EXERCISES 10.6

Symmetries and Polar Graphs

Identify the symmetries of the curves in Exercises 1–12. Then sketch the curves.

1. $r = 1 + \cos\theta$
2. $r = 2 - 2\cos\theta$
3. $r = 1 - \sin\theta$
4. $r = 1 + \sin\theta$
5. $r = 2 + \sin\theta$
6. $r = 1 + 2\sin\theta$
7. $r = \sin(\theta/2)$
8. $r = \cos(\theta/2)$
9. $r^2 = \cos\theta$
10. $r^2 = \sin\theta$
11. $r^2 = -\sin\theta$
12. $r^2 = -\cos\theta$

Graph the lemniscates in Exercises 13–16. What symmetries do these curves have?

13. $r^2 = 4\cos 2\theta$
14. $r^2 = 4\sin 2\theta$
15. $r^2 = -\sin 2\theta$
16. $r^2 = -\cos 2\theta$

Slopes of Polar Curves

Find the slopes of the curves in Exercises 17–20 at the given points. Sketch the curves along with their tangents at these points.

17. **Cardioid** $r = -1 + \cos\theta;\quad \theta = \pm\pi/2$
18. **Cardioid** $r = -1 + \sin\theta;\quad \theta = 0, \pi$
19. **Four-leaved rose** $r = \sin 2\theta;\quad \theta = \pm\pi/4, \pm 3\pi/4$
20. **Four-leaved rose** $r = \cos 2\theta;\quad \theta = 0, \pm\pi/2, \pi$

Limaçons

Graph the limaçons in Exercises 21–24. Limaçon ("*lee*-ma-sahn") is Old French for "snail." You will understand the name when you graph the limaçons in Exercise 21. Equations for limaçons have the form $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$. There are four basic shapes.

21. **Limaçons with an inner loop**
    a. $r = \frac{1}{2} + \cos\theta$ b. $r = \frac{1}{2} + \sin\theta$
22. **Cardioids**
    a. $r = 1 - \cos\theta$ b. $r = -1 + \sin\theta$
23. **Dimpled limaçons**
    a. $r = \frac{3}{2} + \cos\theta$ b. $r = \frac{3}{2} - \sin\theta$
24. **Oval limaçons**
    a. $r = 2 + \cos\theta$ b. $r = -2 + \sin\theta$

Graphing Polar Inequalities

25. Sketch the region defined by the inequalities $-1 \le r \le 2$ and $-\pi/2 \le \theta \le \pi/2$.
26. Sketch the region defined by the inequalities $0 \le r \le 2\sec\theta$ and $-\pi/4 \le \theta \le \pi/4$.

In Exercises 27 and 28, sketch the region defined by the inequality.

27. $0 \le r \le 2 - 2\cos\theta$
28. $0 \le r^2 \le \cos\theta$

Intersections

29. Show that the point $(2, 3\pi/4)$ lies on the curve $r = 2\sin 2\theta$.
30. Show that $(1/2, 3\pi/2)$ lies on the curve $r = -\sin(\theta/3)$.

Find the points of intersection of the pairs of curves in Exercises 31–38.

31. $r = 1 + \cos\theta,\quad r = 1 - \cos\theta$
32. $r = 1 + \sin\theta,\quad r = 1 - \sin\theta$
33. $r = 2\sin\theta,\quad r = 2\sin 2\theta$
34. $r = \cos\theta,\quad r = 1 - \cos\theta$
35. $r = \sqrt{2},\quad r^2 = 4\sin\theta$
36. $r^2 = \sqrt{2}\sin\theta,\quad r^2 = \sqrt{2}\cos\theta$
37. $r = 1,\quad r^2 = 2\sin 2\theta$
38. $r^2 = \sqrt{2}\cos 2\theta,\quad r^2 = \sqrt{2}\sin 2\theta$

T Find the points of intersection of the pairs of curves in Exercises 39–42.

39. $r^2 = \sin 2\theta,\quad r^2 = \cos 2\theta$
40. $r = 1 + \cos\frac{\theta}{2},\quad r = 1 - \sin\frac{\theta}{2}$
41. $r = 1,\quad r = 2\sin 2\theta$
42. $r = 1,\quad r^2 = 2\sin 2\theta$

T Grapher Explorations

43. Which of the following has the same graph as $r = 1 - \cos\theta$?

a. $r = -1 - \cos\theta$ **b.** $r = 1 + \cos\theta$

Confirm your answer with algebra.

44. Which of the following has the same graph as $r = \cos 2\theta$?

a. $r = -\sin(2\theta + \pi/2)$ **b.** $r = -\cos(\theta/2)$

Confirm your answer with algebra.

45. A rose within a rose Graph the equation $r = 1 - 2\sin 3\theta$.

46. The nephroid of Freeth Graph the nephroid of Freeth:

$$r = 1 + 2\sin\frac{\theta}{2}.$$

47. Roses Graph the roses $r = \cos m\theta$ for $m = 1/3, 2, 3$, and 7.

48. Spirals Polar coordinates are just the thing for defining spirals. Graph the following spirals.

a. $r = \theta$ **b.** $r = -\theta$

c. *A logarithmic spiral:* $r = e^{\theta/10}$

d. *A hyperbolic spiral:* $r = 8/\theta$

e. *An equilateral hyperbola:* $r = \pm 10/\sqrt{\theta}$

(Use different colors for the two branches.)

Theory and Examples

49. (*Continuation of Example 5.*) The simultaneous solution of the equations

$$r^2 = 4\cos\theta \qquad (1)$$

$$r = 1 - \cos\theta \qquad (2)$$

in the text did not reveal the points (0, 0) and $(2, \pi)$ in which their graphs intersected.

a. We could have found the point $(2, \pi)$, however, by replacing the (r, θ) in Equation (1) by the equivalent $(-r, \theta + \pi)$ to obtain

$$\begin{aligned} r^2 &= 4\cos\theta \\ (-r)^2 &= 4\cos(\theta + \pi) \\ r^2 &= -4\cos\theta. \end{aligned} \qquad (3)$$

Solve Equations (2) and (3) simultaneously to show that $(2, \pi)$ is a common solution. (This will still not reveal that the graphs intersect at (0, 0).)

b. The origin is still a special case. (It often is.) Here is one way to handle it: Set $r = 0$ in Equations (1) and (2) and solve each equation for a corresponding value of θ. Since $(0, \theta)$ is the origin for *any* θ, this will show that both curves pass through the origin even if they do so for different θ-values.

50. If a curve has any two of the symmetries listed at the beginning of the section, can anything be said about its having or not having the third symmetry? Give reasons for your answer.

***51.** Find the maximum width of the petal of the four-leaved rose $r = \cos 2\theta$, which lies along the x-axis.

***52.** Find the maximum height above the x-axis of the cardioid $r = 2(1 + \cos\theta)$.