Pocket Book of Integrals and Mathematical Formulas

3rd Edition

Ronald J. Tallarida

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Preface to the Third Edition

This new edition has been enlarged to contain all the material in the second edition, an expanded chapter on statistics that now includes sample size estimations for means and proportions, and a totally new chapter on financial mathematics. In adding this new chapter we have also included a number of tables that aid in performing the calculations on annuities, true interest, amortization schedules, compound interest, systematic withdrawals from interest accounts, etc. The treatment and style of this material reflect the rest of the book, i.e., clear explanations of concepts, relevant formulas, and worked examples. The new financial material includes analyses not readily found in other sources, such as the effect of lump sum payments on amortization schedules and a novel "in-out formula" that calculates current regular deposits to savings in order to allow the start of systematic withdrawals of a specified amount at a later date. While many engineers, mathematicians, and scientists have found much use for this handy pocket book, this new edition extends its usage to them and to the many business persons and individuals who make financial calculations.

R.J.T
Philadelphia

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Preface to the Second Edition

This second edition has been enlarged by the addition of several new topics while preserving its convenient pocket size. New in this edition are the following topics: z-transforms, orthogonal polynomials, Bessel functions, probability and Bayes' rule, a summary of the most common probability distributions (Binomial, Poisson, normal, t, Chi square and F), the error function, and several topics in multivariable calculus that include surface area and volume, the ideal gas laws, and a table of centroids of common plane shapes. A list of physical constants has also been added to this edition.

I am grateful for many valuable suggestions from users of the first edition, especially Lt. Col. W. E. Skeith and his colleagues at the U.S. Air Force Academy.

R.J.T.
Philadelphia, 1992

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Preface to the First Edition

The material of this book has been compiled so that it may serve the needs of students and teachers as well as professional workers who use mathematics. The contents and size make it especially convenient and portable. The widespread availability and low price of scientific calculators have greatly reduced the need for many numerical tables (e.g., logarithms, trigonometric functions, powers, etc.) that make most handbooks bulky. However, most calculators do not give integrals, derivatives, series, and other mathematical formulas and figures that are often needed. Accordingly, this book contains that information in addition to a comprehensive table of integrals. A section on statistics and the accompanying tables, also not readily provided by calculators, have also been included.

The size of the book is comparable to that of many calculators and it is really very much a companion to the calculator and the computer as a source of information for writing one's own programs. To facilitate such use, the author and the publisher have worked together to make the format attractive and clear. Yet, an important requirement in a book of this kind is accuracy. Toward that end we have checked each item against at least two independent sources.

Students and professionals alike will find this book a valuable supplement to standard textbooks, a source for review, and a handy reference for many years.

Ronald J. TaHarida
Philadelphia

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About the Author

Ronald J. Tallarida holds B.S. and M.S. degrees in physics/mathematics and a Ph.D. in pharmacology. His primary appointment is as Professor of Pharmacology at Temple University School of Medicine, Philadelphia; he also serves as Adjunct Professor of Biomedical Engineering (Mathematics) at Drexel University in Philadelphia.

Dr. Tallarida received the Lindback Award for Distinguished Teaching in 1964 while in the Drexel mathematics department. As an author and researcher, he has published over 200 works, including 7 books, and has served as a consultant to both industry and government agencies.

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Greek Letters

α	A	Alpha
β	B	Beta
γ	Γ	Gamma
δ	Δ	Delta
ε	E	Epsilon
ζ	Z	Zeta
η	H	Eta
θ	Θ	Theta
ι	I	Iota
κ	K	Kappa
λ	Λ	Lambda
μ	M	Mu
ν	N	Nu
ξ	Ξ	Xi
o	O	Omicron
π	Π	Pi
ρ	P	Rho
σ	Σ	Sigma
τ	T	Tau
υ	Υ	Upsilon
ϕ	Φ	Phi
χ	X	Chi
ψ	Ψ	Psi
ω	Ω	Omega

The Numbers π and e

π	=	3.14159	26535	89793
e	=	2.71828	18284	59045
$\log_{10}e$	=	0.43429	44819	03252
$\log_e 10$	=	2.30258	50929	94046

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Prime Numbers

2	3	5	7	11	13	17	19	23	29
31	37	41	43	47	53	59	61	67	71
73	79	83	89	97	101	103	107	109	113
127	131	137	139	149	151	157	163	167	173
179	181	191	193	197	199	211	223	227	229
233	239	241	251	257	263	269	271	277	281
…				…					…

Important Numbers in Science (Physical Constants)

Avogadro constant (N_A)	6.02×10^{26} kmole^{-1}
Boltzmann constant (k)	1.38×10^{-23} J°K^{-1}
Electron charge (e)	1.602×10^{-19} C
Electron, charge/mass, (e/m_e)	1.760×10^{11} C·kg^{-1}
Electron rest mass (m_e)	9.11×10^{-31} kg (0.511 MeV)
Faraday constant (F)	9.65×10^{4} C·mole^{-1}
Gas constant (R)	8.31×10^{3} J·°K^{-1} kmole^{-1}
Gas (Ideal) normal volume (V_o)	22.4 m^3·kmole^{-1}
Gravitational constant (G)	6.67×10^{-11} N·m^2·kg^{-2}
Hydrogen atom (rest mass) (m_H)	1.673×10^{-27} kg (938.8 Mev)

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Neutron (rest mass) (m_n)	1.675×10^{-27} kg (939.6 MeV)
Planck constant (h)	6.63×10^{-34} J·s
Proton (rest mass) (m_p)	1.673×10^{-27} kg (938.3 MeV)
Speed of light (c)	3.00×10^{8} m·s^{-1}

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