EXERCISES 12.6

Matching Equations with Surfaces

In Exercises 1–12, match the equation with the surface it defines. Also, identify each surface by type (paraboloid, ellipsoid, etc.) The surfaces are labeled (a)–(1).

1. $x^2 + y^2 + 4z^2 = 10$

2. $z^2 + 4y^2 - 4x^2 = 4$

3. $9y^2 + z^2 = 16$

4. $y^2 + z^2 = x^2$

5. $x = y^2 - z^2$

6. $x = -y^2 - z^2$

7. $x^2 + 2z^2 = 8$

8. $z^2 + x^2 - y^2 = 1$

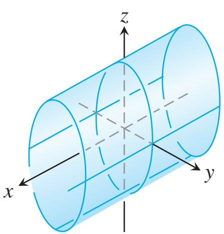
9. $x = z^2 - y^2$

10. $z = -4x^2 - y^2$

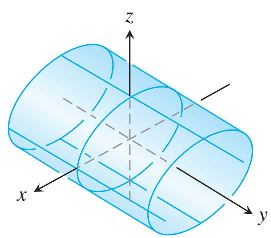
11. $x^2 + 4z^2 = y^2$

12. $9x^2 + 4y^2 + 2z^2 = 36$

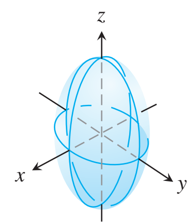
a.



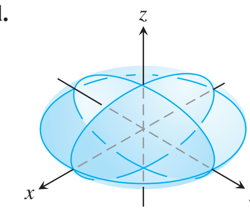
b.



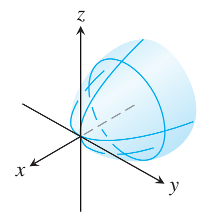
c.



d.



e.



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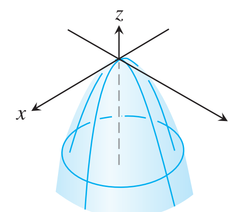
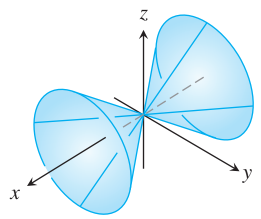
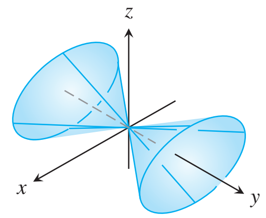


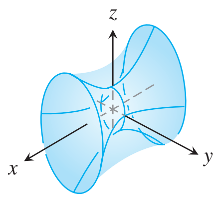
g.



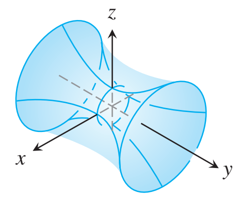
h.



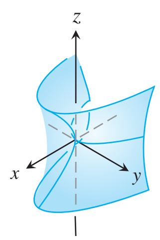
i.



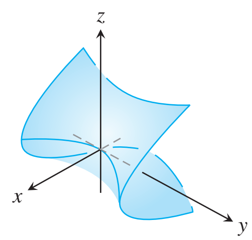
j.



k.



l.



Drawing

Sketch the surfaces in Exercises 13–76.

CYLINDERS

13. $x^2 + y^2 = 4$
14. $x^2 + z^2 = 4$
15. $z = y^2 - 1$
16. $x = y^2$
17. $x^2 + 4z^2 = 16$
18. $4x^2 + y^2 = 36$
19. $z^2 - y^2 = 1$
20. $yz = 1$

ELLIPSOIDS

21. $9x^2 + y^2 + z^2 = 9$
22. $4x^2 + 4y^2 + z^2 = 16$
23. $4x^2 + 9y^2 + 4z^2 = 36$
24. $9x^2 + 4y^2 + 36z^2 = 36$

PARABOLOIDS

25. $z = x^2 + 4y^2$
26. $z = x^2 + 9y^2$
27. $z = 8 - x^2 - y^2$
28. $z = 18 - x^2 - 9y^2$
29. $x = 4 - 4y^2 - z^2$
30. $y = 1 - x^2 - z^2$

CONES

31. $x^2 + y^2 = z^2$
32. $y^2 + z^2 = x^2$
33. $4x^2 + 9z^2 = 9y^2$
34. $9x^2 + 4y^2 = 36z^2$

HYPERBOLOIDS

35. $x^2 + y^2 - z^2 = 1$
36. $y^2 + z^2 - x^2 = 1$
37. $(y^2/4) + (z^2/9) - (x^2/4) = 1$
38. $(x^2/4) + (y^2/4) - (z^2/9) = 1$
39. $z^2 - x^2 - y^2 = 1$
40. $(y^2/4) - (x^2/4) - z^2 = 1$
41. $x^2 - y^2 - (z^2/4) = 1$
42. $(x^2/4) - y^2 - (z^2/4) = 1$

HYPERBOLIC PARABOLOIDS

43. $y^2 - x^2 = z$
44. $x^2 - y^2 = z$

ASSORTED

45. $x^2 + y^2 + z^2 = 4$
46. $4x^2 + 4y^2 = z^2$
47. $z = 1 + y^2 - x^2$
48. $y^2 - z^2 = 4$
49. $y = -(x^2 + z^2)$
50. $z^2 - 4x^2 - 4y^2 = 4$
51. $16x^2 + 4y^2 = 1$
52. $z = x^2 + y^2 + 1$
53. $x^2 + y^2 - z^2 = 4$
54. $x = 4 - y^2$
55. $x^2 + z^2 = y$
56. $z^2 - (x^2/4) - y^2 = 1$
57. $x^2 + z^2 = 1$
58. $4x^2 + 4y^2 + z^2 = 4$
59. $16y^2 + 9z^2 = 4x^2$
60. $z = x^2 - y^2 - 1$
61. $9x^2 + 4y^2 + z^2 = 36$
62. $4x^2 + 9z^2 = y^2$
63. $x^2 + y^2 - 16z^2 = 16$
64. $z^2 + 4y^2 = 9$
65. $z = -(x^2 + y^2)$
66. $y^2 - x^2 - z^2 = 1$
67. $x^2 - 4y^2 = 1$
68. $z = 4x^2 + y^2 - 4$
69. $4y^2 + z^2 - 4x^2 = 4$
70. $z = 1 - x^2$
71. $x^2 + y^2 = z$
72. $(x^2/4) + y^2 - z^2 = 1$
73. $yz = 1$
74. $36x^2 + 9y^2 + 4z^2 = 36$
75. $9x^2 + 16y^2 = 4z^2$
76. $4z^2 - x^2 - y^2 = 4$

Theory and Examples

77. **a.** Express the area A of the cross-section cut from the ellipsoid

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$$

by the plane $z = c$ as a function of c. (The area of an ellipse with semiaxes a and b is πab.)

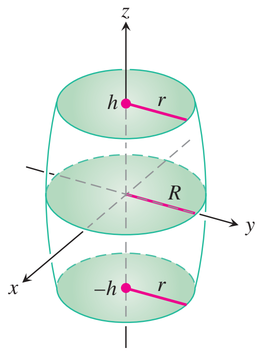
b. Use slices perpendicular to the z-axis to find the volume of the ellipsoid in part (a).

c. Now find the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Does your formula give the volume of a sphere of radius a if $a = b = c$?

78. The barrel shown here is shaped like an ellipsoid with equal pieces cut from the ends by planes perpendicular to the z-axis. The cross-sections perpendicular to the z-axis are circular. The barrel is $2h$ units high, its midsection radius is R, and its end radii are both r. Find a formula for the barrel's volume. Then check two things. First, suppose the sides of the barrel are straightened to turn the barrel into a cylinder of radius R and height $2h$. Does your formula give the cylinder's volume? Second, suppose $r = 0$ and $h = R$ so the barrel is a sphere. Does your formula give the sphere's volume?



79. Show that the volume of the segment cut from the paraboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$$

by the plane $z = h$ equals half the segment's base times its altitude. (Figure 12.49 shows the segment for the special case $h = c$.)

80. **a.** Find the volume of the solid bounded by the hyperboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

and the planes $z = 0$ and $z = h$, $h > 0$.

b. Express your answer in part (a) in terms of h and the areas A_0 and A_h of the regions cut by the hyperboloid from the planes $z = 0$ and $z = h$.

c. Show that the volume in part (a) is also given by the formula

$$V = \frac{h}{6}(A_0 + 4A_m + A_h),$$

where A_m is the area of the region cut by the hyperboloid from the plane $z = h/2$.

81. If the hyperbolic paraboloid $(y^2/b^2) - (x^2/a^2) = z/c$ is cut by the plane $y = y_1$, the resulting curve is a parabola. Find its vertex and focus.

82. Suppose you set $z = 0$ in the equation

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Jz + K = 0$$

to obtain a curve in the xy-plane. What will the curve be like? Give reasons for your answer.

83. Every time we found the trace of a quadric surface in a plane parallel to one of the coordinate planes, it turned out to be a conic section. Was this mere coincidence? Did it have to happen? Give reasons for your answer.

84. Suppose you intersect a quadric surface with a plane that is *not* parallel to one of the coordinate planes. What will the trace in the plane be like? Give reasons for your answer.

T Computer Grapher Explorations

Plot the surfaces in Exercises 85–88 over the indicated domains. If you can, rotate the surface into different viewing positions.

85. $z = y^2, \quad -2 \le x \le 2, \quad -0.5 \le y \le 2$

86. $z = 1 - y^2, \quad -2 \le x \le 2, \quad -2 \le y \le 2$

87. $z = x^2 + y^2, \quad -3 \le x \le 3, \quad -3 \le y \le 3$

88. $z = x^2 + 2y^2$ over

a. $-3 \le x \le 3, \quad -3 \le y \le 3$

b. $-1 \le x \le 1, \quad -2 \le y \le 3$

c. $-2 \le x \le 2, \quad -2 \le y \le 2$

d. $-2 \le x \le 2, \quad -1 \le y \le 1$

COMPUTER EXPLORATIONS

Surface Plots

Use a CAS to plot the surfaces in Exercises 89–94. Identify the type of quadric surface from your graph.

89. $\frac{x^2}{9} + \frac{y^2}{36} = 1 - \frac{z^2}{25}$

90. $\frac{x^2}{9} - \frac{z^2}{9} = 1 - \frac{y^2}{16}$

91. $5x^2 = z^2 - 3y^2$

92. $\frac{y^2}{16} = 1 - \frac{x^2}{9} + z$

93. $\frac{x^2}{9} - 1 = \frac{y^2}{16} + \frac{z^2}{2}$

94. $y - \sqrt{4 - z^2} = 0$