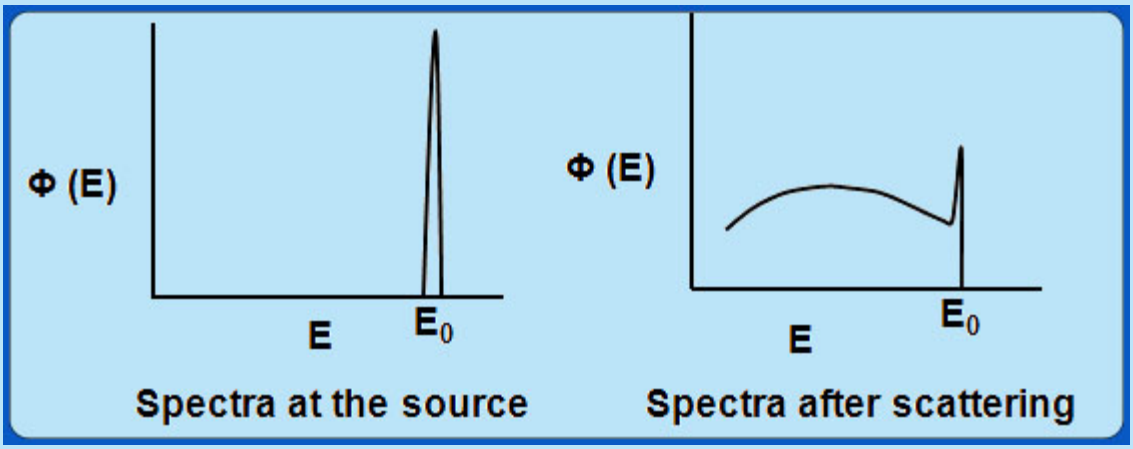
- If we know the flux on a spherical reference surface, the further variation of intensity in the domain $R_{ref} < r < R$ can be expressed as

$$\Rightarrow \phi = \frac{R_{ref}^{\ 2}}{r^2}\phi_{ref}e^{-\mu(r-R_{ref})} = \frac{S}{4\pi r^2}e^{-\mu(r-R_{ref})}$$

- We have seen previously seen that Betas can be assumed to have specific ionisation decreasing exponentially.
- This implies that the flux (with $E = E_{max}$) can be assumed to decrease exponentially with distance and hence the uncollided flux can be computed using the governing equation just derived.
- However, for Gammas, experiments have indicated that the above model underpredicts the dose rate and hence needs some correction.

Build-up Factors

- To have more realistic computations, a factor called a build-up factor has been introduced.
- The spectrum of gamma from a source is typically monoenergetic, whereas, after scattering, the same have a continuous energy spectra as shown in the figures



- To compute the exposure rate in such cases, we need to perform the following integral

$$\dot{X}(r) = \int_0^{E_0} \phi(E,r) E dE \frac{\mu}{\rho}$$

- The computations required to find spectra are complex.
- Such computations have been carried out and the result is finally expressed as

$$\dot{X}(r) = \dot{X}(0) B_p(\mu r) e^{-\mu r}$$

- In the above equation $B_p(\mu r)$ is an empirical parameter obtained from the integral.
- We had derived for a point source the intensity (flux) at a distance r from the source as

$$\Rightarrow \phi_u(r) = \frac{S}{4\pi r^2} e^{-\mu r}$$

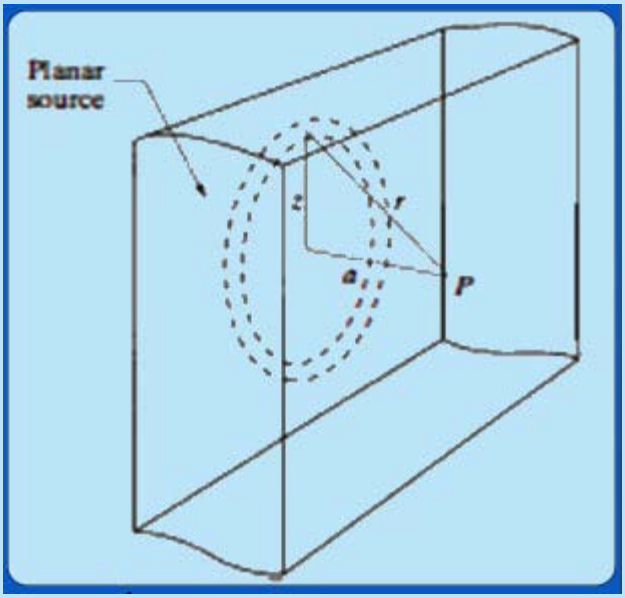
- We can define a buildup flux, as the modified flux for computation of dose as,

$$\Rightarrow \phi_b(r) = \phi_u(r) B_p(\mu r) = \frac{S}{4\pi r^2} e^{-\mu r} B_p(\mu r)$$

- The values of $B_p(\mu r)$ are tabulated in standard texts and handbooks.

Isotropic infinite planar source

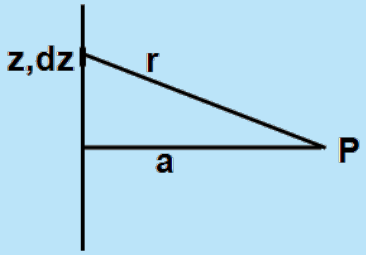
- The aim is to compute the build-up flux at the point p which at a distance a from the source.
- The uncollided flux from a ring of radius z and thickness dz is



$$\Rightarrow d\phi_u(a) = \frac{S'' 2\pi z dz}{4\pi r^2} e^{-\mu r}$$

- Since $r^2 = z^2 + a^2$

$$\Rightarrow r dr = z dz$$



- Thus, the build-up flux due to the whole planar source can be obtained as

$$\Rightarrow \phi_b(a) = \int_a^\infty \frac{S'' 2\pi r dr}{4\pi r^2} B_p(\mu r) e^{-\mu r}$$

- The Build-up factor can be fitted into the following for obtaining analytical solutions

$$B_p(\mu r) = A_1 e^{-\alpha_1 \mu r} + A_2 e^{-\alpha_2 \mu r}$$

- This is referred to as Taylor's expression for build up factor.
- Since $B_p(0) = 1$, we can write

$$A_1 + A_2 = 1$$

- Hence, we can write $B_p(\mu r) = A e^{-\alpha_1 \mu r} + (1 - A) e^{-\alpha_2 \mu r}$

- The values of α_1 , α_2 and A are tabulated in text books and hand books.
-

Substituting the Taylor form for build-up factor, we can write,

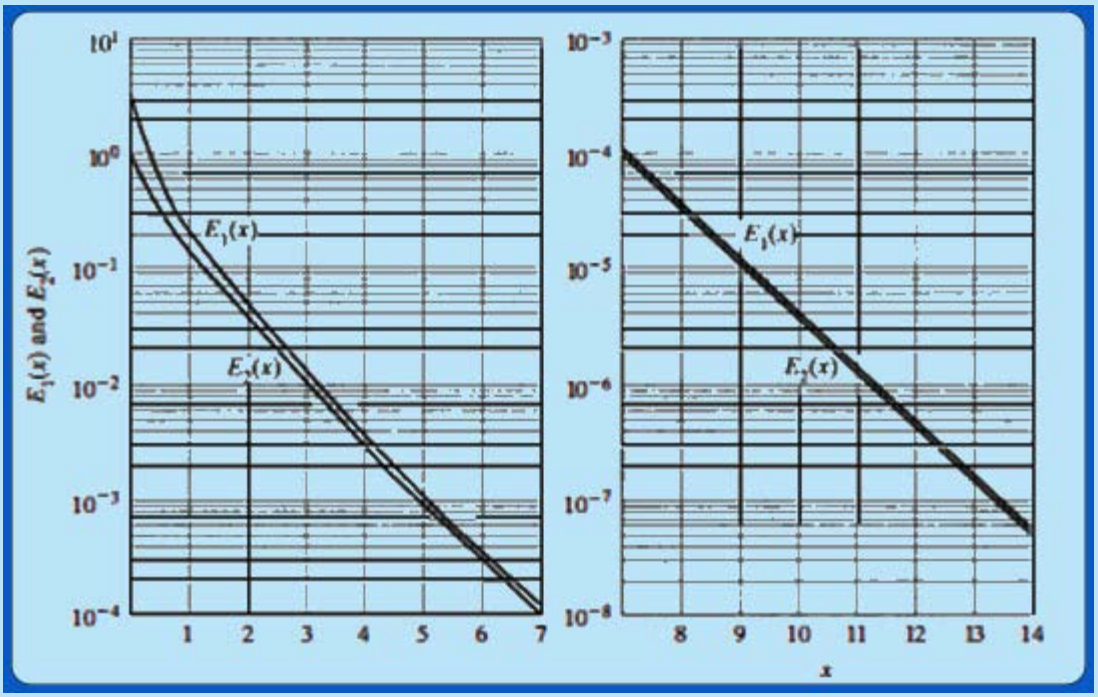
$$\Rightarrow \phi_b(a) = \frac{S''}{2}\int_a^\infty \frac{dr}{r}\left(Ae^{-\mu r(1+\alpha_1)} + (1-A)e^{-\mu r(1+\alpha_2)}\right)$$

- Since closed form solution for the integral is not possible, these are expressed in the form of E_n functions.
- $E_n(x)$ is defined as

$$\Rightarrow E_n(x) = x^{n-1}\int_x^\infty \frac{e^{-t}dt}{t}$$

E_n Functions

- The value of $E_1(x)$ and $E_2(x)$ have been tabulated and their variations are shown below



- Beyond x = 14, the function can be approximated by

$$\Rightarrow E_n(x) = e^{-t}\left(\frac{1}{x+n} + \frac{n}{(x+n)^3}\right)$$

- Thus,

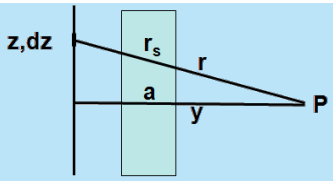
$$\phi_b(a) = \frac{S''}{2}\left(\int_{\mu a(1+\alpha_1)}^{\infty} A\frac{d\mu(1+\alpha_1)}{\mu(1+\alpha_1)r}(e^{-\mu r(1+\alpha_1)}) + \int_{\mu a(1+\alpha_2)}^{\infty}(1-A)\frac{d\mu(1+\alpha_2)}{\mu(1+\alpha_2)r}(e^{-\mu r(1+\alpha_2)})\right)$$

$$\Rightarrow \phi_b(a) = \frac{S''}{2}\left(AE_1(\mu a(1+\alpha_1)) + (1-A)E_1(\mu a(1+\alpha_2))\right)$$

- Now let us consider the same case with absorption only in part of the slab and the rest does not interact.

- The uncollided flux from a ring of radius z and thickness dz is

$$\Rightarrow d\phi_u(y) = \frac{S'' 2\pi z dz}{4\pi r^2} e^{-\mu r}$$



- Since $\Rightarrow \frac{r}{y} = \frac{r_s}{a}$ $\Rightarrow d\phi_s(y) = \frac{S'' r dr}{2r^2} e^{-\frac{\mu r a}{y}} B_p(\frac{\mu r a}{y})$

$$\Rightarrow \phi_s(y) = \frac{S''}{2} \int_{\mu a}^{\infty} \frac{dr}{r} e^{-\frac{\mu r a}{y}} (Ae^{-\frac{\mu r a}{y}\alpha_1} + (1-A)e^{-\frac{\mu r a}{y}\alpha_2})$$

- As done earlier, splitting the two terms and using the definition of E_n function, we can write

$$\Rightarrow \phi_s(y) = \frac{S''}{2}\left(AE_1(\mu a(1+\alpha_1)) + (1-A)E_1(\mu a(1+\alpha_2))\right)$$

- This is same as the previous solution.
- This is also consistent with the beam solution as the intensity will not drop in non-interacting media.

Arbitrary Source distribution

- The build-up flux for any arbitrary source can be computed numerically as follows
  - Divide the volume into convenient sub-volumes.
  - From any sub-volume i, the build up flux can be computed at a point of interest as,

$$\Rightarrow \phi_b = \frac{S_i}{4\pi r_i^2} e^{-\mu r_i} B_p(\mu r_i)$$

where, r_i is the distance between the reference point and the centroid of the sub-volume.

  - Sum this up for all the sub-volumes.

Dose from Internally Deposited Source

- Often radioactive nuclei gets ingested and deposited in the body due to the bio-chemistry of the body.
- It is assumed that the source distributes uniformly in the organ where it is deposited.
- Assuming that the source density is S''', and the organ is much bigger than the range of the radiation, it can be assumed that the organ is infinite.
- This would imply absorption per unit volume is same as the source density.
- Hence the dose rate shall be S'''E/ϱ.
- The absorbed material will be removed from the body by the biological excretion mechanisms.
- In addition, due to radio-active decay, the concentration or the source density shall also reduce.
- Thus the dose rate will decay with time and we can write the following expressions

$$\frac{d\dot{D}}{dt} = -(\lambda_B + \lambda_R)\dot{D} \qquad \Rightarrow \dot{D}(t) = \dot{D}(0)e^{-t(\lambda_B+\lambda_R)}$$

- In the above expression λ_B and λ_R are decay constants for the biological and radioactive decay mechanisms.
- Often we define $\lambda_{eff} = \lambda_B + \lambda_R$
- As half life gives a good feel for appreciating the decay of dose, we can define $T_{0.5\text{-eff}}$ as follows

$$\frac{\lambda_{eff}}{0.693} = \frac{\lambda_B}{0.693} + \frac{\lambda_R}{0.693} \qquad \Rightarrow \frac{1}{T_{0.5-eff}} = \frac{1}{T_{0.5-B}} + \frac{1}{T_{0.5-R}}$$

- The effective half life will be smaller than the lower of the biological and radioactive half-lives.
- The total dose due to the isotope can be computed as

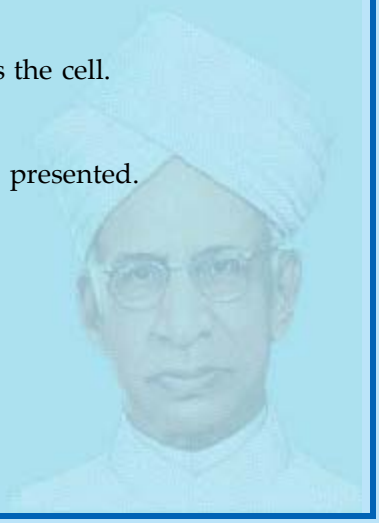
$$D(\infty) = \int_0^{\infty} \dot{D}(t)dt$$

Objectives

objectives

In this lecture you will learn the following

- The objective of this lecture is to summarise the effect of radiation on health of humans.
- It begins with the history of adverse health effects on humans.
- Then we study the human cell and appreciate how radiation affects the cell.
- Some appreciation of cancer is presented.
- Then the allowable dose for humans as per the ICRP guidelines are presented.
- Finally effects of large exposure to radiation is summarised.

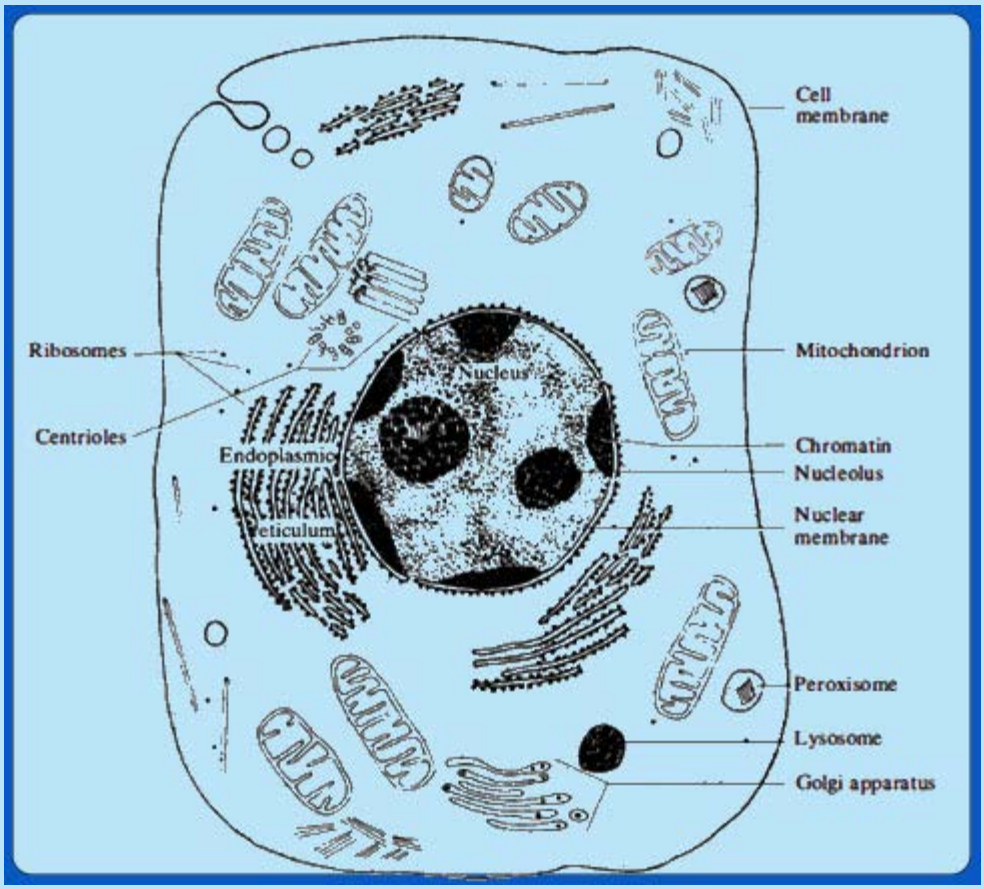


History of Radiation Effects

- Late 1800: Discovery of radioactivity Becquerel, and Madam Curie suffered from reddening of skin called Erythema.
- 1897: Many were misusing X rays and several cases of X-ray burns were reported.
- 1911: 94 cases were X-ray induced tumor were reported, out of which 50 were radiologists.
- 1922: 100 radiologists died of cancer.
- All of the above led to the fact that external exposure was harmful.
- Similar conclusions were drawn from internal exposure.
- 1915-30: It was fashionable to take radium and thorium injections as tonics.
- 1928-45: Many patients were injected ThO_2 as a contrast material for X-ray images.
- Many of the above suffered from Cancer.
- Many radium paint workers suffered from cancer as they ingested Ra unintentionally.
- Thus there was overwhelming evidence that both internal and external exposure is hazardous.

Cell Biology

- Human tissues are made of cells.
- Most cells are of the order of 10^{-3} cm in diameter. Some cell like nerve cell can stretch to 1 m!!
- There are about 4×10^{13} cells in an average adult.
- There are two kinds of cells:
  - Somatic Cells (Body cells).
  - Germ Cells (Reproductive Cells).
- Every Cell has a nucleus and the surrounding cytoplasm. There are several components performing different functions (Refer Lamarsh).

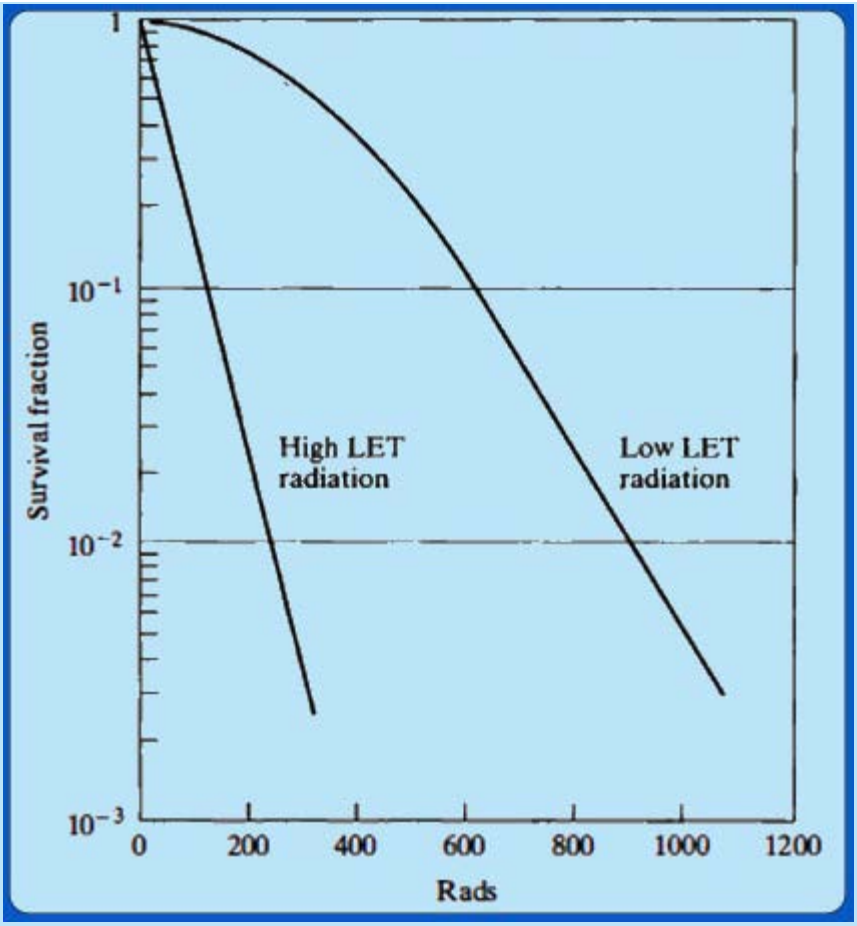


- One of the important constituent of the cell is chromatin. Its role shall be discussed later.
- Cells multiply by cell division. The division of somatic cell is called **Mitosis**.
- Every cell follows a cycle. There are four stages in this cycle. These are denoted by M-G1-S-G2. G1 and G2 refer to time gaps between the Mitosis and Synthesis of DNA phases.
- The chromatin is normally a tangled bunch strands of DNA (De-oxyribose Nucleic Acid) molecules and some nuclear proteins.
- During Mitosis, the chromatin untangles itself and becomes distinct set of bundles called Chromosomes.

- In humans there are 46 chromosomes in somatic cells and 23 chromosomes in germ cells.
- During Mitosis, each chromosome replicates itself.
- Certain segments of DNA molecules in chromosomes provide coding that control the synthesis of proteins. These are called genes.
- Changes brought about in genetic codes can result in inheritable changes.

Radiation Effects

- Effects on cells can be classified as
  - Stochastic.
  - Non-Stochastic.
- Stochastic events cannot be predicted with certainty.
- Cancer and Genetic Mutations are examples of Stochastic effect
- The severity of the cancer is unrelated to the original dose.
- Similarly, there is no reason yet to believe that there is a threshold below which stochastic effects will not be initiated.
- In these cases there is a distinct threshold below which these effects are not seen.
- The damage to the cells are caused by two effects
  - Direct effect- Breakage of Molecules by rupturing bonds.
  - Indirect Effect- Formation of Oxy and Hydroxyl radicals, which react chemically with the cell.
- The latter is claimed to be more important in creating biological effects.
- Whenever a colony of cell is exposed to radiation for a short period of time (Called **Acute Exposure**), a fraction of cells will cease to reproduce.
- This is called **Reproductive Death.**
- By increasing the exposure dose, we can obtain curves as shown in the following figure



- Damage caused has a direct relationship with the absorbed dose.
- Biologists have developed techniques to produce colony of synchronous cells.
- These cells are in the same phase during all stages of growth.
- This provides opportunity to check the radio-sensitivity of the cells at different phases.
- Experiments reveal that the cells are most sensitive during Mitotic phase.
- If they survive mitotic phase, they generally survive. However, they might have been sensitized for cancer to develop later.
- As most cells multiply during embryonic stage, radiation exposure to embryos must be avoided.
- Similarly, it is better to avoid growing individuals to radiation exposure.

Characteristics of Cancer

- Cancer is the second largest killer of humans after heart attacks.
- Cancer is characterized by uncontrolled growth of cell division.
- There are thousands of chemicals and natural products (**carcinogens**) that cause cancer.
- The behaviour of radiation induced cancer is no different than caused by other causes.
- There are two stages to a cancer.
- The first one is called the **initiation phase**, where a lesion (injury) is produced in the DNA of one or more cells.
- For some unknown reason, either due to hormonal or immunological protective agents, these cells do not multiply.
- In the second phase called **Promotion Phase**, the protective mechanism goes awry and permits unrestrained multiplication.
- The promotion can be triggered by viral infections, hormonal changes due to ageing, chemical irritants, etc.
- Due to a two-stage process there is a large latent period. Hence it always becomes difficult to pin point the cause.
- Further the cells that go awry are not the originally affected cells but those that are several generations later.
- This leads to the suggestion that these are caused by chromosomes, that carry the information.
- Several studies, carried out on the data from damage to radiation affected victims, by several agencies have not yet established that the radiation induced effects have been passed on from parent to progeny.
- The data is always clouded by natural causes.

Quantifying Health Risks

- From all the discussions, it should be obvious that estimating health damage can be complex.
- However, for the purpose of regulation of the industry, safety standards have to be evolved that serves as the guiding principle.
- Several international and national bodies in several counties have worked on it and standards have been evolved that are fairly conservative.
- The standards have been constantly upgraded in light of new research findings.

Databases on Biological Effects of Radiation

- There are several sources of information on the biological effects. These are:
  - Occupational Exposure from
    - Medical Personnel.
    - Uranium Miners.
    - Radium Painters.
    - Atomic Energy Workers.
    - Industrial Radiographers.
  - Patients undergoing radiotherapy.
  - Survivors of nuclear accidents from Japan, UK.
- The most comprehensive database is from the study of 86,572 Japanese survivors of atomic bombs.
- 65% of them received dose less than 100 mGy or 10 rads.
- The remaining received greater than 10 rads.
- The findings have played a key role in the formulation of Radiation Protection standards.

History of Radiation Protection

- The first organized action on Radiation Safety was organized by British Roentgen Society in 1915.
- Subsequently, the Society published further recommendations.
- The first international congress on Radiology was held in the year 1925.
- The second international congress on Radiology was held in 1927 and it established the International Radium and X-ray Protection Committee.
- In the year 1950, the Committee was renamed as the International Committee on Radiation Protection (ICRP).

ICRP Recommendations

- Since its inception, ICRP is the leading agency for providing guidance in radiation protection.
- If we summarize the latest state,
  - There is little evidence of any harmful effects at the levels prescribed.
  - However, estimate of consequences needs societal and scientific judgment.
- Let us now see the justification of the above statements.

ICRP – 1959 Recommendations

- In 1959, the initial dose recommendation was based on the concept of **tolerance dose.**
- Such a limit prevents observable harmful effects.
- The dose limits prescribed were
  - Surface skin dose (skin depth of 0.007 cm) of 600 mrems/week.
  - Deep Dose (at a depth of 1 cm) of 300 mrems/week at 1 cm depth.
  - Deep Dose of 5 rems/year for prevention of genetic damage.

ICRP Publication 26 - I

- In 1977, with research data available form the survivors of atomic bombings led to new recommendation called ICRP Publication 26.
- Here **Cancer** was recognized as the major concern.
- Bio-math models assume linear hypothesis and assumes that even a single radiation can cause cancer.
- It was also recognized that different organs have different tolerance. Hence the concept of Dose Equivalence was introduced.

Effective Dose

- For converting non-uniformly irradiated organs into a uniformly irradiated whole body, an effective dose was defined.
- The effective dose is computed using the relation,

$$\Rightarrow H_E = \sum_{All\ T} w_T H_T$$

- In the above equation w is the weighting factor for a given tissue, H_T is the dose for a particular tissue and H_E is the effective dose.
- The values for W_T for various organs from ICRP 26 and ICRP 60 are tabulated below

Dose Equivalence

Tissue/Organ	W_T, ICRP 26	W_T, ICRP 60
Gonads	0.25	0.20
Red bone marrow	0.12	0.12
Colon	Not given	0.12
Lung	0.12	0.12
Stomach	Not given	0.12
Bladder	Not given	0.05
Breast	0.15	0.05
Liver	Not given	0.05
Thyroid	0.03	0.05
Esophagus	Not given	0.05
Skin	Not given	0.01
Bone Surface	0.03	0.01
Remainder	0.30	0.05

- The effective whole body dose shall be restricted to 50 mSv (5 rems/year).
- This shall include external as well as internal radiation.
- The continuing study on Japanese survivors suggested that the cancer probability is under estimated.
- ICRP issued a revised document ICRP 60 in 1990.

ICRP Publication 60

- IN ICRP 60, the effective whole body dose has been revised to less than 2 rem averaged over a five year period and less than 5 rem in any given year.
- Apart from individual dose limits, ICRP also prescribes a definition of collective dose for population

$$\Rightarrow H_{Pop} = \sum_{All\ n} n_i H_i$$

- ICRP postulates 500 excess cancer for a collective dose of 10^6 person-rem.
- Even this number is within the variability in number of cancers per year due to natural causes.

Comparision between recommendations of ICRP 26 and ICRP 60 is shown below

Exposure to Occupational Workers

Category of Exposure	ICRP 26	ICRP 60
Individual Dose Limits		
Occupational Exposure including Recovery Operations	50 mSv/year	20 mSv/year averaged over 5 years (though in one year ICRP 26 is OK)
Any organ other than listed below	500 mSv/year	Dropped
Lens of the eye	300 mSv/year	150 mSv/year
Skin	20 Sv in life time	500 mSv/year
Hands and feet	Not given	500 mSv/year
Pregnant woman	< 10 mSv	2 mSv to surface of abdomen or 1 mSv from intake of nuclides

- **Exposure to public is typically 1/10 of above except for general whole body dose 1 mSv for ICRP-60**.

Emergency Exposure

Category of Exposure	ICRP 26	ICRP 60
Emergency Exposure		
Occupational Exposure for life saving and other rescue operations	500 mGy whole body or 5Gy individual organ	~ 500 mSv and 50 Sv for Skin
Public Exposure Food stuff Static Iodine Sheltering Temporary Evacuation Permanent Relocation	5-50 mSv in first year 50-500 mSv to thyroid 5-50 mSv in 2 days 50-500 mSv in 1 week 100 mSv in first year and 1 Sv in life time	Same

ICRP Publication 103 (2007)

- ICRP released a new publication in 2007.
- The recommendations are almost identical to ICRP 60 except for emergency exposure.
- While the general philosophy here is also same but gives wider scope during application.
- For details one should look at higher level texts on Health Physics.

ALARA Principle

- Although dose limits have been worked out for the deterministic effects very scientifically with suitable factor of safety built in, ICRP does not permit deliberate exposure.
- This is because any exposure to radiation is detrimental to health, if linear hypothesis is valid for the behaviour of stochastic processes.
- Molecular biologists have found intracellular mechanisms for the repair of DNA in bacteria.
- Geneticists have observed dose rate dependence on radiogenic mutagenesis.
- The above two points suggest existence of a threshold for stochastic effects.
- Stochastic effects in population have not been seen for dose less than 10 rads.
- However, ICRP notes that absence of proof cannot be taken as the proof of absence.
- Hence ICRP subscribes to linear hypothesis all the way to zero dose.
- Thus setting up a limit on the Maximum Allowable Dose (MAD), one has to pass a value judgment.
- Hence ICRP recommends justification for any exposure.
- ICRP's major guidelines are summarized below.
- No policies shall be adopted unless its introduction has a net positive benefit.
- Hence the decision has to be societal and not a radiation decision.
- All doses have to be optimized to keep it As Low As Reasonably Achievable (ALARA).
- However, dose limits shall not exceed the allowable ones.

Conclusions

- The biggest controversy is to take decisions based on economic principle and this cannot be done unless a price is put on human life.
- Such decisions become necessary when deciding on amount of shield to be provided, etc.
- While one can keep debating inconclusively, the statement made by the ICRP founder Lauriston Taylor as quoted in Cember is given below.
- No one has been identifiably injured by radiation while working within the numerical standard set in 1934. Since then radiation standards have been lowered by a factor of 10. Hence one can conculde that current standards are sufficiently restrictive.

Health Response to Radiation

- The radiation exposure can be divided into two major classes
  - Acute Exposure (Short exposure generally due to accidents or medical treatments).
  - Chronic Exposure (Long term exposure on account of occupation).
- The prediction due to acute large exposure has been fairly well understood and documented.
- These are summarised in the Table given below

Dose	Effect
5-75	Chromosomal aberrations and temporary depression of white blood cell levels in some individuals. No other observable effects.
75-200	Vomiting in 5 to 50% of exposed individuals within a few hours, with fatigue and loss of appetite. Moderate blood changes. Recovery within a few weeks for most symptoms.
200-600	For doses of 300 rems or more, all exposed individuals will exhibit vomiting within 2 hours. Severe blood changes, with hemorrhage and increased susceptibility to infection, particularly at the higher doses. Loss of hair after 2 weeks for doses over 300 rems. Recovery from 1 month to a year for most individuals at the lower end of the dose range; only 20% survive at the upper end of the range.
600-1000	Vomiting within 1 hour. Severe blood changes, hemorrhage, infection, and loss of hair. From 80% to 100% of exposed individuals will succumb within 2 months; those who survive will be convalescent over a long period.

Objectives

In this lecture you will learn the following

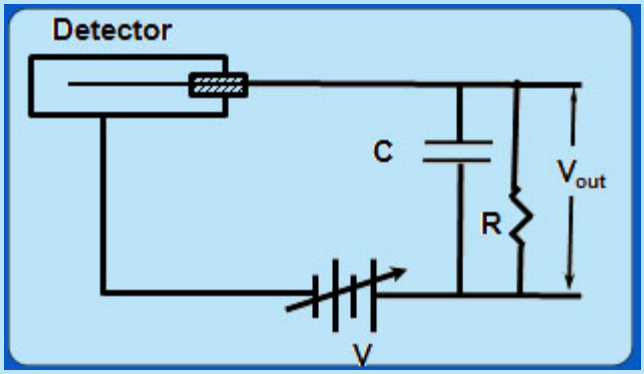
- We shall study the principles used in radiation detectors.

Introduction

- We have seen in the last few lectures that
  - Radiation is harmful if there is excessive exposure.
  - We have studied the laid regulations on acceptable exposure and dose.
  - Hence there is a need to quantify the radiation field in a contaminated area of near sources of radiation to control the exposure.
  - In this lecture we shall see the principles of radiation detection and the associated uncertainty in measurements.
- Radiation is colourless, odourless and tasteless. Has no form or exerts any pressure.
- Hence at low doses, it is not responsive to any of our sense organs.
- Hence, to detect its presence and intensity, external sensors are required.
- Obviously, to detect its presence, we need to exploit some of its interaction modes. Some of the common detectors and the interaction modes they use are summarized in the next table

Effect	Type of Instrument	Media
Electrical	Inonisation Chamber Proportional Counter G.M. Counter Solid-state Detector	Gas Gas Gas Semi-conductor
Chemical	Film Chemical Dosimeter	Photographic Emulsion Solid or Liquid
Light	Scintillation Counter Cerenkov Counter Opticoluminescent Dosimeter	Crystal or Liquid
Thermo-luminesence	Thermo-luminesence Dosimeter	Crystal
Heat	Calorimeter	Solid or Liquid

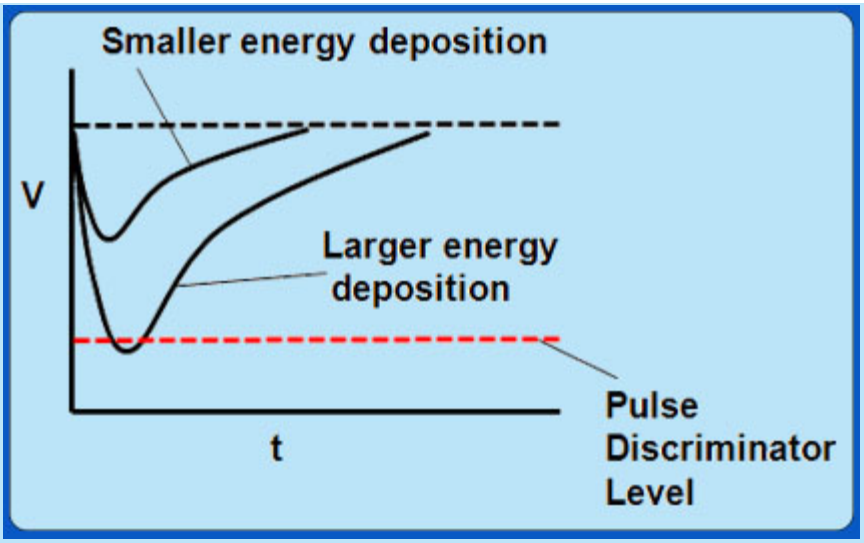
Gas Filled Counters



- Has a outer cathode and a central anode.
- The electric field for a cylindrical system for an applied V is

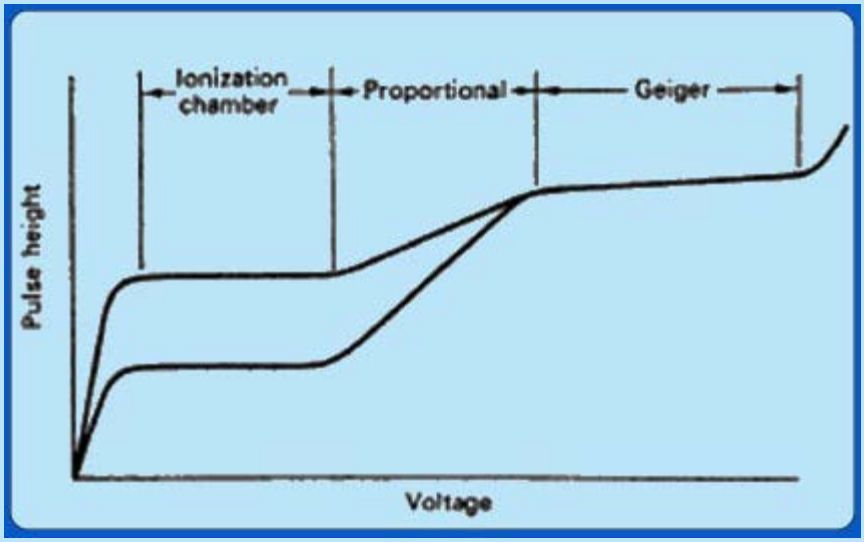
$$E_r = \frac{V_o}{\ln\left(\frac{b}{a}\right)} \times \frac{1}{r}$$

- This implies that the electric field is very high as r approaches 0.
- This makes collection of electrons from ionised gas that much easier.
- Hence a cylindrical geometry is the preferred arrangement in all gas detectors.
- Whenever a radiation passes through the gas, it ionises the radiation and produces a number of electron-ion pairs depending on the energy deposited.
- If the applied voltage is too small, the electrons and ion recombine.
- If the voltage is above a threshold value, the electrons and positive ions move towards anode and cathode respectively.
- This polarisation reduces the electric field and hence a voltage reduction is observed.
- As electrons and positive ions are collected, the field is established back and the voltage rebuilds between the electrode.
- The shape of the curve shall depend on the value of RC in the circuit.



- The electronics can be designed to count the radiation as pulses (counts) or current.
- By using a discriminator, pulses can be digitised.
- Higher the applied voltage, more of the ion-electron pair shall be collected leading to a taller pulse of more current.
- Hence the characteristic of the detector would depend on the applied voltage.
- By applying higher and higher voltage, we can get different kinds of detectors.
- Let us look at the response of gas filled detectors, when the applied voltage is increased.

The General Characteristic



Ionisation Chamber

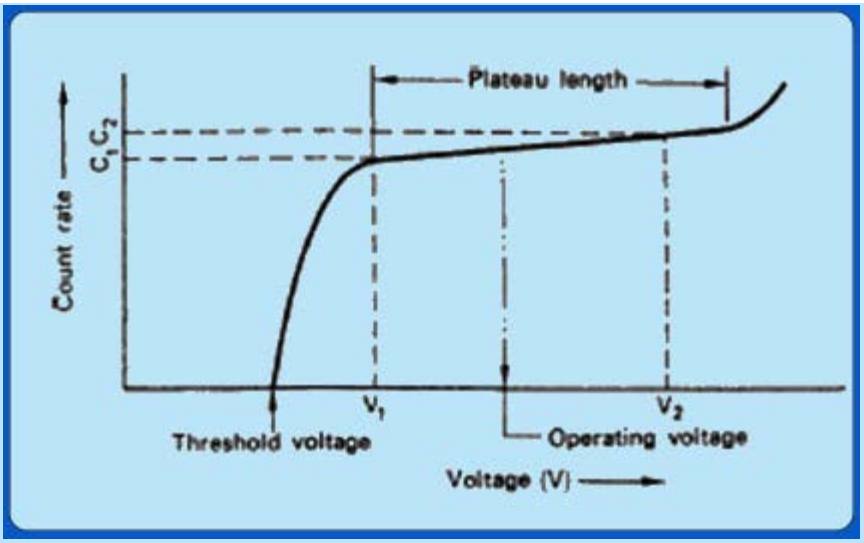
- As the voltage is increased, the recombinations are decreased and the pulse height is increased.
- Once sufficient voltage is applied, all the primary ions are collected and hence the pulse height saturates.
- As the voltage in increased beyond a second threshold, the pulse height starts increasing again.
- This because of secondary ionisation created by the accelerating electrons moving towards the anode.
- This first plateau is called Ionisation Chamber Plateau.
- The voltage at which secondary ionisation begins is a characteristic of the gas, its pressure, the geometry of the detector, etc.
- The saturation current, being directly proportional to the amount of energy deposited, is a measure of the dose.
- Higher the energy deposited, larger is the saturation current.
- Technically, ionisation chamber can discriminate between alphas and betas.
- However, the currents are too feeble in this region.

Proportional Counter

- As the voltage is increased, the pulse height increases and the output is proportional to the energy deposited and hence called proportional counter.
- It may be noticed that the ratio of response for the higher energy to the lower energy keeps decreasing, till both almost give the same response.
- This is the beginning of a new region called the Geiger Muller region or GM region.
- Counters operating in this region are called GM counters.

GM Counter

- At a characteristic voltage, an avalanche of electrons are formed all over the anode, called "Townsend Avalanche".
- At this point the pulse height is constant irrespective of the energy of the radiation that triggers the count.
- If we define a term called **amplification (A)** that represents the ratio of the total ion-electron pairs collected to the number primarily formed by the radiation, its value in GM region reaches ~ 10^6.
- The value of A in the Ionisation chamber plateau is one.
- It gradually increases in the proportional region.
- The reason for the pulse height being independent of the initial number of ion-electron pair is that the amplification is so high that it saturates the detector, cripples the electric field and prevents further multiplication.
- This plateau is called the GM Plateau.
- It has been argued that as the voltage is increased, the avalanche spreads radially and increases the count rate or the pulse height, though the increase is small.
- Thus, the plateau has a slight positive slope.
- As the voltage is increased, at a characteristic voltage, the gas breaks down and goes into a continuous discharge region.
- The detector will fail if taken to continuous discharge region.
- The slope of the GM Plateau is defined as a percentage increase in count rate per 100 V.
- It is given as $Slope = \frac{C_2 - C_1}{0.5(C_2 + C_1)} \times \frac{100}{(V_2 - V_1)} x100$.

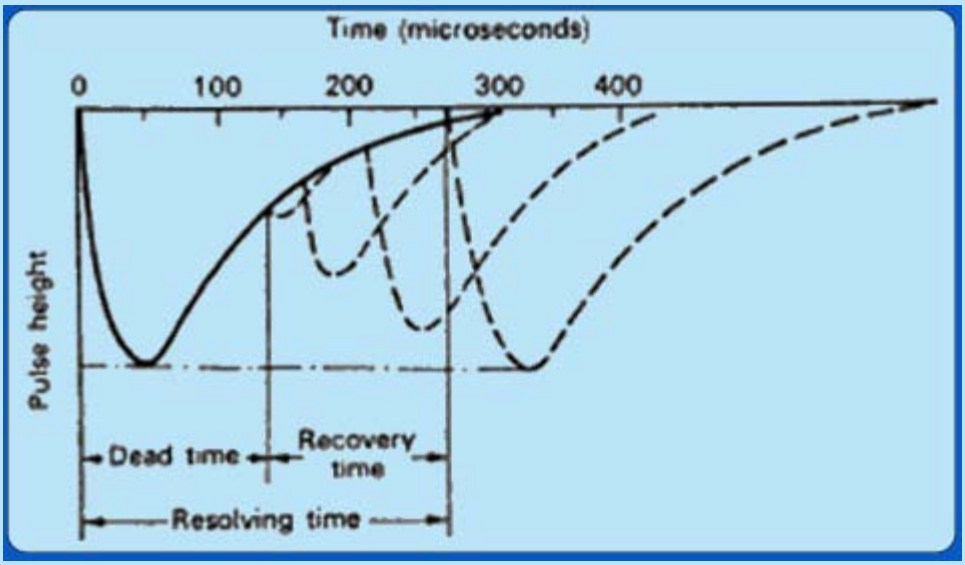

Plateau length
Count rate
C_1C_2
V_1
V_2
Threshold voltage
Operating voltage
Voltage (V)

Quenching

- When the positive ions reach the surface of the detector and hit the cathode, the kinetic energy is mostly dissipated as heat.
- Some however they excite the wall, which results in emitting of UV photons.
- Since at this time the electric field is established, the UV can trigger another avalanche, giving spurious counts. Prevention of spurious counting is called Quenching.
- This can be done electronically by decreasing the applied voltage for a short while after the count is registered.
- Another way is to add an organic gas such as ethyl alcohol that get decomposed by absorbing the UV radiation. One can also add a trace of halogen, which also absorbs energy.
- Since halogen does not dissociate, the life of the detector is longer.

Resolving Time

- Another issue with GM Counter is its dead time or the resolving time.
- Because of the production of the avalanche, the electric field in the detector is crippled, it is dysfunctional till majority of the positive ions are collected.
- During this time, if another radiation is incident on the detector, it will not be counted.
- The reason for this is explained in the following figure



- The resolving time can be estimated as follows:
- Count the number of radiations from a source A. Let the count rate be N_A.
- Similarly count for another source, B, for the same time. Let this be N_B.
- Now count for both A and B together, and let the count rate be N_{AB}.
- It will be noticed that N_{AB} will be < ($N_A + N_B$). This is because the detector is dysfunctional for more time in the third case.
- The resolving time can be obtained from the following algebra.
- If the resolving time is τ, then true counts for A , B and A+B are:

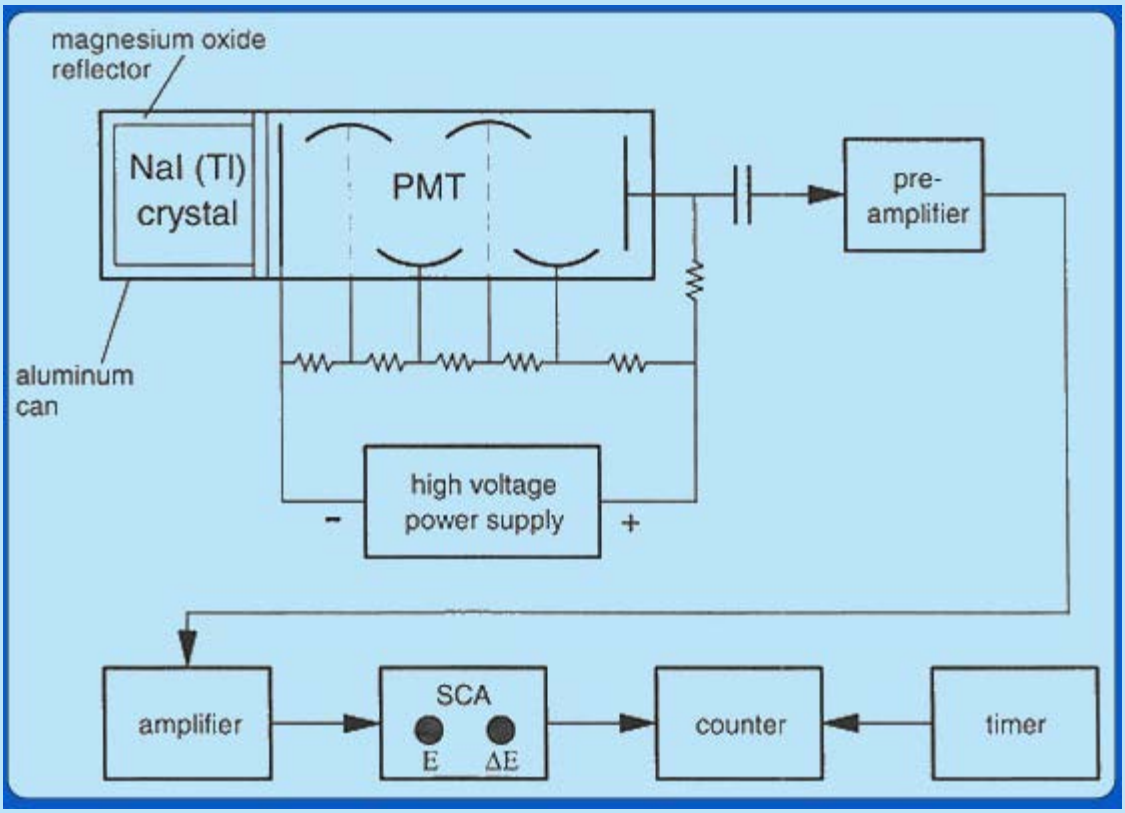
$$n_A = \frac{N_A}{1-\tau N_A} \qquad n_B = \frac{N_B}{1-\tau N_B} \qquad n_{AB} = \frac{N_{AB}}{1-\tau N_{AB}}$$

- Since $n_A + n_B = n_{AB}$, by neglecting higher order of τ we can show that

$$\tau = \frac{N_A + N_B - N_{AB}}{2N_A N_B}$$

Scintillation Detector

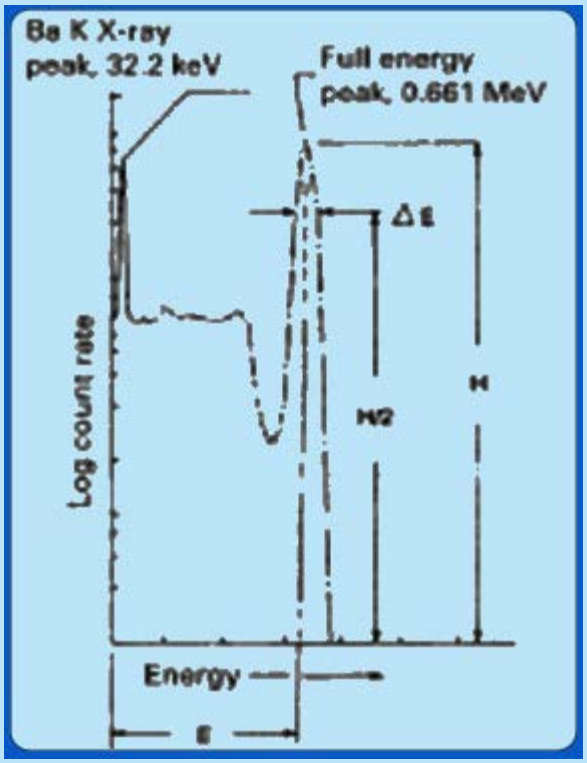
- For low energy radiation, gas filled counters can be ineffective.
- Scintillation counters are widely used to count for gammas.
- Usually, a crystal such as NaI, activated with Thallium (Tl) is used.
- The energy absorbed in this case is converted into light.
- The light is allowed to fall on a photosensitive cathode of a photomultiplier, which emits electrons.
- These electrons are amplified by a series of plates kept at increasing voltages.



- The pulse height can be discriminated using a Single Channel Analyser (SCA).
- The pulses are then appropriately counted for the desired amount of time.

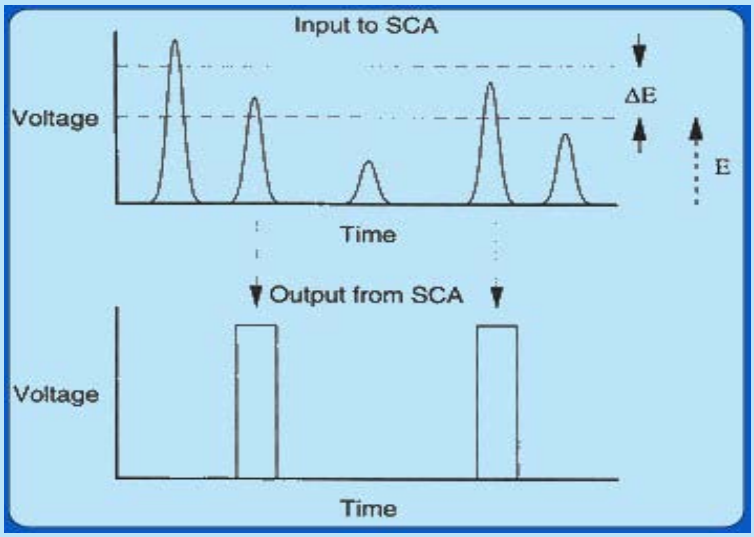
- The Amplification can be very high (10^5 or higher).

- These are highly sensitive and can discriminate energy.
- Since both photoelectric effect and Compton scattering are prevalent with different possibilities, the entire signature is caught.
- Figure shows the spectra of Cs^{137}. The internal conversion of Ba^{137} as well as full energy are caught.



Pulse Height Analysers

- Pulse height analysers are used to make spectroscopic measurements.
- Can be done with Single Channel Analysers or Multi-Channel Analysers.
- The electronics is set so that only pulse that caused by radiation having energy E and E + dE are registered.
- The value of E and ΔE can be adjusted and counts made sequentially with different Es, if we have only one channel.



- In multi-channel analysers several channels set to different E's simultaneously operate and collect data.
- Thus, the entire spectrum can be automatically plotted.

Semi-conductor Detector

- The semi-conductor (Si or Ge) doped with appropriate impurity is used as the ionisation media.
- The knocked out electrons are collected, amplified and counted.
- They have high-speed detection due to very low resolving time (~10^{-9}s).
- They have high sensitivity as very less energy is used to knock off electrons and operate at low voltages.
- Signal conditioning concepts are same.

Neutron Counting

- Unlike gammas and charge particles, a neutron does not ionise directly.
- Usually an isotope, typically Boron (B^{10}) that undergoes an (n,α) reaction is used to generate charge particles, which are then counted.
- Typically BF_3 gas is used.
- Other methods employed are, fission chamber using fissile material, Activation of foils, etc.
- The principle of counting the subsequent radiation is same as the detectors discussed earlier.

Summary

Radiation	Ideal Detector
α	Thin Semiconductor Detectors Proportional Counters
β	Organic Scintillators Geiger Counters Proportional Counters
γ	Inorganic Scintillators Thick Semiconductor Detectors
nuetrons	Plastic Scintillators Proportional Counters (He, BF_3) Lithium Glass Scintillators

Objectives

In this lecture you will learn the following

- We shall solve same problems to illustrate the concepts learnt in last few lectures.

Problem 1

Calculate the thickness of iron that has the same electronic areal density as 3 mm thick piece of aluminium.

Solution 1

The relevant data for the problem are $\varrho_{Al} = 2.7$ g/cm^3, $\varrho_{Fe} = 7.87$ g/cm^3, Atomic Mass of Al = $M_{Al} = 27$ g/mole, Atomic Mass of Fe $M_{Fe} = 55.85$ g/mole, $Z_{Al} = 13$, $Z_{Fe} = 26$

Number density of atoms for an element, i, is given by

$$N_i = \frac{\rho_i N_{Avo}}{M_i}$$

The electronic areal density A_{ei} for the material whose thickness is t_i is obtained as

$$A_{ei} = N_i Z_i t_i = \frac{\rho_i N_{Avo}}{M_i} Z_i t_i$$

Hence the ratio of electronic areal density of two material can be given as

$$A_{e-R} = \frac{\rho_R}{M_R} Z_R t_R$$

Since in the present problem the electronic areal density is same, we can write

$$1 = \frac{\rho_{Al}}{\rho_{Fe}} \frac{M_{Fe}}{M_{Al}} \frac{Z_{Al}}{Z_{fe}} \frac{t_{Al}}{t_{Fe}} \qquad \Rightarrow t_{Fe} = t_{Al} \frac{\rho_{Al}}{\rho_{Fe}} \frac{M_{Fe}}{M_{Al}} \frac{Z_{Al}}{Z_{Fe}}$$

$$\Rightarrow t_{Fe} = \frac{2.7}{7.87} \frac{55.85}{27} \frac{13}{26} 3 = 1.064 \ mm$$

Problem 2

a) In the relativistic framework show that

$$E^2 - E_0^{\,2} = p^2c^2$$

(b) Derive the expression for the Compton Shift.

(c) What is the maximum possible shift in Compton wavelength**?**

Solution 2

(a) Starting from the relativity relations

$$E = mc^2 \qquad E_0 = m_0c^2$$

$$\Rightarrow E^2 - E_0^{\,2} = \left(m^2 - m_0^{\,2}\right)c^4 \qquad (1)$$

Also the mass variation can be expressed as

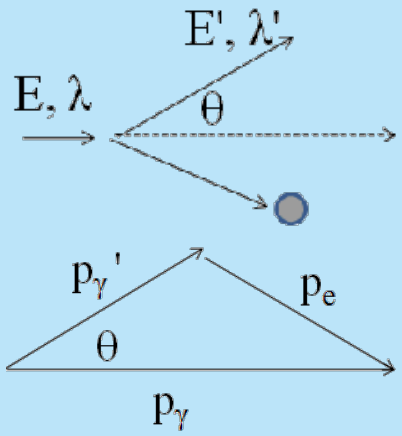
$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \Rightarrow m_0^{\,2} = m^2\left(1-\frac{v^2}{c^2}\right)$$

Substituting the expression of m_0^2 in Eq. (1), we get

$$\Rightarrow E^2 - E_0^{\,2} = m^2\left(1-\left(1-\frac{v^2}{c^2}\right)\right)c^4$$

$$\Rightarrow E^2 - E_0^{\,2} = m^2v^2c^2 = p^2c^2$$

(b) Referring to the sketch showing the Compton scattering on the right, we can draw the momentum conservation triangle as shown



From the triangle identity, we can write

$$p_e^{\,2} = p_\gamma^{\,2} + p_\gamma'^{\,2} - 2p_\gamma p_\gamma' \cos\theta \qquad (2)$$

Conservation of energy implies that

$$E_0 + E_\gamma = E_\gamma' + E_e$$
$$\Rightarrow E_0 + E_\gamma - E_\gamma' = E_e \qquad (3)$$

Using the expressions for Gamma energy and using Eq. (1) we can write

$$E_\gamma = p_\gamma c \qquad E_\gamma' = p_\gamma' c \qquad E_e = \sqrt{E_0^{\,2} + p^2 c^2} \qquad (4)$$

Using Equations (3) and (4) we can write

$$\Rightarrow E_e = E_0 + (p_\gamma - p_\gamma')\, c$$
$$\Rightarrow E_e^{\,2} = E_0^{\,2} + (p_\gamma - p_\gamma')^2 c^2 + 2E_0(p_\gamma - p_\gamma')\, c$$
$$\Rightarrow E_0^{\,2} + p_e^{\,2} c^2 = E_0^{\,2} + (p_\gamma - p_\gamma')^2 c^2 + 2E_0(p_\gamma - p_\gamma')\, c$$
$$\Rightarrow p_e^{\,2} = (p_\gamma - p_\gamma')^2 + \frac{2E_0(p_\gamma - p_\gamma')}{c} \qquad (5)$$

Equating the RHS of Eqs. (2) and (5), we get

$$\Rightarrow p_\gamma^{\,2} + p_\gamma'^{\,2} - 2p_\gamma p_\gamma' \cos\theta = p_\gamma^{\,2} + p_\gamma'^{\,2} - 2p_\gamma p_\gamma' + \frac{2E_0(p_\gamma - p_\gamma')}{c}$$

$$\Rightarrow 2p_\gamma p_\gamma'(1 - \cos\theta) = \frac{2m_0 c^2 (p_\gamma - p_\gamma')}{c}$$

$$\Rightarrow \frac{h}{\lambda}\frac{h}{\lambda'}(1 - \cos\theta) = m_0 ch\left(\frac{1}{\lambda} - \frac{1}{\lambda'_\gamma}\right)$$

$$\Rightarrow (1 - \cos\theta) = \frac{m_0 c}{h}(\lambda' - \lambda) \quad \Rightarrow (\lambda' - \lambda) = (1 - \cos\theta)\frac{h}{m_0 c}$$

(c) The maximum possible shift occurs when θ = 180°

$$\Rightarrow (\lambda' - \lambda) = 2\frac{h}{m_0 c}$$

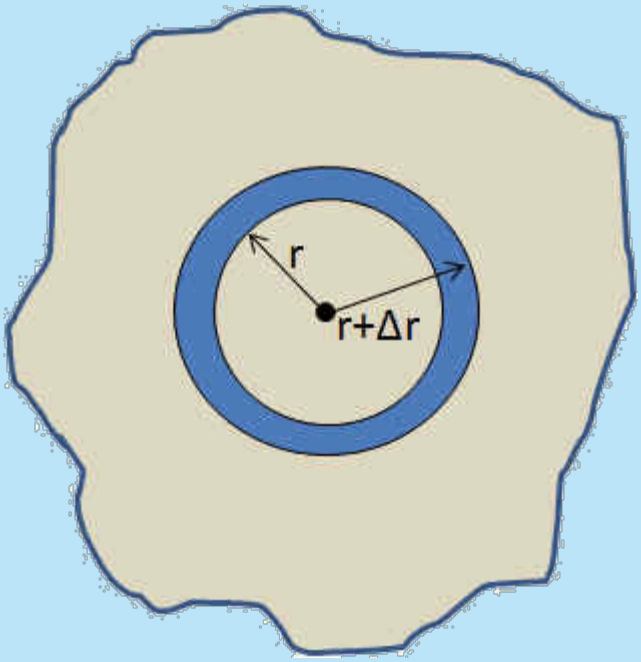
$$= 2\lambda_{Compton} = 2x2.426x10^{-10} cm = 4.952x10^{-10} cm$$

Problem 3

A radioactive substance is dissolved in a large body of water so that S''' γ-rays are emitted per cm^3 per unit time throughout the water. (a) Show that the uncollided flux at any point in the water is given by S'''/μ. (b) Using Taylor form of build up factor, derive an expression the buildup flux.

Solution 3

- Consider the slice of the large body shown in the figure.
- In the slice consider a shell around the point at which we wish to compute the uncollided flux.



- Since the sources are spherically symmetric, the uncollided flux at the centre due to the sources in the shell can be written as

$$\Rightarrow \phi_u = \frac{S'''4\pi r^2 dr}{4\pi r^2} e^{-\mu r} = S''' dr\, e^{-\mu r}$$

- The total flux due to the entire body can be computed as

$$\Rightarrow \phi_u = \int_0^\infty S''' dr\, e^{-\mu r} = -\frac{S'''}{\mu}\left(e^{-\mu r}\right]_0^\infty = \frac{S'''}{\mu}$$

- Using Taylor form of build up factors, the buildup flux due to the shell at the point of interest is

$$\Rightarrow \phi_b = S''' dr\, e^{-\mu r}\left(Ae^{-\mu r \alpha_1} + (1-A)e^{-\mu r \alpha_1}\right)$$

- The total buildup flux due to the entire body can be computed as

$$\Rightarrow \phi_3 = S''' \int_0^\infty A e^{-\mu r(1+\alpha_1)} dr + \int_0^\infty (1-A) e^{-\mu r(1+\alpha_2)} dr$$

$$\Rightarrow \phi_3 = \frac{S'''}{\mu}\left[\left(\frac{A}{(1+\alpha_1)}\right)+\left(\frac{1-A}{(1+\alpha_2)}\right)\right]$$

Problem 4

Consider the explosion of a 20 kiloton nuclear warhead. If there was no attenuation of the γ-rays in the atmosphere and all of the fission γ-rays escape from the warhead, approximately what would be the γ-ray dose received by a person standing 1,000 m from the point of the blast? [Note: Take the energy release per fission to be 200 MeV, of which 7 MeV is in the form of fission γ-rays. Also, 1 kiloton = 2.6×10^{25} MeV.

Solution 4

Energy released = $20 \times 2.6 \times 10^{25}$ MeV = 52×10^{25} MeV

Number of fissions = $52 \times 10^{25}/200 = 26 \times 10^{23}$ fissions

γ-energy released = $26 \times 10^{23} \times 7 = 11.2 \times 10^{24}$ MeV

Assuming the exploding device as a point source and air to be transparent to γ-rays

$$Energy\ Flux, E = \frac{11.2 \times 10^{24}}{4\pi(1000)^2}$$

$$= 8.91 \times 10^{17} MeV/m^2$$

The value of μ/ρ for air for an average γ energy of 2 MeV is taken as 0.0238 cm^2/g = 2.38×10^{-6} m^2/g

$$Exposure, X = 8.91 \times 10^{17} \times 2.38 \times 10^{-6}$$

$$= 2.12 \times 10^{12} \text{MeV/g}$$

$$= 2.12 \times 10^{12} \times 1.602 \times 10^{-6} \text{erg/g}$$

$$= 3.40 \times 10^{6} \text{erg/g}$$

$$= 3.40 \times 10^{6}/87.7 = 3.87 \times 10^{4} \text{R}$$

$$Absorbed\ Dose = 3.87 \times 10^{4} \times 0.95 = 3.87 \times 10^{4} \text{rad}$$

$$Biological\ Dose = 3.87 \times 10^{4} \times 1 = 3.68 \times 10^{4} \text{rem}$$

Problem 5

Whether inhaled or ingested, iodine, in most chemical forms, quickly enters the bloodstream, and much of this element flows to and is taken up by the thyroid gland. Shortly after an injection of iodine, the retention function for the thyroid is given approximately by

$R(t) = 0.3e^{-0.00502t}$

where t is in days. Suppose that a patient is given an injection of 1 μCi of ^{131}I for diagnosis of a thyroid condition. (a) What is the biological half-life of iodine in the thyroid? (b) What total dose will the patient's thyroid receive from the single injection? The mass of the thyroid is 20 g.

Solution 5

(a) From the data it is clear that $\lambda_B = 0.005\ d^{-1}$.

Hence Biological Half life = 0.693/0.005 = 138 d

(b) Half life of ^{131}I = 8.04 d

Hence $\lambda_R = 0.693/8.04 = 0.0862\ d^{-1}$

Hence $\lambda_{eff} = 0.005 + 0.0862 = 0.0912\ d^{-1}$

(b) If C_o is the activity of radioactive specie ingested in micro-Curies and q is the fraction of the total activity absorbed in the organ and E in MeV is the average energy deposited per disintegration, Q is the quality factor for the radiation, m is the mass of the organ in grams which absorbs the entire energy, we can write

$$Dose\ rate, \dot{D}(t) = \frac{3.7x10^4 x1.602x10^{-6} C_0 e^{-\lambda_{eff} t} qE}{m} \text{ erg/g - s}$$

$$= \frac{5.92x10^{-2} C_0 e^{-\lambda_{eff} t} qE}{100m} \text{ rad/s}$$

$$Biological\ Dose\ rate, \dot{H}(t) = \frac{5.92x10^{-4} C_0 e^{-\lambda_{eff} t} qEQ}{m} \text{ rem/s}$$

$$= \frac{5.92x10^{-4} x3600x24 C_0 e^{-\lambda_{eff} t} qEQ}{m} \text{ rem/d}$$

$$= \frac{51.1 C_0 e^{-\lambda_{eff} t} qEQ}{m} \text{ rem/d}$$

The total dose due to the isotope can be computed as

$$H(\infty) = \int_0^\infty \dot{H}(t) dt$$

$$H(\infty) = \frac{51.1 C_o qEQ}{m} \int_0^\infty e^{-\lambda_{eff} t} dt = \frac{51.1 C_o qEQ}{m\lambda_{eff}} \left(1 - e^{-\lambda_{eff} t}\right)_0^\infty$$

$$H(\infty) = \frac{51.1 C_o qEQ}{m\lambda_{eff}} rem$$

For the given problem

λ_{eff} = 0.0912 d^{-1}

C_0 = 1 μCi

From literature data q = 0.3, EQ = 0.23 MeV

m = 20 g

Substituting these in the above equation,

H(∞) = 1.93 rem.

Objectives

In this lecture you will learn the following

- We shall learn about the safety philosophy employed in nuclear reactors.
- The entire spectrum of measures employed, often termed as "Defense in Depth", is explained.

Nuclear Reactor Characteristics

- Has large amount of fission products (Activity order of 10^{19} Bq).
- Harmful to human health.
- Long lived.
- Hence is feared by the general Public.
- Peak temperature that can be reached in the fuel is technically very large.
- This is due to radioactive decay of fission products.
- Nuclear reactor produces heat even after the plant has been shut down.

Key Issues

- Radiation safety during operation of NPPs.
- Prevention of accidental release of radioactivity.
- Radioactive waste management.

Safety Philosophy

- Provide Defense in Depth to maintain integrity of systems designed for achieving three safety functions
  - Confinement of radioactivity
    - Achieved by providing multiple fission product barriers.
  - Control of reactivity
    - Achieved by redundancy in reactor shut down system.
  - Cooling to reactor fuel
    - Achieved by providing an elaborate (ECCS) Emergency Core Cooling System.

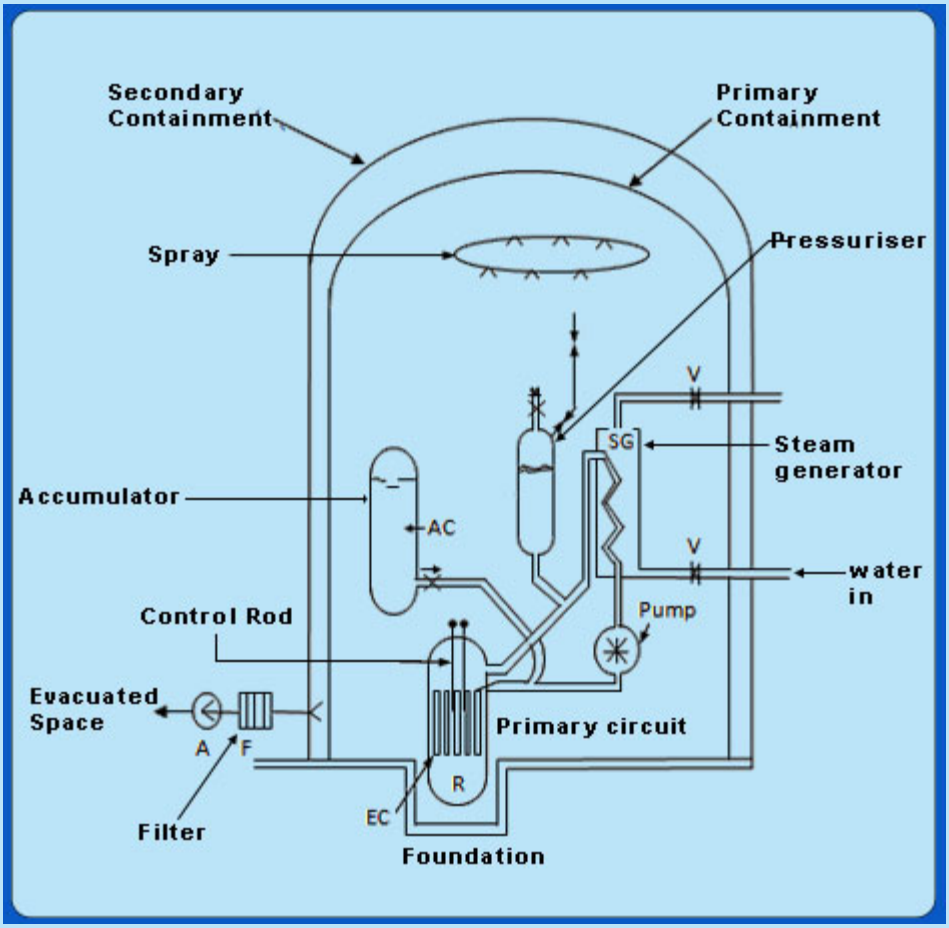
Defense in Depth

- It implies several levels of defense one backing another. These can be:
  - Prevention of Deviation from Normal Operation ensured by Robust and Conservative design.
  - Detect deviations and intercept deviations by providing suitable Control and Protection Systems.
  - Control the Consequences of deviations (accidents) by providing engineered safety features and inherent safety characteristics.
  - Mitigate or Limit damage to Public due to consequences of accidents by having Disaster Management Plans (Emergency Preparedness).

Defense in Depth

The following figure shows various safety systems in a nuclear reactor

Safety Systems



The safety systems and their intent are summarised below

1. Control rod - To shut down the reactor during emergency.
2. Accumulator - This pressurised tank will inject water into the reactor during emergency.
3. Primary Containment - To confine radioactive species that may be released.
4. Spray - To reduce the pressure in containment by condensing steam that may be release during accident.
5. Filtered vent - To trap fission products and vent the air to a vaccum building.

Safety in Design

- Stringent quality control measures in design, construction, operation and maintenance as well as regulation
  - Use of proven engineering practices by using approved codes.
  - Designs independently checked for any mistakes in assumptions, calculations and inferences.
  - Selection of standard and proven materials for construction.
  - Employing well established testing and quality assurance methods.
- Employment of Redundancies
  - Failure of single component shall not compromise safety.
  - Adequate redundancy to meet Reliability Targets.
- Employment of Diversity
  - Different principles of operation (Gravity, electrical, pneumatic, hydraulic).
  - Different manufactures.
- Employment of Independence
  - Physical separation of process and safety systems to ensure that damage one will not damage the other.
  - Functional separation to ensure no sharing of components or support systems.
- Design must be such that failure of a system leads to a safe state.
- Testability (Preferably on line) and Inspectability be built in design.
- Human Factors in Design
  - Fault tolerant.
  - Availability of diagnostic aids.
  - Interlocks to avoid errors in operation.
- Safety analyses of Postulated Initiating Events (PIE) as a part of the Design procedure.
- There are multiple barriers to present radioactive release.
- The first barrier for fission products in the fuel itself.
- So long as the fuel is kept cold, the fission products cannot escape.
- The second barrier is the clad in which the fuel pellets are housed.
- The third barrier in the primary system, which is also leak tight.
- The fourth and ultimate barrier is the leak tight containment building.
- Some reactors have two containment buildings called primary and secondary containment.
- A spray system is provided to condense steam incase of pipe breaks.
- In addition to provide water into the core during any pipe breaks an accumulator system is provided.
- This consists of a tank full of water kept under Nitrogen pressure.
- If the system pressure falls, this tank will inject water into the reactor.
- If any parameter exceeds allowable value, a reactor is shut down by control rod drop, which

falls under gravity.

- Often reactors are provided with secondary shut down system, which injects chemical poison into the reactor to shut fission reactions.
- Newer reactors have filtered vent system to protect containment over pressurisation.

Categories of PIEs

- Analysis is carried out for several Postulated Initiating events(PIEs).
- Analysis includes both probabilistic as well as deterministic approaches.
- Some examples of PIEs are given in the following table

Category	Frequency (f) [per reactor year]	Example
1	≥ 1	Process transients
2	$10^{-2} \leq f \leq 1$	Reactivity anomalies
3	$10^{-4} \leq f \leq 10^{-2}$	Feed water line break
4	$10^{-6} \leq f \leq 10^{-4}$	Station Blackout
Beyond 4	$f \leq 10^{-6}$	Loss of Coolant Accident (LOCA) + failure of both shut down systems

Levels of Probablistic Safety Analysis

Level	Safety Criterion	Objective
1	Core Damage Frequency (System Analysis)	Reduce probability of the accident
2	Radioactive release to public (Investigate Containment Availability)	Reduce radioactive release to public
3	Consequence Analysis	Minimise Societal Risk (Fatalities, Land contamination)

- From these analysis, if the probability of damage is not acceptable, more safety systems are incorporated.

Safety in Construction

- Design Quality Assurance (QA) procedures to ensure that construction meets the quality requirements envisaged in the design.
- Periodic review of Violations, lessons learnt and modification of QA procedure.

Safety in Operation

- Develop safe operating procedures.
- Select quality manpower, train, qualify and re-qualify in periodic intervals.
- Provide operational experience of off normal situations through simulators.
- Carry out periodic in-service inspection to check health of the equipment.
- Carry out periodic reviews in operating procedures in light of any surprises during operation.
- Continuous monitoring of radioactive releases.
- Continuous monitoring of radioactive dose to personnel ensure absence of contamination.

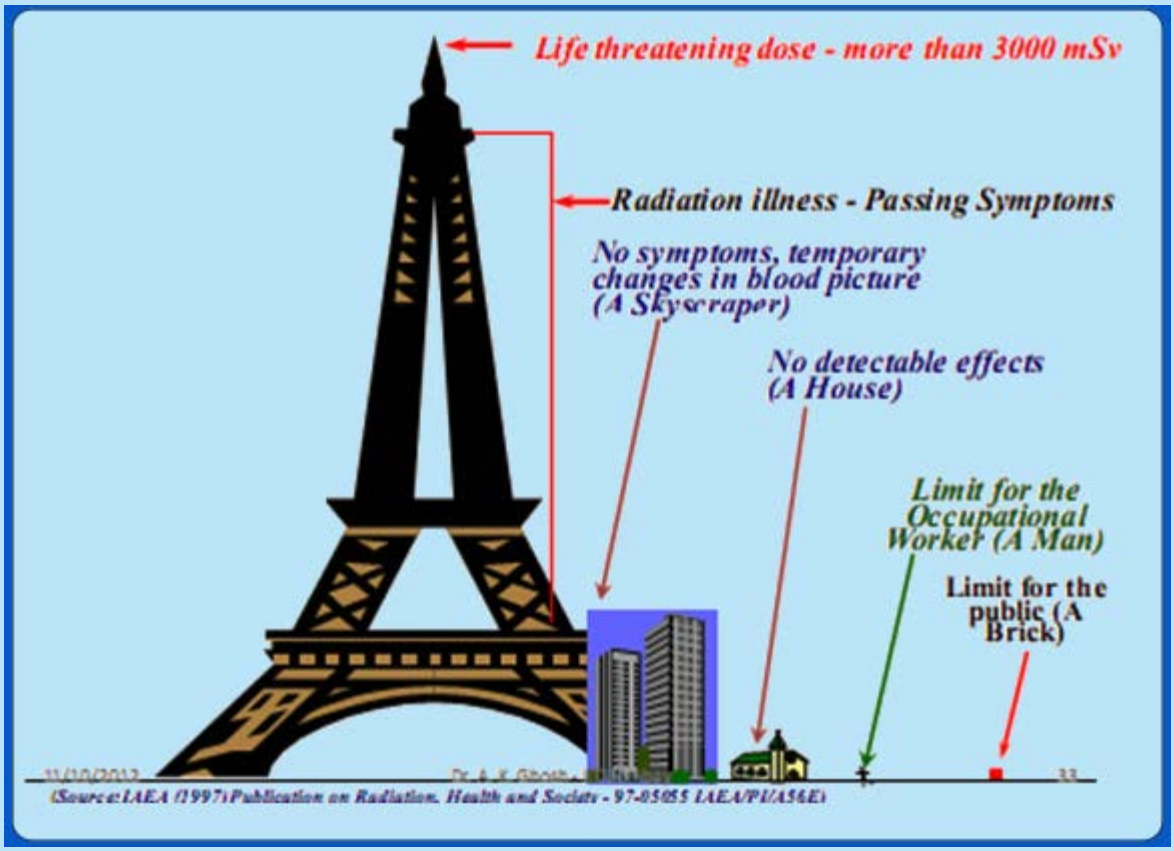
Safety Through Regulation

- Establishment of an independent regulatory authority and having laws to enforce nuclear utilities to follow the guidelines issued by the authority.
- The regulator is given powers to exercise regulatory control at all stages, viz., siting, design, construction, operation and eventual dismantling.
- The regulator is given the responsibility to generate regulatory documents, codes and guides for the utility to follow at all stages.
- The regulator is expected to have an independent monitoring mechanism to ensure that all guidelines are followed.
- The authority is obliged to get all documents prepared by the utility that the regulatory procedures are followed and maintain these records.
- The authority shall conduct periodic regulatory meetings with experts and utility operators who will satisfy themselves that all is in order.
- The regulator is expected to keep a watch on developments around the globe through exchange meetings.
- The regulator shall analyse all events of significance and issue regulatory guidelines that may require modifications in design, operating procedure, or both.
- The regulator shall ascertain that all new guidelines are followed in a timely manner.

Ultimate Safety

- The doctrine of Defense in Depth has been primarily responsible for the relatively safe record of this industry as summarised in the following two figures.

Some Facts on Safety



Average Environmental Dose at 1.6km for the last seven years (micro Sv/Year)