EXERCISES 1.2

Increments and Distance

In Exercises 1–4, a particle moves from A to B in the coordinate plane. Find the increments Δx and Δy in the particle's coordinates. Also find the distance from A to B.

1. $A(-3, 2)$, $B(-1, -2)$
2. $A(-1, -2)$, $B(-3, 2)$
3. $A(-3.2, -2)$, $B(-8.1, -2)$
4. $A(\sqrt{2}, 4)$, $B(0, 1.5)$

Describe the graphs of the equations in Exercises 5–8.

5. $x^2 + y^2 = 1$
6. $x^2 + y^2 = 2$
7. $x^2 + y^2 \le 3$
8. $x^2 + y^2 = 0$

Slopes, Lines, and Intercepts

Plot the points in Exercises 9–12 and find the slope (if any) of the line they determine. Also find the common slope (if any) of the lines perpendicular to line AB.

9. $A(-1, 2)$, $B(-2, -1)$
10. $A(-2, 1)$, $B(2, -2)$
11. $A(2, 3)$, $B(-1, 3)$
12. $A(-2, 0)$, $B(-2, -2)$

In Exercises 13–16, find an equation for (a) the vertical line and (b) the horizontal line through the given point.

13. $(-1, 4/3)$
14. $\left(\sqrt{2}, -1.3\right)$
15. $\left(0, -\sqrt{2}\right)$
16. $(-\pi, 0)$

In Exercises 17–30, write an equation for each line described.

17. Passes through $(-1, 1)$ with slope -1

18. Passes through $(2, -3)$ with slope $1/2$
19. Passes through $(3, 4)$ and $(-2, 5)$
20. Passes through $(-8, 0)$ and $(-1, 3)$
21. Has slope $-5/4$ and y-intercept 6
22. Has slope $1/2$ and y-intercept -3
23. Passes through $(-12, -9)$ and has slope 0
24. Passes through $(1/3, 4)$, and has no slope
25. Has y-intercept 4 and x-intercept -1
26. Has y-intercept -6 and x-intercept 2
27. Passes through $(5, -1)$ and is parallel to the line $2x + 5y = 15$
28. Passes through $\left(-\sqrt{2}, 2\right)$ parallel to the line $\sqrt{2}x + 5y = \sqrt{3}$
29. Passes through $(4, 10)$ and is perpendicular to the line $6x - 3y = 5$
30. Passes through $(0, 1)$ and is perpendicular to the line $8x - 13y = 13$

In Exercises 31–34, find the line's x- and y-intercepts and use this information to graph the line.

31. $3x + 4y = 12$
32. $x + 2y = -4$
33. $\sqrt{2}x - \sqrt{3}y = \sqrt{6}$
34. $1.5x - y = -3$

35. Is there anything special about the relationship between the lines $Ax + By = C_1$ and $Bx - Ay = C_2$ ($A \ne 0, B \ne 0$)? Give reasons for your answer.

36. Is there anything special about the relationship between the lines $Ax + By = C_1$ and $Ax + By = C_2$ ($A \ne 0, B \ne 0$)? Give reasons for your answer.

Increments and Motion

37. A particle starts at $A(-2, 3)$ and its coordinates change by increments $\Delta x = 5$, $\Delta y = -6$. Find its new position.

38. A particle starts at $A(6, 0)$ and its coordinates change by increments $\Delta x = -6$, $\Delta y = 0$. Find its new position.

39. The coordinates of a particle change by $\Delta x = 5$ and $\Delta y = 6$ as it moves from $A(x, y)$ to $B(3, -3)$. Find x and y.

40. A particle started at $A(1, 0)$, circled the origin once counterclockwise, and returned to $A(1, 0)$. What were the net changes in its coordinates?

Circles

In Exercises 41–46, find an equation for the circle with the given center $C(h, k)$ and radius a. Then sketch the circle in the xy-plane. Include the circle's center in your sketch. Also, label the circle's x- and y-intercepts, if any, with their coordinate pairs.

41. $C(0, 2)$, $a = 2$

42. $C(-3, 0)$, $a = 3$

43. $C(-1, 5)$, $a = \sqrt{10}$

44. $C(1, 1)$, $a = \sqrt{2}$

45. $C\left(-\sqrt{3}, -2\right)$, $a = 2$

46. $C(3, 1/2)$, $a = 5$

Graph the circles whose equations are given in Exercises 47–52. Label each circle's center and intercepts (if any) with their coordinate pairs.

47. $x^2 + y^2 + 4x - 4y + 4 = 0$

48. $x^2 + y^2 - 8x + 4y + 16 = 0$

49. $x^2 + y^2 - 3y - 4 = 0$

50. $x^2 + y^2 - 4x - (9/4) = 0$

51. $x^2 + y^2 - 4x + 4y = 0$

52. $x^2 + y^2 + 2x = 3$

Parabolas

Graph the parabolas in Exercises 53–60. Label the vertex, axis, and intercepts in each case.

53. $y = x^2 - 2x - 3$

54. $y = x^2 + 4x + 3$

55. $y = -x^2 + 4x$

56. $y = -x^2 + 4x - 5$

57. $y = -x^2 - 6x - 5$

58. $y = 2x^2 - x + 3$

59. $y = \frac{1}{2}x^2 + x + 4$

60. $y = -\frac{1}{4}x^2 + 2x + 4$

Inequalities

Describe the regions defined by the inequalities and pairs of inequalities in Exercises 61–68.

61. $x^2 + y^2 > 7$

62. $x^2 + y^2 < 5$

63. $(x - 1)^2 + y^2 \le 4$

64. $x^2 + (y - 2)^2 \ge 4$

65. $x^2 + y^2 > 1$, $x^2 + y^2 < 4$

66. $x^2 + y^2 \le 4$, $(x + 2)^2 + y^2 \le 4$

67. $x^2 + y^2 + 6y < 0$, $y > -3$

68. $x^2 + y^2 - 4x + 2y > 4$, $x > 2$

69. Write an inequality that describes the points that lie inside the circle with center $(-2, 1)$ and radius $\sqrt{6}$.

70. Write an inequality that describes the points that lie outside the circle with center $(-4, 2)$ and radius 4.

71. Write a pair of inequalities that describe the points that lie inside or on the circle with center $(0, 0)$ and radius $\sqrt{2}$, and on or to the right of the vertical line through $(1, 0)$.

72. Write a pair of inequalities that describe the points that lie outside the circle with center $(0, 0)$ and radius 2, and inside the circle that has center $(1, 3)$ and passes through the origin.

Intersecting Lines, Circles, and Parabolas

In Exercises 73–80, graph the two equations and find the points in which the graphs intersect.

73. $y = 2x$, $x^2 + y^2 = 1$

74. $x + y = 1$, $(x - 1)^2 + y^2 = 1$

75. $y - x = 1$, $y = x^2$

76. $x + y = 0$, $y = -(x - 1)^2$

77. $y = -x^2$, $y = 2x^2 - 1$

78. $y = \frac{1}{4}x^2$, $y = (x - 1)^2$

79. $x^2 + y^2 = 1$, $(x - 1)^2 + y^2 = 1$

80. $x^2 + y^2 = 1$, $x^2 + y = 1$

Applications

81. Insulation   By measuring slopes in the accompanying figure, estimate the temperature change in degrees per inch for (a) the gypsum wallboard; (b) the fiberglass insulation; (c) the wood sheathing.
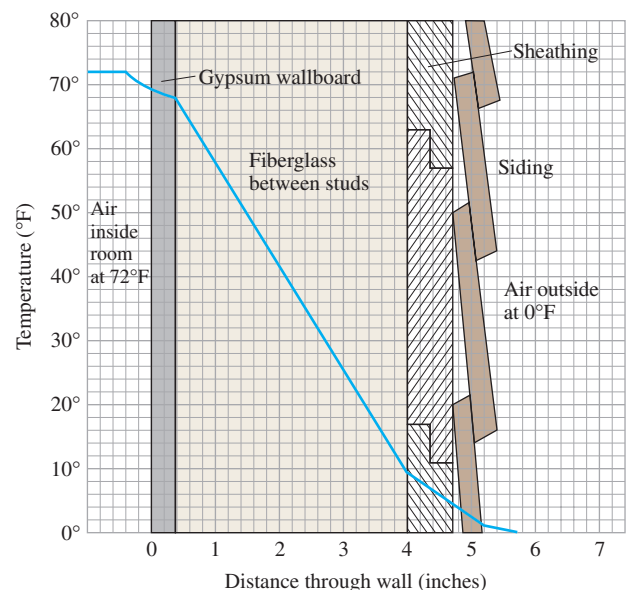
The temperature changes in the wall in Exercises 81 and 82.

82. Insulation   According to the figure in Exercise 81, which of the materials is the best insulator? the poorest? Explain.

83. Pressure under water   The pressure p experienced by a diver under water is related to the diver's depth d by an equation of the form $p = kd + 1$ (k a constant). At the surface, the pressure is 1 atmosphere. The pressure at 100 meters is about 10.94 atmospheres. Find the pressure at 50 meters.

84. Reflected light   A ray of light comes in along the line $x + y = 1$ from the second quadrant and reflects off the x-axis (see the accompanying figure). The angle of incidence is equal to the angle of reflection. Write an equation for the line along which the departing light travels.
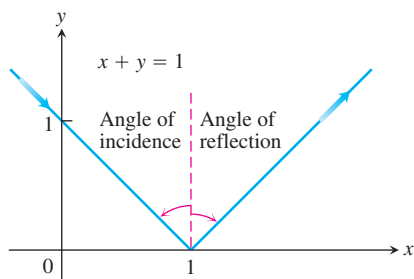
The path of the light ray in Exercise 84. Angles of incidence and reflection are measured from the perpendicular.

85. Fahrenheit vs. Celsius   In the FC-plane, sketch the graph of the equation

$$C = \frac{5}{9}(F - 32)$$

linking Fahrenheit and Celsius temperatures. On the same graph sketch the line $C = F$. Is there a temperature at which a Celsius thermometer gives the same numerical reading as a Fahrenheit thermometer? If so, find it.

86. The Mt. Washington Cog Railway   Civil engineers calculate the slope of roadbed as the ratio of the distance it rises or falls to the distance it runs horizontally. They call this ratio the **grade** of the roadbed, usually written as a percentage. Along the coast, commercial railroad grades are usually less than 2%. In the mountains, they may go as high as 4%. Highway grades are usually less than 5%.

     The steepest part of the Mt. Washington Cog Railway in New Hampshire has an exceptional 37.1% grade. Along this part of the track, the seats in the front of the car are 14 ft above those in the rear. About how far apart are the front and rear rows of seats?

Theory and Examples

87. By calculating the lengths of its sides, show that the triangle with vertices at the points $A(1, 2)$, $B(5, 5)$, and $C(4, -2)$ is isosceles but not equilateral.

88. Show that the triangle with vertices $A(0, 0)$, $B\left(1, \sqrt{3}\right)$, and $C(2, 0)$ is equilateral.

89. Show that the points $A(2, -1)$, $B(1, 3)$, and $C(-3, 2)$ are vertices of a square, and find the fourth vertex.

90. The rectangle shown here has sides parallel to the axes. It is three times as long as it is wide, and its perimeter is 56 units. Find the coordinates of the vertices A, B, and C.
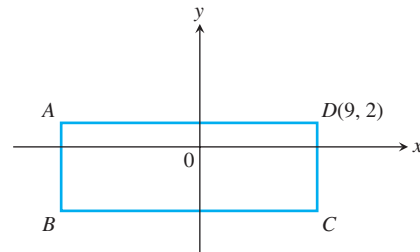
91. Three different parallelograms have vertices at $(-1, 1)$, $(2, 0)$, and $(2, 3)$. Sketch them and find the coordinates of the fourth vertex of each.

92. A 90° rotation counterclockwise about the origin takes $(2, 0)$ to $(0, 2)$, and $(0, 3)$ to $(-3, 0)$, as shown in the accompanying figure. Where does it take each of the following points?

     a. $(4, 1)$ **b.** $(-2, -3)$ **c.** $(2, -5)$

     d. $(x, 0)$ **e.** $(0, y)$ **f.** (x, y)

     g. What point is taken to $(10, 3)$?
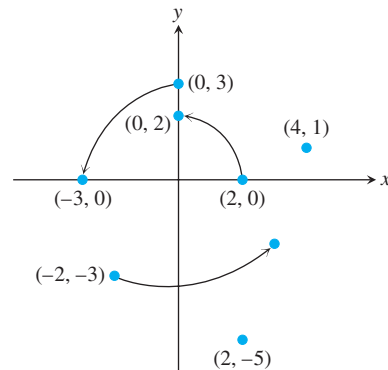
93. For what value of k is the line $2x + ky = 3$ perpendicular to the line $4x + y = 1$? For what value of k are the lines parallel?

94. Find the line that passes through the point $(1, 2)$ and through the point of intersection of the two lines $x + 2y = 3$ and $2x - 3y = -1$.

95. Midpoint of a line segment   Show that the point with coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

is the midpoint of the line segment joining $P(x_1, y_1)$ to $Q(x_2, y_2)$.

96. The distance from a point to a line We can find the distance from a point $P(x_0, y_0)$ to a line $L: Ax + By = C$ by taking the following steps (there is a somewhat faster method in Section 12.5):

1. Find an equation for the line M through P perpendicular to L.

2. Find the coordinates of the point Q in which M and L intersect.

3. Find the distance from P to Q.

Use these steps to find the distance from P to L in each of the following cases.

a. $P(2, 1), \quad L: y = x + 2$

b. $P(4, 6), \quad L: 4x + 3y = 12$

c. $P(a, b), \quad L: x = -1$

d. $P(x_0, y_0), \quad L: Ax + By = C$