EXERCISES 10.5

Polar Coordinate Pairs

1. Which polar coordinate pairs label the same point?

a. $(3, 0)$ **b.** $(-3, 0)$ **c.** $(2, 2\pi/3)$

d. $(2, 7\pi/3)$ **e.** $(-3, \pi)$ **f.** $(2, \pi/3)$

g. $(-3, 2\pi)$ **h.** $(-2, -\pi/3)$

2. Which polar coordinate pairs label the same point?

a. $(-2, \pi/3)$ **b.** $(2, -\pi/3)$ **c.** (r, θ)

d. $(r, \theta + \pi)$ **e.** $(-r, \theta)$ **f.** $(2, -2\pi/3)$

g. $(-r, \theta + \pi)$ **h.** $(-2, 2\pi/3)$

3. Plot the following points (given in polar coordinates). Then find all the polar coordinates of each point.

a. $(2, \pi/2)$ **b.** $(2, 0)$

c. $(-2, \pi/2)$ **d.** $(-2, 0)$

4. Plot the following points (given in polar coordinates). Then find all the polar coordinates of each point.

a. $(3, \pi/4)$ **b.** $(-3, \pi/4)$

c. $(3, -\pi/4)$ **d.** $(-3, -\pi/4)$

Polar to Cartesian Coordinates

5. Find the Cartesian coordinates of the points in Exercise 1.

6. Find the Cartesian coordinates of the following points (given in polar coordinates).

a. $\left(\sqrt{2}, \pi/4\right)$ **b.** $(1, 0)$

c. $(0, \pi/2)$ **d.** $\left(-\sqrt{2}, \pi/4\right)$

e. $(-3, 5\pi/6)$ **f.** $(5, \tan^{-1}(4/3))$

g. $(-1, 7\pi)$ **h.** $\left(2\sqrt{3}, 2\pi/3\right)$

Graphing Polar Equations and Inequalities

Graph the sets of points whose polar coordinates satisfy the equations and inequalities in Exercises 7–22.

7. $r = 2$ **8.** $0 \le r \le 2$

9. $r \ge 1$ **10.** $1 \le r \le 2$

11. $0 \le \theta \le \pi/6, \quad r \ge 0$ **12.** $\theta = 2\pi/3, \quad r \le -2$

13. $\theta = \pi/3, \quad -1 \le r \le 3$ **14.** $\theta = 11\pi/4, \quad r \ge -1$

15. $\theta = \pi/2, \quad r \ge 0$ **16.** $\theta = \pi/2, \quad r \le 0$

17. $0 \le \theta \le \pi, \quad r = 1$ **18.** $0 \le \theta \le \pi, \quad r = -1$

19. $\pi/4 \le \theta \le 3\pi/4, \quad 0 \le r \le 1$

20. $-\pi/4 \le \theta \le \pi/4, \quad -1 \le r \le 1$

21. $-\pi/2 \le \theta \le \pi/2, \quad 1 \le r \le 2$

22. $0 \le \theta \le \pi/2, \quad 1 \le |r| \le 2$

Polar to Cartesian Equations

Replace the polar equations in Exercises 23–48 by equivalent Cartesian equations. Then describe or identify the graph.

23. $r \cos\theta = 2$ **24.** $r \sin\theta = -1$

25. $r \sin\theta = 0$ **26.** $r \cos\theta = 0$

27. $r = 4 \csc\theta$ **28.** $r = -3 \sec\theta$

29. $r \cos\theta + r \sin\theta = 1$ **30.** $r \sin\theta = r \cos\theta$

31. $r^2 = 1$ **32.** $r^2 = 4r \sin\theta$

33. $r = \dfrac{5}{\sin\theta - 2\cos\theta}$ **34.** $r^2 \sin 2\theta = 2$

35. $r = \cot\theta \csc\theta$ **36.** $r = 4 \tan\theta \sec\theta$

37. $r = \csc\theta\, e^{r\cos\theta}$ **38.** $r \sin\theta = \ln r + \ln\cos\theta$

39. $r^2 + 2r^2 \cos\theta \sin\theta = 1$ **40.** $\cos^2\theta = \sin^2\theta$

41. $r^2 = -4r\cos\theta$ **42.** $r^2 = -6r\sin\theta$

43. $r = 8\sin\theta$ **44.** $r = 3\cos\theta$

45. $r = 2\cos\theta + 2\sin\theta$ **46.** $r = 2\cos\theta - \sin\theta$

47. $r \sin\left(\theta + \dfrac{\pi}{6}\right) = 2$ **48.** $r \sin\left(\dfrac{2\pi}{3} - \theta\right) = 5$

Cartesian to Polar Equations

Replace the Cartesian equations in Exercises 49–62 by equivalent polar equations.

49. $x = 7$ **50.** $y = 1$ **51.** $x = y$

52. $x - y = 3$ **53.** $x^2 + y^2 = 4$ **54.** $x^2 - y^2 = 1$

55. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ **56.** $xy = 2$

Exercises

57. $y^2 = 4x$

58. $x^2 + xy + y^2 = 1$

59. $x^2 + (y - 2)^2 = 4$

60. $(x - 5)^2 + y^2 = 25$

61. $(x - 3)^2 + (y + 1)^2 = 4$

62. $(x + 2)^2 + (y - 5)^2 = 16$

Theory and Examples

63. Find all polar coordinates of the origin.

64. Vertical and horizontal lines

a. Show that every vertical line in the xy-plane has a polar equation of the form $r = a \sec \theta$.

b. Find the analogous polar equation for horizontal lines in the xy-plane.