EXERCISES 1.5

Sums, Differences, Products, and Quotients

In Exercises 1 and 2, find the domains and ranges of f, g, $f+g$, and $f \cdot g$.

1. $f(x) = x, \quad g(x) = \sqrt{x-1}$
2. $f(x) = \sqrt{x+1}, \quad g(x) = \sqrt{x-1}$

In Exercises 3 and 4, find the domains and ranges of f, g, f/g, and g/f.

3. $f(x) = 2, \quad g(x) = x^2 + 1$
4. $f(x) = 1, \quad g(x) = 1 + \sqrt{x}$

Composites of Functions

5. If $f(x) = x + 5$ and $g(x) = x^2 - 3$, find the following.
   a. $f(g(0))$ **b.** $g(f(0))$
   c. $f(g(x))$ **d.** $g(f(x))$
   e. $f(f(-5))$ **f.** $g(g(2))$
   g. $f(f(x))$ **h.** $g(g(x))$
6. If $f(x) = x - 1$ and $g(x) = 1/(x+1)$, find the following.
   a. $f(g(1/2))$ **b.** $g(f(1/2))$
   c. $f(g(x))$ **d.** $g(f(x))$
   e. $f(f(2))$ **f.** $g(g(2))$
   g. $f(f(x))$ **h.** $g(g(x))$
7. If $u(x) = 4x - 5$, $v(x) = x^2$, and $f(x) = 1/x$, find formulas for the following.
   a. $u(v(f(x)))$ **b.** $u(f(v(x)))$
   c. $v(u(f(x)))$ **d.** $v(f(u(x)))$
   e. $f(u(v(x)))$ **f.** $f(v(u(x)))$
8. If $f(x) = \sqrt{x}$, $g(x) = x/4$, and $h(x) = 4x - 8$, find formulas for the following.
   a. $h(g(f(x)))$ **b.** $h(f(g(x)))$
   c. $g(h(f(x)))$ **d.** $g(f(h(x)))$
   e. $f(g(h(x)))$ **f.** $f(h(g(x)))$

Let $f(x) = x - 3$, $g(x) = \sqrt{x}$, $h(x) = x^3$, and $j(x) = 2x$. Express each of the functions in Exercises 9 and 10 as a composite involving one or more of f, g, h, and j.

9. **a.** $y = \sqrt{x} - 3$ **b.** $y = 2\sqrt{x}$
   c. $y = x^{1/4}$ **d.** $y = 4x$
   e. $y = \sqrt{(x-3)^3}$ **f.** $y = (2x-6)^3$
10. **a.** $y = 2x - 3$ **b.** $y = x^{3/2}$
    c. $y = x^9$ **d.** $y = x - 6$
    e. $y = 2\sqrt{x-3}$ **f.** $y = \sqrt{x^3 - 3}$

Exercises

11. Copy and complete the following table.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
a.	$x - 7$	$\sqrt{x}$	
b.	$x + 2$	$3x$	
c.		$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
d.	$\frac{x}{x - 1}$	$\frac{x}{x - 1}$	
e.		$1 + \frac{1}{x}$	x
f.	$\frac{1}{x}$		x

12. Copy and complete the following table.

	$g(x)$	$f(x)$	$(f \circ g)(x)$
a.	$\frac{1}{x - 1}$	$\lvert x \rvert$	?
b.	?	$\frac{x - 1}{x}$	$\frac{x}{x + 1}$
c.	?	$\sqrt{x}$	$\lvert x \rvert$
d.	$\sqrt{x}$	?	$\lvert x \rvert$

In Exercises 13 and 14, **(a)** write a formula for $f \circ g$ and $g \circ f$ and find the **(b)** domain and **(c)** range of each.

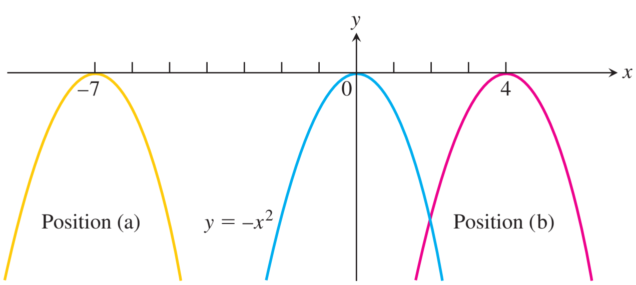
13. $f(x) = \sqrt{x + 1}, \quad g(x) = \frac{1}{x}$

14. $f(x) = x^2, \quad g(x) = 1 - \sqrt{x}$

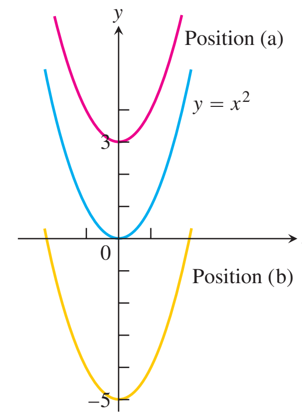
Shifting Graphs

Exercises

15. The accompanying figure shows the graph of $y = -x^2$ shifted to two new positions. Write equations for the new graphs.



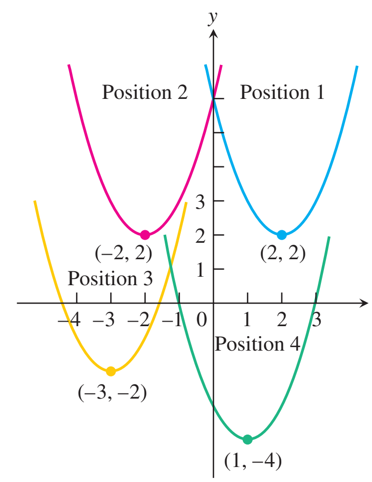
16. The accompanying figure shows the graph of $y = x^2$ shifted to two new positions. Write equations for the new graphs.



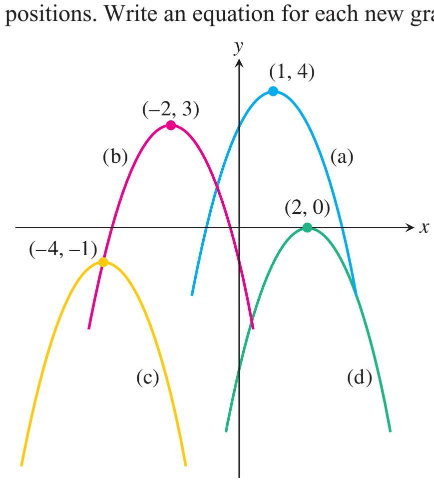
17. Match the equations listed in parts (a)–(d) to the graphs in the accompanying figure.

a. $y = (x - 1)^2 - 4$ **b.** $y = (x - 2)^2 + 2$

c. $y = (x + 2)^2 + 2$ **d.** $y = (x + 3)^2 - 2$



18. The accompanying figure shows the graph of $y = -x^2$ shifted to four new positions. Write an equation for each new graph.



Exercises 19–28 tell how many units and in what directions the graphs of the given equations are to be shifted. Give an equation for the shifted graph. Then sketch the original and shifted graphs together, labeling each graph with its equation.

19. $x^2 + y^2 = 49$ Down 3, left 2

20. $x^2 + y^2 = 25$ Up 3, left 4

21. $y = x^3$ Left 1, down 1

22. $y = x^{2/3}$ Right 1, down 1

23. $y = \sqrt{x}$ Left 0.81

24. $y = -\sqrt{x}$ Right 3

25. $y = 2x - 7$ Up 7

26. $y = \frac{1}{2}(x + 1) + 5$ Down 5, right 1

27. $y = 1/x$ Up 1, right 1

28. $y = 1/x^2$ Left 2, down 1

Graph the functions in Exercises 29–48.

29. $y = \sqrt{x + 4}$

30. $y = \sqrt{9 - x}$

31. $y = |x - 2|$

32. $y = |1 - x| - 1$

33. $y = 1 + \sqrt{x - 1}$

34. $y = 1 - \sqrt{x}$

35. $y = (x + 1)^{2/3}$

36. $y = (x - 8)^{2/3}$

37. $y = 1 - x^{2/3}$

38. $y + 4 = x^{2/3}$

39. $y = \sqrt[3]{x - 1} - 1$

40. $y = (x + 2)^{3/2} + 1$

41. $y = \dfrac{1}{x - 2}$

42. $y = \dfrac{1}{x} - 2$

43. $y = \dfrac{1}{x} + 2$

44. $y = \dfrac{1}{x + 2}$

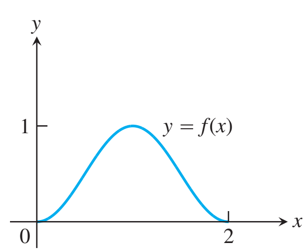
45. $y = \dfrac{1}{(x - 1)^2}$

46. $y = \dfrac{1}{x^2} - 1$

47. $y = \dfrac{1}{x^2} + 1$

48. $y = \dfrac{1}{(x + 1)^2}$

49. The accompanying figure shows the graph of a function $f(x)$ with domain $[0, 2]$ and range $[0, 1]$. Find the domains and ranges of the following functions, and sketch their graphs.



a. $f(x) + 2$

b. $f(x) - 1$

c. $2f(x)$

d. $-f(x)$

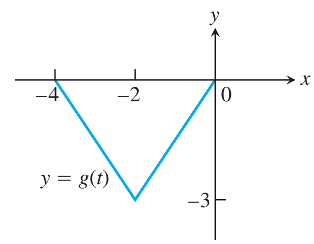
e. $f(x + 2)$

f. $f(x - 1)$

g. $f(-x)$

h. $-f(x + 1) + 1$

50. The accompanying figure shows the graph of a function $g(t)$ with domain $[-4, 0]$ and range $[-3, 0]$. Find the domains and ranges of the following functions, and sketch their graphs.



a. $g(-t)$

b. $-g(t)$

c. $g(t) + 3$

d. $1 - g(t)$

e. $g(-t + 2)$

f. $g(t - 2)$

g. $g(1 - t)$

h. $-g(t - 4)$

Vertical and Horizontal Scaling

Exercises 51–60 tell by what factor and direction the graphs of the given functions are to be stretched or compressed. Give an equation for the stretched or compressed graph.

51. $y = x^2 - 1$, stretched vertically by a factor of 3

52. $y = x^2 - 1$, compressed horizontally by a factor of 2

53. $y = 1 + \dfrac{1}{x^2}$, compressed vertically by a factor of 2

54. $y = 1 + \dfrac{1}{x^2}$, stretched horizontally by a factor of 3

55. $y = \sqrt{x + 1}$, compressed horizontally by a factor of 4

56. $y = \sqrt{x + 1}$, stretched vertically by a factor of 3

57. $y = \sqrt{4 - x^2}$, stretched horizontally by a factor of 2

58. $y = \sqrt{4 - x^2}$, compressed vertically by a factor of 3

59. $y = 1 - x^3$, compressed horizontally by a factor of 3

60. $y = 1 - x^3$, stretched horizontally by a factor of 2

Graphing

In Exercises 61–68, graph each function, not by plotting points, but by starting with the graph of one of the standard functions presented in Figures 1.36–1.38, and applying an appropriate transformation.

61. $y = -\sqrt{2x + 1}$

62. $y = \sqrt{1 - \dfrac{x}{2}}$

63. $y = (x - 1)^3 + 2$

64. $y = (1 - x)^3 + 2$

65. $y = \dfrac{1}{2x} - 1$

66. $y = \dfrac{2}{x^2} + 1$

67. $y = -\sqrt[3]{x}$

68. $y = (-2x)^{2/3}$

69. Graph the function $y = |x^2 - 1|$.

70. Graph the function $y = \sqrt{|x|}$.

Ellipses

Exercises 71–76 give equations of ellipses. Put each equation in standard form and sketch the ellipse.

Exercises

71. $9x^2 + 25y^2 = 225$

72. $16x^2 + 7y^2 = 112$

73. $3x^2 + (y - 2)^2 = 3$

74. $(x + 1)^2 + 2y^2 = 4$

75. $3(x - 1)^2 + 2(y + 2)^2 = 6$

76. $6\left(x + \frac{3}{2}\right)^2 + 9\left(y - \frac{1}{2}\right)^2 = 54$

77. Write an equation for the ellipse $(x^2/16) + (y^2/9) = 1$ shifted 4 units to the left and 3 units up. Sketch the ellipse and identify its center and major axis.

78. Write an equation for the ellipse $(x^2/4) + (y^2/25) = 1$ shifted 3 units to the right and 2 units down. Sketch the ellipse and identify its center and major axis.

Even and Odd Functions

79. Assume that f is an even function, g is an odd function, and both f and g are defined on the entire real line $\mathbb{R}$. Which of the following (where defined) are even? odd?

a. fg **b.** f/g **c.** g/f

d. $f^2 = ff$ **e.** $g^2 = gg$ **f.** $f \circ g$

g. $g \circ f$ **h.** $f \circ f$ **i.** $g \circ g$

80. Can a function be both even and odd? Give reasons for your answer.

T **81.** (*Continuation of Example* 1.) Graph the functions $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1 - x}$ together with their (a) sum, (b) product, (c) two differences, (d) two quotients.

T **82.** Let $f(x) = x - 7$ and $g(x) = x^2$. Graph f and g together with $f \circ g$ and $g \circ f$.