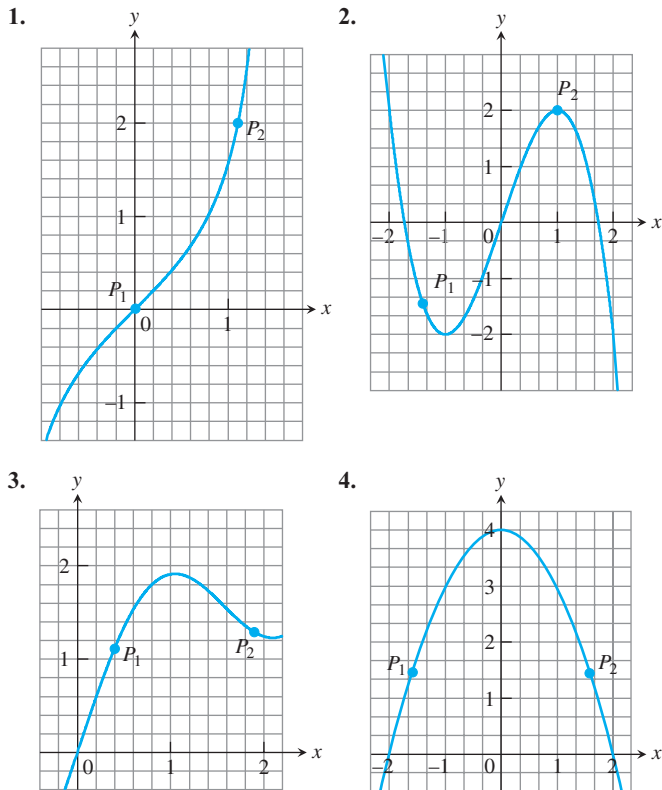


EXERCISES 2.7

Slopes and Tangent Lines

In Exercises 1–4, use the grid and a straight edge to make a rough estimate of the slope of the curve (in y -units per x -unit) at the points P_1 and P_2 . Graphs can shift during a press run, so your estimates may be somewhat different from those in the back of the book.



In Exercises 5–10, find an equation for the tangent to the curve at the given point. Then sketch the curve and tangent together.

5. $y = 4 - x^2$, $(-1, 3)$ 6. $y = (x - 1)^2 + 1$, $(1, 1)$
 7. $y = 2\sqrt{x}$, $(1, 2)$ 8. $y = \frac{1}{x^2}$, $(-1, 1)$
 9. $y = x^3$, $(-2, -8)$ 10. $y = \frac{1}{x^3}$, $(-2, -\frac{1}{8})$

In Exercises 11–18, find the slope of the function's graph at the given point. Then find an equation for the line tangent to the graph there.

11. $f(x) = x^2 + 1$, $(2, 5)$ 12. $f(x) = x - 2x^2$, $(1, -1)$
 13. $g(x) = \frac{x}{x-2}$, $(3, 3)$ 14. $g(x) = \frac{8}{x^2}$, $(2, 2)$
 15. $h(t) = t^3$, $(2, 8)$ 16. $h(t) = t^3 + 3t$, $(1, 4)$
 17. $f(x) = \sqrt{x}$, $(4, 2)$ 18. $f(x) = \sqrt{x+1}$, $(8, 3)$

In Exercises 19–22, find the slope of the curve at the point indicated.

19. $y = 5x^2$, $x = -1$ 20. $y = 1 - x^2$, $x = 2$
 21. $y = \frac{1}{x-1}$, $x = 3$ 22. $y = \frac{x-1}{x+1}$, $x = 0$

Tangent Lines with Specified Slopes

At what points do the graphs of the functions in Exercises 23 and 24 have horizontal tangents?

23. $f(x) = x^2 + 4x - 1$ 24. $g(x) = x^3 - 3x$
 25. Find equations of all lines having slope -1 that are tangent to the curve $y = 1/(x-1)$.
 26. Find an equation of the straight line having slope $1/4$ that is tangent to the curve $y = \sqrt{x}$.

Rates of Change

27. **Object dropped from a tower** An object is dropped from the top of a 100-m-high tower. Its height above ground after t sec is $100 - 4.9t^2$ m. How fast is it falling 2 sec after it is dropped?
 28. **Speed of a rocket** At t sec after liftoff, the height of a rocket is $3t^2$ ft. How fast is the rocket climbing 10 sec after liftoff?
 29. **Circle's changing area** What is the rate of change of the area of a circle ($A = \pi r^2$) with respect to the radius when the radius is $r = 3$?
 30. **Ball's changing volume** What is the rate of change of the volume of a ball ($V = (4/3)\pi r^3$) with respect to the radius when the radius is $r = 2$?

Testing for Tangents

31. Does the graph of

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

have a tangent at the origin? Give reasons for your answer.

32. Does the graph of

$$g(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

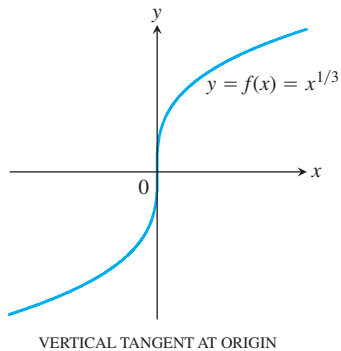
have a tangent at the origin? Give reasons for your answer.

Vertical Tangents

We say that the curve $y = f(x)$ has a **vertical tangent** at the point where $x = x_0$ if $\lim_{h \rightarrow 0} (f(x_0 + h) - f(x_0))/h = \infty$ or $-\infty$.

Vertical tangent at $x = 0$ (see accompanying figure):

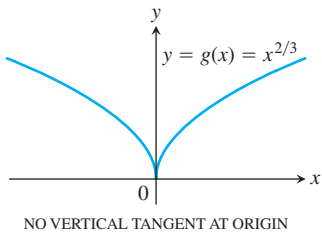
$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0} \frac{h^{1/3} - 0}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h^{2/3}} = \infty \end{aligned}$$



No vertical tangent at $x = 0$ (see next figure):

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{g(0+h) - g(0)}{h} &= \lim_{h \rightarrow 0} \frac{h^{2/3} - 0}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h^{1/3}} \end{aligned}$$

does not exist, because the limit is ∞ from the right and $-\infty$ from the left.



33. Does the graph of

$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

have a vertical tangent at the origin? Give reasons for your answer.

34. Does the graph of

$$U(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

have a vertical tangent at the point $(0, 1)$? Give reasons for your answer.

- T** a. Graph the curves in Exercises 35–44. Where do the graphs appear to have vertical tangents?
b. Confirm your findings in part (a) with limit calculations. But before you do, read the introduction to Exercises 33 and 34.

35. $y = x^{2/5}$

36. $y = x^{4/5}$

37. $y = x^{1/5}$

38. $y = x^{3/5}$

39. $y = 4x^{2/5} - 2x$

40. $y = x^{5/3} - 5x^{2/3}$

41. $y = x^{2/3} - (x-1)^{1/3}$

42. $y = x^{1/3} + (x-1)^{1/3}$

43. $y = \begin{cases} -\sqrt{|x|}, & x \leq 0 \\ \sqrt{x}, & x > 0 \end{cases}$

44. $y = \sqrt{|4-x|}$

COMPUTER EXPLORATIONS

Graphing Secant and Tangent Lines

Use a CAS to perform the following steps for the functions in Exercises 45–48.

- a. Plot $y = f(x)$ over the interval $(x_0 - 1/2) \leq x \leq (x_0 + 3)$.
b. Holding x_0 fixed, the difference quotient

$$q(h) = \frac{f(x_0 + h) - f(x_0)}{h}$$

at x_0 becomes a function of the step size h . Enter this function into your CAS workspace.

- c. Find the limit of q as $h \rightarrow 0$.
d. Define the secant lines $y = f(x_0) + q \cdot (x - x_0)$ for $h = 3, 2,$ and 1 . Graph them together with f and the tangent line over the interval in part (a).

45. $f(x) = x^3 + 2x, \quad x_0 = 0$ 46. $f(x) = x + \frac{5}{x}, \quad x_0 = 1$

47. $f(x) = x + \sin(2x), \quad x_0 = \pi/2$

48. $f(x) = \cos x + 4 \sin(2x), \quad x_0 = \pi$