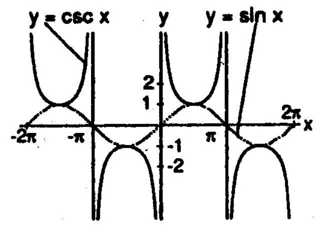
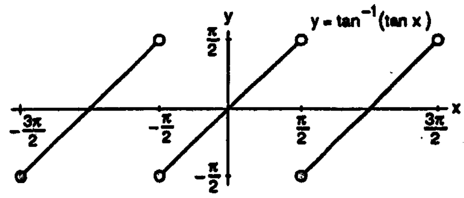
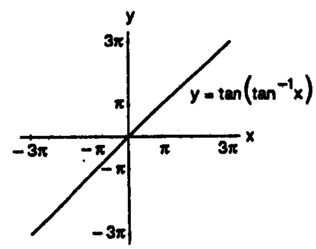
(b) Sin x and csc x are positive in QII and negative in QIII and QIV. Csc x is undefined when sin x is 0. The range of csc x is $(-\infty, -1] \cup [1, \infty)$; the range of sin x is $[-1, 1]$.



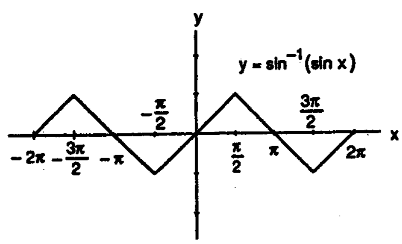
41. (a) Domain: all real numbers except those having the form $\frac{\pi}{2} + k\pi$ where k is an integer.
Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$



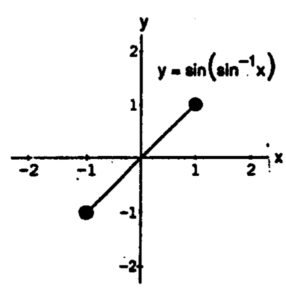
(b) Domain: $-\infty < x < \infty$; Range: $-\infty < y < \infty$
The graph of $y = \tan^{-1}(\tan x)$ is periodic, the graph of $y = \tan(\tan^{-1} x) = x$ for $-\infty \leq x < \infty$.



42. (a) Domain: $-\infty < x < \infty$; Range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$



(b) Domain: $-1 \leq x \leq 1$; Range: $-1 \leq y \leq 1$
The graph of $y = \sin^{-1}(\sin x)$ is periodic; the graph of $y = \sin(\sin^{-1} x) = x$ for $-1 \leq x \leq 1$.



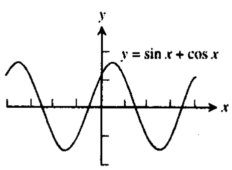
43. The angle $\tan^{-1}(2.5) \approx 1.190$ is the solution to this equation in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Another solution in $0 \leq x < 2\pi$ is $\tan^{-1}(2.5) + \pi \approx 4.332$. The solutions are $x \approx 1.190$ and $x \approx 4.332$.

44. The angle $\cos^{-1}(-0.7) \approx 2.346$ is the solution to this equation in the interval $0 \le x \le \pi$. Since the cosine function is even, the value $-\cos^{-1}(-0.7) \approx -2.346$ is also a solution, so any value of the form $\pm\cos^{-1}(-0.7) + 2k\pi$ is a solution, where k is an integer. In $2\pi \le x, 4\pi$ the solutions are $x = \cos^{-1}(-0.7) + 2\pi \approx 8.629$ and $x = -\cos^{-1}(-0.7) + 4\pi \approx 10.220$.

45. This equation is equivalent to $\cos x = -\frac{1}{3}$, so the solution in the interval $0 \le x \le \pi$ is $y = \cos^{-1}\left(-\frac{1}{3}\right) \approx 1.911$. Since the cosine function is even, the solutions in the interval $-\pi \le x < \pi$ are $x \approx -1.911$ and $x \approx 1.911$.

46. The solutions in the interval $0 \le x < 2\pi$ are $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$. Since $y = \sin x$ has period 2π, the solutions are all of the form $x = \frac{7\pi}{6} + 2k\pi$ or $x = \frac{11\pi}{6} + 2k\pi$, where k is any integer.

47. (a)



The graph is a sine/cosine type graph, but it is shifted and has an amplitude greater than 1.

(b) Amplitude ≈ 1.414 (that is, $\sqrt{2}$)

Period $= 2\pi$

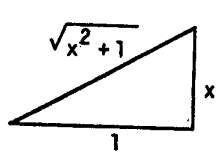
Horizontal shift ≈ -0.785 $\left(\text{that is, } -\frac{\pi}{4}\right)$ or 5.498 $\left(\text{that is, } \frac{7\pi}{4}\right)$ relative to sin x.

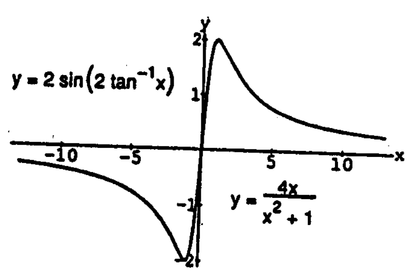
Vertical shift: 0

(c)
$$\sin\left(x + \frac{\pi}{4}\right) = (\sin x)\left(\cos\frac{\pi}{4}\right) + (\cos x)\left(\sin\frac{\pi}{4}\right)$$
$$= (\sin x)\left(\frac{1}{\sqrt{2}}\right) + (\cos x)\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{1}{\sqrt{2}}(\sin x + \cos x)$$

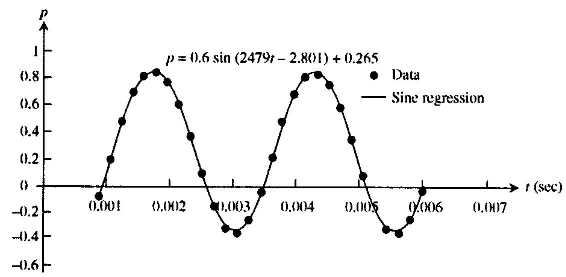
Therefore, $\sin x + \cos x = \sqrt{2}\,\sin\left(x + \frac{\pi}{4}\right)$

48. The graphs are identical for $y = 2\sin\left(2\tan^{-1}x\right)$
$$= 4\left[\sin\left(\tan^{-1}x\right)\right]\left[\cos\left(\tan^{-1}x\right)\right] = 4\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{1}{\sqrt{x^2+1}}\right)$$
$= \frac{4x}{x^2+1}$ from the triangle





49. (a) The sinusoidal regression on the TI-92 Plus calculator gives $p = 0.599 \sin(2479t - 2.801) + 0.265$



(b) The period is approximately $\frac{2\pi}{2479}$ seconds, so the frequency is approximately $\frac{2479}{2\pi} \approx 395$ Hz

50. (a) $b = \frac{2\pi}{12} = \frac{\pi}{6}$

(b) It's half of the difference, so $a = \frac{80-30}{2} = 25$.

(c) $k = \frac{80+30}{2} = 55$

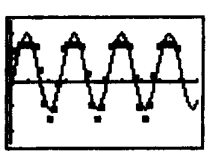
(d) The function should have its minimum at $t = 2$ (when the temperature is 30°F) and its maximum at $t = 8$ (when the temperature is 80°F). The value of h is $\frac{2+8}{2} = 5$. Equation: $y = 25 \sin\left[\frac{\pi}{6}(x-5)\right] + 55$

(e)

[−1, 13] by [−10, 100]

51. (a) Using a graphing calculator with the sinusoidal regression feature, the equation is $y = 3.0014 \sin(0.9996x + 2.0012) + 2.9999$.
(b) $y = 3 \sin(x+2) + 3$

52. (a) Using a graphing calculator with the sinusoidal regression feature, the equation is $y = 1.543 \sin(2468.635x - 0.494) + 0.438$.

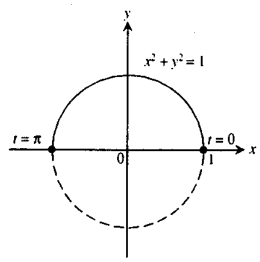


[0, 0.01] by [−2.5, 2.5]

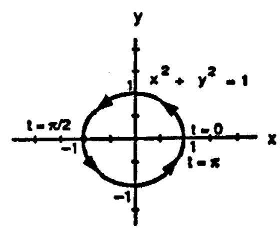
(b) The frequency is 2468.635 radians per second, which is equivalent to $\frac{2468.635}{2\pi} \approx 392.9$ cycles per second (Hz). The note is a "G."

P.6 PARAMETRIC EQUATIONS

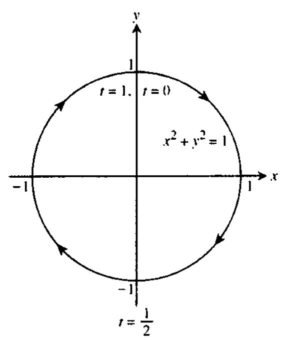
1. $x = \cos t,\ y = \sin t,\ 0 \le t \le \pi$
$\Rightarrow \cos^2 t + \sin^2 t = 1 \Rightarrow x^2 + y^2 = 1$



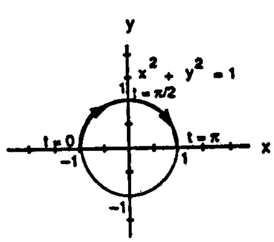
2. $x = \cos 2t,\ y = \sin 2t,\ 0 \le t \le \pi$
$\Rightarrow \cos^2 2t + \sin^2 2t = 1 \Rightarrow x^2 + y^2 = 1$



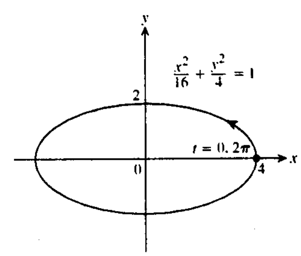
3. $x = \sin(2\pi t),\ y = \cos(2\pi t),\ 0 \le t \le 1$
$\sin^2(2\pi t) + \cos^2(2\pi t) = 1 \Rightarrow x^2 + y^2 = 1$



4. $x = \cos(\pi - t),\ y = \sin(\pi - t),\ 0 \le t \le \pi$
$\Rightarrow \cos^2(\pi - t) + \sin^2(\pi - t) = 1$
$\Rightarrow x^2 + y^2 = 1$

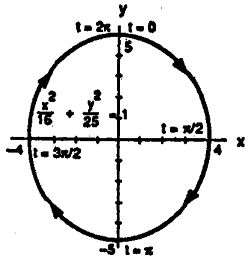


5. $x = 4\cos t,\ y = 2\sin t,\ 0 \le t \le 2\pi$
$\Rightarrow \frac{16\cos^2 t}{16} + \frac{4\sin^2 t}{4} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$

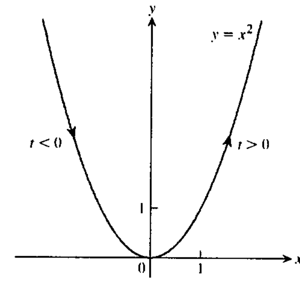


6. $x = 4 \sin t,\ y = 5 \cos t,\ 0 \le t \le 2\pi$

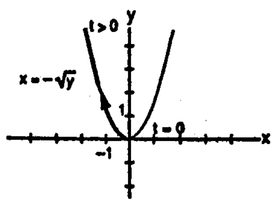
$\Rightarrow \frac{16 \sin^2 t}{16} + \frac{25 \cos^2 t}{25} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1$



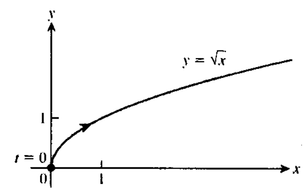
7. $x = 3t,\ y = 9t^2,\ -\infty < t < \infty \Rightarrow y = x^2$



8. $x = -\sqrt{t},\ y = t,\ t \ge 0 \Rightarrow x = -\sqrt{y}$
or $y = x^2,\ x \le 0$

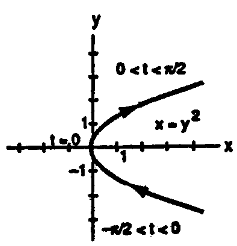


9. $x = t,\ y = \sqrt{t},\ t \ge 0 \Rightarrow y = \sqrt{x}$



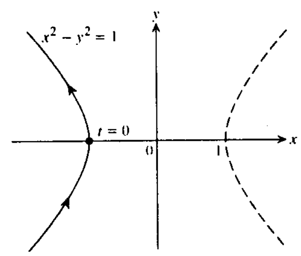
10. $x = \sec^2 t - 1,\ y = \tan t,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$

$\Rightarrow \sec^2 t - 1 = \tan^2 t \Rightarrow x = y^2$



11. $x = -\sec t,\ y = \tan t,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$

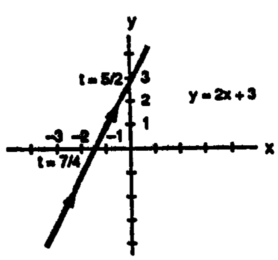
$\Rightarrow \sec^2 t - \tan^2 t = 1 \Rightarrow x^2 - y^2 = 1$



12. $x = 2t - 5,\ y = 4t - 7,\ -\infty < t < \infty$

$\Rightarrow x + 5 = 2t \Rightarrow 2(x + 5) = 4t$

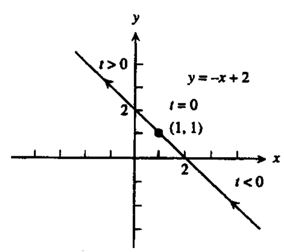
$\Rightarrow y = 2(x + 5) - 7 \Rightarrow y = 2x + 3$



13. $x = 1 - t,\ y = 1 + t,\ -\infty < t < \infty$

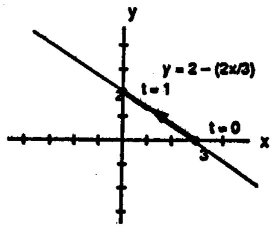
$\Rightarrow 1 - x = t \Rightarrow y = 1 + (1 - x)$

$\Rightarrow y = -x + 2$



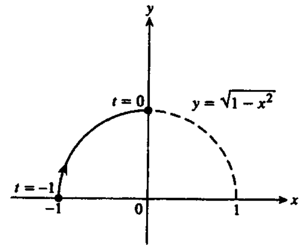
14. $x = 3 - 3t,\ y = 2t,\ 0 \le t \le 1 \Rightarrow \frac{y}{2} = t$

$\Rightarrow x = 3 - 3\left(\frac{y}{2}\right) \Rightarrow 2x = 6 - 3y \Rightarrow y = 2 - \frac{2}{3}x$



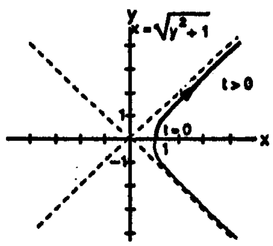
15. $x = t,\ y = \sqrt{1-t^2},\ -1 \le t \le 0$

$\Rightarrow y = \sqrt{1-x^2}$



16. $x = \sqrt{t+1},\ y = \sqrt{t},\ t \ge 0$

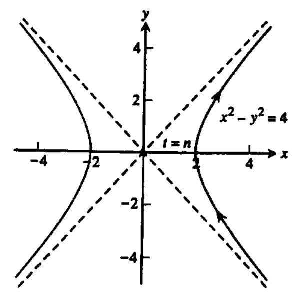
$\Rightarrow y^2 = t \Rightarrow x = \sqrt{y^2+1},\ y \ge 0$



17. $x = e^t + e^{-t},\ y = e^t - e^{-t},\ -\infty < t < \infty$

$(e^t + e^{-t})^2 - (e^t - e^{-t})^2 = (e^{2t} + 2 + e^{-2t})$

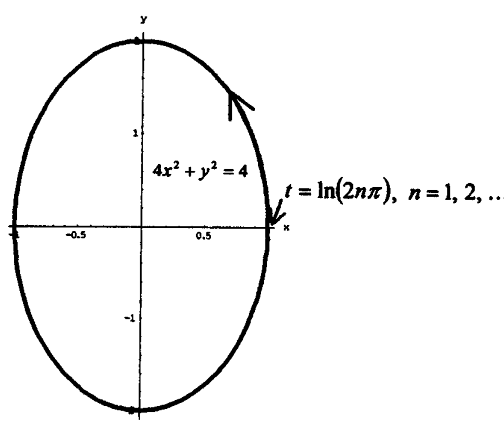
$- (e^{2t} - 2 + e^{-2t}) = 4 \Rightarrow x^2 - y^2 = 4$



18. $x = \cos(e^t),\ y = 2\sin(e^t),\ -\infty < t < \infty$

$\cos^2(e^t) + \sin^2(e^t) = 1 \Rightarrow x^2 + (y/2)^2 = 1$

$\Rightarrow 4x^2 + y^2 = 4$



19. (a) $x = a\cos t,\ y = -a\sin t,\ 0 \le t \le 2\pi$
    (b) $x = a\cos t,\ y = a\sin t,\ 0 \le t \le 2\pi$
    (c) $x = a\cos t,\ y = -a\sin t,\ 0 \le t \le 4\pi$
    (d) $x = a\cos t,\ y = a\sin t,\ 0 \le t \le 4\pi$

20. (a) $x = a\sin t,\ y = b\cos t,\ \frac{\pi}{2} \le t \le \frac{5\pi}{2}$
    (b) $x = a\cos t,\ y = b\sin t,\ 0 \le t \le 2\pi$
    (c) $x = a\sin t,\ y = b\cos t,\ \frac{\pi}{2} \le t \le \frac{9\pi}{2}$
    (d) $x = a\cos t,\ y = b\sin t,\ 0 \le t \le 4\pi$

21. Using $(-1,-3)$ we create the parametric equations $x = -1 + at$ and $y = -3 + bt$, representing a line which goes through $(-1,-3)$ at $t = 0$. We determine a and b so that the line goes through $(4,1)$ when $t = 1$.
Since $4 = -1 + a$, $a = 5$.

Since $1 = -3 + b$, $b = 4$.
Therefore, one possible parametrization is $x = -1 + 5t$, $y = -3 + 4t$, $0 \le t \le 1$.

22. Using $(-1, -3)$ we create the parametric equations $x = -1 + at$ and $y = 3 + bt$, representing a line which goes through $(-1, 3)$ at $t = 0$. We determine a and b so that the line goes through $(3, -2)$ $t = 1$.
Since $3 = -1 + a$, $a = 4$.
Since $-2 = 3 + b$, $b = -5$.
Therefore, one possible parametrization is $x = -1 + 4t$, $y = 3 - 5t$, $0 \le t \le 1$.

23. The lower half of the parabola is given by $x = y^2 + 1$ for $y \le 0$. Substituting t for y, we obtain one possible parametrization $x = t^2 + 1$, $y = t$, $t \le 0$.

24. The vertex of the parabola is at $(-1, -1)$, so the left half of the parabola is given by $y = x^2 + 2x$ for $x \le -1$. Substituting t for x, we obtain one possible parametrization: $x = t$, $y = t^2 + 2t$, $t \le -1$.

25. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at (2, 3) for $t = 0$ and passes through $(-1, -1)$ at $t = 1$. Then $x = f(t)$, where $f(0) = 2$ and $f(1) = -1$.
Since slope $= \frac{\Delta x}{\Delta t} = \frac{-1-2}{1-0} = -3$, $x = f(t) = -3t + 2 = 2 - 3t$. Also, $y = g(t)$, where $g(t) = 3$ and $g(1) = -1$.
Since slope $= \frac{\Delta y}{\Delta t} = \frac{-1-3}{1-0} = -4$, $y = g(t) = -4t + 3 = 3 - 4t$.
One possible parametrization is: $x = 2 - 3t$, $y = 3 - 4t$, $t \ge 0$.

26. For simplicity, we assume that x and y are linear functions of t and that the point (x, y) starts at $(-1, 2)$ for $t = 0$ and passes through $(0, 0)$ at $t = 1$. Then $x = f(t)$, where $f(0) = -1$ and $f(1) = 0$.
Since slope $= \frac{\Delta x}{\Delta t} = \frac{0-(-1)}{1-0} = 1$, $x = f(t) = 1t + (-1) = -1 + t$. Also, $y = g(t)$, where $g(0) = 2$ and $g(1) = 0$.
Since slope $= \frac{\Delta y}{\Delta t} = \frac{0-2}{1-0} = -2$, $y = g(t) = -2t + 2 = 2 - 2t$.
One possible parametrization is: $x = -1 + t$, $y = 2 - 2t$, $t \ge 0$.

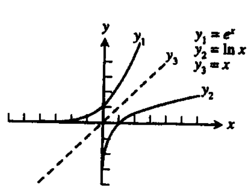
27. Graph (c). Window: $[-4, 4]$ by $[-3, 3]$, $0 \le t \le 2\pi$

28. Graph (a). Window: $[-2, 2]$ by $[-2, 2]$, $0 \le t \le 2\pi$

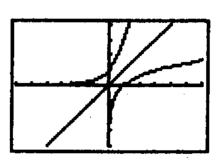
29. Graph (d). Window: $[-10, 10]$ by $[-10, 10]$, $0 \le t \le 2\pi$

30. Graph (b). Window: $[-15, 15]$ by $[-15, 15]$, $0 \le t \le 2\pi$

31. Graph of f: $x_1 = t$, $y_1 = e^t$
Graph of f^{-1}: $x_2 = e^t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

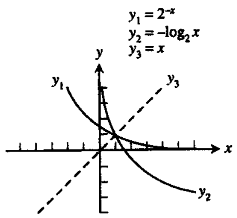


32. Graph of f: $x_1 = t$, $y_1 = 3^t$
Graph of f^{-1}: $x_2 = 3^t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

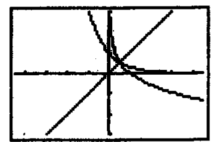


[−6, 6] by [−4, 4]

33. Graph of f: $x_1 = t$, $y_1 = 2^{-t}$
Graph of f^{-1}: $x_2 = 2^{-t}$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

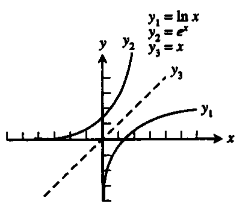


34. Graph of f: $x_1 = t$, $y_1 = 3^{-t}$
Graph of f^{-1}: $x_2 = 3^{-t}$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

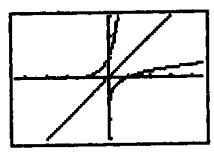


$[-4.5, 4.5]$ by $[-3, 3]$

35. Graph of f: $x_1 = t$, $y_1 = \ln t$
Graph of f^{-1}: $x_2 = \ln t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

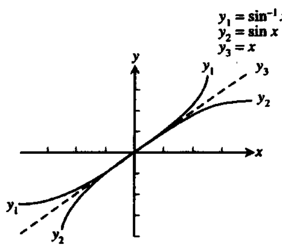


36. Graph of f: $x_1 = t$, $y_1 = \log t$
Graph of f^{-1}: $x_2 = \log t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$

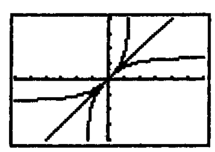


$[-4.5, 4.5]$ by $[-3, 3]$

37. Graph of f: $x_1 = t$, $y_1 = \sin^{-1} t$
Graph of f^{-1}: $x_2 = \sin^{-1} t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$



38. Graph of f: $x_1 = t$, $y_1 = \tan^{-1} t$
Graph of f^{-1}: $x_2 = \tan^{-1} t$, $y_2 = t$
Graph of $y = x$: $x_3 = t$, $y_3 = t$



$[-6, 6]$ by $[-4, 4]$

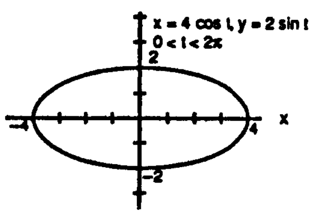
39. The graph is in Quadrant I when $0 < y < 2$, which corresponds to $1 < t < 3$. To confirm, note that $x(1) = 2$ and $x(3) = 0$.

40. The graph is in Quadrant II when $2 < y \leq 4$, which corresponds to $3 < t \leq 5$. To confirm, note that $x(3) = 0$ and $x(5) = -2$.

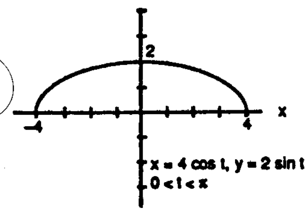
41. The graph is in Quadrant III when $-6 \leq y < -4$, which corresponds to $-5 \leq t < -3$. To confirm, note that $x(-5) = -2$ and $x(-3) = 0$.

42. The graph is in Quadrant IV when $-4 < y < 0$, which corresponds to $-3 < t < 1$. To confirm, note that $x(-3) = 0$ and $x(1) = 2$.

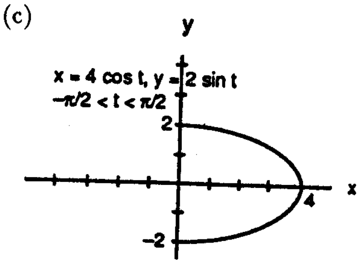
43. (a)



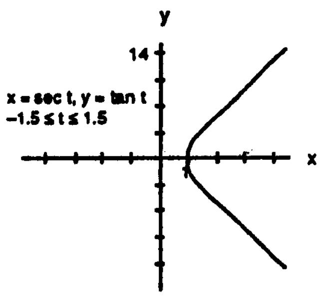
(b)



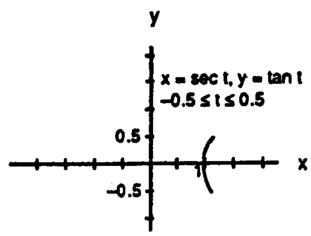
(c)



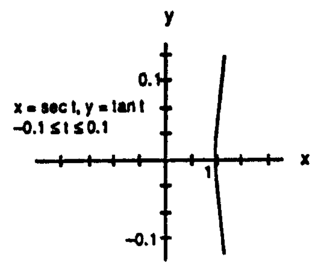
44. (a)



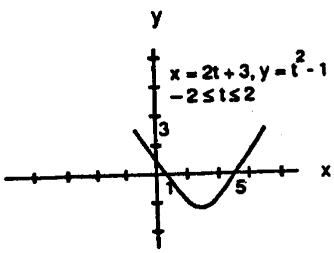
(b)



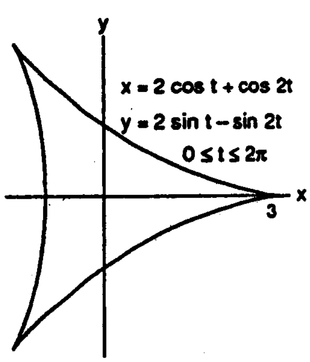
(c)



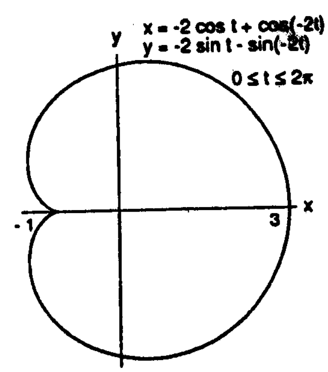
45.



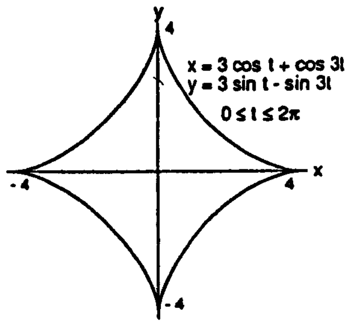
46. (a)



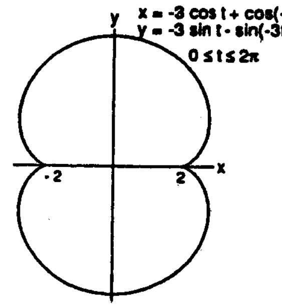
(b)



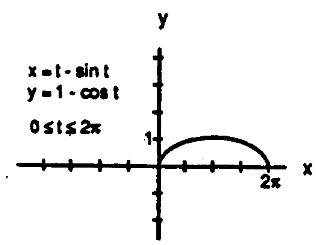
47. (a)



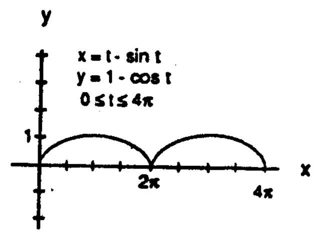
(b)



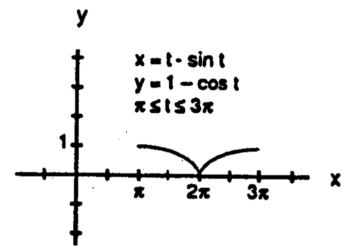
48. (a)



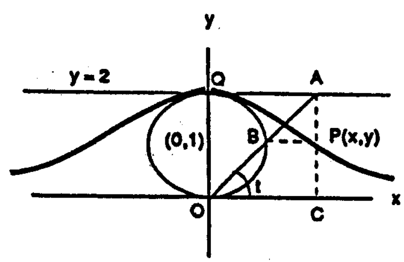
(b)



(c)



49. Extend the vertical line through A to the x-axis and let C be the point of intersection. Then OC = AQ = x and $\tan t = \frac{2}{OC} = \frac{2}{x} \Rightarrow x = \frac{2}{\tan t} = 2 \cot t$; $\sin t = \frac{2}{OA}$ $\Rightarrow OA = \frac{2}{\sin t}$; and $(AB)(OA) = (AQ)^2 \Rightarrow AB\left(\frac{2}{\sin t}\right) = x^2$ $\Rightarrow AB\left(\frac{2}{\sin t}\right) = \left(\frac{2}{\tan t}\right)^2 \Rightarrow AB = \frac{2 \sin t}{\tan^2 t}$. Next $y = 2 - AB \sin t \Rightarrow y = 2 - \left(\frac{2 \sin t}{\tan^2 t}\right) \sin t =$



$2 - \frac{2 \sin^2 t}{\tan^2 t} = 2 - 2 \cos^2 t = 2 \sin^2 t$. Therefore let $x = 2 \cot t$ and $y = 2 \sin^2 t$, $0 < t < \pi$.

50. (a) $x = x_0 + (x_1 - x_0)t$ and $y = y_0 + (y_1 - y_0)t \Rightarrow t = \frac{x - x_0}{x_1 - x_0} \Rightarrow y = y_0 + (y_1 - y_0)\left(\frac{x - x_0}{x_1 - x_0}\right)$

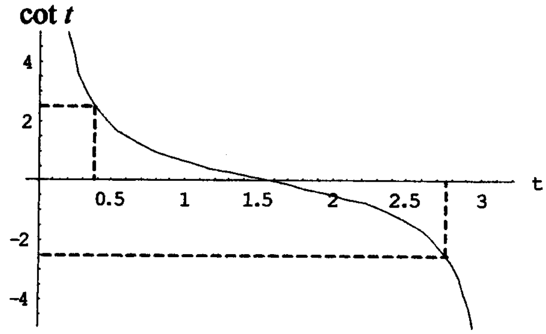
$\Rightarrow y - y_0 = \left(\frac{y_1 - y_0}{x_1 - x_0}\right)(x - x_0)$ which is an equation of the line through the points (x_0, y_0) and (x_1, y_1)

(b) Let $x_0 = y_0 = 0$ in (a) $\Rightarrow x = x_1 t$, $y = y_1 t$ (the answer is not unique)

(c) Let $(x_0, y_0) = (-1, 0)$ and $(x_1, y_1) = (0, 1)$ or let $(x_0, y_0) = (0, 1)$ and $(x_1, y_1) = (-1, 0)$ in part (a) $\Rightarrow x = -1 + t$, $y = t$ or $x = -t$, $y = 1 - t$ (the answer is not unique)

51. (a) $-5 \le x \le 5 \Rightarrow -5 \le 2 \cot t \le 5 \Rightarrow -\frac{5}{2} \le \cot t \le \frac{5}{2}$

The graph of cot t shows where to look for the limits on t.



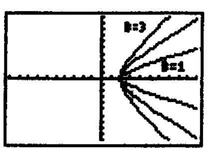
$\tan^{-1}\left(\frac{2}{5}\right) \leq t \leq \pi + \tan^{-1}\left(-\frac{2}{5}\right) \Rightarrow 0.381 \leq t \leq 2.761$

The curve is traced from right to left and extends infinitely in both directions from the origin.

(b) For $-\frac{\pi}{2} < t < \frac{\pi}{2}$, the curve is the same as that which is given. It first traces from the vertex at $(0, 2)$ to the left extreme point in the window, and then from the right extreme point in the window to the vertex point. For $0 < t < \frac{\pi}{2}$, only the right half of the curve appears, and it traces from the right extreme of the window to the vertex at $(0, 2)$ and terminates there. For $\frac{\pi}{2} < t < \pi$, only the left half of the curve appears, and it traces from the vertex to the left extreme of the window.

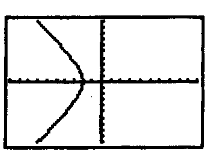
(c) For $x = -2 \cot t$, the curve traces from left to right rather than from right to left. For $x = 2 \cot(\pi - t)$, the curve traces from right to left, as it does with the original parametrization.

52. (a) The resulting graph appears to be the right half of a hyperbola in the first and fourth quadrants. The parameter a determines the x-intercept. The parameter b determines the shape of the hyperbola. If b is smaller, the graph has less steep slopes and appears "sharper." If b is larger, the slopes are steeper and the graph appears more "blunt." The graphs for a = 2 and b = 1, 2, and 3 are shown.



[−10, 10] by [−10, 10]

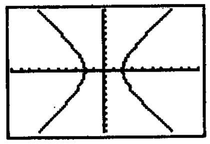
(b)



[−10, 10] by [−10, 10]

This appears to be the left half of the same hyperbola.

(c)



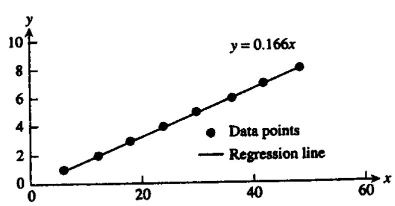
One must be careful because both sec t and tan t are discontinuous at these points. This might cause the grapher to include extraneous lines (the asymptotes of the hyperbola) in its graph. The extraneous lines can be avoided by using the grapher's dot mode instead of connected mode.

(d) Note that $\sec^2 t - \tan^2 t = 1$ by a standard trigonometric identity. Substituting $\frac{x}{a}$ for sec t and $\frac{y}{b}$ for tan t gives $\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$.

(e) This changes the orientation of the hyperbola. In this case, b determines the y-intercept of the hyperbola, and a determines the shape. The parameter interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ gives the upper half of the hyperbola. The parameter interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ gives the lower half. The same values of t cause discontinuities and may add extraneous lines to the graph. Substituting $\frac{y}{b}$ for sec t and $\frac{x}{a}$ for tan t in the identity $\sec^2 t - \tan^2 t = 1$ gives $\left(\frac{y}{b}\right)^2 - \left(\frac{x}{a}\right)^2 = 1$.

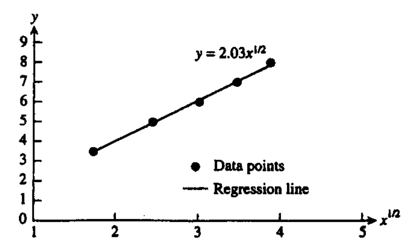
P.7 MODELING CHANGE

1. (a)



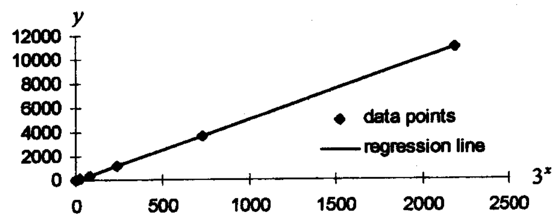
The graph supports the assumption that y is proportional to x. The constant of proportionality is estimated from the slope of the regression line, which is 0.166.

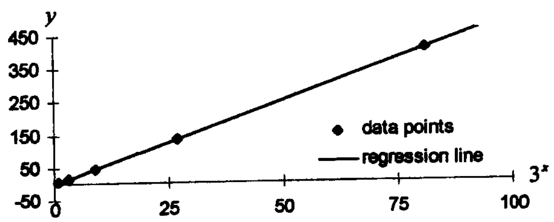
(b)



The graph supports the assumption that y is proportional to $x^{1/2}$. The constant of proportionality is estimated from the slope of the regression line, which is 2.03.

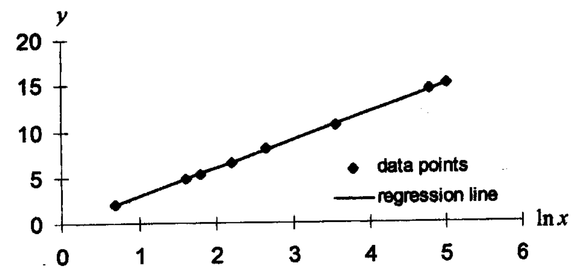
(c) Because of the wide range of values of the data, two graphs are needed to observe all of the points in relation to the regression line.





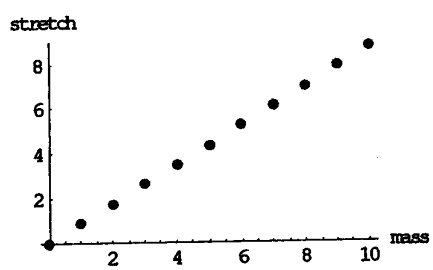
The graphs support the assumption that y is proportional to 3^x. The constant of proportionality is estimated from the slope of the regression line, which is 5.00.

(d)

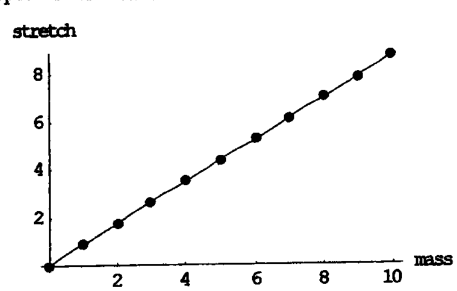


The graph supports the assumption that y is proportional to ln x. The constant of proportionality is estimated from the slope of the regression line, which is 2.99.

2. (a) Plot the data to see if there is a recognizable pattern.

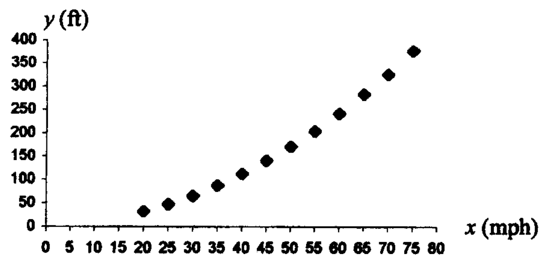


The data clearly suggests a linear relationship. The line of best fit, or the regression line, is $s = 0.8742m$ where s is the stretch in the spring and m is the mass. Now we superimpose the regression line on the graph of the data.

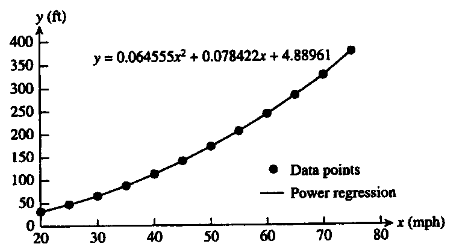


(b) The model fits the data very well.
(c) When $m = 13$, the model gives a stretch of $s = 0.8742(13) = 11.365$. Since this data point is outside the range of the data that the model is based upon, one should feel uncomfortable with this prediction of the stretch without further experimental verification.

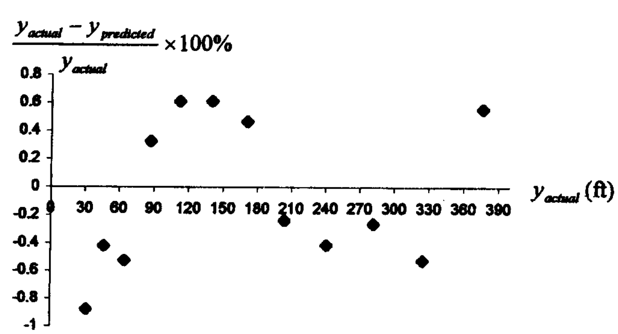
3. First, plot the braking distance versus the speed.



The shape of the graph suggests either a power function or an exponential function to describe the relationship. First, try to fit a quadratic function. Using quadratic regression on the TI-92 Plus calculator gives $y = 0.064555x^2 + 0.078422x + 4.88961$.



The quadratic regression fits the data well as seen by the following plot of the relative errors versus the actual stopping distance.

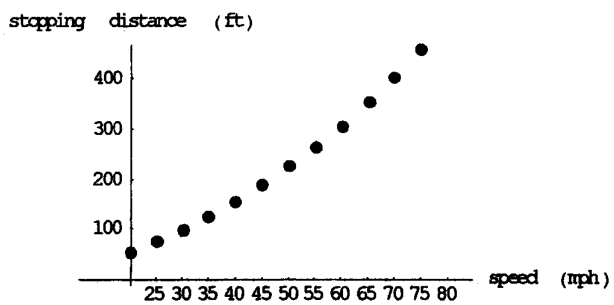


The largest relative error is less than 1%.

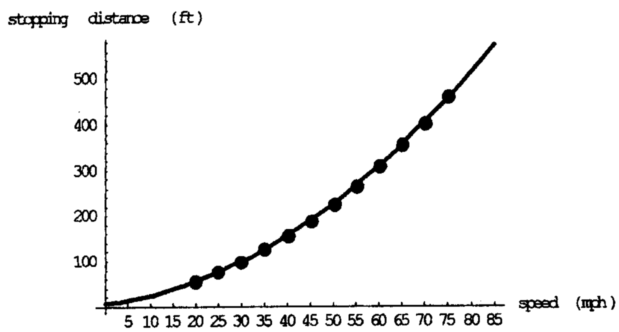
4. The following table gives the total stopping distance (reaction distance + braking distance) for automobile speeds ranging from 20 to 75 miles per hour.

speed	20	25	30	35	40	45	50	55	60	65	70	75
stopping distance	54	75	98	126	156	190	226	265	307	354	402	459

Plot the total stopping distance versus speed.



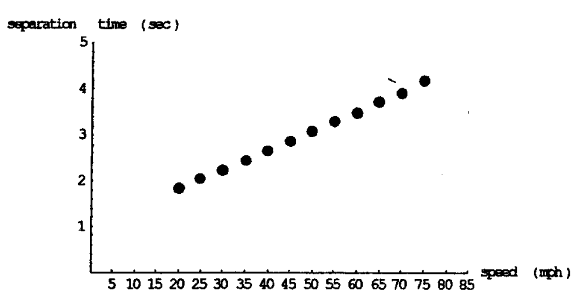
The graph suggests a possible quadratic relationship. Quadratic regression on the data gives $d = 0.0646v^2 + 1.181v + 5.040$ where d is the total stopping distance in feet and v is the travel speed in miles per hour. Now superimpose the quadratic regression on the graph of the data.



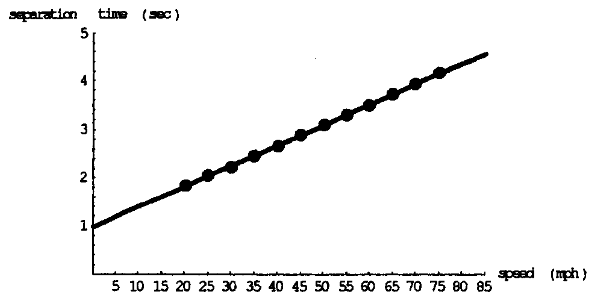
The quadratic regression fits the data very well. To test the 2-second "rule of thumb," calculate the time the vehicle will travel the distance d when it is traveling at speed v. (Don't forget to convert mph into ft/sec using 60 mph = 88 ft/sec.) The following table gives separation times versus travel speed.

v (mph)	20	25	30	35	40	45	50	55	60	65	70	75
t (sec)	1.84	2.05	2.23	2.45	2.66	2.88	3.08	3.29	3.49	3.71	3.92	4.17

Plot the data.

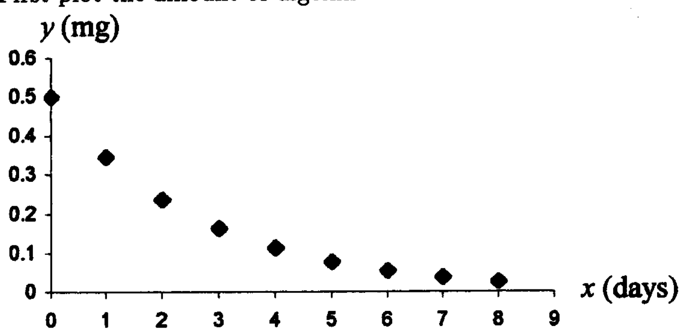


The graph suggests a linear relationship and a linear regression gives $t = 0.042v + 0.983$. Now superimpose the linear regression function on the graph of the data.

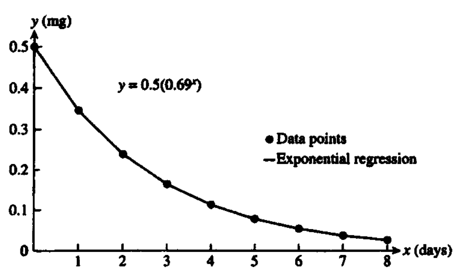


Based on the preceding analysis, a better rule of thumb would be to keep a minimum separation time of 2 seconds and add 1 sec for every 20 mph increment of speed above 20 mph. So, for example, if you are traveling at 40 mph your separation should be $2+1(1)=3$ seconds, at 60 mph your separation should be $2+2(1)=4$ seconds, at 80 mph it should be $2+3(1)=5$ seconds, and so on.

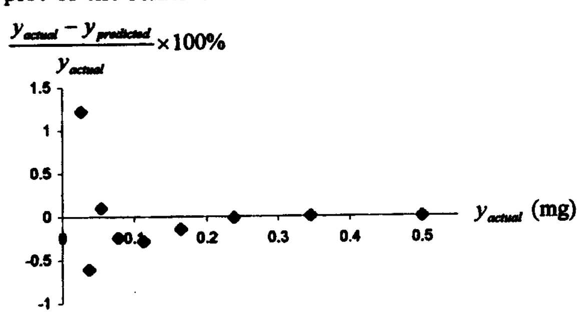
5. (a) First plot the amount of digoxin in the blood versus time.



The graph suggests that the amount decays exponentially with time. The exponential regression on the TI-92 Plus calculator gives $y = 0.5(0.69^x) = 0.5e^{-0.371x}$.

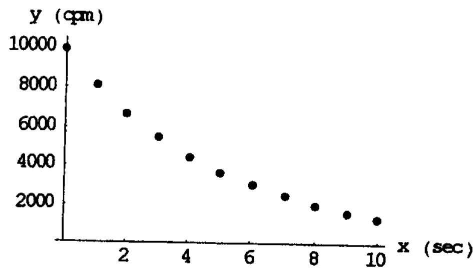


(b) The exponential function fits the data very well as demonstrated by the graph above and the following is a plot of the relative error versus the actual amount in the blood.

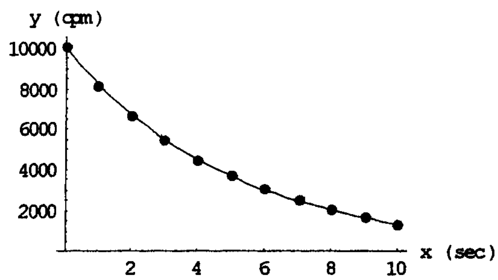


(c) $y(12) = 0.5e^{-0.371(12)} = 0.00583$, therefore, the model predicts that after 12 hours, the amount of digoxin in the blood will be less than 0.006 mg.

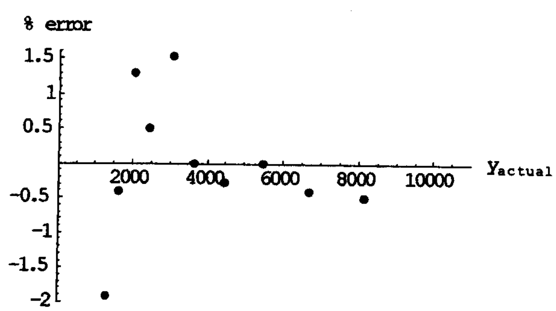
6. (a) Plot the data.



An exponential regression on the TI-92 Plus calculator gives $y = 10{,}037e^{-0.2005x}$. Superimpose the regression function on the graph of the data.



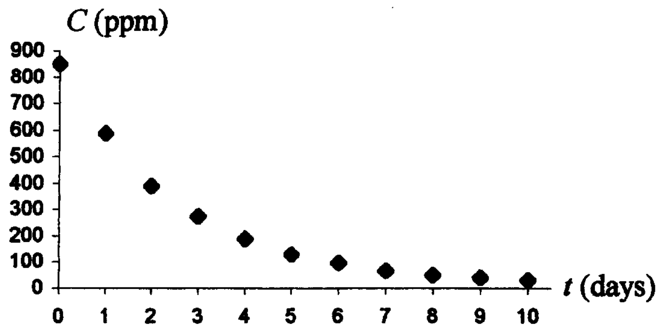
(b) The exponential function fits the data very well as indicated by the graph above. The following is a graph of the relative error, $\frac{y_{predicted} - y_{actual}}{y_{actual}} \times 100\%$, versus y_{actual}.



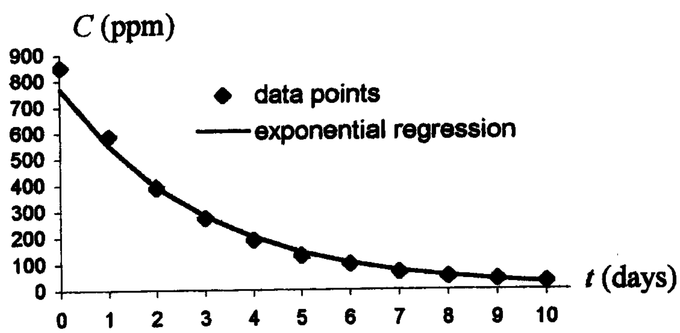
The largest relative error is less than 2% in magnitude.

(c) $500 = 10{,}037e^{-0.2005x} \Rightarrow -0.2005x = \ln\left(\frac{500}{10{,}037}\right) \Rightarrow x = 15.0$ minutes.

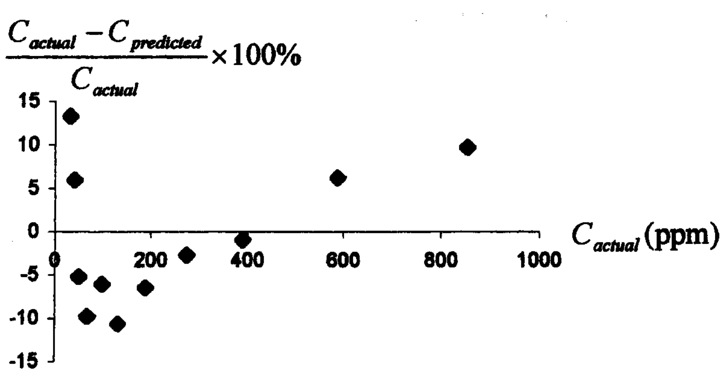
7. (a) First, plot a graph of the blood concentration versus time. Let t represent the elapsed time in days and C the blood concentration in parts per million.



The graph suggests that the amount decays exponentially with time. The exponential regression function on the TI-92 Plus calculator gives $C = 770(0.7146^t) = 770e^{-0.336t}$.



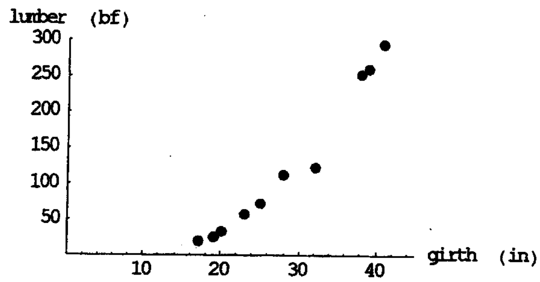
(b) The exponential function appears to capture a trend for this data. The following graph shows the relative errors in the model estimates.



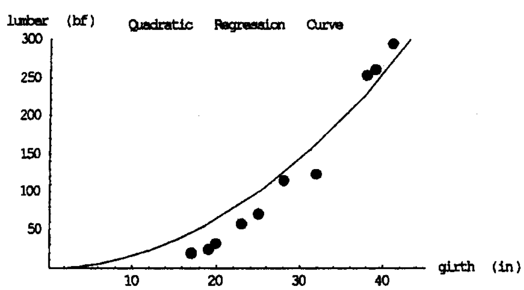
The relative errors in the predicted values are as large as 13.2% and the errors are large for small as well as large blood concentrations. The pattern of the residual errors does not suggest an obvious improvement of the model.

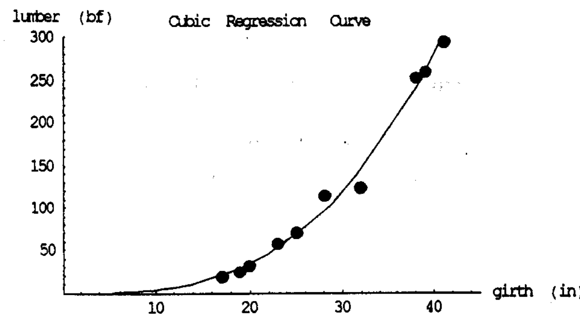
(c) $10 = 770e^{-0.336t} \Rightarrow t = -\frac{1}{0.336}\ln\left(\frac{10}{770}\right) = 12.93$ days. Therefore, the model predicts that the blood concentration will fall below 10 ppm after 12 days and 22 hours.

8. Plot the data.



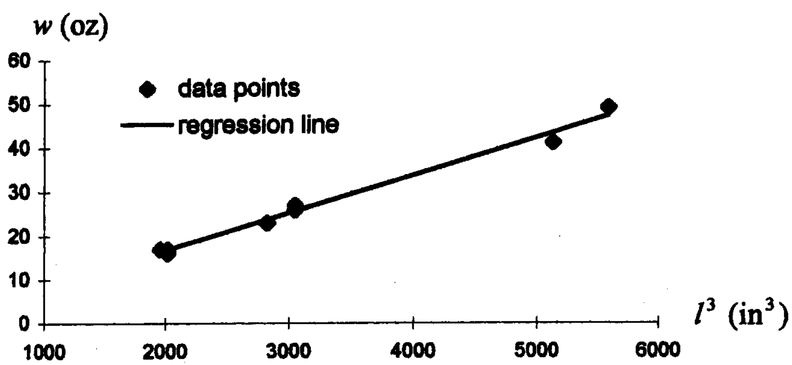
(a) and (b) The quadratic regression function is $y = 0.1579x^2$ where x represents the girth in inches and y the amount of usable lumber in board feet. The cubic regression function is $y = 0.00436x^3$. Superimpose the two regression functions on the graph of the data.



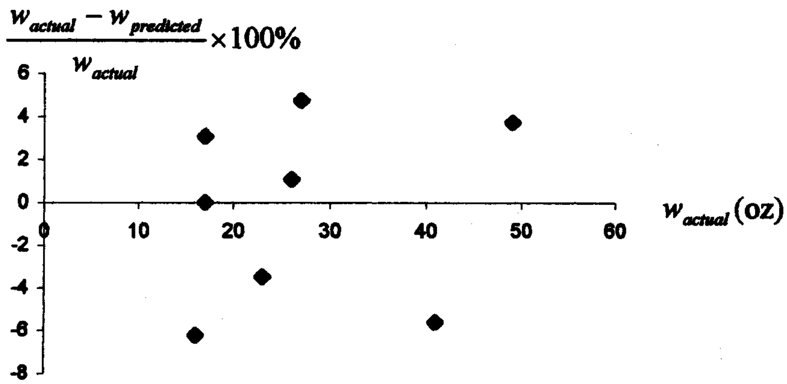


The graphs show that the cubic relationship provides the better model.
Explanation of the model: The unit of board feet is a measure of the volume and, if a tree is modeled as a right circular cone, its volume would be $y = \frac{1}{3}\pi r^2 h$. The girth is the circumference of the tree near the base so that $x = 2\pi r \Rightarrow r = \frac{x}{2\pi}$. If, in addition, we assume that as a tree grows the proportion $\frac{h}{r} = k$, a constant, then we have that $y = \frac{1}{3}\pi\left(\frac{x}{2\pi}\right)^2(kr) = \frac{1}{3}\pi\left(\frac{x}{2\pi}\right)^2\left(\frac{kx}{2\pi}\right) = \frac{k}{24\pi}x^3$, which shows that $y = 0.00436x^3$ is a rational model.

9.

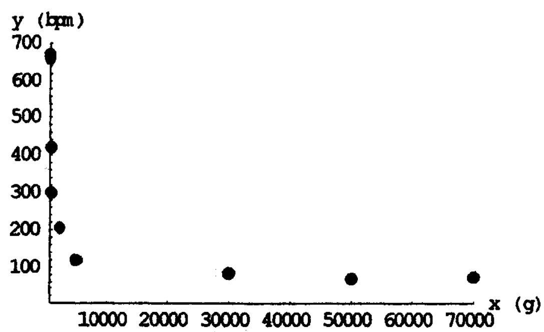


The slope of the regression line is 0.008435, so the model that estimates the weight as a function of L is $w = 0.008435L^3$. The model fits the data reasonably well as demonstrated by the following plot of the relative errors in the weight estimates by the model.

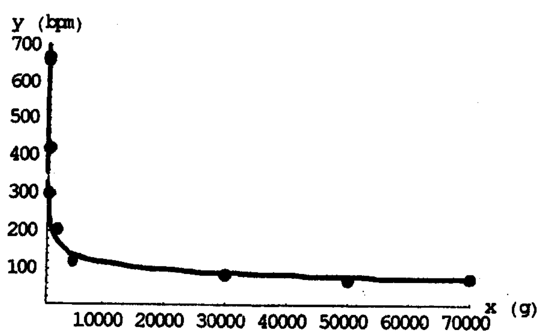


The relative error in the estimated values is always less than 7%.

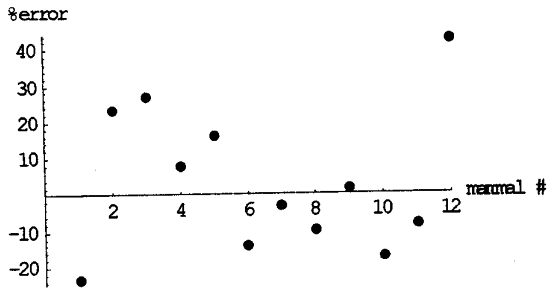
10. The following plot of the data does not include the ox and elephant. However, the data for these mammals are used in the analysis that follows.



There does appear to be a trend. After trying regressions with n = 1, 2, 3, 4, 5, the best fit was found with n = 4. The following graph superimposes the regression function $y = 1150x^{-1/4}$ on the data points.



To test the model, calculate the relative errors (i.e., $\frac{y_{predicted} - y_{actual}}{y_{actual}} \times 100\%$) for all of the mammals in the sample set. These are shown in the following graph.



The errors appear to be random and the largest relative errors are for the two larger animals (i.e., the ox and the elephant) with magnitudes of 92% and 43%, respectively. The model appears to capture a trend in the data, which could be useful in understanding the relationship between mammal size and heart rate; however, it probably would not be useful as a predictive tool.

11. Graph (c). For some drugs, the rate of elimination is proportional to the concentration of the drug in the blood-stream. Graph (c) matches this behavior because the graph falls faster at higher concentrations.

12. Graph (d) or graph (f). Often times, when we begin to learn a new subject, we master the basics quickly at first, but then as the subject becomes more intricate our proficiency increases more slowly. This learning behavior would be described by graph (d). Some subjects have high overhead in terms of learning basic skills and so our proficiency increases slowly at first but, as we acquire the basic skills, our proficiency increases more rapidly. Then, as we reach our intellectual capacity or as our interest wanes, our proficiency will increase more slowly. Graph (f) would match this learning pattern.

13. Graph (c). The rate of decay of radioactive Carbon-14 is proportional to the amount of Carbon-14 present in the artwork. Graph (c) matches this behavior because the graph falls faster at higher amounts.

14. (a) Graph (e). At first, the water velocity is high but as the tank drains the velocity will decrease. When the water level in the tank is high the discharge velocity will decrease slowly, but as time progresses and the water level drops, the discharge velocity will decrease more rapidly.
    (b) Graph (c). Assuming the tank is an upright circular cylinder, the rate at which the water level in the tank falls will be proportional to the rate at which the volume of water in the tank decreases. Also, the rate at which the volume decreases will be proportional to the discharge velocity. Therefore, when the discharge velocity is high at the start, the rate at which the volume decreases will be high and so will the rate of decrease in the water depth. As the discharge velocity decreases, the rate at which the water depth drops will also decrease. This behavior is depicted by graph (c).

15. (a) One possibility: If an item sells for \$p and x is the number of items sold, then the revenue from sales will be $y = px$, and the graph of the revenue function looks like graph (a).
    (b) One possibility: If y is the number of deer in a very large game reserve with unlimited resources to support the deer and x represents the number of years elapsed, then the population would exhibit unconstrained growth over time. In this situation, the population can be modeled by an exponential growth function like $y = y_0 e^{kx}$, where y_0 is the initial deer population, k is a constant, and the growth of the function looks like graph (b).
    (c) One possibility: If y represents the selling price per unit that can be realized for a certain commodity, say grape jelly for example, and x represents the availability of the commodity, then the unit selling price for the commodity is often times inversely proportional to its availability. This relationship can be modeled with a function of the form $y = \frac{y_0}{(x+1)^{\alpha}}$, where y_0 is the unit selling price when no product is available, α is a positive constant, and the graph of the function looks like graph (c).

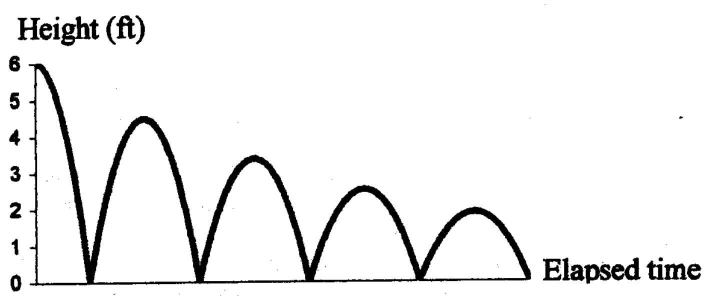
(d) One possibility: Let y represent the speed of your car and x represent the amount of time after you punch the accelerator. At first you will rapidly accelerate but, as the car picks up speed, the rate of acceleration (i.e., the rate at which the car speeds up) decreases. This can be modeled by a function like $y = y_{new} + (y_0 - y_{new})e^{-kx}$, where y_0 is the speed you were traveling when you stepped on it, y_{new} is the new speed you achieve when you are done accelerating, and k is a positive constant (determined in part by the size of your engine and how good your traction is). The graph of this function looks like graph(d).

(e) One possibility: Let y represent the amount you owe on your credit card and x represent the number of monthly payments you have made. At first the amount you owe decreases slowly because most of your payment goes toward paying the monthly interest charge. But, as the amount you owe decreases, the interest charge decreases and your payment makes a bigger difference toward reducing the debt. This can be modeled with a function like the one represented by graph (c).

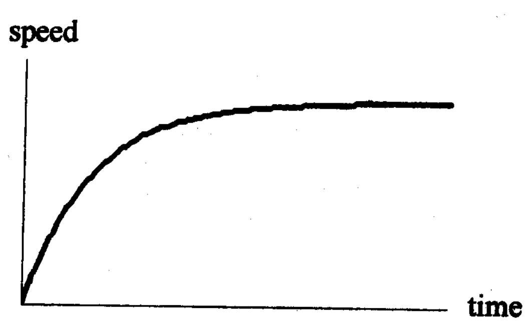
(f) One possibility: Let y represent the number of people in your school who have the flu and let x represent the number of days that have elapsed after the first person gets sick. At first the flu doesn't spread very quickly because there are only a few sick people to pass it on. But, as more people get sick the disease spreads more rapidly. The most volatile mixture is when half the people are sick, because then there are a lot of sick people to spread the disease and a lot of uninfected people who can still catch it. As time continues and more people get sick, there are fewer and fewer people available to catch the flu and the spread of the disease begins to slow down. This behavior can be modeled with a function like the one represented by graph (f).

16. The intensity of light will probable decrease linearly as the number of layers of plastic increases. If I_0 is the intensity with no layers of plastic, then the relationship would be $I = I_0 - kn$, where n is the number of layers and k is a constant.

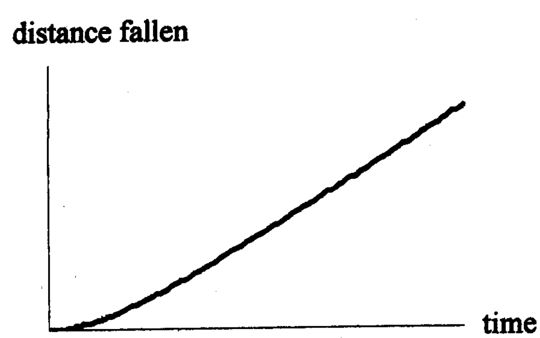
17.



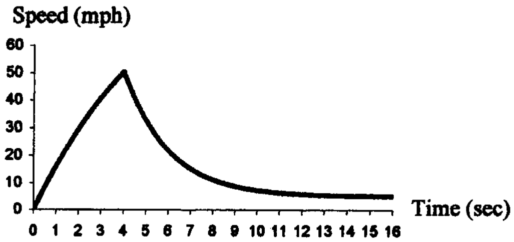
18. (a)



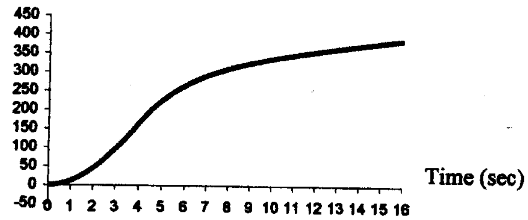
(b)



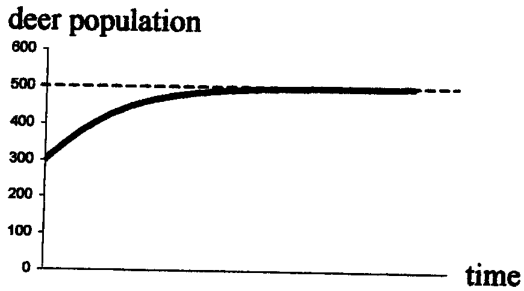
19. (a)



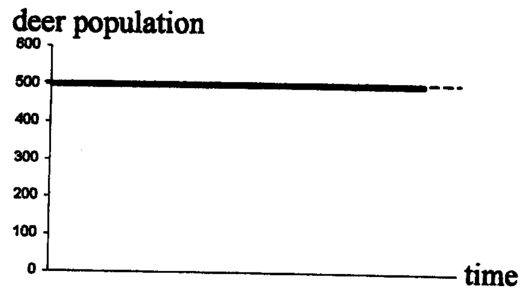
(b)



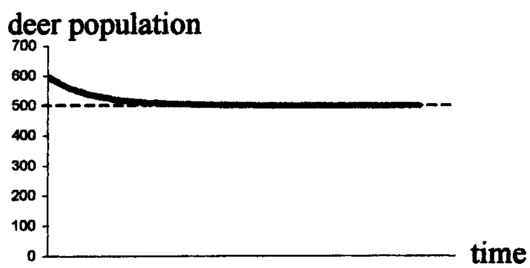
20. (a)



(b)



(c)



21. (a) The graph could represent the angle that a pendulum makes with the vertical as it swings back and forth. The variable y represents the angle and x represents time. Because of friction, the amplitude of the oscillation decays, as depicted by the graph. When y is positive, the pendulum is on one side of the vertical and when y is negative, the pendulum is on the other side.

(b) The graph could represent the angle the playground swing makes with the vertical as a child "pumps" on the swing to get it going. The variable y represents the angle and x represents time. Because the child puts mechanical energy into the system (swing + child), the amplitude of the oscillation grows with time, as depicted by the graph. When y is positive, the swing is on one side of the vertical and when y is negative, the swing is on the other side.

22. Answers will vary. An example follows.

(f) I would like to study the effect that the geometric configuration of a group of four light poles and fixtures would have on the illumination intensity on the ground. More specifically, if four light poles are arranged in a square, how is the light intensity on the ground at the center of the square (where the light intensity is assumed to be minimum) affected by the spacing of the poles? To determine the effect of the pole spacing, I will need to design an experiment to measure the intensity of light at the center of the square pattern as the pole spacing is varied. After collecting the data I would then try to find a mathematical model to fit the data. The parking lot designer could use this model to determine the maximum pole spacing, given the minimum required light intensity on the ground. In addition to the spacing of the poles, some other variables that affect the illumination are the type, size and brightness of the light sources, shadowing by other objects, and the height of the poles.

PRELIMINARY CHAPTER PRACTICE EXERCISES

1. $y = 3(x - 1) + (-6)$
   $y = 3x - 9$

2. $y = -\frac{1}{2}(x + 1) + 2$

   $y = -\frac{1}{2}x + \frac{3}{2}$

3. $x = 0$

4. $m = \frac{-2 - 6}{1 - (-3)} = \frac{-8}{4} = -2$

   $y = -2(x + 3) + 6$

   $y = -2x$

5. $y = 2$

6. $m = \frac{5 - 3}{-2 - 3} = \frac{2}{-5} = -\frac{2}{5}$

   $y = -\frac{2}{5}(x - 3) + 3$

   $y = -\frac{2}{5}x + \frac{21}{5}$

7. $y = -3x + 3$

8. Since $2x - y = -2$ is equivalent to $y = 2x + 2$, the slope of the given line (and hence the slope of the desired line) is 2.
   $y = 2(x - 3) + 1$
   $y = 2x - 5$

9. Since $4x + 3y = 12$ is equivalent to $y = -\frac{4}{3}x + 4$, the slope of the given line (and hence the slope of the desired line) is $-\frac{4}{3}$.

   $y = -\frac{4}{3}(x - 4) - 12$

   $y = -\frac{4}{3}x - \frac{20}{3}$

10. Since $3x - 5y = 1$ is equivalent to $y = \frac{3}{5}x - \frac{1}{5}$, the slope of the given line is $\frac{3}{5}$ and the slope of the perpendicular line is $-\frac{5}{3}$.

$y = -\frac{5}{3}(x+2) - 3$

$y = -\frac{5}{3}x - \frac{19}{3}$

11. Since $\frac{1}{2}x + \frac{1}{3}y = 1$ is equivalent to $y = -\frac{3}{2}x + 3$, the slope of the given line is $-\frac{3}{2}$ and the slope of the perpendicular line is $\frac{2}{3}$.

$y = \frac{2}{3}(x+1) + 2$

$y = \frac{2}{3}x + \frac{8}{3}$

12. The line passes through $(0, -5)$ and $(3, 0)$.

$m = \frac{0-(-5)}{3-0} = \frac{5}{3}$

$y = \frac{5}{3}x - 5$

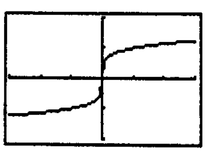
13. The area is $A = \pi r^2$ and the circumference is $C = 2\pi r$. Thus, $r = \frac{C}{2\pi} \Rightarrow A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$.

14. The surface area is $S = 4\pi r^2 \Rightarrow r = \left(\frac{S}{4\pi}\right)^{1/2}$. The volume is $V = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$. Substitution into the formula for surface area gives $S = 4\pi r^2 = 4\pi\left(\frac{3V}{4\pi}\right)^{2/3}$.

15. The coordinates of a point on the parabola are (x, x^2). The angle of inclination θ joining this point to the origin satisfies the equation $\tan\theta = \frac{x^2}{x} = x$. Thus the point has coordinates $(x, x^2) = (\tan\theta, \tan^2\theta)$.

16. $\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{h}{500} \Rightarrow h = 500 \tan\theta$ ft.

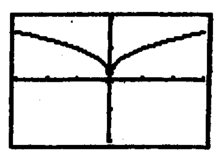
17.



[−3, 3] by [−2, 2]

Symmetric about the origin.

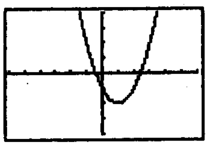
18.



[−3, 3] by [−2, 2]

Symmetric about the y-axis.

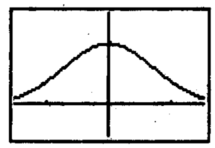
19.



$[-6, 6]$ by $[-4, 4]$

Neither

20.



$[-1.5, 1.5]$ by $[-0.5, 1.5]$

Symmetric about the y-axis.

21. $y(-x) = (-x)^2 + 1 = x^2 + 1 = y(x)$
Even

22. $y(-x) = (-x)^5 - (-x) = -x^5 + x^3 + x = -y(x)$
Odd

23. $y(-x) = 1 - \cos(-x) = 1 - \cos x = y(x)$
Even

24. $y(-x) = \sec(-x) \tan(-x)$

$$= \frac{\sin(-x)}{\cos^2(-x)} = \frac{-\sin x}{\cos^2 x}$$

$$= -\sec x \tan x = -y(x)$$

Odd

25. $y(-x) = \dfrac{(-x)^4 + 1}{(-x)^3 - 2(-x)} = \dfrac{x^4 + 1}{-x^3 + 2x} = -\dfrac{x^4 + 1}{x^3 - 2x} = -y(x)$

Odd

26. $y(-x) = 1 - \sin(-x) = 1 + \sin x$
Neither even nor odd

27. $y(-x) = -x + \cos(-x) = -x + \cos x$
Neither even nor odd

28. $y(-x) = \sqrt{(-x)^4 - 1} = \sqrt{x^4 - 1} = y(x)$
Even

29. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) Since $|x|$ attains all nonnegative values, the range is $[-2, \infty)$.

30. (a) Since the square root requires $1 - x \geq 0$, the domain is $(-\infty, 1]$.

(b) Since $\sqrt{1 - x}$ attains all nonnegative values, the range is $[-2, \infty)$.

31. (a) Since the square root requires $16 - x^2 \geq 0$, the domain is $[-4, 4]$

(b) For values of x in the domain, $0 \leq 16 - x^2 \leq 16$, so $0 \leq \sqrt{16 - x^2} \leq 4$. The range is $[0, 4]$.

32. (a) The function is defined for all value os x, so the domain is $(-\infty, \infty)$.

(b) Since 3^{2-x} attains all possible values, the range is $(1, \infty)$.

33. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) Since $2e^{-x}$ attains all positive values, the range is $(-3, \infty)$.

34. (a) The function is equivalent to $y = \tan 2x$, so we require $2x \neq \frac{k\pi}{2}$ for odd integers k. The domain is given by $x \neq \frac{k\pi}{4}$ for odd integers k.

(b) Since the tangent function attains all values, the range is $(-\infty, \infty)$.

35. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The sine function attains values from -1 to 1, so $-2 \leq 2\sin(3x + \pi) \leq 2$, and hence $-3 \leq 2\sin(3x + \pi) - 1 \leq 1$. The range is $[-3, 1]$.

36. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The function is equivalent to $y = \sqrt[5]{x^2}$, which attains all nonnegative values. The range is $[0, \infty)$.

37. (a) The logarithm requires $x - 3 > 0$, so the domain is $(3, \infty)$.

(b) The logarithm attains all real values, so the range is $(-\infty, \infty)$.

38. (a) The function is defined for all values of x, so the domain is $(-\infty, \infty)$.

(b) The cube root attains all real values, so the range is $(-\infty, \infty)$.

39. (a) The function is defined for $-4 \leq x \leq 4$, so the domain is $[-4, 4]$.

(b) The function is equivalent to $y = \sqrt{|x|}$, $-4 \leq x \leq 4$, which attains values from 0 to 2 for x in the domain. The range is $[0, 2]$.

40. (a) The function is defined for $-2 \leq x \leq 2$, so the domain is $[-2, 2]$.

(b) The range is $[-1, 1]$.

41. First piece: Line through $(0, 1)$ and $(1, 0)$

$$m = \frac{0-1}{1-0} = \frac{-1}{1} = -1$$

$y = -x + 1$ or $1 - x$

Second piece: Line through $(1, 1)$ and $(2, 0)$

$$m = \frac{0-1}{2-1} = \frac{-1}{1} = -1$$

$y = -(x - 1) + 1$

$y = -x + 2$ or $2 - x$

$$f(x) = \begin{cases} 1 - x, & 0 \leq x < 1 \\ 2 - x, & 1 \leq x \leq 2 \end{cases}$$

42. First piece: Line through $(0, 0)$ and $(2, 5)$

$$m = \frac{5-0}{2-0} = \frac{5}{2}$$

$y = \frac{5}{2}x$

Second piece: Line through $(2, 5)$ and $(4, 0)$

$$m = \frac{0-5}{4-2} = \frac{-5}{2} = -\frac{5}{2}$$

$y = -\frac{5}{2}(x - 2) + 5$

$y = -\frac{5}{2}x + 10$ or $10 - \frac{5x}{2}$

$$f(x) = \begin{cases} \frac{5x}{2}, & 0 \leq x < 2 \\ 10 - \frac{5x}{2}, & 2 \leq x \leq 4 \end{cases}$$

(Note: $x = 2$ can be included on either piece.)

43. (a) $(f \circ g)(-1) = f(g(-1)) = f\left(\frac{1}{\sqrt{-1+2}}\right) = f(1) = \frac{1}{1} = 1$

(b) $(g \circ f)(2) = g(f(2)) = g\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1/2+2}} = \frac{1}{\sqrt{2.5}}$ or $\sqrt{\frac{2}{5}}$

(c) $(f \circ f)(x) = f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{1/x} = x,\ x \neq 0$

(d) $(g \circ g)(x) = g(g(x)) = g\left(\frac{1}{\sqrt{x+2}}\right) = \frac{1}{\sqrt{1/\sqrt{x+2}+2}}$

$$= \frac{\sqrt[4]{x+2}}{\sqrt{1+2\sqrt{x+2}}}$$

44. (a) $(f \circ g)(-1) = f(g(-1))$

$= f(\sqrt[3]{-1+1})$

$= f(0) = 2 - 0 = 2$

(b) $(g \circ f)(2) g(f(2)) = g(2-2) = g(0) = \sqrt[3]{0+1} = 1$

(c) $(f \circ f)(x) = f(f(x)) = f(2-x) = 2-(2-x) = x$

(d) $(g \circ g)(x) = g(g(x)) = g(\sqrt[3]{x+1}) = \sqrt[3]{\sqrt[3]{x+1}+1}$

45. (a) $(f \circ g)(x) = f(g(x))$

$= f(\sqrt{x+2})$

$= 2 - (\sqrt{x+2})^2$

$= -x,\ x \geq -2$

$(g \circ f)(x) = g(f(x))$

$= g(2 - x^2)$

$= \sqrt{(2-x^2)+2} = \sqrt{4-x^2}$

(b) Domain of $f \circ g$: $[-2, \infty)$

Domain of $g \circ f$: $[-2, 2]$

(c) Range of $f \circ g$: $(-\infty, 2]$

Range of $g \circ f$: $[0, 2]$

46. (a) $(f \circ g)(x) = f(g(x))$

$= f(\sqrt{1-x})$

$= \sqrt{\sqrt{1-x}}$

$= \sqrt[4]{1-x}$

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sqrt{1-\sqrt{x}}$

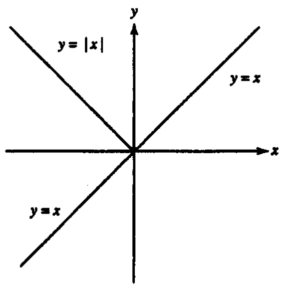
(b) Domain of $f \circ g$: $(-\infty, 1]$

Domain of $g \circ f$: $[0, 1]$

(c) Range of $f \circ g$: $[0, \infty)$

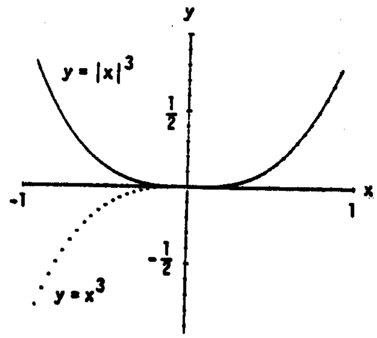
Range of $g \circ f$: $[0, 1]$

47.



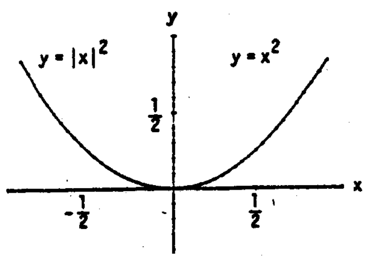
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

48.



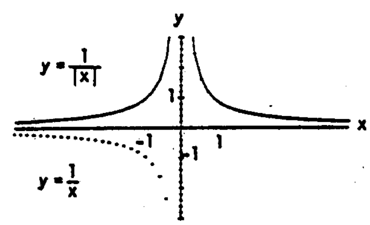
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

49.



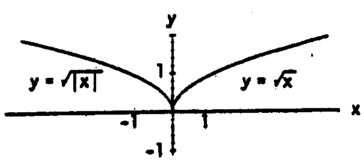
It does not change the graph.

50.



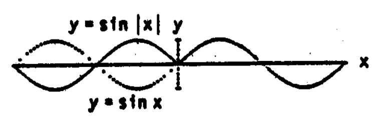
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

51.



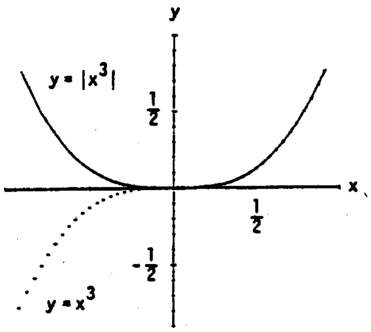
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

52.



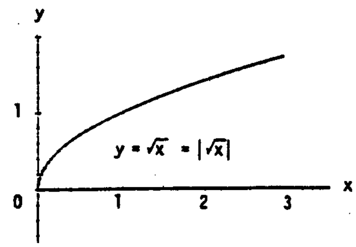
The graph of $f_2(x) = f_1(|x|)$ is the same as the graph of $f_1(x)$ to the right of the y-axis. The graph of $f_2(x)$ to the left of the y-axis is the reflection of $y = f_1(x)$, $x \geq 0$ across the y-axis.

53.



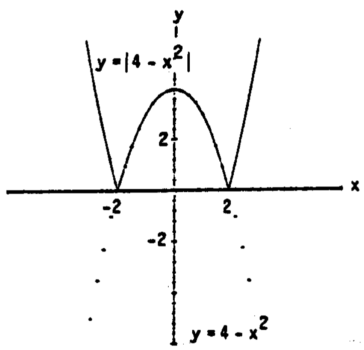
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

54.



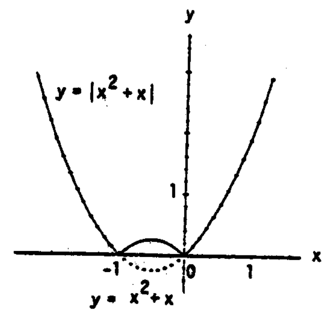
It does not change the graph.

55.



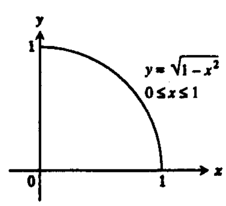
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

56.



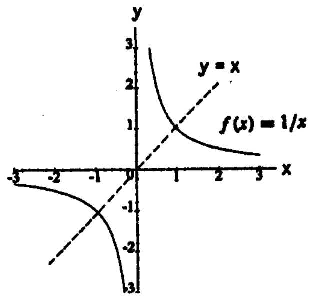
Whenever $g_1(x)$ is positive, the graph of $y = g_2(x) = |g_1(x)|$ is the same as the graph of $y = g_1(x)$. When $g_1(x)$ is negative, the graph of $y = g_2(x)$ is the reflection of the graph of $y = g_1(x)$ across the x-axis.

57. (a) The graph is symmetric about $y = x$.



(b) $y = \sqrt{1-x^2} \Rightarrow y^2 = 1 - x^2 \Rightarrow x^2 = 1 - y^2 \Rightarrow x = \sqrt{1-y^2} \Rightarrow y = \sqrt{1-x^2} = f^{-1}(x)$

58. The graph is symmetric about $y = x$.



(b) $y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \Rightarrow y = \frac{1}{x} = f^{-1}(x)$

59. (a) $y = 2 - 3x \rightarrow 3x = 2 - y \rightarrow x = \frac{2-y}{3}$.

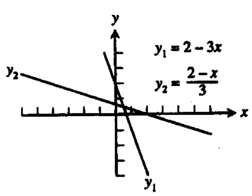
Interchange x and y: $y = \frac{2-x}{3} \rightarrow f^{-1}(x) = \frac{2-x}{3}$

Verify.

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f\left(\frac{2-x}{3}\right) = 2 - 3\left(\frac{2-x}{3}\right) = 2 - (2-x) = x$

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}(2-3x) = \frac{2-(2-3x)}{3} = \frac{3x}{3} = x$

(b)



60. (a) $y = (x+2)^2,\ x \geq -2 \rightarrow \sqrt{y} = x + 2 \rightarrow x = \sqrt{y} - 2$.

Interchange x and y: $y = \sqrt{x} - 2 \rightarrow f^{-1}(x) = \sqrt{x} - 2$

Verify.

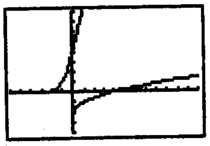
For $x \geq 0$ (the domain of f^{-1})

$(f \circ f^{-1})(x) = f(f^{-1}(x)) = f(\sqrt{x} - 2) = [(\sqrt{x} - 2) + 2]^2 = (\sqrt{x})^2 = x$

For $x \geq -2$ (the domain of f),

$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = f^{-1}((x+2)^2) = \sqrt{(x+2)^2} - 2 = |x+2| - 2 = (x+2) - 2 = x$

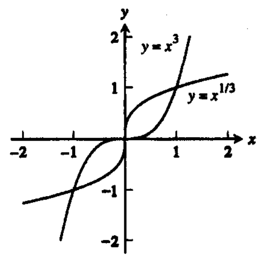
(b)



[−6, 12] by [−4, 8]

61. (a) $f(g(x)) = \left(\sqrt[3]{x}\right)^3 = x$, $g(f(x)) = \sqrt[3]{x^3} = x$

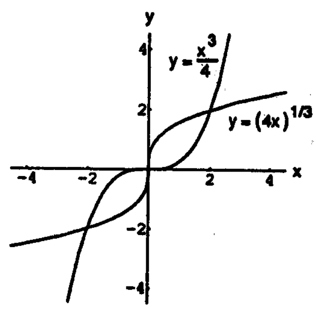
(b)



62. (a) $h(k(x)) = \frac{1}{4}\left((4x)^{1/3}\right)^3 = x$,

$k(h(x)) = \left(4 \cdot \frac{x^3}{4}\right)^{1/3} = x$

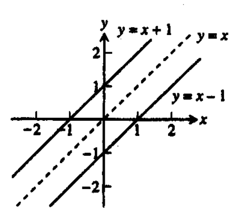
(b)



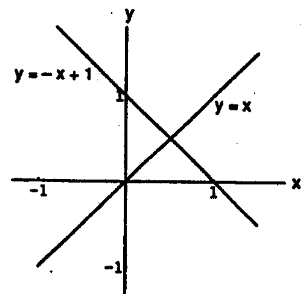
63. (a) $y = x + 1 \Rightarrow x = y - 1 \Rightarrow f^{-1}(x) = x - 1$

(b) $y = x + b \Rightarrow x = y - b \Rightarrow f^{-1}(x) = x - b$

(c) Their graphs will be parallel to one another and lie on opposite sides of the line $y = x$ equidistant from that line.



64. (a) $y = -x + 1 \Rightarrow x = -y + 1 \Rightarrow f^{-1}(x) = 1 - x$;
the lines intersect at a right angle
(b) $y = -x + b \Rightarrow x = -y + b \Rightarrow f^{-1}(x) = b - x$;
the lines intersect at a right angle
(c) f is its own inverse



65. $x = 2.71828182846$ (using a TI-92 Plus calculator).

66. $e^{\ln x} = x$ and $\ln(e^x) = x$ for all $x > 0$

67. (a) $e^{\ln 7.2} = 7.2$ (b) $e^{-\ln x^2} = \frac{1}{e^{\ln x^2}} = \frac{1}{x^2}$ (c) $e^{\ln x - \ln y} = e^{\ln(x/y)} = \frac{x}{y}$

68. (a) $e^{\ln(x^2+y^2)} = x^2 + y^2$ (b) $e^{-\ln 0.3} = \frac{1}{e^{\ln 0.3}} = \frac{1}{0.3} = \frac{10}{3}$ (c) $e^{\ln \pi x - \ln 2} = e^{\ln(\pi x/2)} = \frac{\pi x}{2}$

69. (a) $2 \ln \sqrt{e} = 2 \ln e^{1/2} = (2)\left(\frac{1}{2}\right) \ln e = 1$ (b) $\ln(\ln e^e) = \ln(e \ln e) = \ln e = 1$
(c) $\ln e^{\left(-x^2-y^2\right)} = \left(-x^2 - y^2\right) \ln e = -x^2 - y^2$

70. (a) $\ln\left(e^{\sec\theta}\right) = (\sec\theta)(\ln e) = \sec\theta$ (b) $\ln e^{\left(e^x\right)} = \left(e^x\right)(\ln e) = e^x$
(c) $\ln\left(e^{2\ln x}\right) = \ln\left(e^{\ln x^2}\right) = \ln x^2 = 2 \ln x$

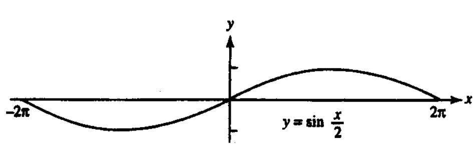
71. Using a calculator, $\sin^{-1}(0.6) \approx 0.6435$ radians or $36.8699°$.

72. Using a calculator, $\tan^{-1}(-2.3) \approx -1.1607$ radians or $-66.5014°$.

73. Since $\cos\theta = \frac{3}{7}$ and $0 \le \theta \le \pi$, $\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{3}{7}\right)^2} = \sqrt{\frac{40}{49}} = \frac{\sqrt{40}}{7}$. Therefore,
$\sin\theta = \frac{\sqrt{40}}{7}$, $\cos\theta = \frac{3}{7}$, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{40}}{3}$, $\cot\theta = \frac{1}{\tan\theta}, = \frac{3}{\sqrt{40}}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{7}{3}$, $\csc\theta = \frac{1}{\sin\theta} = \frac{7}{\sqrt{40}}$

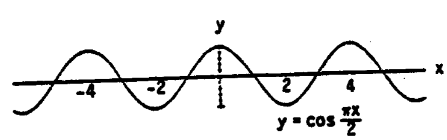
74. (a) Note that $\sin^{-1}(-0.2) \approx -0.2014$. In $[0, 2\pi)$, the solutions are $x = \pi - \sin^{-1}(-0.2) \approx 3.3430$ and $x = \sin^{-1}(-0.2) + 2\pi \approx 6.0818$.
(b) Since the period of sin x is 2π, the solutions are $x \approx 3.3430 + 2k\pi$ and $x \approx 6.0818 + 2k\pi$, k any integer.

75.



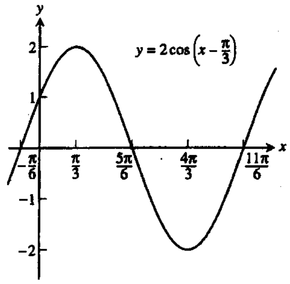
period $= 4\pi$

76.



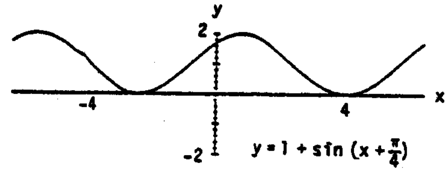
period $= 4$

77.



period $= 2\pi$

78.



period $= 2\pi$

79. (a) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{b}{2} \Rightarrow b = 2 \sin \frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$. By the theorem of Pythagoras, $a^2 + b^2 = c^2 \Rightarrow a = \sqrt{c^2 - b^2} = \sqrt{4-3} = 1$.

(b) $\sin B = \sin \frac{\pi}{3} = \frac{b}{c} = \frac{2}{c} \Rightarrow c = \frac{2}{\sin \frac{\pi}{3}} = \frac{2}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{4}{\sqrt{3}}$. Thus, $a = \sqrt{c^2 - b^2} = \sqrt{\left(\frac{4}{\sqrt{3}}\right)^2 - (2)^2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$.

80. (a) $\sin A = \frac{a}{c} \Rightarrow a = c \sin A$

(b) $\tan A = \frac{a}{b} \Rightarrow a = b \tan A$

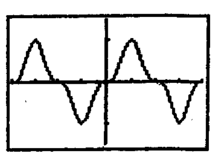
81. (a) $\tan B = \frac{b}{a} \Rightarrow a = \frac{b}{\tan B}$

(b) $\sin A = \frac{a}{c} \Rightarrow c = \frac{a}{\sin A}$

82. (a) $\sin A = \frac{a}{c}$

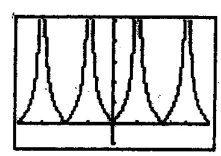
(c) $\sin A = \frac{a}{c} = \frac{\sqrt{c^2 - b^2}}{c}$

83. Since sin x has period 2π, $\sin^3(x + 2\pi) = \sin^3(x)$. This function has period 2π. A graph shows that no smaller number works for the period.



[−2π, 2π] by [−1.5, 1.5]

84. Since tan x has period π, $|\tan(x + \pi)| = |\tan x|$. This function has period π. A graph shows that no smaller number works for the period.



[−2π, 2π] by [−1, 5]

85. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$

86. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$

87. $\sin \frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin \frac{\pi}{4} \cos \frac{\pi}{3} + \cos \frac{\pi}{4} \sin \frac{\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$

88. $\cos \frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos \frac{\pi}{4} \cos \frac{2\pi}{3} - \sin \frac{\pi}{4} \sin \frac{2\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2} + \sqrt{6}}{4}$

89. (a) $\frac{\pi}{6}$ (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{3}$

90. (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{5\pi}{6}$

91. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$

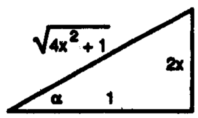
92. (a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{6}$ (c) $\frac{\pi}{3}$

93. $\sec\left(\cos^{-1}\frac{1}{2}\right) = \sec\left(\frac{\pi}{3}\right) = 2$

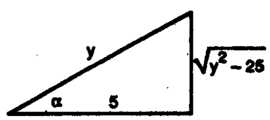
94. $\cot\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = \cot\left(-\frac{\pi}{3}\right) = -\frac{1}{\sqrt{3}}$

95. $\tan\left(\sec^{-1} 1\right) + \sin\left(\csc^{-1}(-2)\right) = \tan\left(\cos^{-1}\frac{1}{1}\right) + \sin\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan(0) + \sin\left(-\frac{\pi}{6}\right) = 0 + \left(-\frac{1}{2}\right) = -\frac{1}{2}$

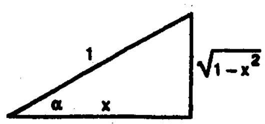
96. $\sec\left(\tan^{-1} 1 + \csc^{-1} 1\right) = \sec\left(\frac{\pi}{4} + \sin^{-1}\frac{1}{1}\right) = \sec\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$

97. $\alpha = \tan^{-1} 2x$ indicates the diagram 

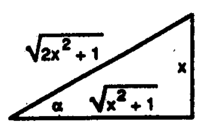
$\Rightarrow \sec\left(\tan^{-1} 2x\right) = \sec\alpha = \sqrt{4x^2+1}$

98. $\alpha = \sec^{-1}\frac{y}{5}$ indicates the diagram 

$\Rightarrow \tan\left(\sec^{-1}\frac{y}{5}\right) = \tan\alpha = \frac{\sqrt{y^2-25}}{5}$

99. $\alpha = \cos^{-1} x$ indicates the diagram 

$\Rightarrow \tan\left(\cos^{-1} x\right) = \tan\alpha = \frac{\sqrt{1-x^2}}{x}$

100. $\alpha = \tan^{-1}\frac{x}{\sqrt{x^2+1}}$ indicates the diagram 

$\Rightarrow \sin\left(\tan^{-1}\frac{x}{\sqrt{x^2+1}}\right) = \sin\alpha = \frac{x}{\sqrt{2x^2+1}}$

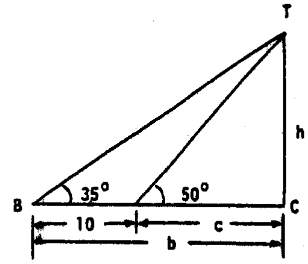
101. (a) Defined; there is an angle whose tangent is 2.
(b) Not defined; there is no angle whose cosine is 2.

102. (a) Not defined; there is no angle whose cosecant is $\frac{1}{2}$.
(b) Defined; there is an angle whose cosecant is 2.

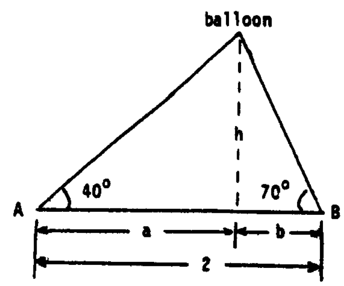
103. (a) Not defined; there is no angle whose secant is 0.
(b) Not defined; there is no angle whose sine is $\sqrt{2}$.

104. (a) Defined; there is an angle whose cotangent is $-\frac{1}{2}$.
(b) Not defined; there is no angle whose cosine is -5.

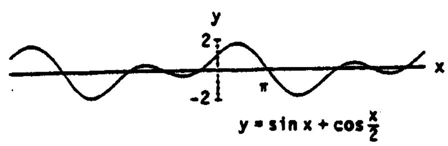
105. Let h = height of vertical pole, and let b and c denote the distances of points B and C from the base of the pole, measured along the flat ground, respectively. Then, $\tan 50^\circ = \frac{h}{c}$, $\tan 35^\circ = \frac{h}{b}$, and $b - c = 10$. Thus, $h = c \tan 50^\circ$ and $h = b \tan 35^\circ = (c + 10)\tan 35^\circ$
$\Rightarrow c \tan 50^\circ = (c + 10)\tan 35^\circ \Rightarrow c\,(\tan 50^\circ - \tan 35^\circ) = 10 \tan 35^\circ$
$\Rightarrow c = \frac{10 \tan 35^\circ}{\tan 50^\circ - \tan 35^\circ} \Rightarrow h = c \tan 50^\circ = \frac{10 \tan 35^\circ \tan 50^\circ}{\tan 50^\circ - \tan 35^\circ}$
≈ 16.98 m.



106. Let h = height of balloon above ground. From the figure at the right, $\tan 40^\circ = \frac{h}{a}$, $\tan 70^\circ = \frac{h}{b}$, and $a + b = 2$. Thus,
$h = b \tan 70^\circ \Rightarrow h = (2 - a)\tan 70^\circ$ and $h = a \tan 40^\circ$
$\Rightarrow (2 - a)\tan 70^\circ = a \tan 40^\circ \Rightarrow a(\tan 40^\circ + \tan 70^\circ) = 2 \tan 70^\circ$
$\Rightarrow a = \frac{2 \tan 70^\circ}{\tan 40^\circ + \tan 70^\circ} \Rightarrow h = a \tan 40^\circ = \frac{2 \tan 70^\circ \tan 40^\circ}{\tan 40^\circ + \tan 70^\circ}$
≈ 1.3 km.

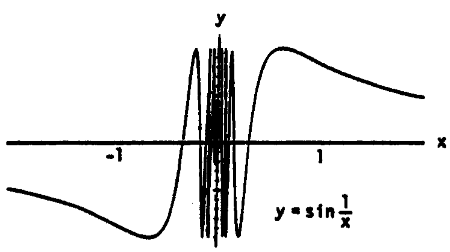


107. (a)



(b) The period appears to be 4π.
(c) $f(x + 4\pi) = \sin(x + 4\pi) + \cos\left(\frac{x + 4\pi}{2}\right) = \sin(x + 2\pi) + \cos\left(\frac{x}{2} + 2\pi\right) = \sin x + \cos \frac{x}{2}$
since the period of sine and cosine is 2π. Thus, f(x) has period 4π.

108. (a)

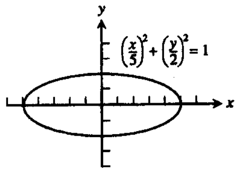


(b) $D = (-\infty, 0) \cup (0, \infty)$; $R = [-1, 1]$
(c) f is not periodic. Suppose f has period p. Then $f\left(\frac{1}{2\pi} + kp\right) = f\left(\frac{1}{2\pi}\right) = \sin 2\pi = 0$ for all integers k.
Choose k so large that $\frac{1}{2\pi} + kp > \frac{1}{\pi} \Rightarrow 0 < \frac{1}{(1/2\pi) + kp} < \pi$. But then $f\left(\frac{1}{2\pi} + kp\right) = \sin\left(\frac{1}{(1/2\pi) + kp}\right) > 0$

which is a contradiction. Thus f has no period, as claimed.

109. (a) Substituting $\cos t = \frac{x}{5}$ and $\sin t = \frac{y}{2}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation $\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$. The entire ellipse is traced by the curve.

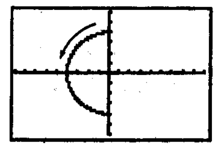
(b)



Initial point: $(5,0)$
Terminal point: $(5,0)$
The ellipse is traced exactly once in a counterclockwise direction starting and ending at the point $(5,0)$.

110. (a) Substituting $\cos t = \frac{x}{4}$ and $\sin t = \frac{y}{4}$ in the identity $\cos^2 t + \sin^2 t = 1$ gives the Cartesian equation $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$, or $x^2 + y^2 = 16$. The left half of the circle is traced by the parametrized curve.

(b)

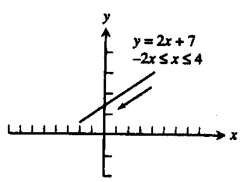


$[-9, 9]$ by $[-6, 6]$

Initial point: $(0,4)$
Terminal point: None (since the endpoint $\frac{3\pi}{2}$ is not included in the t-interval)
The semicircle is traced in a counterclockwise direction starting at $(0,4)$ and extending to, but not including, $(0,-4)$.

111. (a) Substituting $t = 2 - x$ into $y = 11 - 2t$ gives the Cartesian equation $y = 11 - 2(2 - x)$, or $y = 2x + 7$. The part of the line from $(4,15)$ to $(-2,3)$ is traced by the parametrized curve.

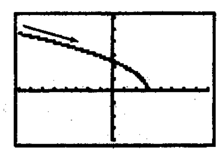
(b)



Initial point: $(4,15)$
Terminal point: $(-2,3)$
The line segment is traced from right to left starting at $(4,15)$ and ending at $(-2,3)$.

112. (a) Substituting $t = x - 1$ into $y = (t-1)^2$ gives the Cartesian equation $y = (x - 1 - 1)^2$, or $y = (x-2)^2$. The part of the parabola for $x \leq 2$ is traced by the parametrized curve.

(b)



$[-8, 8]$ by $[-4, 6]$

Initial point: None
Terminal point: $(3,0)$
The curve is traced from left to right ending at the point $(3,0)$.

113. (a) For simplicity, we assume that x and y are linear functions of t, and that the point (x,y) starts at $(-2,5)$ for $t = 0$ and ends at $(4,3)$ for $t = 1$. Then $x = f(t)$, where $f(0) = -2$ and $f(1) = 4$. Since $\text{slope} = \frac{\Delta x}{\Delta t} = \frac{4-(-2)}{1-0} = 6$, $x = f(t) = 6t - 2 = -2 + 6t$. Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 3$.

Since slope $= \frac{\Delta y}{\Delta t} = \frac{3-5}{1-0} = -2$, $y = g(t) = -2t + 5 = 5 - 2t$. One possible parametrization is: $x = -2 + 6t$, $y = 5 - 2t$, $0 \le t \le 1$.

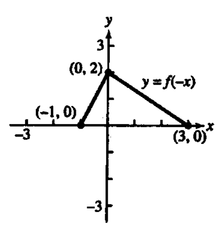
114. For simplicity, we assume that x and y are linear functions of t, and that the point (x, y) passes through $(-3, -2)$ for $t = 0$ and $(4, -1)$ for $t = 1$. Then $x = f(t)$, where $f(0) = -3$ and $f(1) = 4$. Since slope $= \frac{\Delta x}{\Delta t} = \frac{4-(-3)}{1-0} = 7$, $x = f(t) = 7t - 3 = -3 + 7t$. Also, $y = g(t)$, where $g(0) = -2$ and $g(1) = -1$. Since slope $= \frac{\Delta y}{\Delta t} = \frac{-1-(-2)}{1-0} = 1$, $y = g(t) = t - 2 = -2 + t$. One possible parametrization is: $x = -3 + 7t$, $y = -2 + t$, $-\infty < t < \infty$.

115. For simplicity, we assume that x and y are linear functions of t, and that the point (x, y) starts at (2, 5) for $t = 0$ and passes through $(-1, 0)$ for $t = 1$. Then $x = f(t)$, where $f(0) = 2$ and $f(1) = -1$. Since slope $= \frac{\Delta x}{\Delta t} = \frac{-1-2}{1-0} = -3$, $x = f(t) = -3t + 2 = 2 - 3t$. Also, $y = g(t)$, where $g(0) = 5$ and $g(1) = 0$. Since slope $= \frac{\Delta y}{\Delta t} = \frac{0-5}{1-0} = -5$, $y = g(t) = -5t + 5 = 5 - 5t$. One possible parametrization is: $x = 2 - 3t$, $y = 5 - 5t$, $t \ge 0$.

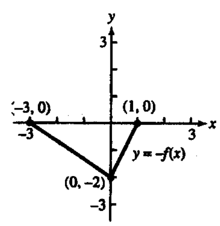
116. One possible parametrization is: $x = t$, $y = t(t-4)$, $t \le 2$.

PRELIMINARY CHAPTER ADDITIONAL EXERCISES–THEORY, EXAMPLES, APPLICATIONS

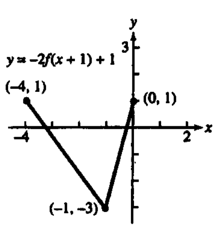
1. (a) The given graph is reflected about the y-axis.



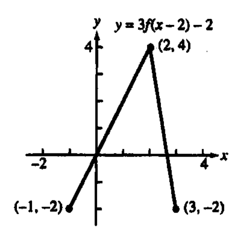
(b) The given graph is reflected about the x-axis.



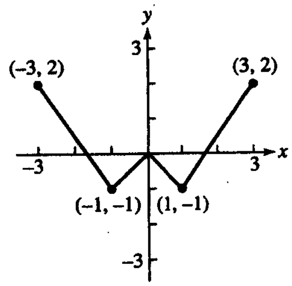
(c) The given graph is shifted left 1 unit, stretched vertically by a factor of 2, reflected about the x-axis, and then shifted upward 1 unit.



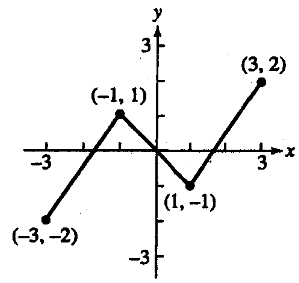
(d) The given graph is shifted right 2 units, stretched vertically by a factor of 3, and then shifted downward 2 units.



2. (a)

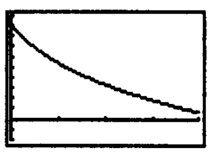


(b)



3. (a) $y = 100{,}000 - 10{,}000x,\ 0 \le x \le 10$
   (b)
$$\begin{aligned} y &= 55{,}000 \\ 100{,}000 - 10{,}000x &= 55{,}000 \\ -10{,}000x &= -45{,}000 \\ x &= 4.5 \end{aligned}$$
   The value is \$55,000 after 4.5 years.

4. (a) $f(0) = 90$ units
   (b) $f(2) = 90 - 52 \ln 3 \approx 32.8722$ units
   (c)

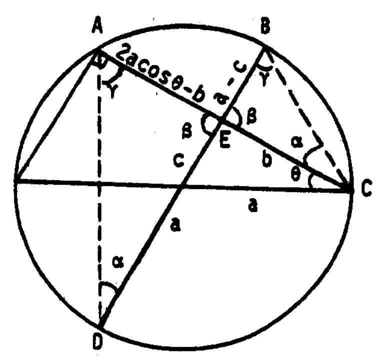


[0, 4] by [−20, 100]

5. $1500(1.08)^t = 5000 \to 1.08^t = \frac{5000}{1500} = \frac{10}{3} \to \ln(1.08)^t = \ln \frac{10}{3} \to t \ln 1.08 = \ln \frac{10}{3} \to t = \frac{\ln(10/3)}{\ln 1.08} \approx 15.6439$

   It will take about 15.6439 years. (If the bank only pays interest at the end of the year, it will take 16 years.)

6. The angles labeled γ in the accompanying figure are equal since both angles subtend arc CD. Similarly, the two angles labeled α are equal since they both subtend arc AB. Thus, triangles AED and BEC are similar which implies $\frac{a-c}{b} = \frac{2a\cos\theta - b}{a+c} \Rightarrow (a-c)(a+c) = b(2a\cos\theta - b)$ $\Rightarrow a^2 - c^2 = 2ab\cos\theta - b^2 \Rightarrow c^2 = a^2 + b^2 - 2ab\cos\theta$.



7. As in the proof of the law of sines of Section P.5, Exercise 35, $ah = bc \sin A = ab \sin C = ac \sin B$ $\Rightarrow$ the area of ABC $= \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}ah = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$.

8. (a) The coordinates of P are $\left(\frac{a+0}{2}, \frac{b+0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. Thus the slope of OP $= \frac{\Delta y}{\Delta x} = \frac{b/2}{a/2} = \frac{b}{a}$.

   (b) The slope of AB $= \frac{b-0}{0-a} = -\frac{b}{a}$. The line segments AB and OP are perpendicular when the product of their slopes is $-1 = \left(\frac{b}{a}\right)\left(-\frac{b}{a}\right) = -\frac{b^2}{a^2}$. Thus, $b^2 = a^2 \Rightarrow a = b$ (since both are positive). Therefore, AB is perpendicular to OP when $a = b$.

9. Triangle ABD is an isosceles right triangle with its right angle at B and an angle of measure $\frac{\pi}{4}$ at A. We therefore have $\frac{\pi}{4} = \angle DAB = \angle DAE + \angle CAB = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$.

10. $\ln x^{(x^x)} = x^x \ln x$ and $\ln\left(x^x\right)^x = x \ln x^x = x^2 \ln x$; then, $x^x \ln x = x^2 \ln x \Rightarrow x^x = x^2 \Rightarrow x \ln x = 2 \ln x$ $\Rightarrow x = 2$. Therefore, $x^{(x^x)} = \left(x^x\right)^x$ when $x = 2$.

11. (a) If f is even, then $f(x) = f(-x)$ and $h(-x) = g(f(-x)) = g(f(x)) = h(x)$.
    If f is odd, then $f(-x) = -f(x)$ and $h(-x) = g(f(-x)) = g(-f(x)) = g(f(x)) = h(x)$ because g is even.
    If f is neither, then h may not be even. For example, if $f(x) = x^2 + x$ and $g(x) = x^2$, then $h(x) = x^4 + 2x^3 + x^2$ and $h(-x) = x^4 - 2x^3 + x^2 \neq h(x)$. Therefore, h need not be even.

    (b) No, h is not always odd. Let $g(t) = t$ and $f(x) = x^2$. Then, $h(x) = g(f(x)) = f(x) = x^2$ is even although g is odd.
    If f is odd, then $f(-x) = -f(x)$ and $h(-x) = g(f(-x)) = g(-f(x)) = -g(f(x)) = -h(x)$ because g is odd.
    In this case, h is odd. However, if f is even, as in the above counterexample, we see that h need not be odd.

12. $A(t) = A_0 e^{rt}$; $A(t) = 2A_0 \Rightarrow 2A_0 = A_0 e^{rt} \Rightarrow e^{rt} = 2 \Rightarrow rt = \ln 2 \Rightarrow t = \frac{\ln 2}{r} \Rightarrow t \approx \frac{.7}{r} = \frac{70}{100r} = \frac{70}{(r\%)}$

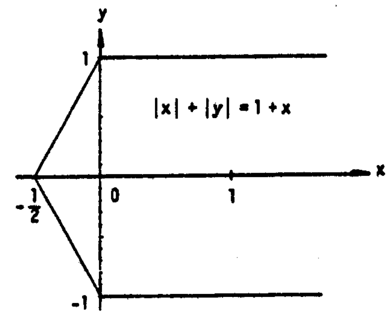
13. There are (infinitely) many such function pairs. For example, $f(x) = 3x$ and $g(x) = 4x$ satisfy $f(g(x)) = f(4x) = 3(4x) = 12x = 4(3x) = g(3x) = g(f(x))$.

14. Yes, there are many such function pairs. For example, if $g(x) = (2x+3)^3$ and $f(x) = x^{1/3}$, then $(f \circ g)(x) = f(g(x)) = f\left((2x+3)^3\right) = \left((2x+3)^3\right)^{1/3} = 2x + 3$.

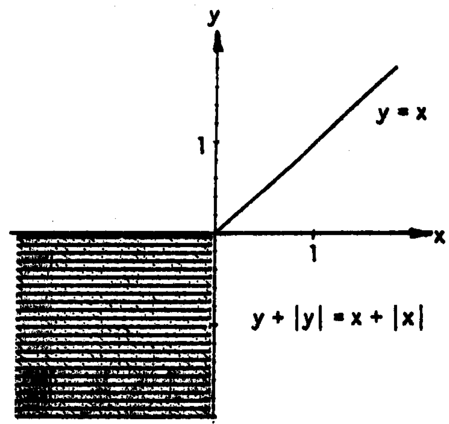
15. If f is odd and defined at x, then $f(-x) = -f(x)$. Thus $g(-x) = f(-x) - 2 = -f(x) - 2$ whereas $-g(x) = -(f(x) - 2) = -f(x) + 2$. Then g cannot be odd because $g(-x) = -g(x) \Rightarrow -f(x) - 2 = -f(x) + 2$ $\Rightarrow 4 = 0$, which is a contradiction. Also, g(x) is not even unless $f(x) = 0$ for all x. On the other hand, if f is even, then $g(x) = f(x) - 2$ is also even: $g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.

16. If g is odd and g(0) is defined, then $g(0) = g(-0) = -g(0)$. Therefore, $2g(0) = 0 \Rightarrow g(0) = 0$.

17. For (x,y) in the 1st quadrant, $|x|+|y| = 1+x \Leftrightarrow x+y = 1+x \Leftrightarrow y = 1$. For (x,y) in the 2nd quadrant, $|x|+|y| = x+1 \Leftrightarrow -x+y = x+1 \Leftrightarrow y = 2x+1$. In the 3rd quadrant, $|x|+|y| = x+1 \Leftrightarrow -x-y = x+1 \Leftrightarrow y = -2x-1$. In the 4th quadrant, $|x|+|y| = x+1 \Leftrightarrow x+(-y) = x+1 \Leftrightarrow y = -1$. The graph is given at the right.



18. We use reasoning similar to Exercise 17.
   (1) 1st quadrant: $y+|y| = x+|x| \Leftrightarrow 2y = 2x \Leftrightarrow y = x$.
   (2) 2nd quadrant: $y+|y| = x+|x| \Leftrightarrow 2y = x+(-x) = 0 \Leftrightarrow y = 0$.
   (3) 3rd quadrant: $y+|y| = x+|x| \Leftrightarrow y+(-y) = x+(-x) \Leftrightarrow 0 = 0$ $\Rightarrow$ <u>all</u> points in the 3rd quadrant satisfy the equation.
   (4) 4th quadrant: $y+|y| = x+|x| \Leftrightarrow y+(-y) = 2x \Leftrightarrow 0 = x$. Combining these results we have the graph given at the right:



19. If f is even and odd, then $f(-x) = -f(x)$ and $f(-x) = f(x) \Rightarrow f(x) = -f(x)$ for all x in the domain of f. Thus $2f(x) = 0 \Rightarrow f(x) = 0$.

20. (a) As suggested, let $E(x) = \frac{f(x)+f(-x)}{2} \Rightarrow E(-x) = \frac{f(-x)+f(-(-x))}{2} = \frac{f(x)+f(-x)}{2} = E(x) \Rightarrow E$ is an even function. Define $O(x) = f(x) - E(x) = f(x) - \frac{f(x)+f(-x)}{2} = \frac{f(x)-f(-x)}{2}$. Then $O(-x) = \frac{f(-x)-f(-(-x))}{2} = \frac{f(-x)-f(x)}{2} = -\left(\frac{f(x)-f(-x)}{2}\right) = -O(x) \Rightarrow O$ is an odd function $\Rightarrow f(x) = E(x) + O(x)$ is the sum of an even and an odd function.

   (b) Part (a) shows that $f(x) = E(x) + O(x)$ is the sum of an even and an odd function. If also $f(x) = E_1(x) + O_1(x)$, where E_1 is even and O_1 is odd, then $f(x) - f(x) = 0 = (E_1(x) + O_1(x)) - (E(x) + O(x))$. Thus, $E(x) - E_1(x) = O_1(x) - O(x)$ for all x in the domain of f (which is the same as the domain of $E - E_1$ and $O - O_1$). Now $(E - E_1)(-x) = E(-x) - E_1(-x) = E(x) - E_1(x)$ (since E and E_1 are even) $= (E - E_1)(x) \Rightarrow E - E_1$ is even. Likewise, $(O_1 - O)(-x) = O_1(-x) - O(-x) = -O_1(x) - (-O(x))$ (since O and O_1 are odd) $= -(O_1(x) - O(x)) = -(O_1 - O)(x) \Rightarrow O_1 - O$ is odd. Therefore, $E - E_1$ and

$O_1 - O$ are both even and odd so they must be zero at each x in the domain of f by Exercise 19. That is, $E_1 = E$ and $O_1 = O$, so the decomposition of f found in part (a) is unique.

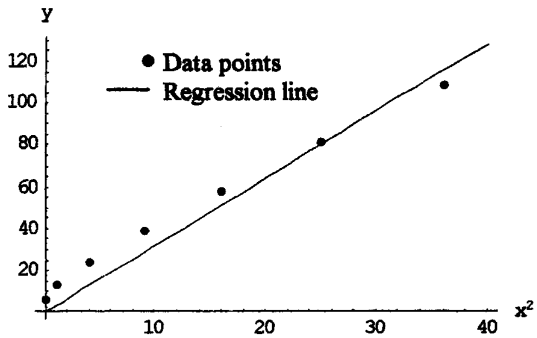
21. If the graph of f(x) passes the horizontal line test, so will the graph of $g(x) = -f(x)$ since it's the same graph reflected about the x-axis.
Alternate answer: If $g(x_1) = g(x_2)$ then $-f(x_1) = -f(x_2)$, $f(x_1) = f(x_2)$, and $x_1 = x_2$ since f is one-to-one.

22. Suppose that $g(x_1) = g(x_2)$. Then $\frac{1}{f(x_1)} = \frac{1}{f(x_2)}$, $f(x_1) = f(x_2)$, and $x_1 = x_2$ since f is one-to-one.

23. (a) The expression $a(b^{c-x}) + d$ is defined for all values of x, so the domain is $(-\infty, \infty)$. Since b^{c-x} attains all positive values, the range is (d, ∞) if $a > 0$ and the range is $(-\infty, d)$ if $a < 0$.
(b) The expression $a \log_b (x - c) + d$ is defined when $x - c > 0$, so the domain is (c, ∞).
Since $a \log_b (x - c) + d$ attains every real value for some value of x, the range is $(-\infty, \infty)$.

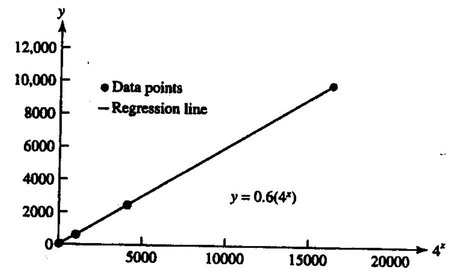
24. (a) Suppose $f(x_1) = f(x_2)$. Then:
$$\frac{ax_1 + b}{cx_1 + d} = \frac{ax_2 + b}{cx_2 + d}$$
$$(ax_1 + b)(cx_2 + d) = (ax_2 + b)(cx_1 + d)$$
$$acx_1x_2 + adx_1 + bcx_2 + bd = acx_1x_2 + adx_2 + bcx_1 + bd$$
$$adx_1 + bcx_2 = adx_2 + bcx_1$$
$$(ad - bc)x_1 = (ad - bc)x_2$$
Since $ad - bc \neq 0$, this means that $x_1 = x_2$.
(b) $y = \frac{ax + b}{cx + d}$
$$cxy + dy = ax + b$$
$$(cy - a)x = -dy + b$$
$$x = \frac{-dy + b}{cy - a}$$
Interchange x and y.
$$y = \frac{-dx + b}{cx - a}$$
$$f^{-1}(x) = \frac{-dx + b}{cx - a}$$
(c) As $x \to \pm\infty$, $f(x) = \frac{ax + b}{cx + d} \to \frac{a}{c}$, so the horizontal asymptote is $y = \frac{a}{c}$ $(c \neq 0)$. Since f(x) is undefined at $x = -\frac{d}{c}$, the vertical asymptote is $x = -\frac{d}{c}$ provided $c \neq 0$.
(d) As $x \to \pm\infty$, $f^{-1}(x) = \frac{-dx + b}{cx - a} \to -\frac{d}{c}$, so the horizontal asymptote is $y = -\frac{d}{c}$ $(c \neq 0)$. Since $f^{-1}(x)$ is undefined at $x = \frac{a}{c}$, the vertical asymptote is $x = \frac{a}{c}$. The horizontal asymptote of f becomes the vertical asymptote of f^{-1} and vice versa due to the reflection of the graph about the line $y = x$.

25. (a)



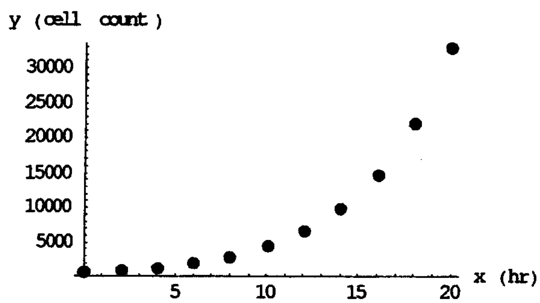
The graph does not support the assumption that $y \propto x^2$

(b)

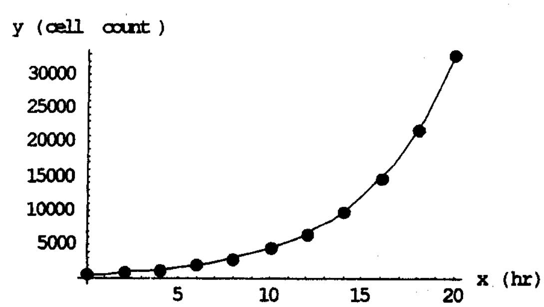


The graph supports the assumption that $y \propto 4^x$. The constant of proportionality is estimated from the slope of the regression line, which is 0.6, therefore, $y = 0.6\left(4^x\right)$.

26. Plot the data.



The graph suggests that an exponential relationship might be appropriate. The exponential regression function on the TI-92 Plus calculator gives $y = 599e^{0.2x}$ and the following graph shows the exponential regression curve superimposed on the graph of the data points.



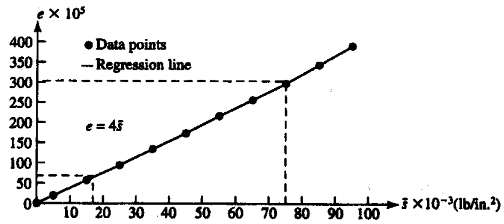
The curve appears to fit the data very well.

The cell count reaches 50,000 when $50{,}000 = 599e^{0.2x} \Rightarrow x = 5 \ln \frac{50{,}000}{599} \approx 22.123$ hours

≈ 22 hours 7.4 minutes.

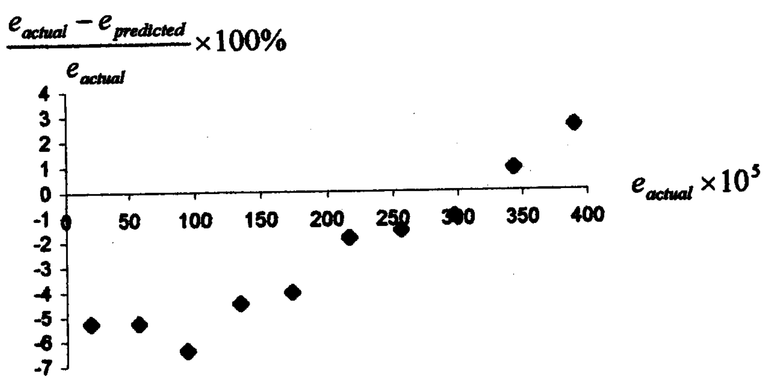
27. (a) Since the elongation of the spring is zero when the stress is $5(10^{-3})(\text{lb/in.}^2)$, the data should be adjusted by subtracting this amount from each of the stress data values. This gives the following table, where $\bar{s} = s - 5(10^{-3})$.

$\bar{s} \times 10^{-3}$	0	5	15	25	35	45	55	65	75	85	95
$e \times 10^5$	0	19	57	94	134	173	216	256	297	343	390



The slope of the graph is $\dfrac{(297-57)(10^5)}{(75-15)(10^{-3})} = 4.00(10^8)$ and the model is $e = 4(10^8)\bar{s}$ or $e = 4(10^8)(s - 5(10^{-3}))$.

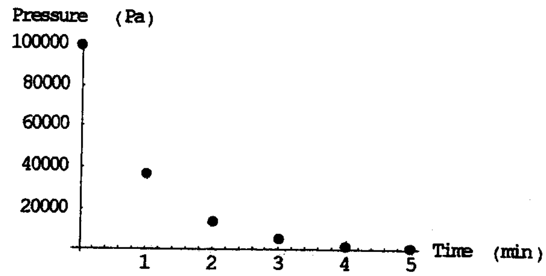
(b) As show in the following graph, the largest relative error is about 6.4%



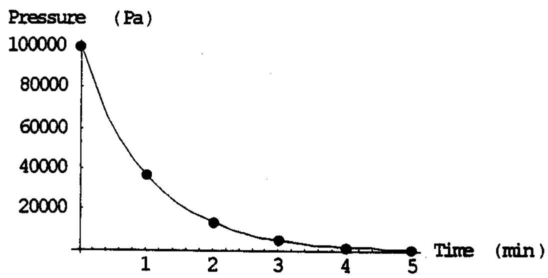
The model fits the data well. There does appear to be a pattern in the errors (i.e., they are not random) indicating that a refinement of the model is possible.

(c) $e = 4(10^8)(200-5)(10^{-3}) = 780(10^5)(\text{in./in.})$. Since $s = 200(10^{-3})(\text{lb/in.}^2)$ is well outside the range of the data used for the model, one should not feel comfortable with this prediction without further testing of the spring.

28. Plot the data.



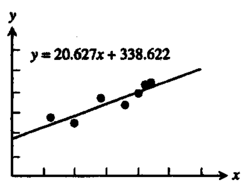
(a) The data suggests a decaying exponential relationship. The exponential regression function on the TI-92 Plus calculator gives $p = 100{,}085e^{-t}$ where p is the pressure in pascals and t is the elapsed time in minutes. The next graph superimposes the exponential regression curve on the data points.



(b) The graph shows that the exponential regression fits the data very well.

(c) The pressure reaches 200 Pa when $200 = 100{,}085e^{-t} \Rightarrow t = -\ln\left(\frac{200}{100{,}085}\right) \approx 6.22$ minutes ≈ 6 minutes 13 seconds.

29. (a) $y = 20.627x + 338.622$



(b) When $x = 30$, $y = 957.445$. According to the regression equation, about 957 degrees will be earned.

(c) The slope is 20.627. It represents the approximate annual increase in the number of doctorates earned by Hispanic Americans per year.

30. (a) $y = 14.60175 \cdot 1.00232^x$

(b) Solving $y = 25$ graphically, we obtain $x \approx 232$. According to the regression equation, the population will reach 25 million in the year 2132.

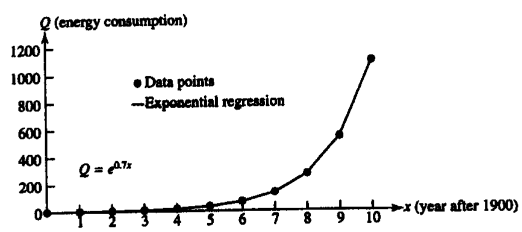
(c) 0.232%

31. (a) The TI-92 Plus calculator gives $f(x) = 2.000268 \sin(2.999187x - 1.000966) + 3.999881$.

(b) $f(x) = 2 \sin(3x - 1) + 4$

32. (a) $y = -590.969 + 152.817 \ln x$, where x is the number of years after 1960.
(b) When $x = 85$, $y \approx 87.94$.
About 87.94 million metric tons were produced.
(c)
$$-590.969 + 152.817 \ln x = 120$$
$$152.817 \ln x = 710.969$$
$$\ln x = \frac{710.969}{152.817}$$
$$x = e^{\frac{710.969}{152.817}} \approx 104.84$$
According to the regression equation, oil production will reach 120 million metric tons when $x \approx 104.84$, in about 2005.

33. (a) The TI-93 Plus calculator gives $Q = 1.00(2.0138^x) = 1.00e^{0.7x}$



(b) For 1996, $x = 9.6 \Rightarrow Q(9.6) = e^{0.7(9.6)} = 828.82$ units of energy consumed that year as estimated by the exponential regression. The exponential regression shows that energy consumption has doubled (i.e., increased by 100%) each decade during the 20th century. The annual rate of increase during this time is $e^{0.7(0.1)} - e^{0.7(0)} = 0.0725 = 7.25\%$.

NOTES: