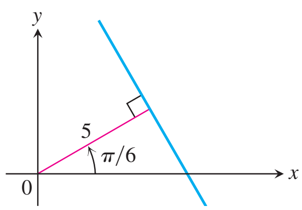
EXERCISES 10.8

Lines

Find polar and Cartesian equations for the lines in Exercises 1–4.

1.



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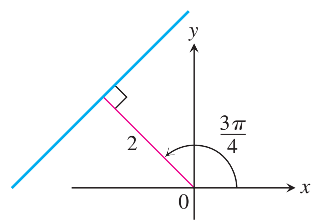

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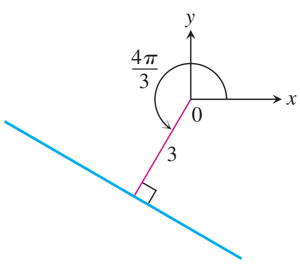

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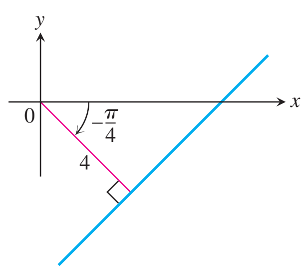


Sketch the lines in Exercises 5–8 and find Cartesian equations for them.

5. $r\cos\left(\theta - \frac{\pi}{4}\right) = \sqrt{2}$

6. $r\cos\left(\theta + \frac{3\pi}{4}\right) = 1$

7. $r\cos\left(\theta - \frac{2\pi}{3}\right) = 3$

8. $r\cos\left(\theta + \frac{\pi}{3}\right) = 2$

Find a polar equation in the form $r\cos(\theta - \theta_0) = r_0$ for each of the lines in Exercises 9–12.

9. $\sqrt{2}x + \sqrt{2}y = 6$

10. $\sqrt{3}x - y = 1$

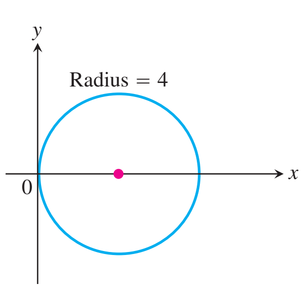
11. $y = -5$

12. $x = -4$

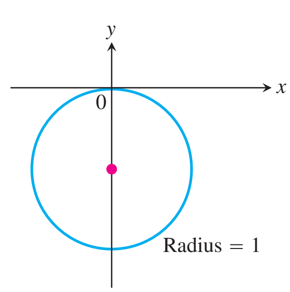
Circles

Find polar equations for the circles in Exercises 13–16.

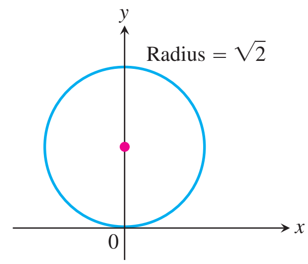
13.



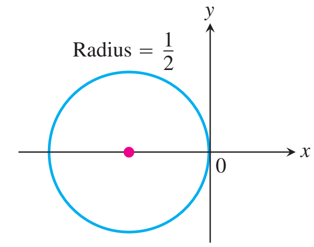
14.



15.



16.



Sketch the circles in Exercises 17–20. Give polar coordinates for their centers and identify their radii.

17. $r = 4\cos\theta$

18. $r = 6\sin\theta$

19. $r = -2\cos\theta$

20. $r = -8\sin\theta$

Find polar equations for the circles in Exercises 21–28. Sketch each circle in the coordinate plane and label it with both its Cartesian and polar equations.

21. $(x - 6)^2 + y^2 = 36$

22. $(x + 2)^2 + y^2 = 4$

23. $x^2 + (y - 5)^2 = 25$

24. $x^2 + (y + 7)^2 = 49$

25. $x^2 + 2x + y^2 = 0$

26. $x^2 - 16x + y^2 = 0$

27. $x^2 + y^2 + y = 0$

28. $x^2 + y^2 - \frac{4}{3}y = 0$

Conic Sections from Eccentricities and Directrices

Exercises 29–36 give the eccentricities of conic sections with one focus at the origin, along with the directrix corresponding to that focus. Find a polar equation for each conic section.

29. $e = 1, \quad x = 2$

30. $e = 1, \quad y = 2$

31. $e = 5, \quad y = -6$

32. $e = 2, \quad x = 4$

33. $e = 1/2, \quad x = 1$

34. $e = 1/4, \quad x = -2$

35. $e = 1/5, \quad y = -10$

36. $e = 1/3, \quad y = 6$

Parabolas and Ellipses

Sketch the parabolas and ellipses in Exercises 37–44. Include the directrix that corresponds to the focus at the origin. Label the vertices with appropriate polar coordinates. Label the centers of the ellipses as well.

37. $r = \dfrac{1}{1 + \cos\theta}$

38. $r = \dfrac{6}{2 + \cos\theta}$

39. $r = \dfrac{25}{10 - 5\cos\theta}$

40. $r = \dfrac{4}{2 - 2\cos\theta}$

41. $r = \dfrac{400}{16 + 8\sin\theta}$

42. $r = \dfrac{12}{3 + 3\sin\theta}$

43. $r = \dfrac{8}{2 - 2\sin\theta}$

44. $r = \dfrac{4}{2 - \sin\theta}$

Graphing Inequalities

Exercises

Sketch the regions defined by the inequalities in Exercises 45 and 46.

45. $0 \le r \le 2\cos\theta$ **46.** $-3\cos\theta \le r \le 0$

T Grapher Explorations

Graph the lines and conic sections in Exercises 47–56.

47. $r = 3\sec(\theta - \pi/3)$ **48.** $r = 4\sec(\theta + \pi/6)$

49. $r = 4\sin\theta$ **50.** $r = -2\cos\theta$

51. $r = 8/(4 + \cos\theta)$ **52.** $r = 8/(4 + \sin\theta)$

53. $r = 1/(1 - \sin\theta)$ **54.** $r = 1/(1 + \cos\theta)$

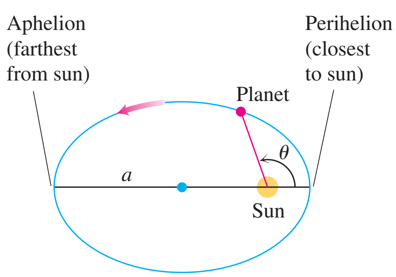
55. $r = 1/(1 + 2\sin\theta)$ **56.** $r = 1/(1 + 2\cos\theta)$

Theory and Examples

57. Perihelion and aphelion A planet travels about its sun in an ellipse whose semimajor axis has length a. (See accompanying figure.)

a. Show that $r = a(1 - e)$ when the planet is closest to the sun and that $r = a(1 + e)$ when the planet is farthest from the sun.

b. Use the data in the table in Exercise 58 to find how close each planet in our solar system comes to the sun and how far away each planet gets from the sun.



58. Planetary orbits In Example 6, we found a polar equation for the orbit of Pluto. Use the data in the table below to find polar equations for the orbits of the other planets.

Planet	Semimajor axis (astronomical units)	Eccentricity
Mercury	0.3871	0.2056
Venus	0.7233	0.0068
Earth	1.000	0.0167
Mars	1.524	0.0934
Jupiter	5.203	0.0484
Saturn	9.539	0.0543
Uranus	19.18	0.0460
Neptune	30.06	0.0082
Pluto	39.44	0.2481

59. a. Find Cartesian equations for the curves $r = 4\sin\theta$ and $r = \sqrt{3}\sec\theta$.

b. Sketch the curves together and label their points of intersection in both Cartesian and polar coordinates.

60. Repeat Exercise 59 for $r = 8\cos\theta$ and $r = 2\sec\theta$.

61. Find a polar equation for the parabola with focus (0, 0) and directrix $r\cos\theta = 4$.

62. Find a polar equation for the parabola with focus (0, 0) and directrix $r\cos(\theta - \pi/2) = 2$.

63. a. The space engineer's formula for eccentricity The space engineer's formula for the eccentricity of an elliptical orbit is

$$e = \frac{r_{\max} - r_{\min}}{r_{\max} + r_{\min}},$$

where r is the distance from the space vehicle to the attracting focus of the ellipse along which it travels. Why does the formula work?

b. Drawing ellipses with string You have a string with a knot in each end that can be pinned to a drawing board. The string is 10 in. long from the center of one knot to the center of the other. How far apart should the pins be to use the method illustrated in Figure 10.5 (Section 10.1) to draw an ellipse of eccentricity 0.2? The resulting ellipse would resemble the orbit of Mercury.

64. Halley's comet (See Section 10.2, Example 1.)

a. Write an equation for the orbit of Halley's comet in a coordinate system in which the sun lies at the origin and the other focus lies on the negative x-axis, scaled in astronomical units.

b. How close does the comet come to the sun in astronomical units? In kilometers?

c. What is the farthest the comet gets from the sun in astronomical units? In kilometers?

In Exercises 65–68, find a polar equation for the given curve. In each case, sketch a typical curve.

65. $x^2 + y^2 - 2ay = 0$ **66.** $y^2 = 4ax + 4a^2$

67. $x\cos\alpha + y\sin\alpha = p$ (α, p constant)

68. $(x^2 + y^2)^2 + 2ax(x^2 + y^2) - a^2y^2 = 0$

COMPUTER EXPLORATIONS

69. Use a CAS to plot the polar equation

$$r = \frac{ke}{1 + e\cos\theta}$$

for various values of k and e, $-\pi \le \theta \le \pi$. Answer the following questions.

a. Take $k = -2$. Describe what happens to the plots as you take e to be 3/4, 1, and 5/4. Repeat for $k = 2$.

b. Take $k = -1$. Describe what happens to the plots as you take e to be $7/6, 5/4, 4/3, 3/2, 2, 3, 5, 10$, and 20. Repeat for $e = 1/2, 1/3, 1/4, 1/10$, and $1/20$.

c. Now keep $e > 0$ fixed and describe what happens as you take k to be $-1, -2, -3, -4$, and -5. Be sure to look at graphs for parabolas, ellipses, and hyperbolas.

70. Use a CAS to plot the polar ellipse

$$r = \frac{a(1 - e^2)}{1 + e\cos\theta}$$

for various values of $a > 0$ and $0 < e < 1, -\pi \leq \theta \leq \pi$.

a. Take $e = 9/10$. Describe what happens to the plots as you let a equal $1, 3/2, 2, 3, 5$, and 10. Repeat with $e = 1/4$.

b. Take $a = 2$. Describe what happens as you take e to be $9/10, 8/10, 7/10, \ldots, 1/10, 1/20$, and $1/50$.