EXERCISES 11.5

Determining Convergence or Divergence

Which of the series in Exercises 1–26 converge, and which diverge? Give reasons for your answers. (When checking your answers, remember there may be more than one way to determine a series' convergence or divergence.)

1. $\sum_{n=1}^{\infty} \frac{n^{\sqrt{2}}}{2^n}$

2. $\sum_{n=1}^{\infty} n^2 e^{-n}$

3. $\sum_{n=1}^{\infty} n! e^{-n}$

4. $\sum_{n=1}^{\infty} \frac{n!}{10^n}$

5. $\sum_{n=1}^{\infty} \frac{n^{10}}{10^n}$

6. $\sum_{n=1}^{\infty} \left(\frac{n-2}{n}\right)^n$

7. $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{1.25^n}$

8. $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n}$

9. $\sum_{n=1}^{\infty} \left(1-\frac{3}{n}\right)^n$

10. $\sum_{n=1}^{\infty} \left(1-\frac{1}{3n}\right)^n$

11. $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

12. $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$

13. $\sum_{n=1}^{\infty} \left(\frac{1}{n}-\frac{1}{n^2}\right)$

14. $\sum_{n=1}^{\infty} \left(\frac{1}{n}-\frac{1}{n^2}\right)^n$

15. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$

16. $\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$

17. $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!}$

18. $\sum_{n=1}^{\infty} e^{-n}(n^3)$

19. $\sum_{n=1}^{\infty} \frac{(n+3)!}{3!n!3^n}$

20. $\sum_{n=1}^{\infty} \frac{n2^n(n+1)!}{3^n n!}$

21. $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$

22. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

23. $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^n}$

24. $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^{(n/2)}}$

25. $\sum_{n=1}^{\infty} \frac{n! \ln n}{n(n+2)!}$

26. $\sum_{n=1}^{\infty} \frac{3^n}{n^3 2^n}$

Which of the series $\sum_{n=1}^{\infty} a_n$ defined by the formulas in Exercises 27–38 converge, and which diverge? Give reasons for your answers.

27. $a_1 = 2, \quad a_{n+1} = \frac{1+\sin n}{n} a_n$

28. $a_1 = 1, \quad a_{n+1} = \frac{1+\tan^{-1} n}{n} a_n$

29. $a_1 = \frac{1}{3}, \quad a_{n+1} = \frac{3n-1}{2n+5} a_n$

30. $a_1 = 3, \quad a_{n+1} = \frac{n}{n+1} a_n$

31. $a_1 = 2, \quad a_{n+1} = \frac{2}{n} a_n$

32. $a_1 = 5, \quad a_{n+1} = \frac{\sqrt[n]{n}}{2} a_n$

33. $a_1 = 1, \quad a_{n+1} = \frac{1+\ln n}{n} a_n$

34. $a_1 = \frac{1}{2}, \quad a_{n+1} = \frac{n+\ln n}{n+10} a_n$

35. $a_1 = \frac{1}{3}, \quad a_{n+1} = \sqrt[n]{a_n}$

36. $a_1 = \frac{1}{2}, \quad a_{n+1} = (a_n)^{n+1}$

37. $a_n = \frac{2^n n! n!}{(2n)!}$

38. $a_n = \frac{(3n)!}{n!(n+1)!(n+2)!}$

Which of the series in Exercises 39–44 converge, and which diverge? Give reasons for your answers.

39. $\sum_{n=1}^{\infty} \frac{(n!)^n}{(n^n)^2}$

40. $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{(n^2)}}$

41. $\sum_{n=1}^{\infty} \frac{n^n}{2^{(n^2)}}$

42. $\sum_{n=1}^{\infty} \frac{n^n}{(2^n)^2}$

43. $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot \cdots \cdot (2n-1)}{4^n 2^n n!}$

44. $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot \cdots \cdot (2n-1)}{[2 \cdot 4 \cdot \cdots \cdot (2n)](3^n+1)}$

Theory and Examples

45. Neither the Ratio nor the Root Test helps with p-series. Try them on

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

and show that both tests fail to provide information about convergence.

46. Show that neither the Ratio Test nor the Root Test provides information about the convergence of

$$\sum_{n=2}^{\infty} \frac{1}{(\ln n)^p} \qquad (p \text{ constant}).$$

47. Let $a_n = \begin{cases} n/2^n, & \text{if } n \text{ is a prime number} \\ 1/2^n, & \text{otherwise.} \end{cases}$

Does $\sum a_n$ converge? Give reasons for your answer.