Chapter 10 Practice Exercises

Graphing Conic Sections

Sketch the parabolas in Exercises 1–4. Include the focus and directrix in each sketch.

1. $x^2 = -4y$

2. $x^2 = 2y$

3. $y^2 = 3x$

4. $y^2 = -(8/3)x$

Find the eccentricities of the ellipses and hyperbolas in Exercises 5–8. Sketch each conic section. Include the foci, vertices, and asymptotes (as appropriate) in your sketch.

5. $16x^2 + 7y^2 = 112$

6. $x^2 + 2y^2 = 4$

7. $3x^2 - y^2 = 3$

8. $5y^2 - 4x^2 = 20$

Shifting Conic Sections

Exercises 9–14 give equations for conic sections and tell how many units up or down and to the right or left each curve is to be shifted. Find an equation for the new conic section and find the new foci, vertices, centers, and asymptotes, as appropriate. If the curve is a parabola, find the new directrix as well.

9. $x^2 = -12y$, right 2, up 3

10. $y^2 = 10x$, left 1/2, down 1

11. $\frac{x^2}{9} + \frac{y^2}{25} = 1$, left 3, down 5

12. $\frac{x^2}{169} + \frac{y^2}{144} = 1$, right 5, up 12

13. $\frac{y^2}{8} - \frac{x^2}{2} = 1$, right 2, up $2\sqrt{2}$

14. $\frac{x^2}{36} - \frac{y^2}{64} = 1$, left 10, down 3

Identifying Conic Sections

Identify the conic sections in Exercises 15–22 and find their foci, vertices, centers, and asymptotes (as appropriate). If the curve is a parabola, find its directrix as well.

15. $x^2 - 4x - 4y^2 = 0$

16. $4x^2 - y^2 + 4y = 8$

17. $y^2 - 2y + 16x = -49$

18. $x^2 - 2x + 8y = -17$

19. $9x^2 + 16y^2 + 54x - 64y = -1$

20. $25x^2 + 9y^2 - 100x + 54y = 44$

21. $x^2 + y^2 - 2x - 2y = 0$

22. $x^2 + y^2 + 4x + 2y = 1$

Using the Discriminant

What conic sections or degenerate cases do the equations in Exercises 23–28 represent? Give a reason for your answer in each case.

23. $x^2 + xy + y^2 + x + y + 1 = 0$

24. $x^2 + 4xy + 4y^2 + x + y + 1 = 0$

25. $x^2 + 3xy + 2y^2 + x + y + 1 = 0$

26. $x^2 + 2xy - 2y^2 + x + y + 1 = 0$

27. $x^2 - 2xy + y^2 = 0$

28. $x^2 - 3xy + 4y^2 = 0$

Rotating Conic Sections

Identify the conic sections in Exercises 29–32. Then rotate the coordinate axes to find a new equation for the conic section that has no cross product term. (The new equations will vary with the size and direction of the rotations used.)

29. $2x^2 + xy + 2y^2 - 15 = 0$

30. $3x^2 + 2xy + 3y^2 = 19$

31. $x^2 + 2\sqrt{3}\,xy - y^2 + 4 = 0$

32. $x^2 - 3xy + y^2 = 5$

Identifying Parametric Equations in the Plane

Exercises 33–36 give parametric equations and parameter intervals for the motion of a particle in the xy-plane. Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation and indicate the direction of motion and the portion traced by the particle.

33. $x = (1/2)\tan t$, $y = (1/2)\sec t$; $-\pi/2 < t < \pi/2$

34. $x = -2\cos t$, $y = 2\sin t$; $0 \le t \le \pi$

35. $x = -\cos t$, $y = \cos^2 t$; $0 \le t \le \pi$

36. $x = 4\cos t$, $y = 9\sin t$; $0 \le t \le 2\pi$

Graphs in the Polar Plane

Sketch the regions defined by the polar coordinate inequalities in Exercises 37 and 38.

37. $0 \le r \le 6\cos\theta$

38. $-4\sin\theta \le r \le 0$

Match each graph in Exercises 39–46 with the appropriate equation (a)–(l). There are more equations than graphs, so some equations will not be matched.

a. $r = \cos 2\theta$

b. $r\cos\theta = 1$

c. $r = \frac{6}{1 - 2\cos\theta}$

d. $r = \sin 2\theta$

e. $r = \theta$

f. $r^2 = \cos 2\theta$

g. $r = 1 + \cos\theta$

h. $r = 1 - \sin\theta$

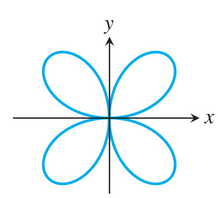
i. $r = \frac{2}{1 - \cos\theta}$

j. $r^2 = \sin 2\theta$

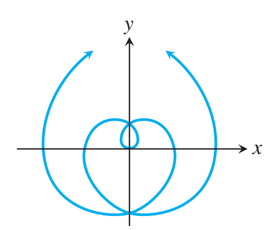
k. $r = -\sin\theta$

l. $r = 2\cos\theta + 1$

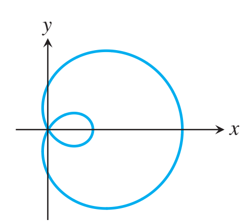
39. Four-leaved rose



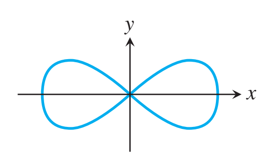
40. Spiral



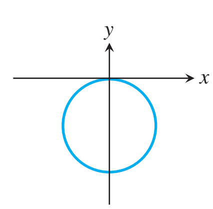
41. Limaçon



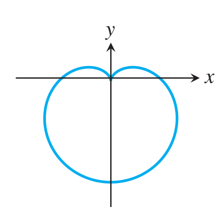
42. Lemniscate



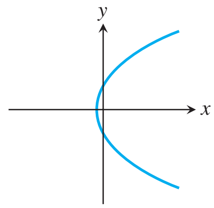
43. Circle



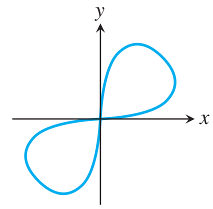
44. Cardioid



45. Parabola



46. Lemniscate



Intersections of Graphs in the Polar Plane

Find the points of intersection of the curves given by the polar coordinate equations in Exercises 47–54.

47. $r = \sin\theta, \quad r = 1 + \sin\theta$

48. $r = \cos\theta, \quad r = 1 - \cos\theta$

49. $r = 1 + \cos\theta, \quad r = 1 - \cos\theta$

50. $r = 1 + \sin\theta, \quad r = 1 - \sin\theta$

51. $r = 1 + \sin\theta, \quad r = -1 + \sin\theta$

52. $r = 1 + \cos\theta, \quad r = -1 + \cos\theta$

53. $r = \sec\theta, \quad r = 2\sin\theta$

54. $r = -2\csc\theta, \quad r = -4\cos\theta$

Polar to Cartesian Equations

Sketch the lines in Exercises 55–60. Also, find a Cartesian equation for each line.

55. $r\cos\left(\theta + \dfrac{\pi}{3}\right) = 2\sqrt{3}$

56. $r\cos\left(\theta - \dfrac{3\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

57. $r = 2\sec\theta$

58. $r = -\sqrt{2}\sec\theta$

59. $r = -(3/2)\csc\theta$

60. $r = \left(3\sqrt{3}\right)\csc\theta$

Find Cartesian equations for the circles in Exercises 61–64. Sketch each circle in the coordinate plane and label it with both its Cartesian and polar equations.

61. $r = -4\sin\theta$

62. $r = 3\sqrt{3}\sin\theta$

63. $r = 2\sqrt{2}\cos\theta$

64. $r = -6\cos\theta$

Cartesian to Polar Equations

Find polar equations for the circles in Exercises 65–68. Sketch each circle in the coordinate plane and label it with both its Cartesian and polar equations.

65. $x^2 + y^2 + 5y = 0$

66. $x^2 + y^2 - 2y = 0$

67. $x^2 + y^2 - 3x = 0$

68. $x^2 + y^2 + 4x = 0$

Conic Sections in Polar Coordinates

Sketch the conic sections whose polar coordinate equations are given in Exercises 69–72. Give polar coordinates for the vertices and, in the case of ellipses, for the centers as well.

69. $r = \dfrac{2}{1 + \cos\theta}$

70. $r = \dfrac{8}{2 + \cos\theta}$

71. $r = \dfrac{6}{1 - 2\cos\theta}$

72. $r = \dfrac{12}{3 + \sin\theta}$

Exercises 73–76 give the eccentricities of conic sections with one focus at the origin of the polar coordinate plane, along with the directrix for that focus. Find a polar equation for each conic section.

73. $e = 2, \quad r\cos\theta = 2$

74. $e = 1, \quad r\cos\theta = -4$

75. $e = 1/2, \quad r\sin\theta = 2$

76. $e = 1/3, \quad r\sin\theta = -6$

Area, Length, and Surface Area in the Polar Plane

Find the areas of the regions in the polar coordinate plane described in Exercises 77–80.

77. Enclosed by the limaçon $r = 2 - \cos\theta$

78. Enclosed by one leaf of the three-leaved rose $r = \sin 3\theta$

79. Inside the "figure eight" $r = 1 + \cos 2\theta$ and outside the circle $r = 1$

80. Inside the cardioid $r = 2(1 + \sin\theta)$ and outside the circle $r = 2\sin\theta$

Find the lengths of the curves given by the polar coordinate equations in Exercises 81–84.

81. $r = -1 + \cos\theta$

82. $r = 2\sin\theta + 2\cos\theta, \quad 0 \le \theta \le \pi/2$

83. $r = 8\sin^3(\theta/3), \quad 0 \le \theta \le \pi/4$

84. $r = \sqrt{1 + \cos 2\theta}, \quad -\pi/2 \le \theta \le \pi/2$

Find the areas of the surfaces generated by revolving the polar coordinate curves in Exercises 85 and 86 about the indicated axes.

85. $r = \sqrt{\cos 2\theta}, \quad 0 \le \theta \le \pi/4, \quad x\text{-axis}$

86. $r^2 = \sin 2\theta, \quad y\text{-axis}$

Theory and Examples

87. Find the volume of the solid generated by revolving the region enclosed by the ellipse $9x^2 + 4y^2 = 36$ about **(a)** the x-axis, **(b)** the y-axis.

88. The "triangular" region in the first quadrant bounded by the x-axis, the line $x = 4$, and the hyperbola $9x^2 - 4y^2 = 36$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

89. A ripple tank is made by bending a strip of tin around the perimeter of an ellipse for the wall of the tank and soldering a flat bottom onto this. An inch or two of water is put in the tank and you drop a marble into it, right at one focus of the ellipse. Ripples radiate outward through the water, reflect from the strip around the edge of the tank, and a few seconds later a drop of water spurts up at the second focus. Why?

90. **LORAN** A radio signal was sent simultaneously from towers A and B, located several hundred miles apart on the northern California coast. A ship offshore received the signal from A 1400 microseconds before receiving the signal from B. Assuming that the signals traveled at the rate of 980 ft/microsecond, what can be said about the location of the ship relative to the two towers?

91. On a level plane, at the same instant, you hear the sound of a rifle and that of the bullet hitting the target. What can be said about your location relative to the rifle and target?

92. Archimedes spirals The graph of an equation of the form $r = a\theta$, where a is a nonzero constant, is called an *Archimedes spiral*. Is there anything special about the widths between the successive turns of such a spiral?

93. a. Show that the equations $x = r\cos\theta$, $y = r\sin\theta$ transform the polar equation

$$r = \frac{k}{1 + e\cos\theta}$$

into the Cartesian equation

$$(1 - e^2)x^2 + y^2 + 2kex - k^2 = 0.$$

b. Then apply the criteria of Section 10.3 to show that

$$\begin{aligned} e = 0 &\Rightarrow \text{circle.} \\ 0 < e < 1 &\Rightarrow \text{ellipse.} \\ e = 1 &\Rightarrow \text{parabola.} \\ e > 1 &\Rightarrow \text{hyperbola.} \end{aligned}$$

94. A satellite orbit A satellite is in an orbit that passes over the North and South Poles of the earth. When it is over the South Pole it is at the highest point of its orbit, 1000 miles above the earth's surface. Above the North Pole it is at the lowest point of its orbit, 300 miles above the earth's surface.

a. Assuming that the orbit is an ellipse with one focus at the center of the earth, find its eccentricity. (Take the diameter of the earth to be 8000 miles.)

b. Using the north–south axis of the earth as the x-axis and the center of the earth as origin, find a polar equation for the orbit.