Chapter 11 Questions to Guide Your Review

1. What is an infinite sequence? What does it mean for such a sequence to converge? To diverge? Give examples.
2. What is a nondecreasing sequence? Under what circumstances does such a sequence have a limit? Give examples.
3. What theorems are available for calculating limits of sequences? Give examples.
4. What theorem sometimes enables us to use l'Hôpital's Rule to calculate the limit of a sequence? Give an example.
5. What six sequence limits are likely to arise when you work with sequences and series?
6. What is an infinite series? What does it mean for such a series to converge? To diverge? Give examples.
7. What is a geometric series? When does such a series converge? Diverge? When it does converge, what is its sum? Give examples.
8. Besides geometric series, what other convergent and divergent series do you know?
9. What is the nth-Term Test for Divergence? What is the idea behind the test?
10. What can be said about term-by-term sums and differences of convergent series? About constant multiples of convergent and divergent series?
11. What happens if you add a finite number of terms to a convergent series? A divergent series? What happens if you delete a finite number of terms from a convergent series? A divergent series?
12. How do you reindex a series? Why might you want to do this?
13. Under what circumstances will an infinite series of nonnegative terms converge? Diverge? Why study series of nonnegative terms?
14. What is the Integral Test? What is the reasoning behind it? Give an example of its use.
15. When do p-series converge? Diverge? How do you know? Give examples of convergent and divergent p-series.
16. What are the Direct Comparison Test and the Limit Comparison Test? What is the reasoning behind these tests? Give examples of their use.
17. What are the Ratio and Root Tests? Do they always give you the information you need to determine convergence or divergence? Give examples.
18. What is an alternating series? What theorem is available for determining the convergence of such a series?
19. How can you estimate the error involved in approximating the sum of an alternating series with one of the series' partial sums? What is the reasoning behind the estimate?
20. What is absolute convergence? Conditional convergence? How are the two related?
21. What do you know about rearranging the terms of an absolutely convergent series? Of a conditionally convergent series? Give examples.
22. What is a power series? How do you test a power series for convergence? What are the possible outcomes?
23. What are the basic facts about
    a. term-by-term differentiation of power series?
    b. term-by-term integration of power series?
    c. multiplication of power series?
    Give examples.
24. What is the Taylor series generated by a function $f(x)$ at a point $x = a$? What information do you need about f to construct the series? Give an example.
25. What is a Maclaurin series?

26. Does a Taylor series always converge to its generating function? Explain.

27. What are Taylor polynomials? Of what use are they?

28. What is Taylor's formula? What does it say about the errors involved in using Taylor polynomials to approximate functions? In particular, what does Taylor's formula say about the error in a linearization? A quadratic approximation?

29. What is the binomial series? On what interval does it converge? How is it used?

30. How can you sometimes use power series to solve initial value problems?

31. How can you sometimes use power series to estimate the values of nonelementary definite integrals?

32. What are the Taylor series for $1/(1 - x)$, $1/(1 + x)$, e^x, $\sin x$, $\cos x$, $\ln(1 + x)$, $\ln[(1 + x)/(1 - x)]$, and $\tan^{-1} x$? How do you estimate the errors involved in replacing these series with their partial sums?

33. What is a Fourier series? How do you calculate the Fourier coefficients $a_0, a_1, a_2, \ldots$ and $b_1, b_2, \ldots$ for a function $f(x)$ defined on the interval $[0, 2\pi]$?

34. State the theorem on convergence of the Fourier series for $f(x)$ when f and f' are piecewise continuous on $[0, 2\pi]$.