EXERCISES 13.6

Reminder: When a calculation involves the gravitational constant G, express force in newtons, distance in meters, mass in kilograms, and time in seconds.

1. **Period of *Skylab 4*** Since the orbit of *Skylab 4* had a semimajor axis of $a = 6808$ km, Kepler's third law with M equal to Earth's mass should give the period. Calculate it. Compare your result with the value in Table 13.2.

2. **Earth's velocity at perihelion** Earth's distance from the sun at perihelion is approximately 149,577,000 km, and the eccentricity of Earth's orbit about the sun is 0.0167. Find the velocity v_0 of Earth in its orbit at perihelion. (Use Equation (15).)

3. **Semimajor axis of *Proton I*** In July 1965, the USSR launched *Proton I*, weighing 12,200 kg (at launch), with a perigee height of 183 km, an apogee height of 589 km, and a period of 92.25 min. Using the relevant data for the mass of Earth and the gravitational constant G, find the semimajor axis a of the orbit from Equation (3). Compare your answer with the number you get by adding the perigee and apogee heights to the diameter of the Earth.

4. **Semimajor axis of *Viking I*** The *Viking I* orbiter, which surveyed Mars from August 1975 to June 1976, had a period of 1639 min. Use this and the mass of Mars, 6.418×10^{23} kg, to find the semimajor axis of the *Viking I* orbit.

5. **Average diameter of Mars** (*Continuation of Exercise 4.*) The *Viking I* orbiter was 1499 km from the surface of Mars at its closest point and 35,800 km from the surface at its farthest point. Use this information together with the value you obtained in Exercise 4 to estimate the average diameter of Mars.

6. **Period of *Viking 2*** The *Viking 2* orbiter, which surveyed Mars from September 1975 to August 1976, moved in an ellipse whose semimajor axis was 22,030 km. What was the orbital period? (Express your answer in minutes.)

7. **Geosynchronous orbits** Several satellites in Earth's equatorial plane have nearly circular orbits whose periods are the same as Earth's rotational period. Such orbits are *geosynchronous* or *geostationary* because they hold the satellite over the same spot on the Earth's surface.

   a. Approximately what is the semimajor axis of a geosynchronous orbit? Give reasons for your answer.

   b. About how high is a geosynchronous orbit above Earth's surface?

   c. Which of the satellites in Table 13.2 have (nearly) geosynchronous orbits?

8. The mass of Mars is 6.418×10^{23} kg. If a satellite revolving about Mars is to hold a stationary orbit (have the same period as

the period of Mars's rotation, which is 1477.4 min), what must the semimajor axis of its orbit be? Give reasons for your answer.

Exercises

9. **Distance from Earth to the moon** The period of the moon's rotation about Earth is 2.36055×10^6 sec. About how far away is the moon?

10. **Finding satellite speed** A satellite moves around Earth in a circular orbit. Express the satellite's speed as a function of the orbit's radius.

11. **Orbital period** If T is measured in seconds and a in meters, what is the value of T^2/a^3 for planets in our solar system? For satellites orbiting Earth? For satellites orbiting the moon? (The moon's mass is 7.354×10^{22} kg.)

12. **Type of orbit** For what values of v_0 in Equation (15) is the orbit in Equation (16) a circle? An ellipse? A parabola? A hyperbola?

13. **Circular orbits** Show that a planet in a circular orbit moves with a constant speed. (*Hint:* This is a consequence of one of Kepler's laws.)

14. Suppose that $\mathbf{r}$ is the position vector of a particle moving along a plane curve and dA/dt is the rate at which the vector sweeps out area. Without introducing coordinates, and assuming the necessary derivatives exist, give a geometric argument based on increments and limits for the validity of the equation

$$\frac{dA}{dt} = \frac{1}{2}|\mathbf{r} \times \dot{\mathbf{r}}|.$$

15. **Kepler's third law** Complete the derivation of Kepler's third law (the part following Equation (34)).

In Exercises 16 and 17, two planets, planet A and planet B, are orbiting their sun in circular orbits with A being the inner planet and B being farther away from the sun. Suppose the positions of A and B at time t are

$$\mathbf{r}_A(t) = 2\cos(2\pi t)\mathbf{i} + 2\sin(2\pi t)\mathbf{j}$$

and

$$\mathbf{r}_B(t) = 3\cos(\pi t)\mathbf{i} + 3\sin(\pi t)\mathbf{j},$$

respectively, where the sun is assumed to be located at the origin and distance is measured in astronomical units. (Notice that planet A moves faster than planet B.)

The people on planet A regard their planet, not the sun, as the center of their planetary system (their solar system).

16. Using planet A as the origin of a new coordinate system, give parametric equations for the location of planet B at time t. Write your answer in terms of $\cos(\pi t)$ and $\sin(\pi t)$.

T 17. Using planet A as the origin, graph the path of planet B.

This exercise illustrates the difficulty that people before Kepler's time, with an earth-centered (planet A) view of our solar system, had in understanding the motions of the planets (i.e., planet B = Mars). See D. G. Saari's article in the *American Mathematical Monthly*, Vol. 97 (Feb. 1990), pp. 105–119.

18. Kepler discovered that the path of Earth around the sun is an ellipse with the sun at one of the foci. Let $\mathbf{r}(t)$ be the position vector from the center of the sun to the center of Earth at time t. Let $\mathbf{w}$ be the vector from Earth's South Pole to North Pole. It is known that $\mathbf{w}$ is constant and not orthogonal to the plane of the ellipse (Earth's axis is tilted). In terms of $\mathbf{r}(t)$ and $\mathbf{w}$, give the mathematical meaning of (i) perihelion, (ii) aphelion, (iii) equinox, (iv) summer solstice, (v) winter solstice.