(c) $\dfrac{|E_T|}{\text{True Value}} \times 100 = 0\%$

II. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6}$;

$\sum mf(x_i) = 36 \Rightarrow S = \frac{1}{6}(36) = 6$;

$f^{(4)}(x) = 0 \Rightarrow M = 0 \Rightarrow |E_S| = 0$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	1	1	1	1
x_1	3/2	2	4	8
x_2	2	3	2	6
x_3	5/2	4	4	16
x_4	3	5	1	5

(b) $\int_1^3 (2x-1)\,dx = 6 \Rightarrow |E_S| = \int_1^3 (2x-1)\,dx - S = 6 - 6 = 0$

(c) $\dfrac{|E_S|}{\text{True Value}} \times 100 = 0\%$

3. $\int_{-1}^{1} (x^2+1)\,dx$

I. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{1-(-1)}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{2} = \frac{1}{4}$;

$\sum mf(x_i) = 11 \Rightarrow T = \frac{1}{4}(11) = 2.75$;

$f(x) = x^2 + 1 \Rightarrow f'(x) = 2x \Rightarrow f''(x) = 2 \Rightarrow M = 2$

$\Rightarrow |E_T| \le \frac{1-(-1)}{12}\left(\frac{1}{2}\right)^2(2) = \frac{1}{12}$ or 0.08333

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	−1	2	1	2
x_1	−1/2	5/4	2	5/2
x_2	0	1	2	2
x_3	1/2	5/4	2	5/2
x_4	1	2	1	2

(b) $\int_{-1}^{1} (x^2+1)\,dx = \left[\frac{x^3}{3} + x\right]_{-1}^{1} = \left(\frac{1}{3}+1\right) - \left(-\frac{1}{3}-1\right) = \frac{8}{3} \Rightarrow E_T = \int_{-1}^{1} (x^2+1)\,dx - T = \frac{8}{3} - \frac{11}{4} = -\frac{1}{12}$

$\Rightarrow |E_T| = \left|-\frac{1}{12}\right| \approx 0.08333$

(c) $\dfrac{|E_T|}{\text{True Value}} \times 100 = \left(\dfrac{\frac{1}{12}}{\frac{8}{3}}\right) \times 100 \approx 3\%$

II. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6}$;

$\sum mf(x_i) = 16 \Rightarrow S = \frac{1}{6}(16) = \frac{8}{3} = 2.66667$;

$f^{(3)}(x) = 0 \Rightarrow f^{(4)}(x) = 0 \Rightarrow M = 0 \Rightarrow |E_S| = 0$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	−1	2	1	2
x_1	−1/2	5/4	4	5
x_2	0	1	2	2
x_3	1/2	5/4	4	5
x_4	1	2	1	2

(b) $\int_{-1}^{1}(x^2+1)\,dx=\left[\frac{x^3}{3}+x\right]_{-1}^{1}=\frac{8}{3}\Rightarrow|E_S|=\int_{-1}^{1}(x^2+1)\,dx-S=\frac{8}{3}-\frac{8}{3}=0$

(c) $\frac{|E_S|}{\text{True Value}}\times 100=0\%$

4. $\int_{-2}^{0}(x^2-1)\,dx$

I. (a) For $n=4$, $h=\frac{b-a}{n}=\frac{0-(-2)}{4}=\frac{2}{4}=\frac{1}{2}\Rightarrow\frac{h}{2}=\frac{1}{4}$;

$\sum mf(x_i)=3\Rightarrow T=\frac{1}{4}(3)=\frac{3}{4}$;

$f(x)=x^2-1\Rightarrow f'(x)=2x+1\Rightarrow f''(x)=2\Rightarrow M=2$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	−2	3	1	3
x_1	−3/2	5/4	2	5/2
x_2	−1	0	2	0
x_3	−1/2	−3/4	2	−3/2
x_4	0	−1	1	−1

$\Rightarrow|E_T|\le\frac{0-(-2)}{12}\left(\frac{1}{2}\right)^2(2)=\frac{1}{12}=0.08333$

(b) $\int_{-2}^{0}(x^2-1)\,dx=\left[\frac{x^3}{3}-x\right]_{-2}^{0}=0-\left(-\frac{8}{3}+2\right)=\frac{2}{3}\Rightarrow E_T=\int_{-2}^{0}(x^2-1)\,dx-T=\frac{2}{3}-\frac{2}{4}=-\frac{1}{12}$

$\Rightarrow|E_T|=\frac{1}{12}$

(c) $\frac{|E_T|}{\text{True Value}}\times 100=\left(\frac{\frac{1}{12}}{\frac{2}{3}}\right)\times 100\approx 13\%$

II. (a) For $n=4$, $h=\frac{b-a}{n}=\frac{0-(-2)}{4}=\frac{2}{4}=\frac{1}{2}\Rightarrow\frac{h}{3}=\frac{1}{6}$;

$\sum mf(x_i)=4\Rightarrow S=\frac{1}{6}(4)=\frac{2}{3}$;

$f^{(3)}(x)=0\Rightarrow f^{(4)}(x)=0\Rightarrow M=0\Rightarrow|E_S|=0$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	−2	3	1	3
x_1	−3/2	5/4	4	5
x_2	−1	0	2	0
x_3	−1/2	−3/4	4	−3
x_4	0	−1	1	−1

(b) $\int_{-2}^{0}(x^2-1)\,dx=\frac{2}{3}\Rightarrow|E_S|=\int_{-2}^{0}(x^2-1)\,dx-S=\frac{2}{3}-\frac{2}{3}=0$

(c) $\frac{|E_S|}{\text{True Value}}\times 100=0\%$

5. $\int_0^2 (t^3+t)\,dt$

I. (a) For n = 4, $h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{2} = \frac{1}{4}$;

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	1/2	5/8	2	5/4
t_2	1	2	2	4
t_3	3/2	39/8	2	39/4
t_4	2	10	1	10

$\sum mf(t_i) = 25 \Rightarrow T = \frac{1}{4}(25) = \frac{25}{4}$;

$f(t) = t^3 + t \Rightarrow f'(t) = 3t^2 + 1 \Rightarrow f''(t) = 6t \Rightarrow M = 12$

$= f''(2) \Rightarrow |E_T| \le \frac{2-0}{12}\left(\frac{1}{2}\right)^2(12) = \frac{1}{2}$

(b) $\int_0^2 (t^3+t)\,dt = \left[\frac{t^4}{4} + \frac{t^2}{2}\right]_0^2 = \left(\frac{2^4}{4} + \frac{2^2}{2}\right) - 0 = 6 \Rightarrow |E_T| = \int_0^2 (t^3+t)\,dt - T = 6 - \frac{25}{4} = -\frac{1}{4} \Rightarrow |E_T| = \frac{1}{4}$

(c) $\frac{|E_T|}{\text{True Value}} \times 100 = \frac{\left|-\frac{1}{4}\right|}{6} \times 100 \approx 4\%$

II. (a) For n = 4, $h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6}$;

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	1/2	5/8	4	5/2
t_2	1	2	2	4
t_3	3/2	39/8	4	39/2
t_4	2	10	1	10

$\sum mf(t_i) = 36 \Rightarrow S = \frac{1}{6}(36) = 6$;

$f^{(3)}(t) = 6 \Rightarrow f^{(4)}(t) = 0 \Rightarrow M = 0 \Rightarrow |E_S| = 0$

(b) $\int_0^2 (t^3+t)\,dt = 6 \Rightarrow |E_S| = \int_0^2 (t^3+t)\,dt - S = 6 - 6 = 0$

(c) $\frac{|E_S|}{\text{True Value}} \times 100 = 0\%$

6. $\int_{-1}^1 (t^3+1)\,dt$

I. (a) For n = 4, $h = \frac{b-a}{n} = \frac{1-(-1)}{4} = \frac{2}{4} = \frac{1}{2} \Rightarrow \frac{h}{2} = \frac{1}{4}$;

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	−1	0	1	0
t_1	−1/2	7/8	2	7/4
t_2	0	1	2	2
t_3	1/2	9/8	2	9/4
t_4	1	2	1	2

$\sum mf(t_i) = 8 \Rightarrow T = \frac{1}{4}(8) = 2$;

$f(t) = t^3 + 1 \Rightarrow f'(t) = 3t^2 \Rightarrow f''(t) = 6t \Rightarrow M = 6$

$= f''(1) \Rightarrow |E_T| \le \frac{1-(-1)}{12}\left(\frac{1}{2}\right)^2(6) = \frac{1}{4}$

(b) $\int_{-1}^{1}(t^3+1)\,dt=\left[\frac{t^4}{4}+t\right]_{-1}^{1}=\left(\frac{1^4}{4}+1\right)-\left(\frac{(-1)^4}{4}+(-1)\right)=2\Rightarrow|E_T|=\int_{-1}^{1}(t^3+1)\,dt-T=2-2=0$

(c) $\frac{|E_T|}{\text{True Value}}\times 100=0\%$

II. (a) For $n=4,\ h=\frac{b-a}{n}=\frac{1-(-1)}{4}=\frac{2}{4}=\frac{1}{2}\Rightarrow\frac{h}{3}=\frac{1}{6}$;

$\sum mf(t_i)=12\Rightarrow S=\frac{1}{6}(12)=2$;

$f^{(3)}(t)=6\Rightarrow f^{(4)}(t)=0\Rightarrow M=0\Rightarrow|E_S|=0$

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	−1	0	1	0
t_1	−1/2	7/8	4	7/2
t_2	0	1	2	2
t_3	1/2	9/8	4	9/2
t_4	1	2	1	2

(b) $\int_{-1}^{1}(t^3+1)\,dt=2\Rightarrow|E_S|=\int_{-1}^{1}(t^3+1)\,dt-S=2-2=0$

(c) $\frac{|E_S|}{\text{True Value}}\times 100=0\%$

7. $\int_{1}^{2}\frac{1}{s^2}\,ds$

I. (a) For $n=4,\ h=\frac{b-a}{n}=\frac{2-1}{4}=\frac{1}{4}\Rightarrow\frac{h}{2}=\frac{1}{8}$;

$\sum mf(s_i)=\frac{179{,}573}{44{,}100}\Rightarrow T=\frac{1}{8}\left(\frac{179{,}573}{44{,}100}\right)=\frac{179{,}573}{352{,}800}$

≈ 0.50899; $f(s)=\frac{1}{s^2}\Rightarrow f'(s)=-\frac{2}{s^3}\Rightarrow f''(s)=\frac{6}{s^4}$

$\Rightarrow M=6=f''(1)\Rightarrow|E_T|\le\frac{2-1}{12}\left(\frac{1}{4}\right)^2(6)=\frac{1}{32}=0.03125$

	s_i	$f(s_i)$	m	$mf(s_i)$
s_0	1	1	1	1
s_1	5/4	16/25	2	32/25
s_2	3/2	4/9	2	8/9
s_3	7/4	16/49	2	32/49
s_4	2	1/4	1	1/4

(b) $\int_{1}^{2}\frac{1}{s^2}\,ds=\int_{1}^{2}s^{-2}\,ds=\left[-\frac{1}{s}\right]_1^2=-\frac{1}{2}-\left(-\frac{1}{1}\right)=\frac{1}{2}\Rightarrow E_T=\int_{1}^{2}\frac{1}{s^2}\,ds-T=\frac{1}{2}-0.50899=-0.00899$

$\Rightarrow|E_T|=0.00899$

(c) $\frac{|E_T|}{\text{True Value}}\times 100=\frac{0.00899}{0.5}\times 100\approx 2\%$

II. (a) For $n=4,\ h=\frac{b-a}{n}=\frac{2-1}{4}=\frac{1}{4}\Rightarrow\frac{h}{3}=\frac{1}{12}$;

$\sum mf(s_i)=\frac{264{,}821}{44{,}100}\Rightarrow S=\frac{1}{12}\left(\frac{264{,}821}{44{,}100}\right)=\frac{264{,}821}{529{,}200}$

≈ 0.50042; $f^{(3)}(s)=-\frac{24}{s^5}\Rightarrow f^{(4)}(s)=\frac{120}{s^6}$

	s_i	$f(s_i)$	m	$mf(s_i)$
s_0	1	1	1	1
s_1	5/4	16/25	4	64/25
s_2	3/2	4/9	2	8/9
s_3	7/4	16/49	4	64/49
s_4	2	1/4	1	1/4

$\Rightarrow M = 120 = f^{(4)}(1) \Rightarrow |E_S| \le \frac{2-1}{180}\left(\frac{1}{4}\right)^4(120) = \frac{1}{384} \approx 0.002604$

(b) $\int_1^2 \frac{1}{s^2}\,ds = \frac{1}{2} \Rightarrow E_S = \int_1^2 \frac{1}{s^2}\,ds - S = \frac{1}{2} - 0.050041 = -0.00041 \Rightarrow |E_S| = 0.00041$

(c) $\frac{|E_S|}{\text{True Value}} \times 100 = \frac{0.0004}{0.5} \times 100 \approx 0.08\%$

8. $\int_2^4 \frac{1}{(s-1)^2}\,ds$

I. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2} \Rightarrow \frac{h}{2} = \frac{1}{4}$;

	s_i	$f(s_i)$	m	$mf(s_i)$
s_0	2	1	1	1
s_1	5/2	4/9	2	8/9
s_2	3	1/4	2	1/2
s_3	7/2	4/25	2	8/25
s_4	4	1/9	1	1/9

$\sum mf(s_i) = \frac{1269}{450} \Rightarrow T = \frac{1}{4}\left(\frac{1269}{450}\right) = \frac{1269}{1800} = 0.70500;$

$f(s) = (s-1)^{-2} \Rightarrow f'(s) = -\frac{2}{(s-1)^3}$

$\Rightarrow f''(s) = \frac{6}{(s-1)^4} \Rightarrow M = 6 = f''(2)$

$\Rightarrow |E_T| \le \frac{4-2}{12}\left(\frac{1}{2}\right)^2(6) = \frac{1}{4} = 0.25$

(b) $\int_2^4 \frac{1}{(s-1)^2}\,ds = \left[\frac{-1}{(s-1)}\right]_2^4 = \left(\frac{-1}{4-1}\right) - \left(\frac{-1}{2-1}\right) = \frac{2}{3} \Rightarrow E_T = \int_2^4 \frac{1}{(s-1)^2}\,ds - T = \frac{2}{3} - 0.705 \approx -0.03833$

$\Rightarrow |E_T| \approx 0.03833$

(c) $\frac{|E_T|}{\text{True Value}} \times 100 = \frac{0.03833}{\left(\frac{2}{3}\right)} \times 100 \approx 6\%$

II. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{3-1}{4} = \frac{1}{2} \Rightarrow \frac{h}{3} = \frac{1}{6}$;

	s_i	$f(s_i)$	m	$mf(s_i)$
s_0	2	1	1	1
s_1	5/2	4/9	4	16/9
s_2	3	1/4	2	1/2
s_3	7/2	4/25	4	16/25
s_4	4	1/9	1	1/9

$\sum mf(s_i) = \frac{1813}{450} \Rightarrow S = \frac{1}{6}\left(\frac{1813}{450}\right) = \frac{1813}{2700} \approx 0.67148;$

$f^{(3)}(s) = \frac{-24}{(s-1)^5} \Rightarrow f^{(4)}(s) = \frac{120}{(s-1)^6} \Rightarrow M = 120$

$= f^{(4)}(2) \Rightarrow |E_S| \le \frac{4-2}{180}\left(\frac{1}{2}\right)^4(120) = \frac{1}{12} \approx 0.08333$

(b) $\int_2^4 \frac{1}{(s-1)^2}\,ds = \frac{2}{3} \Rightarrow E_S = \int_2^4 \frac{1}{(s-1)^2}\,ds - S \approx \frac{2}{3} - 0.67148 = -0.00481 \Rightarrow |E_S| \approx 0.00481$

(c) $\frac{|E_S|}{\text{True Value}} \times 100 = \frac{0.00481}{\left(\frac{2}{3}\right)} \times 100 \approx 1\%$

9. $\int_0^{\pi} \sin t \, dt$

I. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{\pi - 0}{4} = \frac{\pi}{4} \Rightarrow \frac{h}{2} = \frac{\pi}{8}$;

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	$\pi/4$	$\sqrt{2}/2$	2	$\sqrt{2}$
t_2	$\pi/2$	1	2	2
t_3	$3\pi/4$	$\sqrt{2}/2$	2	$\sqrt{2}$
t_4	π	0	1	0

$\sum mf(t_i) = 2 + 2\sqrt{2} \approx 4.8284 \Rightarrow T = \frac{\pi}{8}(2 + 2\sqrt{2})$

≈ 1.89612; $f(t) = \sin t \Rightarrow f'(t) = \cos t \Rightarrow f''(t) = -\sin t$

$\Rightarrow M = 1 = |f''(0)| \Rightarrow |E_T| \le \frac{\pi - 0}{12}\left(\frac{\pi}{4}\right)^2(1) = \frac{\pi^3}{192} \approx 0.16149$

(b) $\int_0^{\pi} \sin t \, dt = [-\cos t]_0^{\pi} = (-\cos \pi) - (-\cos 0) = 2 \Rightarrow |E_T| = \int_0^{\pi} \sin t \, dt - T \approx 2 - 1.89612 = 0.10388$

(c) $\frac{|E_T|}{\text{True Value}} \times 100 = \frac{0.10388}{2} \times 100 \approx 5\%$

II. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{\pi - 0}{4} = \frac{\pi}{4} \Rightarrow \frac{h}{3} = \frac{\pi}{12}$;

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	$\pi/4$	$\sqrt{2}/2$	4	$2\sqrt{2}$
t_2	$\pi/2$	1	2	2
t_3	$3\pi/4$	$\sqrt{2}/2$	4	$2\sqrt{2}$
t_4	π	0	1	0

$\sum mf(t_i) = 2 + 4\sqrt{2} \approx 7.6569 \Rightarrow S = \frac{\pi}{12}(2 + 4\sqrt{2})$

≈ 2.00456; $f^{(3)}(t) = -\cos t \Rightarrow f^{(4)}(t) = \sin t$

$\Rightarrow M = 1 = f^{(4)}(0) \Rightarrow |E_S| \le \frac{\pi - 0}{180}\left(\frac{\pi}{4}\right)^4(1) \approx 0.00664$

(b) $\int_0^{\pi} \sin t \, dt = 2 \Rightarrow E_S = \int_0^{\pi} \sin t \, dt - S \approx 2 - 2.00456 = -0.00456 \Rightarrow |E_S| \approx 0.00456$

(c) $\frac{|E_S|}{\text{True Value}} \times 100 = \frac{0.00456}{2} \times 100 \approx 0.23\%$

10. $\int_0^{1} \sin \pi t \, dt$

I. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} \Rightarrow \frac{h}{2} = \frac{1}{8}$;

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	1/4	$\sqrt{2}/2$	2	$\sqrt{2}$
t_2	1/2	1	2	2
t_3	3/4	$\sqrt{2}/2$	2	$\sqrt{2}$
t_4	1	0	1	0

$\sum mf(t_i) = 2 + 2\sqrt{2} \approx 4.828 \Rightarrow T = \frac{1}{8}(2 + 2\sqrt{2})$

≈ 0.60355; $f(t) = \sin \pi t \Rightarrow f'(t) = \pi \cos \pi t$

$\Rightarrow f''(t) = -\pi^2 \sin \pi t \Rightarrow M = \pi^2 = |f''(0)|$

$\Rightarrow |E_T| \le \frac{1-0}{12}\left(\frac{1}{4}\right)^2(\pi^2) \approx 0.05140$

(b) $\int_0^1 \sin \pi t \, dt = \left[-\frac{1}{\pi}\cos \pi t\right]_0^1 = \left(-\frac{1}{\pi}\cos \pi\right) - \left(-\frac{1}{\pi}\cos 0\right) = \frac{2}{\pi} \approx 0.63662 \Rightarrow |E_T| = \int_0^1 \sin \pi t \, dt - T$

$\approx \frac{2}{\pi} - 0.60355 = 0.03307$

(c) $\frac{|E_T|}{\text{True Value}} \times 100 = \frac{0.03307}{\left(\frac{2}{\pi}\right)} \times 100 \approx 5\%$

II. (a) For $n = 4$, $h = \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} \Rightarrow \frac{h}{3} = \frac{1}{12}$;

	t_i	$f(t_i)$	m	$mf(t_i)$
t_0	0	0	1	0
t_1	1/4	$\sqrt{2}/2$	4	$2\sqrt{2}$
t_2	1/2	1	2	2
t_3	3/4	$\sqrt{2}/2$	4	$2\sqrt{2}$
t_4	1	0	1	0

$\sum mf(t_i) = 2 + 4\sqrt{2} \approx 7.65685 \Rightarrow S = \frac{1}{12}(2 + 4\sqrt{2})$

≈ 0.63807; $f^{(3)}(t) = -\pi^3 \cos \pi t \Rightarrow f^{(4)}(t) = \pi^4 \sin \pi t$

$\Rightarrow M = \pi^4 = f^{(4)}(0) \Rightarrow |E_S| \le \frac{1-0}{180}\left(\frac{1}{4}\right)^4(\pi^4) \approx 0.00211$

(b) $\int_0^1 \sin \pi t \, dt = \frac{2}{\pi} \approx 0.63662 \Rightarrow E_S = \int_0^1 \sin \pi t \, dt - S \approx \frac{2}{\pi} - 0.63807 = -0.00145 \Rightarrow |E_S| \approx 0.00145$

(c) $\frac{|E_S|}{\text{True Value}} \times 100 = \frac{0.00145}{\left(\frac{2}{\pi}\right)} \times 100 \approx 0\%$

11. (a) $n = 8 \Rightarrow h = \frac{1}{8} \Rightarrow \frac{h}{2} = \frac{1}{16}$;

$\sum mf(x_i) = 1(0.0) + 2(0.12402) + 2(0.24206) + 2(0.34763) + 2(0.43301) + 2(0.48789) + 2(0.49608)$
$+ 2(0.42361) + 1(0) = 5.1086 \Rightarrow T = \frac{1}{16}(5.1086) = 0.31929$

(b) $n = 8 \Rightarrow h = \frac{1}{8} \Rightarrow \frac{h}{3} = \frac{1}{24}$;

$\sum mf(x_i) = 1(0.0) + 4(0.12402) + 2(0.24206) + 4(0.34763) + 2(0.43301) + 4(0.48789) + 2(0.49608)$
$+ 4(0.42361) + 1(0) = 7.8749 \Rightarrow S = \frac{1}{24}(7.8749) = 0.32812$

(c) Let $u = 1 - x^2 \Rightarrow du = -2x \, dx \Rightarrow -\frac{1}{2} du = x \, dx$; $x = 0 \Rightarrow u = 1$, $x = 1 \Rightarrow u = 0$

$\int_0^1 x\sqrt{1-x^2} \, dx = \int_1^0 \sqrt{u}\left(-\frac{1}{2} du\right) = \frac{1}{2}\int_0^1 u^{1/2} \, du = \left[\frac{1}{2}\left(\frac{u^{3/2}}{\frac{3}{2}}\right)\right]_0^1 = \left[\frac{1}{3}u^{3/2}\right]_0^1 = \frac{1}{3}\left(\sqrt{1}\right)^3 - \frac{1}{3}\left(\sqrt{0}\right)^3 = \frac{1}{3}$;

$E_T = \int_0^1 x\sqrt{1-x^2} \, dx - T \approx \frac{1}{3} - 0.31929 = 0.01404$; $E_S = \int_0^1 x\sqrt{1-x^2} \, dx - S \approx \frac{1}{3} - 0.32812 = 0.00521$

12. (a) $n = 8 \Rightarrow h = \frac{3}{8} \Rightarrow \frac{h}{2} = \frac{3}{16}$;

$\sum mf(\theta_i) = 1(0) + 2(0.09334) + 2(0.18429) + 2(0.27075) + 2(0.35112) + 2(0.42443) + 2(0.49026)$
$+ 2(0.58466) + 1(0.6) = 5.3977 \Rightarrow T = \frac{3}{16}(5.3977) = 1.01207$

(b) $n=8 \Rightarrow h=\frac{3}{8} \Rightarrow \frac{h}{3}=\frac{1}{8}$;

$\sum mf(\theta_i) = 1(0)+4(0.09334)+2(0.18429)+4(0.27075)+2(0.35112)+4(0.42443)+2(0.49026)$
$+4(0.58466)+1(0.6) = 8.14406 \Rightarrow S=\frac{1}{8}(8.14406)=1.01801$

(c) Let $u=16+\theta^2 \Rightarrow du=2\theta\, d\theta \Rightarrow \frac{1}{2}du=\theta\, d\theta$; $\theta=0 \Rightarrow u=16$, $\theta=3 \Rightarrow u=16+3^2=25$

$$\int_0^3 \frac{\theta}{\sqrt{16+\theta^2}}\,d\theta=\int_{16}^{25}\frac{1}{\sqrt{u}}\left(\frac{1}{2}\,du\right)=\frac{1}{2}\int_{16}^{25}u^{-1/2}\,du=\left[\frac{1}{2}\left(\frac{u^{1/2}}{\frac{1}{2}}\right)\right]_{16}^{25}=\sqrt{25}-\sqrt{16}=1;$$

$$E_T=\int_0^3\frac{\theta}{\sqrt{16+\theta^2}}\,d\theta-T\approx 1-1.01207=-0.01207;\ E_S=\int_0^3\frac{\theta}{\sqrt{16+\theta^2}}\,d\theta-S\approx 1-0.01801=-0.01801$$

13. (a) $n=8 \Rightarrow h=\frac{\pi}{8} \Rightarrow \frac{h}{2}=\frac{\pi}{16}$;

$\sum mf(t_i)=1(0.0)+2(0.99138)+2(1.26906)+2(1.05961)+2(0.75)+2(0.48821)+2(0.28946)+2(0.13429)$
$+1(0)=9.96402 \Rightarrow T=\frac{\pi}{16}(9.96402)\approx 1.95643$

(b) $n=8 \Rightarrow h=\frac{\pi}{8} \Rightarrow \frac{h}{3}=\frac{\pi}{24}$;

$\sum mf(t_i)=1(0.0)+4(0.99138)+2(1.26906)+4(1.05961)+2(0.75)+4(0.48821)+2(0.28946)+4(0.13429)$
$+1(0)=15.311 \Rightarrow S\approx\frac{\pi}{24}(15.311)\approx 2.00421$

(c) Let $u=2+\sin t \Rightarrow du=\cos t\,dt$; $t=-\frac{\pi}{2} \Rightarrow u=2+\sin\left(-\frac{\pi}{2}\right)=1$, $t=\frac{\pi}{2}\Rightarrow u=2+\sin\frac{\pi}{2}=3$

$$\int_{-\pi/2}^{\pi/2}\frac{3\cos t}{(2+\sin t)^2}\,dt=\int_1^3\frac{3}{u^2}\,du=3\int_1^3 u^{-2}\,du=\left[3\left(\frac{u^{-1}}{-1}\right)\right]_1^3=3\left(-\frac{1}{3}\right)-3\left(-\frac{1}{1}\right)=2;$$

$$E_T=\int_{-\pi/2}^{\pi/2}\frac{3\cos t}{(2+\sin t)^2}\,dt-T\approx 2-1.95643=0.04357;\ E_S=\int_{-\pi/2}^{\pi/2}\frac{3\cos t}{(2+\sin t)^2}\,dt-S$$

$\approx 2-2.00421=-0.00421$

14. (a) $n=8 \Rightarrow h=\frac{\pi}{32} \Rightarrow \frac{h}{2}=\frac{\pi}{64}$;

$\sum mf(y_i)=1(2.0)+2(1.51606)+2(1.18237)+2(0.93998)+2(0.75402)+2(0.60145)+2(0.46364)$
$+2(0.31688)+1(0)=13.5488 \Rightarrow T\approx\frac{\pi}{64}(13.5488)=0.66508$

(b) $n=8 \Rightarrow h=\frac{\pi}{32} \Rightarrow \frac{h}{3}=\frac{\pi}{96}$;

$\sum mf(y_i)=1(2.0)+4(1.51606)+2(1.18237)+4(0.93988)+2(0.75402)+4(0.60145)+2(0.46364)$
$+4(0.31688)+1(0)=20.29754 \Rightarrow S\approx\frac{\pi}{96}(20.29754)=0.66424$

(c) Let $u=\cot y \Rightarrow du=-\csc^2 y\,dy$; $y=\frac{\pi}{4}\Rightarrow u=1$, $y=\frac{\pi}{2}\Rightarrow u=0$

$$\int_{\pi/4}^{\pi/2} (\csc^2 y)\sqrt{\cot y}\, dy = \int_1^0 \sqrt{u}\,(-du) = \int_0^1 u^{1/2}\, du = \left[\frac{u^{3/2}}{\frac{3}{2}}\right]_0^1 = \tfrac{2}{3}(\sqrt{1})^3 - \tfrac{2}{3}(\sqrt{0})^3 = \tfrac{2}{3};$$

$$E_T = \int_{\pi/4}^{\pi/2} (\csc^2 y)\sqrt{\cot y}\, dy - T \approx \tfrac{2}{3} - 0.66508 = 0.00159;\ E_S = \int_{\pi/4}^{\pi/2} (\csc^2 y)\sqrt{\cot y}\, dy - S$$

$$\approx \tfrac{2}{3} - 0.66424 = 0.00243$$

15. $\frac{5}{2}(6.0 + 2(8.2) + 2(9.1 + \ldots + 2(12.7) + 13.0)(30) = 15{,}990 \text{ ft}^3$

16. (a) Using the Trapezoid Rule, $h = 200 \Rightarrow \frac{h}{2} = \frac{200}{2} = 100$;

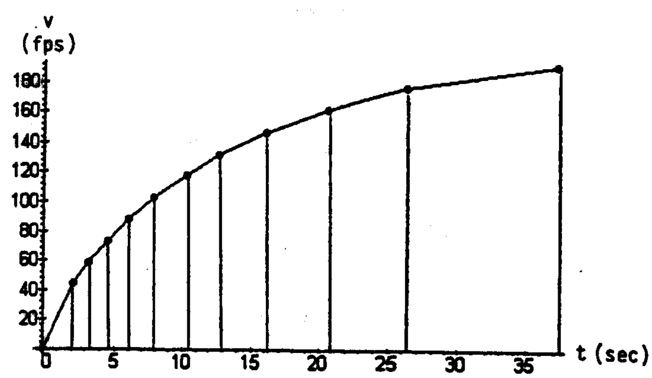
$\sum mf(x_i) = 13{,}180 \Rightarrow \text{Area} \approx 100(13{,}180)$

$= 1{,}318{,}000 \text{ ft}^2$. Since the average depth = 20 ft we obtain Volume $\approx 20\,(\text{Area}) \approx 26{,}360{,}000 \text{ ft}^3$.

(b) The number of fish $= \frac{\text{Volume}}{1000} = 26{,}360$ (to the nearest fish) $\Rightarrow$ Maximum to be caught = 75% of 26,360 = 19,770 $\Rightarrow$ Number of licenses $= \frac{19{,}770}{20} = 988$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	200	520	2	1040
x_2	400	800	2	1600
x_3	600	1000	2	2000
x_4	800	1140	2	2280
x_4	1000	1160	2	2320
x_6	1200	1110	2	2220
x_7	1400	860	2	1720
x_8	1600	0	1	0

17. Use the conversion 30 mph = 44 fps (ft per sec) since time is measured in seconds. The distance traveled as the car accelerates from, say, 40 mph = 58.67 fps to 50 mph = 73.33 fps in $(4.5 - 3.2) = 1.3$ sec is the area of the trapezoid (see figure) associated with that time interval: $\frac{1}{2}(58.67 + 73.33)(1.3) = 85.8$ ft. The total distance traveled by the Ford Mustang Cobra is the sum of all these eleven trapezoids (using $\frac{\Delta t}{2}$ and the table below):



$s = (44)(1.1) + (102.67)(0.5) + (132)(0.65) + (161.33)(0.7) + (190.67)(0.95) + (220)(1.2) + (249.33)(1.25)$
$\quad + (278.67)(1.65) + (308)(2.3) + (337.33)(2.8) + (366.67)(5.45) = 5166.33 \text{ ft} \approx 0.978 \text{ mi}$

v (mph)	0	30	40	50	60	70	80	90	100	110	120	130
v (fps)	0	44	58.67	73.33	88	102.67	117.33	132	146.67	161.33	176	190.67
t (sec)	0	2.2	3.2	4.5	5.9	7.8	10.2	12.7	16	20.6	26.2	37.1
$\Delta t/2$	0	1.1	0.5	0.65	0.7	0.95	1.2	1.25	1.65	2.3	2.8	5.45

18. Using Simpson's Rule, $h = \frac{b-a}{n} = \frac{24-0}{6} = \frac{24}{6} = 4$;

$\sum my_i = 350 \Rightarrow S = \frac{4}{3}(350) = \frac{1400}{3} \approx 466.7 \text{ in.}^2$

	x_i	y_i	m	my_i
x_0	0	0	1	0
x_1	4	18.75	4	75
x_2	8	24	2	48
x_3	12	26	4	104
x_4	16	24	2	48
x_5	20	18.75	4	75
x_6	24	0	1	0

19. Using Simpson's Rule, $h = 1 \Rightarrow \frac{h}{3} = \frac{1}{3}$;

$\sum my_i = 33.6 \Rightarrow$ Cross Section Area $\approx \frac{1}{3}(33.6) = 11.2 \text{ ft}^2$. Let x be the length of the tank. Then the Volume V $=$ (Cross Sectional Area) $x = 11.2x$. Now 5000 lb of gasoline at 42 lb/ft^3 $\Rightarrow V = \frac{5000}{42} = 119.05 \text{ ft}^3$ $\Rightarrow 119.05 = 11.2x \Rightarrow x \approx 10.63$ ft

	x_i	y_i	m	my_i
x_0	0	1.5	1	1.5
x_1	1	1.6	4	6.4
x_2	2	1.8	2	3.6
x_3	3	1.9	4	7.6
x_4	4	2.0	2	4.0
x_5	5	2.1	4	8.4
x_6	6	2.1	1	2.1

20. Using Simpson's Rule, $h = 24 \Rightarrow \frac{h}{2} = 12$; $12[0.019 + 2(0.020) + 2(0.021) + \ldots + 2(0.031) + 0.035] = 4.2$ L

21. $n = 2 \Rightarrow h = \frac{2-0}{2} = 1 \Rightarrow \frac{h}{3} = \frac{1}{3}$;

$\sum mf(x_i) = 12 \Rightarrow S = \frac{1}{3}(12) = 4$;

$$\int_0^2 x^3\,dx = \left[\frac{x^4}{4}\right]_0^2 = \frac{2^4}{4} - \frac{0^4}{4} = 4$$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	1	1	4	4
x_2	2	8	1	8

22. (a) $|E_S| \le \frac{b-a}{180}(h^4)M$; $n = 4 \Rightarrow h = \frac{\frac{\pi}{2}-0}{4} = \frac{\pi}{8}$; $|f^{(4)}| \le 1 \Rightarrow M = 1 \Rightarrow |E_S| \le \frac{\left(\frac{\pi}{2}-0\right)}{180}\left(\frac{\pi}{8}\right)^4(1) \approx 0.00021$

(b) $h = \frac{\pi}{8} \Rightarrow \frac{h}{3} = \frac{\pi}{24}$;

$\sum mf(x_i) = 10.472087048$

$\Rightarrow S = \frac{\pi}{24}(10.472087048) \approx 1.37079$

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	1	1	1
x_1	$\pi/8$	0.974495358	4	3.897981432
x_2	$\pi/4$	0.900316316	2	1.800632632
x_3	$3\pi/8$	0.784213303	4	3.136853212
x_4	$\pi/2$	0.636619772	1	0.636619772

(c) $\approx \left(\frac{0.00021}{1.37079}\right) \times 100 \approx 0.015\%$

23. (a) $h = \frac{b-a}{n} = \frac{1-0}{10} = 0.1 \Rightarrow \text{erf}(1) = \frac{2}{\sqrt{\pi}}\left(\frac{0.1}{3}\right)(y_0 + 4y_1 + 2y_2 + 4y_3 + \ldots + 4y_9 + y_{10})$

$= \frac{2}{30\sqrt{\pi}}\left(e^0 + 4e^{-0.01} + 2e^{-0.04} + 4e^{-0.09} + \ldots + 4e^{-0.81} + e^{-1}\right) \approx 0.8427$

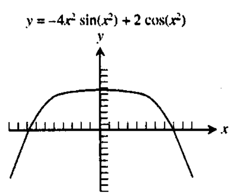
(b) $|E_s| \le \frac{1-0}{180}(0.1)^4(12) \approx 6.7 \times 10^{-6}$

24. The average of the 13 discrete temperatures gives equal weight to the low values at the end.

25. (a) $f'(x) = 2x\cos(x^2)$

$f''(x) = 2x \cdot -2x\sin(x^2) + 2\cos(x^2) = -4x^2\sin(x^2) + 2\cos(x^2)$

(b)



(c) The graph shows that $-3 \le f''(x) \le 2$ so $|f''(x)| \le 3$ for $-1 \le x \le 1$.

(d) $|E_T| \le \frac{1-(-1)}{12}(h^2)(3) = \frac{h^2}{2}$

(e) For $0 < h \le 0.1$, $|E_T| \le \frac{h^2}{2} \le \frac{0.1^2}{2} = 0.005 < 0.01$

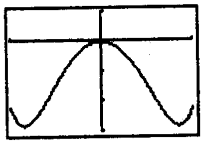
(f) $n \ge \frac{1-(-1)}{h} \ge \frac{2}{0.1} = 20$

26. (a) $f'''(x) = -4x^2 \cdot 2x\cos(x^2) - 8x\sin(x^2) - 4x\sin(x^2) = -8x^3\cos(x^2) - 12x\sin(x^2)$

$f^{(4)}(x) = -8x^3 \cdot -2x\sin(x^2) - 24x^2\cos(x^2) - 12x \cdot 2x\cos(x^2) - 12\sin(x^2)$

$= (16x^4 - 12)\sin(x^2) - 48x^2\cos(x^2)$

(b)



[−1, 1] by [−30, 10]

(c) The graph shows that $-30 \le f^{(4)}(x) \le 0$ so $\left|f^{(4)}(x)\right| \le 30$ for $-1 \le x \le 1$.

(d) $|E_T| \le \frac{1-(-1)}{180}(h^4)(30) = \frac{h^4}{3}$

(e) For $0 < h \le 0.4$, $|E_S| \le \frac{h^4}{3} \le \frac{0.4^4}{3} \approx 0.00853 < 0.01$

(f) $n \ge \frac{1-(-1)}{h} \ge \frac{2}{0.4} = 5$

Exercises 27-30 were done using the nInt( ) command on a TI-92 Plus calculator. Your answer may be different depending on your calculator.

27. 3.1415927

28. 1.0894294

29. 1.3707622

30. 0.82811633

31. (a) $T_{10} \approx 1.983523538$
$T_{100} \approx 1.999835504$
$T_{1000} \approx 1.999998355$

(b)

n	$\lvert E_T\rvert = 2 - T_n$
10	$0.016476462 = 1.6476462 \times 10^{-2}$
100	1.64496×10^{-4}
1000	1.645×10^{-6}

(c) $\left|E_{T_{10n}}\right| \approx 10^{-2}\left|E_{T_n}\right|$

(d) $b - a = \pi$, $h^2 = \frac{\pi^2}{n^2}$, $M = 1 \Rightarrow \left|E_{T_n}\right| \le \frac{\pi}{12}\left(\frac{\pi^2}{n^2}\right) = \frac{\pi^3}{12n^2}$ and $\left|E_{T_{10n}}\right| \le \frac{\pi^3}{12(10n)^2} = 10^{-2}\left|E_{T_n}\right|$

32. (a) $S_{10} \approx 2.000109517$
$S_{100} \approx 2.000000011$
$S_{1000} \approx 2.000000000$

(b)

n	$\lvert E_S \rvert = 2 - S_n$
10	1.09517×10^{-4}
100	1.1×10^{-8}
1000	0

(c) $\left|E_{S_{10n}}\right| \approx 10^{-4}\left|E_{S_n}\right|$

(d) $b - a = \pi,\ h^4 = \frac{\pi^4}{n^4},\ M = 1 \Rightarrow \left|E_{S_n}\right| \le \frac{\pi}{180}\left(\frac{\pi^4}{n^4}\right) = \frac{\pi^5}{180n^4}$ and $\left|E_{S_{10n}}\right| \le \frac{\pi^5}{180(10n)^4} = 10^{-4}\left|E_{S_n}\right|$

CHAPTER 4 PRACTICE EXERCISES

1. $\int (x^3 + 5x - 7)\,dx = \frac{x^4}{4} + \frac{5x^2}{2} - 7x + C$

2. $\int \left(8t^3 - \frac{t^2}{2} + t\right) dt = \frac{8t^4}{4} - \frac{t^3}{6} + \frac{t^2}{2} + C = 2t^4 - \frac{t^3}{6} + \frac{t^2}{2} + C$

3. $\int \left(3\sqrt{t} + \frac{4}{t^2}\right) dt = \int \left(3t^{1/2} + 4t^{-2}\right) dt = \frac{3t^{3/2}}{\left(\frac{3}{2}\right)} + \frac{4t^{-1}}{-1} + C = 2t^{3/2} - \frac{4}{t} + C$

4. $\int \left(\frac{1}{2\sqrt{t}} - \frac{3}{t^4}\right) dt = \int \left(\frac{1}{2}t^{-1/2} - 3t^{-4}\right) dt = \frac{1}{2}\left(\frac{t^{1/2}}{\frac{1}{2}}\right) - \frac{3t^{-3}}{(-3)} + C = \sqrt{t} + \frac{1}{t^3} + C$

5. Let $u = r^2 + 5 \Rightarrow du = 2r\,dr \Rightarrow \frac{1}{2}\,du = r\,dr$

$$\int \frac{r\,dr}{(r^2+5)^2} = \int \frac{\left(\frac{1}{2}\right) du}{u^2} = \frac{1}{2}\int u^{-2}\,du = \frac{1}{2}\left(\frac{u^{-1}}{-1}\right) + C = -\frac{1}{2}u^{-1} + C = -\frac{1}{2(r^2+5)} + C$$

6. Let $u = r^3 - \sqrt{2} \Rightarrow du = 3r^2\,dr \Rightarrow 2\,du = 6r^2\,dr$

$$\int \frac{6r^2\,dr}{(r^3 - \sqrt{2})^3} = \int \frac{2\,du}{u^3} = 2\int u^{-3}\,du = 2\left(\frac{u^{-2}}{-2}\right) + C = -u^{-2} + C = -\frac{1}{(r^3 - \sqrt{2})^2} + C$$

7. Let $u = 2 - \theta^2 \Rightarrow du = -2\theta\,d\theta \Rightarrow -\frac{1}{2}\,du = \theta\,d\theta$

$$\int 3\theta\sqrt{2 - \theta^2}\,d\theta = \int \sqrt{u}\left(-\frac{3}{2}\,du\right) = -\frac{3}{2}\int u^{1/2}\,du = -\frac{3}{2}\left(\frac{u^{3/2}}{\frac{3}{2}}\right) + C = -u^{3/2} + C = -(2 - \theta^2)^{3/2} + C$$

8. Let $u = 73 + \theta^3 \Rightarrow du = 3\theta^2\, d\theta \Rightarrow \frac{1}{27}\, du = \frac{\theta^2}{9}\, d\theta$

$$\int \frac{\theta^2}{9\sqrt{73+\theta^3}}\, d\theta = \int \frac{1}{\sqrt{u}}\left(\frac{1}{27}\, du\right) = \frac{1}{27}\int u^{-1/2}\, du = \frac{1}{27}\left(\frac{u^{1/2}}{\frac{1}{2}}\right) + C = \frac{2}{27}u^{1/2} + C = \frac{2}{27}\sqrt{73+\theta^3} + C$$

9. $\int e^x \sin(e^x)\, dx = \int \sin u\, du$, where $u = e^x$ and $du = e^x\, dx$

$= -\cos u + C = -\cos(e^x) + C$

10. $\int e^t \cos(3e^t - 2)\, dt = \frac{1}{3}\int \cos u\, du$, where $u = 3e^t - 2$ and $du = 3e^t\, dt$

$= \frac{1}{3}\sin u + C = \frac{1}{3}\sin(3e^t - 2) + C$

11. $\int e^x \sec^2(e^x - 7)\, dx = \int \sec^2 u\, du$, where $u = e^x - 7$ and $du = e^x\, dx$

$= \tan u + C = \tan(e^x - 7) + C$

12. $\int e^y \csc(e^y + 1)\cot(e^y + 1)\, dy = \int \csc u \cot u\, du$, where $u = e^y + 1$ and $du = e^y\, dy$

$= -\csc u + C = -\csc(e^y + 1) + C$

13. $\int \frac{\tan(\ln v)}{v}\, dv = \int \tan u\, du = \int \frac{\sin u}{\cos u}\, du$, where $u = \ln v$ and $du = \frac{1}{v}\, dv$

$= -\ln|\cos u| + C = -\ln|\cos(\ln v)| + C$

14. Let $u = \frac{\theta}{3} \Rightarrow du = \frac{1}{3}\, d\theta \Rightarrow 3\, du = d\theta$

$$\int \sec\frac{\theta}{3}\tan\frac{\theta}{3}\, d\theta = \int (\sec u \tan u)(3\, du) = 3\sec u + C = 3\sec\frac{\theta}{3} + C$$

15. Let $u = \frac{x}{4} \Rightarrow du = \frac{1}{4}\, dx \Rightarrow 4\, du = dx$

$$\int \sin^2\frac{x}{4}\, dx = \int (\sin^2 u)(4\, du) = \int 4\left(\frac{1-\cos 2u}{2}\right) du = 2\int (1-\cos 2u)\, du = 2\left(u - \frac{\sin 2u}{2}\right) + C$$

$$= 2u - \sin 2u + C = 2\left(\frac{x}{4}\right) - \sin 2\left(\frac{x}{4}\right) + C = \frac{x}{2} - \sin\frac{x}{2} + C$$

16. $\int \frac{(\ln x)^{-3}}{x}\, dx = \int u^{-3}\, du$, where $u = \ln x$ and $du = \frac{1}{x}\, dx$

$= \frac{u^{-2}}{-2} + C = -\frac{1}{2}(\ln x)^{-2} + C$

17. $\int \frac{1}{r}\csc^2(1 + \ln r)\, dr = \int \csc^2 u\, du$, where $u = 1 + \ln r$ and $du = \frac{1}{r}\, dr$

$= -\cot u + C = -\cot(1 + \ln r) + C$

18. $\int \frac{\cos(1 - \ln v)}{v}\, dv = -\int \cos u\, du$, where $u = 1 - \ln v$ and $du = -\frac{1}{v}\, dv$

$= -\sin u + C = -\sin(1 - \ln v) + C$

19. $\int x3^{x^2}\, dx = \frac{1}{2}\int 3^u\, du$, where $u = x^2$ and $du = 2x\, dx$

$= \frac{1}{2\ln 3}(3^u) + C = \frac{1}{2\ln 3}\left(3^{x^2}\right) + C$

20. $\int 2^{\tan x}\sec^2 x\, dx = \int 2^u\, du$, where $u = \tan x$ and $du = \sec^2 x\, dx$

$= \frac{1}{\ln 2}(2^u) + C = \frac{2^{\tan x}}{\ln 2} + C$

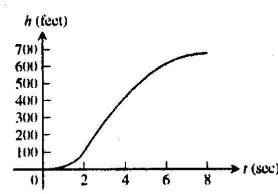
21. (a) Each time subinterval is of length $\Delta t = 0.4$ sec. The distance traveled over each subinterval, using the midpoint rule, is $\Delta h = \frac{1}{2}(v_i + v_{i+1})\Delta t$, where v_i is the velocity at the left, and v_{i+1} the velocity at the right, endpoint of the subinterval. We then add Δh to the height attained so far at the left endpoint v_i to arrive at the height associated with velocity v_{i+1} at the right endpoint. Using this methodology we build the following table based on the figure in the text:

t (sec)	0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0	4.4	4.8	5.2	5.6	6.0
v (fps)	0	10	25	55	100	190	180	170	155	140	130	120	105	90	80	65
h (ft)	0	2	9	25	56	114	188	258	323	382	436	486	531	570	604	633

t (sec)	6.4	6.8	7.2	7.6	8.0
v (fps)	52	40	30	15	0
h (ft)	656	674	688	697	700

NOTE: Your table values may vary slightly from ours depending on the v-values you read from the graph. Remember that some shifting of the graph occurs in the printing process. The total height attained is about 700 ft.

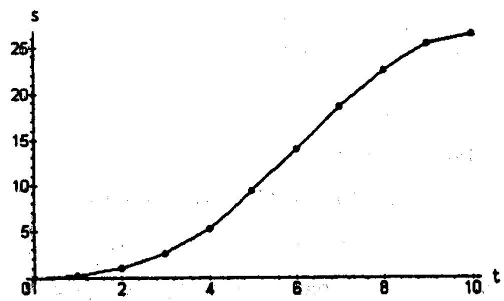
(b) The graph is based on the table in part (a).



22. (a) Each time subinterval is of length $\Delta t = 1$ sec. The distance traveled over each subinterval, using the midpoint rule, is $\Delta s = \frac{1}{2}(v_i + v_{i+1})\Delta t$, where v_i is the velocity at the left, and v_{i+1} the velocity at the right, endpoint of the subinterval. We then add Δs to the distance attained so far at the left endpoint v_i to arrive at the distance associated with velocity v_{i+1} at the right endpoint. Using this methodology we build the table given below based on the figure in the text, obtaining approximately 26 m for the total distance traveled:

t (sec)	0	1	2	3	4	5	6	7	8	9	10
v (m/sec)	0	0.5	1.0	2	3.5	4.5	4.8	4.5	3.5	2	0
s (m)	0	0.25	1.00	2.5	5.25	9.25	13.9	18.55	22.55	25.3	26.3

(b) The graph shows the distance traveled by the moving body as a function of time for $0 \le t \le 10$.



23. Let $u = 2x - 1 \Rightarrow du = 2\,dx \Rightarrow \frac{1}{2}\,du = dx$; $x = 1 \Rightarrow u = 1$, $x = 5 \Rightarrow u = 9$

$$\int_1^5 (2x-1)^{-1/2}\,dx = \int_1^9 u^{-1/2}\left(\frac{1}{2}\,du\right) = \left[u^{1/2}\right]_1^9 = 3 - 1 = 2$$

24. Let $u = x^2 - 1 \Rightarrow du = 2x\,dx \Rightarrow \frac{1}{2}\,du = x\,dx$; $x = 1 \Rightarrow u = 0$, $x = 3 \Rightarrow u = 8$

$$\int_1^3 x\left(x^2-1\right)^{1/3}\,dx = \int_0^8 u^{1/3}\left(\frac{1}{2}\,du\right) = \left[\frac{3}{8}u^{4/3}\right]_0^8 = \frac{3}{8}(16-0) = 6$$

25. Let $u = \frac{x}{2} \Rightarrow 2\,du = dx$; $x = -\pi \Rightarrow u = -\frac{\pi}{2}$, $x = 0 \Rightarrow u = 0$

$$\int_{-\pi}^0 \cos\left(\frac{x}{2}\right)dx = \int_{-\pi/2}^0 (\cos u)(2\,du) = [2\sin u]_{-\pi/2}^0 = 2\sin 0 - 2\sin\left(-\frac{\pi}{2}\right) = 2(0-(-1)) = 2$$

26. Let $u = \sin x \Rightarrow du = \cos x\,dx$; $x = 0 \Rightarrow u = 0$, $x = \frac{\pi}{2} \Rightarrow u = 1$

$$\int_0^{\pi/2} (\sin x)(\cos x)\,dx = \int_0^1 u\,du = \left[\frac{u^2}{2}\right]_0^1 = \frac{1}{2}$$

27. (a) $\int_{-2}^{2} f(x)\,dx = \frac{1}{3}\int_{-2}^{2} 3\,f(x)\,dx = \frac{1}{3}(12) = 4$ (b) $\int_{2}^{5} f(x)\,dx = \int_{-2}^{5} f(x)\,dx - \int_{-2}^{2} f(x)\,dx = 6 - 4 = 2$

(c) $\int_{5}^{-2} g(x)\,dx = -\int_{-2}^{5} g(x)\,dx = -2$ (d) $\int_{-2}^{5} (-\pi\,g(x))\,dx = -\pi\int_{-2}^{5} g(x)\,dx = -\pi(2) = -2\pi$

(e) $\int_{-2}^{5} \left(\frac{f(x)+g(x)}{5}\right)dx = \frac{1}{5}\int_{-2}^{5} f(x)\,dx + \frac{1}{5}\int_{-2}^{5} g(x)\,dx = \frac{1}{5}(6) + \frac{1}{5}(2) = \frac{8}{5}$

28. (a) $\int_{0}^{2} g(x)\,dx = \frac{1}{7}\int_{0}^{2} 7\,g(x)\,dx = \frac{1}{7}(7) = 1$ (b) $\int_{1}^{2} g(x)\,dx = \int_{0}^{2} g(x)\,dx - \int_{0}^{1} g(x)\,dx = 1 - 2 = -1$

(c) $\int_{2}^{0} f(x)\,dx = -\int_{0}^{2} f(x)\,dx = -\pi$ (d) $\int_{0}^{2} \sqrt{2}\,f(x)\,dx = \sqrt{2}\int_{0}^{2} f(x)\,dx = \sqrt{2}\,(\pi) = \pi\sqrt{2}$

(e) $\int_{0}^{2} [g(x) - 3\,f(x)]\,dx = \int_{0}^{2} g(x)\,dx - 3\int_{0}^{2} f(x)\,dx = 1 - 3\pi$

29. $\int_{-1}^{1} (3x^2 - 4x + 7)\,dx = \left[x^3 - 2x^2 + 7x\right]_{-1}^{1} = \left[1^3 - 2(1)^2 + 7(1)\right] - \left[(-1)^3 - 2(-1)^2 + 7(-1)\right] = 6 - (-10) = 16$

30. $\int_{0}^{1} (8s^3 - 12s^2 + 5)\,ds = \left[2s^4 - 4s^3 + 5s\right]_{0}^{1} = \left[2(1)^4 - 4(1)^3 + 5(1)\right] - 0 = 3$

31. $\int_{1}^{4} \left(\frac{x}{8} + \frac{1}{2x}\right)dx = \frac{1}{2}\int_{1}^{4} \left(\frac{1}{4}x + \frac{1}{x}\right)dx = \frac{1}{2}\left[\frac{1}{8}x^2 + \ln|x|\right]_{1}^{4} = \frac{1}{2}\left[\left(\frac{16}{8} + \ln 4\right) - \left(\frac{1}{8} + \ln 1\right)\right] = \frac{15}{16} + \frac{1}{2}\ln 4$

$= \frac{15}{16} + \ln\sqrt{4} = \frac{15}{16} + \ln 2$

32. $\int_{1}^{8} \left(\frac{2}{3x} - \frac{8}{x^2}\right)dx = \frac{2}{3}\int_{1}^{8} \left(\frac{1}{x} - 12x^{-2}\right)dx = \frac{2}{3}\left[\ln|x| + 12x^{-1}\right]_{1}^{8} = \frac{2}{3}\left[\left(\ln 8 + \frac{12}{8}\right) - (\ln 1 + 12)\right]$

$= \frac{2}{3}\left(\ln 8 + \frac{3}{2} - 12\right) = \frac{2}{3}\left(\ln 8 - \frac{21}{2}\right) = \frac{2}{3}(\ln 8) - 7 = \ln\left(8^{2/3}\right) - 7 = \ln 4 - 7$

33. $\int_{1}^{4} \frac{dt}{t\sqrt{t}} = \int_{1}^{4} \frac{dt}{t^{3/2}} = \int_{1}^{4} t^{-3/2}\,dt = \left[-2t^{-1/2}\right]_{1}^{4} = \frac{-2}{\sqrt{4}} - \frac{(-2)}{\sqrt{1}} = 1$

34. Let $x = 1 + \sqrt{u} \Rightarrow dx = \frac{1}{2}u^{-1/2}\,du \Rightarrow 2\,dx = \frac{du}{\sqrt{u}}$; $u = 1 \Rightarrow x = 2$, $u = 4 \Rightarrow x = 3$

$$\int_1^4 \frac{(1+\sqrt{u})^{1/2}}{\sqrt{u}}\,du = \int_2^3 x^{1/2}(2\,dx) = \left[2\left(\frac{2}{3}\right)x^{3/2}\right]_2^3 = \frac{4}{3}(3^{3/2}) - \frac{4}{3}(2^{3/2}) = 4\sqrt{3} - \frac{8}{3}\sqrt{2} = \frac{4}{3}(3\sqrt{3} - 2\sqrt{2})$$

35. Let $u = 2x + 1 \Rightarrow du = 2\,dx \Rightarrow 18\,du = 36\,dx$; $x = 0 \Rightarrow u = 1$, $x = 1 \Rightarrow u = 3$

$$\int_0^1 \frac{36\,dx}{(2x+1)^3} = \int_1^3 18u^{-3}\,du = \left[\frac{18u^{-2}}{-2}\right]_1^3 = \left[\frac{-9}{u^2}\right]_1^3 = \left(\frac{-9}{3^2}\right) - \left(\frac{-9}{1^2}\right) = 8$$

36. Let $u = 7 - 5r \Rightarrow du = -5\,dr \Rightarrow -\frac{1}{5}\,du = dr$; $r = 0 \Rightarrow u = 7$, $r = 1 \Rightarrow u = 2$

$$\int_0^1 \frac{dr}{\sqrt[3]{(7-5r)^2}} = \int_0^1 (7-5r)^{-2/3}\,dr = \int_7^2 u^{-2/3}\left(-\frac{1}{5}\,du\right) = -\frac{1}{5}\left[3u^{1/3}\right]_7^2 = \frac{3}{5}\left(\sqrt[3]{7} - \sqrt[3]{2}\right)$$

37. $\int_0^{\ln 5} e^r(3e^r+1)^{-3/2}\,dr = \frac{1}{3}\int_4^{16} u^{-3/2}\,du$, where $u = 3e^r + 1$, $du = 3e^r$; $r = 0 \Rightarrow u = 4$, $r = \ln 5 \Rightarrow u = 16$

$$= -\frac{2}{3}\left[u^{-1/2}\right]_4^{16} = -\frac{2}{3}\left(16^{-1/2} - 4^{-1/2}\right) = \left(-\frac{2}{3}\right)\left(\frac{1}{4} - \frac{1}{2}\right) = \left(-\frac{2}{3}\right)\left(-\frac{1}{4}\right) = \frac{1}{6}$$

38. $\int_0^{\ln 9} e^\theta(e^\theta - 1)^{1/2}\,d\theta = \int_0^8 u^{1/2}\,du$, where $u = e^\theta - 1$, $du = e^\theta\,d\theta$; $\theta = 0 \Rightarrow u = 0$, $\theta = \ln 9 \Rightarrow u = 8$

$$= \frac{2}{3}\left[u^{3/2}\right]_0^8 = \frac{2}{3}\left(8^{3/2} - 0^{3/2}\right) = \frac{2}{3}\left(2^{9/2} - 0\right) = \frac{2^{11/2}}{3} = \frac{32\sqrt{2}}{3}$$

39. $\int_e^{e^2} \frac{1}{x\sqrt{\ln x}}\,dx = \int_e^{e^2} (\ln x)^{-1/2}\,\frac{1}{x}\,dx = \int_1^2 u^{-1/2}\,du$, where $u = \ln x$, $du = \frac{1}{x}\,dx$; $x = e \Rightarrow u = 1$, $x = e^2 \Rightarrow u = 2$

$$= 2\left[u^{1/2}\right]_1^2 = 2\left(\sqrt{2} - 1\right) = 2\sqrt{2} - 2$$

40. Let $u = 4t - \frac{\pi}{4} \Rightarrow du = 4\,dt \Rightarrow \frac{1}{4}\,du = dt$; $t = 0 \Rightarrow u = -\frac{\pi}{4}$, $t = \frac{\pi}{4} \Rightarrow u = \frac{3\pi}{4}$

$$\int_0^{\pi/4} \cos^2\left(4t - \frac{\pi}{4}\right)dt = \int_{-\pi/4}^{3\pi/4} (\cos^2 u)\left(\frac{1}{4}\,du\right) = \frac{1}{4}\left[\frac{u}{2} + \frac{\sin 2u}{4}\right]_{-\pi/4}^{3\pi/4} = \frac{1}{4}\left(\frac{3\pi}{8} + \frac{\sin\left(\frac{3\pi}{2}\right)}{4}\right) - \frac{1}{4}\left(-\frac{\pi}{8} + \frac{\sin\left(-\frac{\pi}{2}\right)}{4}\right)$$

$$= \frac{\pi}{8} - \frac{1}{16} + \frac{1}{16} = \frac{\pi}{8}$$

41. $\int_1^e \frac{1}{x}(1 + 7\ln x)^{-1/3}\,dx = \frac{1}{7}\int_1^8 u^{-1/3}\,du$, where $u = 1 + 7\ln x$, $du = \frac{7}{x}\,dx$, $x = 1 \Rightarrow u = 1$, $x = e \Rightarrow u = 8$

$$= \frac{3}{14}\left[u^{2/3}\right]_1^8 = \frac{3}{14}\left(8^{2/3} - 1^{2/3}\right) = \left(\frac{3}{14}\right)(4-1) = \frac{9}{14}$$

42. $\int_{\pi/4}^{3\pi/4} \csc^2 x\, dx = [-\cot x]_{\pi/4}^{3\pi/4} = \left(-\cot \frac{3\pi}{4}\right) - \left(-\cot \frac{\pi}{4}\right) = 2$

43. $\int_{-2}^{2} \frac{3}{4+3t^2}\, dt = \sqrt{3} \int_{-2}^{2} \frac{\sqrt{3}}{2^2 + \left(\sqrt{3}t\right)^2}\, dt = \sqrt{3} \int_{-2\sqrt{3}}^{2\sqrt{3}} \frac{1}{2^2 + u^2}\, du$, where $u = \sqrt{3}t$, $du = \sqrt{3}\, dt$;
$t = -2 \Rightarrow u = -2\sqrt{3}$, $t = 2 \Rightarrow u = 2\sqrt{3}$

$= \sqrt{3}\left[\frac{1}{2} \tan^{-1}\left(\frac{u}{2}\right)\right]_{-2\sqrt{3}}^{2\sqrt{3}} = \frac{\sqrt{3}}{2}[\tan^{-1}(\sqrt{3}) - \tan^{-1}(-\sqrt{3})] = \frac{\sqrt{3}}{2}\left[\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right] = \frac{\pi}{\sqrt{3}}$

44. $\int_{1}^{3} \frac{[\ln (v+1)]^2}{v+1}\, dv = \int_{1}^{3} [\ln (v+1)]^2 \frac{1}{v+1}\, dv = \int_{\ln 2}^{\ln 4} u^2\, du$, where $u = \ln (v+1)$, $du = \frac{1}{v+1} dv$;
$v = 1 \Rightarrow u = \ln 2$, $v = 3 \Rightarrow u = \ln 4$;

$= \frac{1}{3}[u^3]_{\ln 2}^{\ln 4} = \frac{1}{3}[(\ln 4)^3 - (\ln 2)^3] = \frac{1}{3}[(2 \ln 2)^3 - (\ln 2)^3] = \frac{(\ln 2)^3}{3}(8-1) = \frac{7}{3}(\ln 2)^3$

45. $\int_{-\pi/3}^{0} \sec x \tan x\, dx = [\sec x]_{-\pi/3}^{0} = \sec 0 - \sec\left(-\frac{\pi}{3}\right) = 1 - 2 = -1$

46. $\int_{\pi/4}^{3\pi/4} \csc z \cot z\, dz = [-\csc z]_{\pi/4}^{3\pi/4} = \left(-\csc \frac{3\pi}{4}\right) - \left(-\csc \frac{\pi}{4}\right) = -\sqrt{2} + \sqrt{2} = 0$

47. $\int_{\sqrt{2}/3}^{2/3} \frac{1}{|y|\sqrt{9y^2-1}}\, dy = \int_{\sqrt{2}/3}^{2/3} \frac{3}{|3y|\sqrt{(3y)^2-1}}\, dy = \int_{\sqrt{2}}^{2} \frac{1}{|u|\sqrt{u^2-1}}\, du$, where $u = 3y$, $du = 3\, dy$;
$y = \frac{\sqrt{2}}{3} \Rightarrow u = \sqrt{2}$, $y = \frac{2}{3} \Rightarrow u = 2$

$= \left[\sec^{-1} u\right]_{\sqrt{2}}^{2} = \left[\sec^{-1} 2 - \sec^{-1} \sqrt{2}\right] = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

48. Let $u = 1 - x^2 \Rightarrow du = -2x\, dx \Rightarrow -du = 2x\, dx$; $x = -1 \Rightarrow u = 0$, $x = 1 \Rightarrow u = 0$
$\int_{-1}^{1} 2x \sin\left(1-x^2\right) dx = \int_{0}^{0} -\sin u\, du = 0$

49. Let $u = 1 + 3 \sin^2 x \Rightarrow du = 6 \sin x \cos x\, dx \Rightarrow \frac{1}{2}\, du = 3 \sin x \cos x\, dx$; $x = 0 \Rightarrow u = 1$, $x = \frac{\pi}{2}$
$\Rightarrow u = 1 + 3 \sin^2 \frac{\pi}{2} = 4$

$\int_{0}^{\pi/2} \frac{3 \sin x \cos x}{\sqrt{1+3 \sin^2 x}}\, dx = \int_{1}^{4} \frac{1}{\sqrt{u}}\left(\frac{1}{2}\, du\right) = \int_{1}^{4} \frac{1}{2}u^{-1/2}\, du = \left[\frac{1}{2}\left(\frac{u^{1/2}}{\frac{1}{2}}\right)\right]_{1}^{4} = \left[u^{1/2}\right]_{1}^{4} = 4^{1/2} - 1^{1/2} = 1$

50. $\int_{-2}^{-1} \frac{2}{v^2+4v+5}\,dv = 2\int_{-2}^{-1} \frac{1}{1+(v^2+4v+4)}\,dv = 2\int_{-2}^{-1} \frac{1}{1+(v+2)^2}\,dv = 2\int_{0}^{1} \frac{1}{1+u^2}\,du,$

where $u = v+2$, $du = dv$; $v=-2 \Rightarrow u=0$, $v=-1 \Rightarrow u=1$

$= 2\left[\tan^{-1} u\right]_0^1 = 2\left(\tan^{-1} 1 - \tan^{-1} 0\right) = 2\left(\frac{\pi}{4}-0\right) = \frac{\pi}{2}$

51. $\int_{-1}^{1} \frac{3}{4v^2+4v+4}\,dv = \frac{3}{4}\int_{-1}^{1} \frac{1}{\frac{3}{4}+\left(v^2+v+\frac{1}{4}\right)}\,dv = \frac{3}{4}\int_{-1}^{1} \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2+\left(v+\frac{1}{2}\right)^2}\,dv = \frac{3}{4}\int_{-1/2}^{3/2} \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2+u^2}\,du$

where $u = v+\frac{1}{2}$, $du = dv$; $v=-1 \Rightarrow u=-\frac{1}{2}$, $v=1 \Rightarrow u=\frac{3}{2}$

$= \frac{3}{4}\left[\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2u}{\sqrt{3}}\right)\right]_{-1/2}^{3/2} = \frac{\sqrt{3}}{2}\left[\tan^{-1}\sqrt{3} - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right] = \frac{\sqrt{3}}{2}\left[\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right] = \frac{\sqrt{3}}{2}\left(\frac{2\pi}{6}+\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\cdot\frac{\pi}{2}$

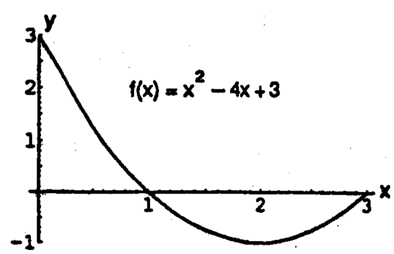
$= \frac{\sqrt{3}\pi}{4}$

52. Let $u = \sin\sqrt{t} \Rightarrow du = \left(\cos\sqrt{t}\right)\left(\frac{1}{2}t^{-1/2}\right)dt = \frac{\cos\sqrt{t}}{2\sqrt{t}}\,dt \Rightarrow 2\,du = \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt$; $t = \frac{\pi^2}{36} \Rightarrow u = \sin\frac{\pi}{6} = \frac{1}{2}$,

$t = \frac{\pi^2}{4} \Rightarrow u = \sin\frac{\pi}{2} = 1$

$\int_{\pi^2/36}^{\pi^2/4} \frac{\cos\sqrt{t}}{\sqrt{t\sin\sqrt{t}}}\,dt = \int_{1/2}^{1} \frac{1}{\sqrt{u}}(2\,du) = 2\int_{1/2}^{1} u^{-1/2}\,du = \left[4\sqrt{u}\right]_{1/2}^{1} = 4\sqrt{1} - 4\sqrt{\frac{1}{2}} = 2\left(2-\sqrt{2}\right)$

53. $x^2-4x+3=0 \Rightarrow (x-3)(x-1)=0 \Rightarrow x=3$ or $x=1$;



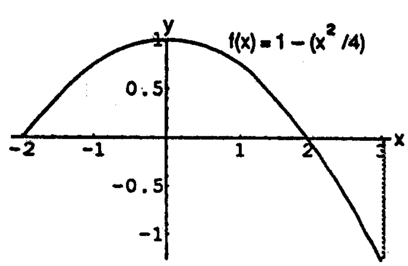
Area $= \int_0^1 \left(x^2-4x+3\right)dx - \int_1^3 \left(x^2-4x+3\right)dx$

$= \left[\frac{x^3}{3}-2x^2+3x\right]_0^1 - \left[\frac{x^3}{3}-2x^2+3x\right]_1^3$

$= \left[\left(\frac{1^3}{3}-2(1)^2+3(1)\right)-0\right]$

$-\left[\left(\frac{3^3}{3}-2(3)^2+3(3)\right)-\left(\frac{1^3}{3}-2(1)^2+3(1)\right)\right] = \left(\frac{1}{3}+1\right)-\left[0-\left(\frac{1}{3}+1\right)\right] = \frac{8}{3}$

54. $1-\frac{x^2}{4}=0 \Rightarrow 4-x^2-0 \Rightarrow x=\pm 2$;



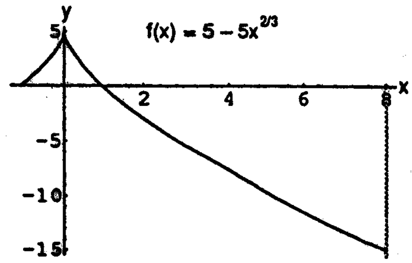
Area $= \int_{-2}^{2} \left(1-\frac{x^2}{4}\right)dx - \int_{2}^{3} \left(1-\frac{x^2}{4}\right)dx$

$= \left[x-\frac{x^3}{12}\right]_{-2}^{2} - \left[x-\frac{x^3}{12}\right]_{2}^{3}$

$$=\left[\left(2-\frac{2^3}{12}\right)-\left(-2-\frac{(-2)^3}{12}\right)\right]$$
$$-\left[\left(3-\frac{3^3}{12}\right)-\left(2-\frac{2^3}{12}\right)\right]=\left[\frac{4}{3}-\left(-\frac{4}{3}\right)\right]-\left(\frac{3}{4}-\frac{4}{3}\right)=\frac{13}{4}$$

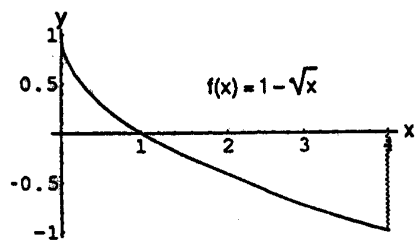
55. $5-5x^{2/3}=0 \Rightarrow 1-x^{2/3}=0 \Rightarrow x=\pm 1$;

$$\text{Area}=\int_{-1}^{1}\left(5-5x^{2/3}\right)dx-\int_{1}^{8}\left(5-5x^{2/3}\right)dx$$
$$=\left[5x-3x^{5/3}\right]_{-1}^{1}-\left[5x-3x^{5/3}\right]_{1}^{8}$$
$$=\left[\left(5(1)-3(1)^{5/3}\right)-\left(5(-1)-3(-1)^{5/3}\right)\right]$$
$$-\left[\left(5(8)-3(8)^{5/3}\right)-\left(5(1)-3(1)^{5/3}\right)\right]=[2-(-2)]-[(40-96)-2]=62$$



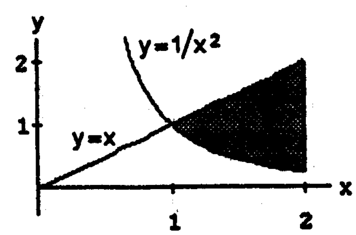
56. $1-\sqrt{x}=0 \Rightarrow x=1$;

$$\text{Area}=\int_{0}^{1}\left(1-\sqrt{x}\right)dx-\int_{1}^{4}\left(1-\sqrt{x}\right)dx$$
$$=\left[x-\frac{2}{3}x^{3/2}\right]_{0}^{1}-\left[x-\frac{2}{3}x^{3/2}\right]_{1}^{4}$$
$$=\left[\left(1-\frac{2}{3}(1)^{3/2}\right)-0\right]-\left[\left(4-\frac{2}{3}(4)^{3/2}\right)-\left(1-\frac{2}{3}(1)^{3/2}\right)\right]$$
$$=\frac{1}{3}-\left[\left(4-\frac{16}{3}\right)-\frac{1}{3}\right]=2$$



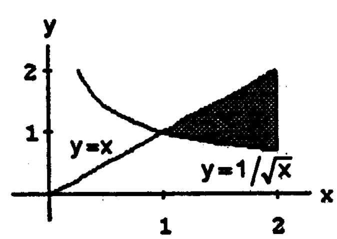
57. $f(x)=x,\ g(x)=\frac{1}{x^2},\ a=1,\ b=2 \Rightarrow A=\int_a^b [f(x)-g(x)]\,dx$

$$=\int_1^2\left(x-\frac{1}{x^2}\right)dx=\left[\frac{x^2}{2}+\frac{1}{x}\right]_1^2=\left(\frac{4}{2}+\frac{1}{2}\right)-\left(\frac{1}{2}+1\right)=1$$



58. $f(x)=x,\ g(x)=\frac{1}{\sqrt{x}},\ a=1,\ b=2 \Rightarrow A=\int_a^b [f(x)-g(x)]\,dx$

$$=\int_1^2\left(x-\frac{1}{\sqrt{x}}\right)dx=\left[\frac{x^2}{2}-2\sqrt{x}\right]_1^2=\left(\frac{4}{2}-2\sqrt{2}\right)-\left(\frac{1}{2}-2\right)$$
$$=\frac{7-4\sqrt{2}}{2}$$



59. $f(x) = (1-\sqrt{x})^2$, $g(x) = 0$, $a = 0$, $b = 1 \Rightarrow A = \int_a^b [f(x) - g(x)]\,dx = \int_0^1 (1-\sqrt{x})^2\,dx = \int_0^1 (1 - 2\sqrt{x} + x)\,dx$

$$= \int_0^1 (1 - 2x^{1/2} + x)\,dx = \left[x - \tfrac{4}{3}x^{3/2} + \tfrac{x^2}{2}\right]_0^1 = 1 - \tfrac{4}{3} + \tfrac{1}{2} = \tfrac{1}{6}(6 - 8 + 3) = \tfrac{1}{6}$$

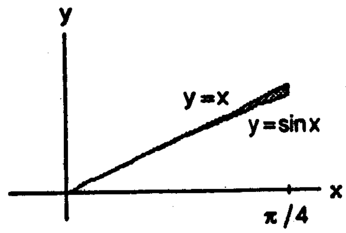
60. $f(x) = (1-x^3)^2$, $g(x) = 0$, $a = 0$, $b = 1 \Rightarrow A = \int_a^b [f(x) - g(x)]\,dx = \int_0^1 (1-x^3)^2\,dx = \int_0^1 (1 - 2x^3 + x^6)\,dx$

$$= \left[x - \tfrac{x^4}{2} + \tfrac{x^7}{7}\right]_0^1 = 1 - \tfrac{1}{2} + \tfrac{1}{7} = \tfrac{9}{14}$$

61. $f(x) = x$, $g(x) = \sin x$, $a = 0$, $b = \frac{\pi}{4}$

$$\Rightarrow A = \int_a^b [f(x) - g(x)]\,dx = \int_0^{\pi/4} (x - \sin x)\,dx$$

$$= \left[\tfrac{x^2}{2} + \cos x\right]_0^{\pi/4} = \left(\tfrac{\pi^2}{32} + \tfrac{\sqrt{2}}{2}\right) - 1$$

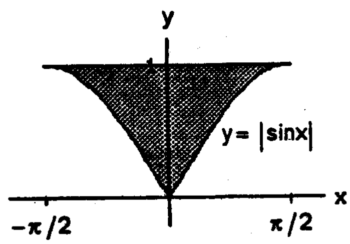


62. $f(x) = 1$, $g(x) = |\sin x|$, $a = -\frac{\pi}{2}$, $b = \frac{\pi}{2}$

$$\Rightarrow A = \int_a^b [f(x) - g(x)]\,dx = \int_{-\pi/2}^{\pi/2} (1 - |\sin x|)\,dx$$

$$= \int_{-\pi/2}^{0} (1 + \sin x)\,dx + \int_0^{\pi/2} (1 - \sin x)\,dx$$

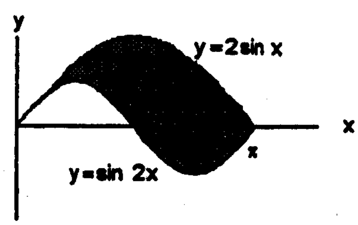
$$= 2\int_0^{\pi/2} (1 - \sin x)\,dx = 2[x + \cos x]_0^{\pi/2} = 2\left(\tfrac{\pi}{2} - 1\right) = \pi - 2$$



63. $a = 0$, $b = \pi$, $f(x) - g(x) = 2\sin x - \sin 2x$

$$\Rightarrow A = \int_0^{\pi} (2\sin x - \sin 2x)\,dx = \left[-2\cos x + \tfrac{\cos 2x}{2}\right]_0^{\pi}$$

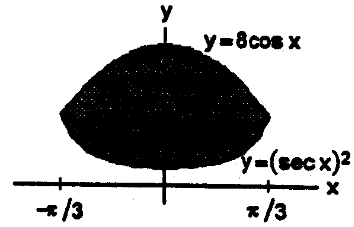
$$= \left[-2\cdot(-1) + \tfrac{1}{2}\right] - \left(-2\cdot 1 + \tfrac{1}{2}\right) = 4$$



64. $a = -\frac{\pi}{3}$, $b = \frac{\pi}{3}$, $f(x) - g(x) = 8\cos x - \sec^2 x$

$\Rightarrow A = \int_{-\pi/3}^{\pi/3} (8\cos x - \sec^2 x)\,dx = [8\sin x - \tan x]_{-\pi/3}^{\pi/3}$

$= \left(8\cdot\frac{\sqrt{3}}{2} - \sqrt{3}\right) - \left(-8\cdot\frac{\sqrt{3}}{2} + \sqrt{3}\right) = 6\sqrt{3}$



65. $f(x) = x^3 - 3x^2 = x^2(x-3) \Rightarrow f'(x) = 3x^2 - 6x = 3x(x-2) \Rightarrow f' = +++\underset{0}{|}----\underset{2}{|}+++$

$\Rightarrow f(0) = 0$ is a maximum and $f(2) = -4$ is a minimum. Then $A = -\int_0^3 (x^3 - 3x^2)\,dx = -\left[\frac{x^4}{4} - x^3\right]_0^3$

$= -\left(\frac{81}{4} - 27\right) = \frac{27}{4}$

66. $A = \int_0^1 \left(1 - x^{1/3}\right)^3 dx = \int_0^1 \left(1 - 3x^{1/3} + 3x^{2/3} - x\right) dx = \left[x - \frac{9}{4}x^{4/3} + \frac{9}{5}x^{5/3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{20}$

67. $y = \int \frac{x^2+1}{x^2}\,dx = \int \left(1 + x^{-2}\right) dx = x - x^{-1} + C = x - \frac{1}{x} + C$; $y = -1$ when $x = 1 \Rightarrow 1 - \frac{1}{1} + C = -1$

$\Rightarrow C = -1 \Rightarrow y = x - \frac{1}{x} - 1$

68. $\frac{dy}{dx} = e^{-x-y-2} \Rightarrow \frac{dy}{dx} = e^{-x-2}\cdot e^{-y} \Rightarrow e^y\,dy = e^{-x-2}\,dx = \int e^y\,dy = \int e^{-x-2}\,dx \Rightarrow e^y = -e^{-x-2} + C$; $x = 0$

and $y = -2 \Rightarrow e^{-2} = -e^{-2} + C \Rightarrow C = 2e^{-2} \Rightarrow e^y = -e^{-x-2} + 2e^{-2} \Rightarrow \ln\left(e^y\right) = \ln\left(-e^{-x-2} + 2e^{-2}\right)$

$\Rightarrow y = \ln\left(-e^{-x-2} + 2e^{-2}\right)$

69. $\frac{dr}{dt} = \int \left(15\sqrt{t} + \frac{3}{\sqrt{t}}\right) dt = \int \left(15t^{1/2} + 3t^{-1/2}\right) dt = 10t^{3/2} + 6t^{1/2} + C$; $\frac{dr}{dt} = 8$ when $t = 1$

$\Rightarrow 10(1)^{3/2} + 6(1)^{1/2} + C = 8 \Rightarrow C = -8$. Thus $\frac{dr}{dt} = 10t^{3/2} + 6t^{1/2} - 8 \Rightarrow r = \int \left(10t^{3/2} + 6t^{1/2} - 8\right) dt$

$= 4t^{5/2} + 4t^{3/2} - 8t + C$; $r = 0$ when $t = 1 \Rightarrow 4(1)^{5/2} + 4(1)^{3/2} - 8(1) + C_1 = 0 \Rightarrow C_1 = 0$. Therefore,

$r = 4t^{5/2} + 4t^{3/2} - 8t$

70. $\frac{d^2r}{dt^2} = \int -\cos t\,dt = -\sin t + C$; $r'' = 0$ when $t = 0 \Rightarrow -\sin 0 + C = 0 \Rightarrow C = 0$. Thus, $\frac{d^2r}{dt^2} = -\sin t$

$\Rightarrow \frac{dr}{dt} = \int -\sin t\,dt = \cos t + C_1$; $r' = 0$ when $t = 0 \Rightarrow 1 + C_1 = 0 \Rightarrow C_1 = -1$. Then $\frac{dr}{dt} = \cos t - 1$

$\Rightarrow r = \int (\cos t - 1)\,dt = \sin t - t + C_2$; $r = -1$ when $t = 0 \Rightarrow 0 - 0 + C_2 = -1$. Therefore, $r = \sin t - t - 1$

71. $y = x^2 + \int_1^x \frac{1}{t}\,dt \Rightarrow \frac{dy}{dx} = 2x + \frac{1}{x} \Rightarrow \frac{d^2y}{dx^2} = 2 - \frac{1}{x^2}$; $y(1) = 1 + \int_1^1 \frac{1}{t}\,dt = 1$ and $y'(1) = 2 + 1 = 3$

72. $y = \int_0^x \left(1 + 2\sqrt{\sec t}\right) dt \Rightarrow \frac{dy}{dx} = 1 + 2\sqrt{\sec x} \Rightarrow \frac{d^2y}{dx^2} = 2\left(\frac{1}{2}\right)(\sec x)^{-1/2}(\sec x \tan x) = \sqrt{\sec x}\,(\tan x)$;

$x = 0 \Rightarrow y = \int_0^0 \left(1 + 2\sqrt{\sec t}\right) dt = 0$ and $x = 0 \Rightarrow \frac{dy}{dx} = 1 + 2\sqrt{\sec 0} = 3$

73. $y = \int_5^x \frac{\sin t}{t}\,dt - 3 \Rightarrow \frac{dy}{dx} = \frac{\sin x}{x}$; $x = 5 \Rightarrow y = \int_5^5 \frac{\sin t}{t}\,dt - 3 = -3$

74. $y = \int_{-1}^x \sqrt{2 - \sin^2 t}\,dt + 2$ so that $\frac{dy}{dx} = \sqrt{2 - \sin^2 x}$; $x = -1 \Rightarrow y = \int_{-1}^{-1} \sqrt{2 - \sin^2 t}\,dt + 2 = 2$

75. (a) $\text{av}(f) = \frac{1}{1-(-1)} \int_{-1}^1 (mx + b)\,dx = \frac{1}{2}\left[\frac{mx^2}{2} + bx\right]_{-1}^1 = \frac{1}{2}\left[\left(\frac{m(1)^2}{2} + b(1)\right) - \left(\frac{m(-1)^2}{2} + b(-1)\right)\right] = \frac{1}{2}(2b) = b$

(b) $\text{av}(f) = \frac{1}{k-(-k)} \int_{-k}^k (mx + b)\,dx = \frac{1}{2k}\left[\frac{mx^2}{2} + bx\right]_{-k}^k = \frac{1}{2k}\left[\left(\frac{m(k)^2}{2} + b(k)\right) - \left(\frac{m(-k)^2}{2} + b(-k)\right)\right]$

$= \frac{1}{2k}(2bk) = b$

76. (a) $y_{av} = \frac{1}{3-0} \int_0^3 \sqrt{3x}\,dx = \frac{1}{3} \int_0^3 \sqrt{3}\,x^{1/2}\,dx = \frac{\sqrt{3}}{3}\left[\frac{2}{3}x^{3/2}\right]_0^3 = \frac{\sqrt{3}}{3}\left[\frac{2}{3}(3)^{3/2} - \frac{2}{3}(0)^{3/2}\right] = \frac{\sqrt{3}}{3}\left(2\sqrt{3}\right) = 2$

(b) $y_{av} = \frac{1}{a-0} \int_0^a \sqrt{ax}\,dx = \frac{1}{a} \int_0^a \sqrt{a}\,x^{1/2}\,dx = \frac{\sqrt{a}}{a}\left[\frac{2}{3}x^{3/2}\right]_0^a = \frac{\sqrt{a}}{a}\left(\frac{2}{3}(a)^{3/2} - \frac{2}{3}(0)^{3/2}\right) = \frac{\sqrt{a}}{a}\left(\frac{2}{3}a\sqrt{a}\right) = \frac{2}{3}a$

77. $f'_{av} = \frac{1}{b-a} \int_a^b f'(x)\,dx = \frac{1}{b-a}[f(x)]_a^b = \frac{1}{b-a}[f(b) - f(a)] = \frac{f(b) - f(a)}{b-a}$ so the average value of f' over $[a, b]$ is the slope of the secant line joining the points $(a, f(a))$ and $(b, f(b))$.

78. Yes, because the average value of f on $[a, b]$ is $\frac{1}{b-a} \int_a^b f(x)\,dx$. If the length of the interval is 2, then $b - a = 2$ and the average value of the function is $\frac{1}{2} \int_a^b f(x)\,dx$.

79. $\frac{dy}{dx} = \sqrt{2 + \cos^3 x}$

80. $\frac{dy}{dx} = \sqrt{2+\cos^3(7x^2)} \cdot \frac{d}{dx}(7x^2) = 14x\sqrt{2+\cos^3(7x^2)}$

81. $\frac{dy}{dx} = \frac{d}{dx}\left(-\int_1^x \frac{6}{3+t^4}\,dt\right) = -\frac{6}{3+x^4}$

82. $\frac{dy}{dx} = \frac{d}{dx}\left(\int_0^{e^{2x}} \frac{1}{1+t^2}\,dt - \int_0^x \frac{1}{1+t^2}\,dt\right) = \frac{1}{1+\left(e^{2x}\right)^2}\left(2e^{2x}\right) - \frac{1}{1+x^2} = \frac{2e^{2x}}{1+e^{4x}} - \frac{1}{1+x^2}$

83. $h = \frac{b-a}{n} = \frac{\pi - 0}{6} = \frac{\pi}{6} \Rightarrow \frac{h}{2} = \frac{\pi}{12}$;

$\sum_{i=0}^{6} mf(x_i) = 12 \Rightarrow T = \left(\frac{\pi}{12}\right)(12) = \pi$;

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	$\pi/6$	1/2	2	1
x_2	$\pi/3$	3/2	2	3
x_3	$\pi/2$	2	2	4
x_4	$2\pi/3$	3/2	2	3
x_5	$5\pi/6$	1/2	2	1
x_6	π	0	1	0

$\sum_{i=0}^{6} mf(x_i) = 18$ and $\frac{h}{3} = \frac{\pi}{18} \Rightarrow S = \left(\frac{\pi}{18}\right)(18) = \pi$.

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	$\pi/6$	1/2	4	2
x_2	$\pi/3$	3/2	2	3
x_3	$\pi/2$	2	4	8
x_4	$2\pi/3$	3/2	2	3
x_5	$5\pi/6$	1/2	4	2
x_6	π	0	1	0

84. (a) Each interval is 5 min $= \frac{1}{12}$ hour.

$\frac{1}{24}[2.5 + 2(2.4) + 2(2.3) + \ldots + 2(2.4) + 2.3] = \frac{29}{12} \approx 2.42$ gal

(b) $(60\text{ mph})\left(\frac{12}{29}\text{ hours/gal}\right) \approx 24.83$ mi/gal

85. $y_{av} = \frac{1}{365-0}\int_0^{365}\left[37\sin\left(\frac{2\pi}{365}(x-101)\right)+25\right]dx = \frac{1}{365}\left[-37\left(\frac{365}{2\pi}\cos\left(\frac{2\pi}{365}(x-101)\right)+25x\right)\right]_0^{365}$

$= \frac{1}{365}\left[\left(-37\left(\frac{365}{2\pi}\right)\cos\left[\frac{2\pi}{365}(365-101)\right]+25(365)\right)-\left(-37\left(\frac{365}{2\pi}\right)\cos\left[\frac{2\pi}{365}(0-101)\right]+25(0)\right)\right]$

$= -\frac{37}{2\pi}\cos\left(\frac{2\pi}{365}(264)\right)+25+\frac{37}{2\pi}\cos\left(\frac{2\pi}{365}(-101)\right) = -\frac{37}{2\pi}\left(\cos\left(\frac{2\pi}{365}(264)\right)-\cos\left(\frac{2\pi}{365}(-101)\right)\right)+25$

$\approx -\frac{37}{2\pi}(0.16705 - 0.16705) + 25 = 25^\circ F$

86. $av(C_v) = \frac{1}{675-20}\int_{20}^{675}\left[8.27 + 10^{-5}(26T - 1.87T^2)\right] dT = \frac{1}{655}\left[8.27T + \frac{13}{10^5}T^2 - \frac{0.62333}{10^5}T^3\right]_{20}^{675}$

$\approx \frac{1}{655}[(5582.25 + 59.23125 - 1917.03194) - (165.4 + 0.052 - 0.04987)] \approx 5.434;$

$8.27 + 10^{-5}(26T - 1.87T^2) = 5.434 \Rightarrow 1.87T^2 - 26T - 283{,}600 = 0 \Rightarrow T \approx \frac{26 + \sqrt{676 + 4(1.87)(283{,}600)}}{2(1.87)}$

$\approx 396.45^\circ C$

87. Using the trapezoidal rule, $h = 15 \Rightarrow \frac{h}{2} = 7.5$;

$\sum mf(x_i) = 794.8 \Rightarrow \text{Area} \approx (794.8)(7.5) = 5961 \text{ ft}^2$;

The cost is Area $\cdot$ ($\$2.10/\text{ft}^2$) $\approx (5961 \text{ ft}^2)(\$2.10/\text{ft}^2)$

$= \$12{,}518.10 \Rightarrow$ the job cannot be done for \$11,000.

	x_i	$f(x_i)$	m	$mf(x_i)$
x_0	0	0	1	0
x_1	15	36	2	72
x_2	30	54	2	108
x_3	45	51	2	102
x_4	60	49.5	2	99
x_5	75	54	2	108
x_6	90	64.4	2	128.8
x_7	105	67.5	2	135
x_8	120	42	1	42

88. (a) Upper estimate:
$3(5.30 + 5.25 + 5.04 + \ldots + 1.11) = 103.05$ ft
Lower estimate:
$3(5.25 + 5.04 + 4.71 + \ldots + 0) = 87.15$ ft

(b) $\frac{3}{2}[5.30 + 2(5.25) + 2(5.04) + \ldots + 2(1.11) + 0] = 95.1$ ft

89. Yes. The function f, being differentiable on [a, b], is then continuous on [a, b]. The Fundamental Theorem of Calculus says that every continuous function on [a, b] is the derivative of a function on [a, b].

90. The second part of the Fundamental Theorem of Calculus states that if F(x) is an antiderivative of f(x) on [a, b], then $\int_a^b f(x)\,dx = F(b) - F(a)$. In particular, if F(x) is an antiderivative of $\sqrt{1+x^4}$ on [0, 1], then

$\int_0^1 \sqrt{1+x^4}\,dx = F(1) - F(0)$.

91. $y(x) = \int_5^x \frac{\sin t}{t}\,dt + 3$

92. $y' = \cos x - \frac{d}{dx}\int_{\pi}^{x} \cos 2t\, dt = \cos x - \cos 2x \Rightarrow y'' = -\sin x + 2\sin 2x$

Thus, y satisfies condition i.

$y(\pi) = \sin \pi + \int_{\pi}^{\pi} \cos 2t\, dt + 1 = 1$ and $y'(\pi) = \cos \pi - \cos 2\pi = -2$

Thus, y satisfies condition ii.

93. (a) $g(1) = \int_{1}^{1} f(t)\, dt = 0$

(b) $g(3) = \int_{1}^{3} f(t)\, dt = -\frac{1}{2}(2)(1) = -1$

(c) $g(-1) = \int_{1}^{-1} f(t)\, dt = -\int_{-1}^{1} f(t)\, dt = -\frac{1}{4}\pi(2)^2 = -\pi$

(d) $g'(x) = f(x)$; Since $f(x) > 0$ for $-3 < x < 1$ and $f(x) < 0$ for $1 < x < 3$, g(x) has a relative maximum at $x = 1$.

(e) $g'(-1) = f(-1) = 2$
The equation of the tangent line is $y - (-\pi) = 2(x+1)$ or $y = 2x + 2 - \pi$

(f) $g''(x) = f'(x)$, $f'(x) = 0$ at $x = -1$ and f' is not defined at $x = 2$. The inflection points are at $x = -1$ and $x = 2$. Note that $g''(x) = f'(x)$ is undefined at $x = 1$ as well, but since $g''(x) = f'(x)$ is negative on both sides of $x = 1$, $x = 1$ is not an inflection point.

(g) Note that the absolute maximum is $g(1) = 0$ and the absolute minimum is

$g(-3) = \int_{1}^{-3} f(t)\, dt = -\int_{-3}^{1} f(t)\, dt = -\frac{1}{2}\pi(2)^2 = -2\pi.$

The range of g is $[-2\pi, 0]$.

94. (a) Before the chute opens for A, $a = -32$ ft/sec^2. Since the helicopter is hovering, $v_0 = 0$ ft/sec $\Rightarrow v = \int -32\, dt = -32t + v_0 = -32t$. Then $s_0 = 6400$ ft $\Rightarrow s = \int -32t\, dt = -16t^2 + s_0 = -16t^2 + 6400$.
At $t = 4$ sec, $s = -16(4)^2 + 6400 = 6144$ ft when A's chute opens;

(b) For B, $s_0 = 7000$ ft, $v_0 = 0$, $a = -32$ ft/sec^2 $\Rightarrow v = \int -32\, dt = -32t + v_0 = -32t \Rightarrow s = \int -32t\, dt$ $= -16t^2 + s_0 = -16t^2 + 7000$. At $t = 13$ sec, $s = -16(13)^2 + 7000 = 4296$ ft when B's chute opens;

(c) After the chutes open, $v = -16$ ft/sec $\Rightarrow s = \int -16\, dt = -16t + s_0$. For A, $s_0 = 6144$ ft and for B, $s_0 = 4296$ ft. Therefore, for A, $s = -16t + 6144$ and for B, $s = -16t + 4296$. When they hit the ground,

$s = 0 \Rightarrow$ for A, $0 = -16t + 6144 \Rightarrow t = \frac{6144}{16} = 384$ seconds, and for B, $0 = -16t + 4296 \Rightarrow t = \frac{4296}{16}$ $= 268.5$ seconds to hit the ground after the chutes open. Since B's chute opens 54 seconds after A's opens $\Rightarrow$ B hits the ground first.

95. $av(I) = \frac{1}{30}\int_0^{30} (1200 - 40t)\, dt = \frac{1}{30}\left[1200t - 20t^2\right]_0^{30} = \frac{1}{30}\left[\left((1200(30) - 20(30)^2\right) - \left(1200(0) - 20(0)^2\right)\right]$
$= \frac{1}{30}(18{,}000) = 600$; Average Daily Holding Cost $= (600)(\$0.03) = \$18$

96. $av(I) = \frac{1}{14}\int_0^{14} (600 + 600t)\, dt = \frac{1}{14}\left[600t + 300t^2\right]_0^{14} = \frac{1}{14}\left[600(14) + 300(14)^2 - 0\right] = 4800$; Average Daily Holding Cost $= (4800)(\$0.04) = \$192$

97. $av(I) = \frac{1}{30}\int_0^{30} \left(450 - \frac{t^2}{2}\right) dt = \frac{1}{30}\left[450t - \frac{t^3}{6}\right]_0^{30} = \frac{1}{30}\left[450(30) - \frac{30^3}{6} - 0\right] = 300$; Average Daily Holding Cost $= (300)(\$0.02) = \$6$

98. $av(I) = \frac{1}{60}\int_0^{60} \left(600 - 20\sqrt{15t}\right) dt = \frac{1}{60}\int_0^{60} \left(600 - 20\sqrt{15}\, t^{1/2}\right) dt = \frac{1}{60}\left[600t - 20\sqrt{15}\left(\frac{2}{3}\right)t^{3/2}\right]_0^{60}$
$= \frac{1}{60}\left[600(60) - \frac{40\sqrt{15}}{3}(60)^{3/2} - 0\right] = \frac{1}{60}\left(36{,}000 - \left(\frac{320}{3}\right)15^2\right) = 200$; Average Daily Holding Cost
$= (200)(\$0.005) = \$1.00$

CHAPTER 4 ADDITIONAL EXERCISES–THEORY, EXAMPLES, APPLICATIONS

1. (a) Yes, because $\int_0^1 f(x)\, dx = \frac{1}{7}\int_0^1 7f(x)\, dx = \frac{1}{7}(7) = 1$

   (b) No. For example, $\int_0^1 8x\, dx = \left[4x^2\right]_0^1 = 4$, but $\int_0^1 \sqrt{8x}\, dx = \left[2\sqrt{2}\left(\frac{x^{3/2}}{\frac{3}{2}}\right)\right]_0^1 = \frac{4\sqrt{2}}{3}\left(1^{3/2} - 0^{3/2}\right)$
   $= \frac{4\sqrt{2}}{3} \neq \sqrt{4}$

2. (a) True: $\int_5^2 f(x)\, dx = -\int_2^5 f(x)\, dx = -3$

   (b) True: $\int_{-2}^5 [f(x) + g(x)]\, dx = \int_{-2}^5 f(x)\, dx + \int_{-2}^5 g(x)\, dx = \int_{-2}^2 f(x)\, dx + \int_2^5 f(x)\, dx + \int_{-2}^5 g(x)\, dx$
   $= 4 + 3 + 2 = 9$

(c) False: $\int_{-2}^{5} f(x)\,dx = 4+3 = 7 > 2 = \int_{-2}^{5} g(x)\,dx \Rightarrow \int_{-2}^{5} [f(x)-g(x)]\,dx > 0 \Rightarrow \int_{-2}^{5} [g(x)-f(x)]\,dx < 0.$

On the other hand, $f(x) \le g(x) \Rightarrow [g(x)-f(x)] \ge 0 \Rightarrow \int_{-2}^{5} [g(x)-f(x)]\,dx \ge 0$ which would be a contradiction.

3. $y = \frac{1}{a}\int_0^x f(t)\sin a(x-t)\,dt = \frac{1}{a}\int_0^x f(t)\sin ax\cos at\,dt - \frac{1}{a}\int_0^x f(t)\cos ax\sin at\,dt$

$= \frac{\sin ax}{a}\int_0^x f(t)\cos at\,dt - \frac{\cos ax}{a}\int_0^x f(t)\sin at\,dt \Rightarrow \frac{dy}{dx} = \cos ax\int_0^x f(t)\cos at\,dt$

$+ \frac{\sin ax}{a}\left(\frac{d}{dx}\int_0^x f(t)\cos at\,dt\right) + \sin ax\int_0^x f(t)\sin at\,dt - \frac{\cos ax}{a}\left(\frac{d}{dx}\int_0^x f(t)\sin at\,dt\right)$

$= \cos ax\int_0^x f(t)\cos at\,dt + \frac{\sin ax}{a}(f(x)\cos ax) + \sin ax\int_0^x f(t)\sin at\,dt - \frac{\cos ax}{a}(f(x)\sin ax)$

$\Rightarrow \frac{dy}{dx} = \cos ax\int_0^x f(t)\cos at\,dt + \sin ax\int_0^x f(t)\sin at\,dt.$ Next,

$\frac{d^2y}{dx^2} = -a\sin ax\int_0^x f(t)\cos at\,dt + (\cos ax)\left(\frac{d}{dx}\int_0^x f(t)\cos at\,dt\right) + a\cos ax\int_0^x f(t)\sin at\,dt$

$+ (\sin ax)\left(\frac{d}{dx}\int_0^x f(t)\sin at\,dt\right) = -a\sin ax\int_0^x f(t)\cos at\,dt + (\cos ax)f(x)\cos ax$

$+ a\cos ax\int_0^x f(t)\sin at\,dt + (\sin ax)f(x)\sin ax = -a\sin ax\int_0^x f(t)\cos at\,dt + a\cos ax\int_0^x f(t)\sin at\,dt + f(x).$

Therefore, $y'' + a^2y = a\cos ax\int_0^x f(t)\sin at\,dt - a\sin ax\int_0^x f(t)\cos at\,dt + f(x)$

$+ a^2\left(\frac{\sin ax}{a}\int_0^x f(t)\cos at\,dt - \frac{\cos ax}{a}\int_0^x f(t)\sin at\,dt\right) = f(x).$ Note also that $y'(0) = y(0) = 0.$

4. $x = \int_0^y \frac{1}{\sqrt{1+4t^2}}\,dt \Rightarrow \frac{d}{dx}(x) = \frac{d}{dx}\int_0^y \frac{1}{\sqrt{1+4t^2}}\,dt = \frac{d}{dy}\left[\int_0^y \frac{1}{\sqrt{1+4t^2}}\,dt\right]\left(\frac{dy}{dx}\right)$ from the chain rule

$\Rightarrow 1 = \frac{1}{\sqrt{1+4y^2}}\left(\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx} = \sqrt{1+4y^2}.$ Then $\frac{d^2y}{dx^2} = \frac{d}{dx}(\sqrt{1+4y^2}) = \frac{d}{dy}(\sqrt{1+4y^2})\left(\frac{dy}{dx}\right)$

$= \frac{1}{2}\left(1+4y^2\right)^{-1/2}(8y)\left(\frac{dy}{dx}\right) = \frac{4y\left(\frac{dy}{dx}\right)}{\sqrt{1+4y^2}} = \frac{4y\left(\sqrt{1+4y^2}\right)}{\sqrt{1+4y^2}} = 4y$. Thus $\frac{d^2y}{dx^2} = 4y$, and the constant of proportionality is 4.

5. (a) $\int_0^{x^2} f(t)\,dt = x\cos\pi x \Rightarrow \frac{d}{dx}\int_0^{x^2} f(t)\,dt = \cos\pi x - \pi x\sin\pi x \Rightarrow f\left(x^2\right)(2x) = \cos\pi x - \pi x\sin\pi x$

$\Rightarrow f\left(x^2\right) = \frac{\cos\pi x - \pi x\sin\pi x}{2x}$. Thus, $x = 2 \Rightarrow f(4) = \frac{\cos 2\pi - 2\pi\sin 2\pi}{4} = \frac{1}{4}$

(b) $\int_0^{f(x)} t^2\,dt = \left[\frac{t^3}{3}\right]_0^{f(x)} = \frac{1}{3}(f(x))^3 \Rightarrow \frac{1}{3}(f(x))^3 = x\cos\pi x \Rightarrow (f(x))^3 = 3x\cos\pi x \Rightarrow f(x) = \sqrt[3]{3x\cos\pi x}$

$\Rightarrow f(4) = \sqrt[3]{3(4)\cos 4\pi} = \sqrt[3]{12}$

6. $\int_0^a f(x)\,dx = \frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$ and let $F(a) = \int_0^a f(t)\,dt \Rightarrow f(a) = F'(a)$. Now $F(a) = \frac{a^2}{2} + \frac{a}{2}\sin a + \frac{\pi}{2}\cos a$

$\Rightarrow f(a) = F'(a) = a + \frac{1}{2}\sin a + \frac{a}{2}\cos a - \frac{\pi}{2}\sin a \Rightarrow f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \frac{1}{2}\sin\frac{\pi}{2} + \frac{\left(\frac{\pi}{2}\right)}{2}\cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2} = \frac{\pi}{2} + \frac{1}{2} - \frac{\pi}{2} = \frac{1}{2}$

7. $\int_1^b f(x)\,dx = \sqrt{b^2+1} - \sqrt{2} \Rightarrow f(b) = \frac{d}{db}\int_1^b f(x)\,dx = \frac{1}{2}\left(b^2+1\right)^{-1/2}(2b) = \frac{b}{\sqrt{b^2+1}} \Rightarrow f(x) = \frac{x}{\sqrt{x^2+1}}$

8. The derivative of the left side of the equation is: $\frac{d}{dx}\left[\int_0^x\left[\int_0^u f(t)\,dt\right]du\right] = \int_0^x f(t)\,dt$; the derivative of the right side of the equation is: $\frac{d}{dx}\left[\int_0^x f(u)(x-u)\,du\right] = \frac{d}{dx}\int_0^x f(u)\,x\,du - \frac{d}{dx}\int_0^x u\,f(u)\,du$

$= \frac{d}{dx}\left[x\int_0^x f(u)\,du\right] - \frac{d}{dx}\int_0^x u\,f(u)\,du = \int_0^x f(u)\,du + x\left[\frac{d}{dx}\int_0^x f(u)\,du\right] - xf(x) = \int_0^x f(u)\,du + xf(x) - xf(x)$

$= \int_0^x f(u)\,du$. Since each side has the same derivative, they differ by a constant, and since both sides equal 0 when $x = 0$, the constant must be 0. Therefore, $\int_0^x\left[\int_0^u f(t)\,dt\right]du = \int_0^x f(u)(x-u)\,du$.

9. $\frac{dy}{dx} = 3x^2 + 2 \Rightarrow y = \int\left(3x^2+2\right)dx = x^3 + 2x + C$. Then $(1,-1)$ on the curve $\Rightarrow 1^3 + 2(1) + C = -1 \Rightarrow C = -4$

$\Rightarrow y = x^3 + 2x - 4$

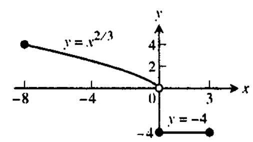
10. The acceleration due to gravity downward is $-32\ \text{ft/sec}^2 \Rightarrow v = \int -32\ dt = -32t + v_0$, where v_0 is the initial velocity $\Rightarrow v = -32t + 32 \Rightarrow s = \int (-32t + 32)\ dt = -16t^2 + 32t + C$. If the release point is $s = 0$, then $C = 0$ $\Rightarrow s = -16t^2 + 32t$. Then $s = 17 \Rightarrow 17 = -16t^2 + 32t \Rightarrow 16t^2 - 32t + 17 = 0$. The discriminant of this quadratic equation is -64 which says there is no real time when $s = 17$ ft. You had better duck.

11. $\int_{-8}^{3} f(x)\ dx = \int_{-8}^{0} x^{2/3}\ dx + \int_{0}^{3} -4\ dx$

$= \left[\frac{3}{5}x^{5/3}\right]_{-8}^{0} + [-4x]_0^3$

$= \left(0 - \frac{3}{5}(-8)^{5/3}\right) + (-4(3) - 0) = \frac{96}{5} - 12$

$= \frac{36}{5}$

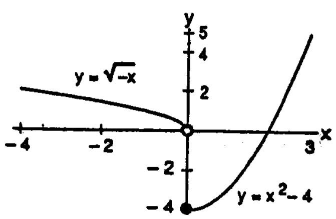


12. $\int_{-4}^{3} f(x)\ dx = \int_{-4}^{0} \sqrt{-x}\ dx + \int_{0}^{3} (x^2 - 4)\ dx$

$= \left[-\frac{2}{3}(-x)^{3/2}\right]_{-4}^{0} + \left[\frac{x^3}{3} - 4x\right]_0^3$

$= \left[0 - \left(-\frac{2}{3}(4)^{3/2}\right)\right] + \left(\frac{3^3}{3} - 4(3)\right) - 0$

$= \frac{16}{3} - 3 = \frac{7}{3}$

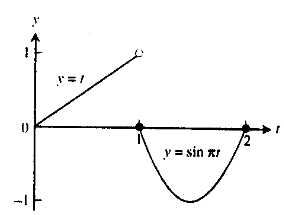


13. $\int_{0}^{2} f(t)\ dt = \int_{0}^{1} t\ dt + \int_{1}^{2} \sin \pi t\ dt$

$= \left[\frac{t^2}{2}\right]_0^1 + \left[-\frac{1}{\pi}\cos \pi t\right]_1^2$

$= \left(\frac{1}{2} - 0\right) + \left[-\frac{1}{\pi}\cos 2\pi - \left(-\frac{1}{\pi}\cos \pi\right)\right]$

$= \frac{1}{2} - \frac{2}{\pi}$



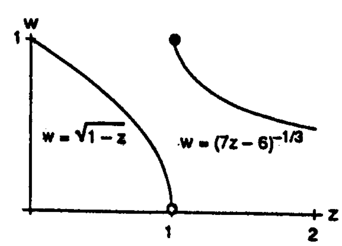
14. $\int_{0}^{2} f(z)\ dz = \int_{0}^{1} \sqrt{1-z}\ dz + \int_{1}^{2} (7z-6)^{-1/3}\ dz$

$= \left[-\frac{2}{3}(1-z)^{3/2}\right]_0^1 + \left[\frac{3}{14}(7z-6)^{2/3}\right]_1^2$

$= \left[-\frac{2}{3}(1-1)^{3/2} - \left(-\frac{2}{3}(1-0)^{3/2}\right)\right]$

$\quad + \left[\frac{3}{14}(7(2)-6)^{2/3} - \frac{3}{14}(7(1)-6)^{2/3}\right]$

$= \frac{2}{3} + \left(\frac{6}{7} - \frac{3}{14}\right) = \frac{55}{42}$

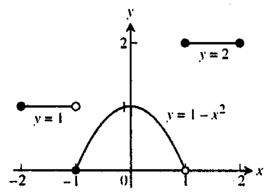


15. $\int_{-2}^{2} f(x)\,dx = \int_{-2}^{-1} dx + \int_{-1}^{1} (1-x^2)\,dx + \int_{1}^{2} 2\,dx$

$= [x]_{-2}^{-1} + \left[x - \frac{x^3}{3}\right]_{-1}^{1} + [2x]_1^2$

$= -1-(-2) + \left(1 - \frac{1^3}{3}\right) - \left(-1 - \frac{(-1)^3}{3}\right) + 2(2) - 2(1)$

$= 1 + \frac{2}{3} - \left(-\frac{2}{3}\right) + 4 - 2 = \frac{13}{3}$

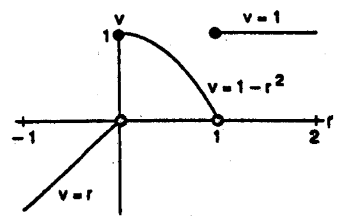


16. $\int_{-1}^{2} h(r)\,dr = \int_{-1}^{0} r\,dr + \int_{0}^{1} (1-r^2)\,dr + \int_{1}^{2} dr$

$= \left[\frac{r^2}{2}\right]_{-1}^{0} + \left[r - \frac{r^3}{3}\right]_{0}^{1} + [r]_1^2$

$= 0 - \frac{(-1)^2}{2} + \left(1 - \frac{1^3}{3}\right) - 0 + 2 - 1$

$= -\frac{1}{2} + \frac{2}{3} + 1 = \frac{7}{6}$



17. Ave. value $= \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-0}\int_0^2 f(x)\,dx = \frac{1}{2}\left[\int_0^1 x\,dx + \int_1^2 (x-1)\,dx\right] = \frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 + \frac{1}{2}\left[\frac{x^2}{2} - x\right]_1^2$

$= \frac{1}{2}\left[\frac{1^2}{2} - 0 + \left(\frac{2^2}{2} - 2\right) - \left(\frac{1^2}{2} - 1\right)\right] = \frac{1}{2}$

18. Ave. value $= \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{3-0}\int_0^3 f(x)\,dx = \frac{1}{3}\left[\int_0^1 dx + \int_1^2 0\,dx + \int_2^3 dx\right] = \frac{1}{3}[1 - 0 + 0 + 3 - 2] = \frac{2}{3}$

19. $f(x) = \int_{1/x}^{x} \frac{1}{t}\,dt \Rightarrow f'(x) = \frac{1}{x}\left(\frac{dx}{dx}\right) - \left(\frac{1}{\frac{1}{x}}\right)\left(\frac{d}{dx}\left(\frac{1}{x}\right)\right) = \frac{1}{x} - x\left(-\frac{1}{x^2}\right) = \frac{1}{x} + \frac{1}{x} = \frac{2}{x}$

20. $f(x) = \int_{\cos x}^{\sin x} \frac{1}{1-t^2}\,dt \Rightarrow f'(x) = \left(\frac{1}{1-\sin^2 x}\right)\left(\frac{d}{dx}(\sin x)\right) - \left(\frac{1}{1-\cos^2 x}\right)\left(\frac{d}{dx}(\cos x)\right) = \frac{\cos x}{\cos^2 x} + \frac{\sin x}{\sin^2 x}$

$= \frac{1}{\cos x} + \frac{1}{\sin x}$

21. $g(y) = \int_{\sqrt{y}}^{2\sqrt{y}} \sin t^2\,dt \Rightarrow g'(y) = \left(\sin\left(2\sqrt{y}\right)^2\right)\left(\frac{d}{dy}\left(2\sqrt{y}\right)\right) - \left(\sin\left(\sqrt{y}\right)^2\right)\left(\frac{d}{dy}\left(\sqrt{y}\right)\right) = \frac{\sin 4y}{\sqrt{y}} - \frac{\sin y}{2\sqrt{y}}$

22. $f(x) = \int_x^{x+3} t(5-t)\,dt \Rightarrow f'(x) = (x+3)(5-(x+3))\left(\frac{d}{dx}(x+3)\right) - x(5-x)\left(\frac{dx}{dx}\right) = (x+3)(2-x) - x(5-x)$
$= 6 - x - x^2 - 5x + x^2 = 6 - 6x$. Thus $f'(x) = 0 \Rightarrow 6 - 6x = 0 \Rightarrow x = 1$. Also, $f''(x) = -6 < 0 \Rightarrow x = 1$ gives a maximum.

NOTES:

TECHNOLOGY NOTES: