EXERCISES 11.11

Finding Fourier Series

In Exercises 1–8, find the Fourier series associated with the given functions. Sketch each function.

1. $f(x) = 1 \quad 0 \le x \le 2\pi$.

2. $f(x) = \begin{cases} 1, & 0 \le x \le \pi \\ -1, & \pi < x \le 2\pi \end{cases}$

3. $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ x - 2\pi, & \pi < x \le 2\pi \end{cases}$

4. $f(x) = \begin{cases} x^2, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$

5. $f(x) = e^x \quad 0 \le x \le 2\pi$.

6. $f(x) = \begin{cases} e^x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$

7. $f(x) = \begin{cases} \cos x, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$

8. $f(x) = \begin{cases} 2, & 0 \le x \le \pi \\ -x, & \pi < x \le 2\pi \end{cases}$

Theory and Examples

Establish the results in Exercises 9–13, where p and q are positive integers.

9. $\displaystyle\int_0^{2\pi} \cos px \, dx = 0$ for all p.

10. $\int_0^{2\pi} \sin px \, dx = 0$ for all p.

11. $\int_0^{2\pi} \cos px \cos qx \, dx = \begin{cases} 0, & \text{if } p \neq q \\ \pi, & \text{if } p = q \end{cases}$.

(*Hint:* $\cos A \cos B = (1/2)[\cos(A + B) + \cos(A - B)]$.)

12. $\int_0^{2\pi} \sin px \sin qx \, dx = \begin{cases} 0, & \text{if } p \neq q \\ \pi, & \text{if } p = q \end{cases}$.

(*Hint:* $\sin A \sin B = (1/2)[\cos(A - B) - \cos(A + B)]$.)

13. $\int_0^{2\pi} \sin px \cos qx \, dx = 0$ for all p and q.

(*Hint*: $\sin A \cos B = (1/2)[\sin(A + B) + \sin(A - B)]$.)

14. Fourier series of sums of functions If f and g both satisfy the conditions of Theorem 24, is the Fourier series of $f + g$ on $[0, 2\pi]$ the sum of the Fourier series of f and the Fourier series of g? Give reasons for your answer.

15. Term-by-term differentiation

a. Use Theorem 24 to verify that the Fourier series for $f(x)$ in Exercise 3 converges to $f(x)$ for $0 < x < 2\pi$.

b. Although $f'(x) = 1$, show that the series obtained by term-by-term differentiation of the Fourier series in part (a) diverges.

16. Use Theorem 24 to find the Value of the Fourier series determined in Exercise 4 and show that $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$.