Chapter 2 Questions to Guide Your Review

1. What is the average rate of change of the function $g(t)$ over the interval from $t = a$ to $t = b$? How is it related to a secant line?
2. What limit must be calculated to find the rate of change of a function $g(t)$ at $t = t_0$?
3. What is an informal or intuitive definition of the limit

$$\lim_{x \to x_0} f(x) = L?$$

Why is the definition "informal"? Give examples.

4. Does the existence and value of the limit of a function $f(x)$ as x approaches x_0 ever depend on what happens at $x = x_0$? Explain and give examples.
5. What function behaviors might occur for which the limit may fail to exist? Give examples.
6. What theorems are available for calculating limits? Give examples of how the theorems are used.

7. How are one-sided limits related to limits? How can this relationship sometimes be used to calculate a limit or prove it does not exist? Give examples.

8. What is the value of $\lim_{\theta \to 0} ((\sin \theta)/\theta)$? Does it matter whether θ is measured in degrees or radians? Explain.

9. What exactly does $\lim_{x \to x_0} f(x) = L$ mean? Give an example in which you find a $\delta > 0$ for a given f, L, x_0, and $\epsilon > 0$ in the precise definition of limit.

10. Give precise definitions of the following statements.

a. $\lim_{x \to 2^-} f(x) = 5$ **b.** $\lim_{x \to 2^+} f(x) = 5$

c. $\lim_{x \to 2} f(x) = \infty$ **d.** $\lim_{x \to 2} f(x) = -\infty$

11. What exactly do $\lim_{x \to \infty} f(x) = L$ and $\lim_{x \to -\infty} f(x) = L$ mean? Give examples.

12. What are $\lim_{x \to \pm\infty} k$ (k a constant) and $\lim_{x \to \pm\infty} (1/x)$? How do you extend these results to other functions? Give examples.

13. How do you find the limit of a rational function as $x \to \pm\infty$? Give examples.

14. What are horizontal, vertical, and oblique asymptotes? Give examples.

15. What conditions must be satisfied by a function if it is to be continuous at an interior point of its domain? At an endpoint?

16. How can looking at the graph of a function help you tell where the function is continuous?

17. What does it mean for a function to be right-continuous at a point? Left-continuous? How are continuity and one-sided continuity related?

18. What can be said about the continuity of polynomials? Of rational functions? Of trigonometric functions? Of rational powers and algebraic combinations of functions? Of composites of functions? Of absolute values of functions?

19. Under what circumstances can you extend a function $f(x)$ to be continuous at a point $x = c$? Give an example.

20. What does it mean for a function to be continuous on an interval?

21. What does it mean for a function to be continuous? Give examples to illustrate the fact that a function that is not continuous on its entire domain may still be continuous on selected intervals within the domain.

22. What are the basic types of discontinuity? Give an example of each. What is a removable discontinuity? Give an example.

23. What does it mean for a function to have the Intermediate Value Property? What conditions guarantee that a function has this property over an interval? What are the consequences for graphing and solving the equation $f(x) = 0$?

24. It is often said that a function is continuous if you can draw its graph without having to lift your pen from the paper. Why is that?

25. What does it mean for a line to be tangent to a curve C at a point P?

26. What is the significance of the formula

$$\lim_{h \to 0} \frac{f(x + h) - f(x)}{h}?$$

Interpret the formula geometrically and physically.

27. How do you find the tangent to the curve $y = f(x)$ at a point (x_0, y_0) on the curve?

28. How does the slope of the curve $y = f(x)$ at $x = x_0$ relate to the function's rate of change with respect to x at $x = x_0$? To the derivative of f at x_0?