EXERCISES 10.4

Parametric Equations for Conics

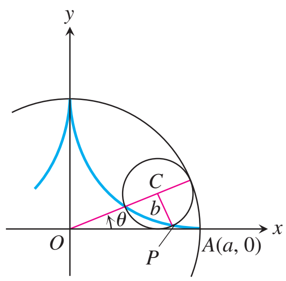
Exercises 1–12 give parametric equations and parameter intervals for the motion of a particle in the xy-plane. Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation. (The graphs will vary with the equation used.) Indicate the portion of the graph traced by the particle and the direction of motion.

Exercises

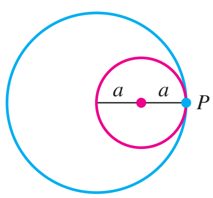
1. $x = \cos t, \quad y = \sin t, \quad 0 \le t \le \pi$
2. $x = \sin(2\pi(1 - t)), \quad y = \cos(2\pi(1 - t)); \quad 0 \le t \le 1$
3. $x = 4\cos t, \quad y = 5\sin t; \quad 0 \le t \le \pi$
4. $x = 4\sin t, \quad y = 5\cos t; \quad 0 \le t \le 2\pi$
5. $x = t, \quad y = \sqrt{t}; \quad t \ge 0$
6. $x = \sec^2 t - 1, \quad y = \tan t; \quad -\pi/2 < t < \pi/2$
7. $x = -\sec t, \quad y = \tan t; \quad -\pi/2 < t < \pi/2$
8. $x = \csc t, \quad y = \cot t; \quad 0 < t < \pi$
9. $x = t, \quad y = \sqrt{4 - t^2}; \quad 0 \le t \le 2$
10. $x = t^2, \quad y = \sqrt{t^4 + 1}; \quad t \ge 0$
11. $x = -\cosh t, \quad y = \sinh t; \quad -\infty < t < \infty$
12. $x = 2\sinh t, \quad y = 2\cosh t; \quad -\infty < t < \infty$
13. **Hypocycloids** When a circle rolls on the inside of a fixed circle, any point P on the circumference of the rolling circle describes a *hypocycloid*. Let the fixed circle be $x^2 + y^2 = a^2$, let the radius of the rolling circle be b, and let the initial position of the tracing point P be $A(a, 0)$. Find parametric equations for the hypocycloid, using as the parameter the angle θ from the positive x-axis to the line joining the circles' centers. In particular, if

$b = a/4$, as in the accompanying figure, show that the hypocycloid is the astroid

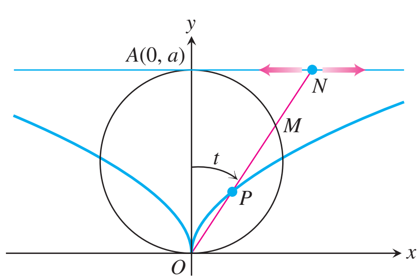
$$x = a\cos^3\theta, \quad y = a\sin^3\theta.$$



14. More about hypocycloids The accompanying figure shows a circle of radius a tangent to the inside of a circle of radius $2a$. The point P, shown as the point of tangency in the figure, is attached to the smaller circle. What path does P trace as the smaller circle rolls around the inside of the larger circle?



15. As the point N moves along the line $y = a$ in the accompanying figure, P moves in such a way that $OP = MN$. Find parametric equations for the coordinates of P as functions of the angle t that the line ON makes with the positive y-axis.



16. Trochoids A wheel of radius a rolls along a horizontal straight line without slipping. Find parametric equations for the curve traced out by a point P on a spoke of the wheel b units from its center. As parameter, use the angle θ through which the wheel turns. The curve is called a *trochoid*, which is a cycloid when $b = a$.

Distance Using Parametric Equations

17. Find the point on the parabola $x = t, y = t^2, -\infty < t < \infty$, closest to the point $(2, 1/2)$. (*Hint:* Minimize the square of the distance as a function of t.)

18. Find the point on the ellipse $x = 2\cos t, y = \sin t, 0 \le t \le 2\pi$ closest to the point $(3/4, 0)$. (*Hint:* Minimize the square of the distance as a function of t.)

T GRAPHER EXPLORATIONS

If you have a parametric equation grapher, graph the following equations over the given intervals.

19. Ellipse $x = 4\cos t, \quad y = 2\sin t,$ over

a. $0 \le t \le 2\pi$ **b.** $0 \le t \le \pi$

c. $-\pi/2 \le t \le \pi/2$.

20. Hyperbola branch $x = \sec t$ (enter as $1/\cos(t)$), $y = \tan t$ (enter as $\sin(t)/\cos(t)$), over

a. $-1.5 \le t \le 1.5$ **b.** $-0.5 \le t \le 0.5$

c. $-0.1 \le t \le 0.1$.

21. Parabola $x = 2t + 3, \quad y = t^2 - 1, \quad -2 \le t \le 2$

22. Cycloid $x = t - \sin t, \quad y = 1 - \cos t,$ over

a. $0 \le t \le 2\pi$ **b.** $0 \le t \le 4\pi$

c. $\pi \le t \le 3\pi$.

23. A nice curve (a deltoid)

$$x = 2\cos t + \cos 2t, \quad y = 2\sin t - \sin 2t; \quad 0 \le t \le 2\pi$$

What happens if you replace 2 with -2 in the equations for x and y? Graph the new equations and find out.

24. An even nicer curve

$$x = 3\cos t + \cos 3t, \quad y = 3\sin t - \sin 3t; \quad 0 \le t \le 2\pi$$

What happens if you replace 3 with -3 in the equations for x and y? Graph the new equations and find out.

25. Three beautiful curves

a. *Epicycloid:*

$$x = 9\cos t - \cos 9t, \quad y = 9\sin t - \sin 9t; \quad 0 \le t \le 2\pi$$

b. *Hypocycloid:*

$$x = 8\cos t + 2\cos 4t, \quad y = 8\sin t - 2\sin 4t; \quad 0 \le t \le 2\pi$$

c. *Hypotrochoid:*

$$x = \cos t + 5\cos 3t, \quad y = 6\cos t - 5\sin 3t; \quad 0 \le t \le 2\pi$$

26. More beautiful curves

a. $x = 6\cos t + 5\cos 3t, \quad y = 6\sin t - 5\sin 3t;$
$0 \le t \le 2\pi$

b. $x = 6\cos 2t + 5\cos 6t, \quad y = 6\sin 2t - 5\sin 6t;$
$0 \le t \le \pi$

c. $x = 6\cos t + 5\cos 3t, \quad y = 6\sin 2t - 5\sin 3t;$
$0 \le t \le 2\pi$

d. $x = 6\cos 2t + 5\cos 6t, \quad y = 6\sin 4t - 5\sin 6t;$
$0 \le t \le \pi$