

EXERCISES 3.6

Derivatives of Rational Powers

Find dy/dx in Exercises 1-10.

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| 1. $y = x^{9/4}$ | 2. $y = x^{-3/5}$ |
| 3. $y = \sqrt[3]{2x}$ | 4. $y = \sqrt[4]{5x}$ |
| 5. $y = 7\sqrt{x+6}$ | 6. $y = -2\sqrt{x-1}$ |
| 7. $y = (2x+5)^{-1/2}$ | 8. $y = (1-6x)^{2/3}$ |
| 9. $y = x(x^2+1)^{1/2}$ | 10. $y = x(x^2+1)^{-1/2}$ |

Find the first derivatives of the functions in Exercises 11-18.

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| 11. $s = \sqrt{t^2}$ | 12. $r = \sqrt[4]{\theta^{-3}}$ |
| 13. $y = \sin[(2t+5)^{-2/3}]$ | 14. $z = \cos[(1-6t)^{2/3}]$ |
| 15. $f(x) = \sqrt{1-\sqrt{x}}$ | 16. $g(x) = 2(2x^{-1/2}+1)^{-1/3}$ |
| 17. $h(\theta) = \sqrt[3]{1+\cos(2\theta)}$ | 18. $k(\theta) = (\sin(\theta+5))^{5/4}$ |

Differentiating Implicitly

Use implicit differentiation to find dy/dx in Exercises 19-32.

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| 19. $x^2y + xy^2 = 6$ | 20. $x^3 + y^3 = 18xy$ |
| 21. $2xy + y^2 = x + y$ | 22. $x^3 - xy + y^3 = 1$ |
| 23. $x^2(x-y)^2 = x^2 - y^2$ | 24. $(3xy+7)^2 = 6y$ |
| 25. $y^2 = \frac{x-1}{x+1}$ | 26. $x^2 = \frac{x-y}{x+y}$ |
| 27. $x = \tan y$ | 28. $xy = \cot(xy)$ |
| 29. $x + \tan(xy) = 0$ | 30. $x + \sin y = xy$ |
| 31. $y \sin\left(\frac{1}{y}\right) = 1 - xy$ | 32. $y^2 \cos\left(\frac{1}{y}\right) = 2x + 2y$ |

Find $dr/d\theta$ in Exercises 33-36.

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| 33. $\theta^{1/2} + r^{1/2} = 1$ | 34. $r - 2\sqrt{\theta} = \frac{3}{2}\theta^{2/3} + \frac{4}{3}\theta^{3/4}$ |
| 35. $\sin(r\theta) = \frac{1}{2}$ | 36. $\cos r + \cot \theta = r\theta$ |

Second Derivatives

In Exercises 37-42, use implicit differentiation to find dy/dx and then d^2y/dx^2 .

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| 37. $x^2 + y^2 = 1$ | 38. $x^{2/3} + y^{2/3} = 1$ |
| 39. $y^2 = x^2 + 2x$ | 40. $y^2 - 2x = 1 - 2y$ |
| 41. $2\sqrt{y} = x - y$ | 42. $xy + y^2 = 1$ |
| 43. If $x^3 + y^3 = 16$, find the value of d^2y/dx^2 at the point $(2, 2)$. | |
| 44. If $xy + y^2 = 1$, find the value of d^2y/dx^2 at the point $(0, -1)$. | |

Slopes, Tangents, and Normals

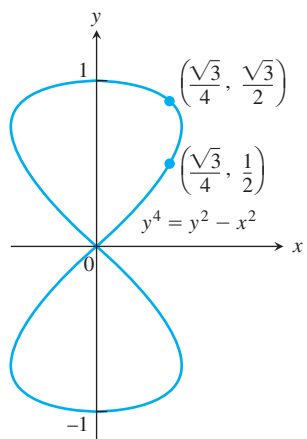
In Exercises 45 and 46, find the slope of the curve at the given points.

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| 45. $y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1)$ |
| 46. $(x^2 + y^2)^2 = (x - y)^2$ at $(1, 0)$ and $(1, -1)$ |

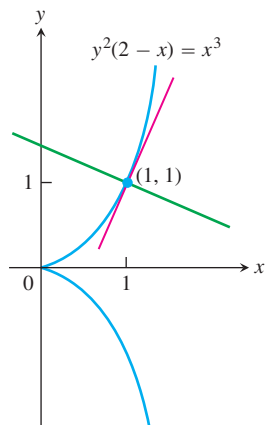
In Exercises 47-56, verify that the given point is on the curve and find the lines that are (a) tangent and (b) normal to the curve at the given point.

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| 47. $x^2 + xy - y^2 = 1$, $(2, 3)$ |
| 48. $x^2 + y^2 = 25$, $(3, -4)$ |
| 49. $x^2y^2 = 9$, $(-1, 3)$ |

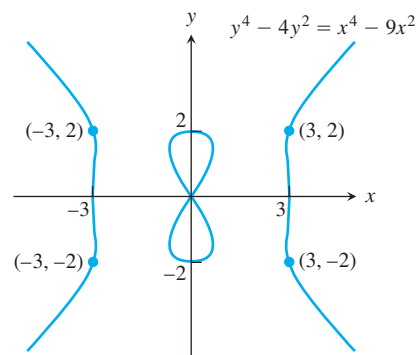
50. $y^2 - 2x - 4y - 1 = 0$, $(-2, 1)$
51. $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$, $(-1, 0)$
52. $x^2 - \sqrt{3}xy + 2y^2 = 5$, $(\sqrt{3}, 2)$
53. $2xy + \pi \sin y = 2\pi$, $(1, \pi/2)$
54. $x \sin 2y = y \cos 2x$, $(\pi/4, \pi/2)$
55. $y = 2 \sin(\pi x - y)$, $(1, 0)$
56. $x^2 \cos^2 y - \sin y = 0$, $(0, \pi)$
57. **Parallel tangents** Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x -axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?
58. **Tangents parallel to the coordinate axes** Find points on the curve $x^2 + xy + y^2 = 7$ (a) where the tangent is parallel to the x -axis and (b) where the tangent is parallel to the y -axis. In the latter case, dy/dx is not defined, but dx/dy is. What value does dx/dy have at these points?
59. **The eight curve** Find the slopes of the curve $y^4 = y^2 - x^2$ at the two points shown here.



60. **The cissoid of Diocles (from about 200 B.C.)** Find equations for the tangent and normal to the cissoid of Diocles $y^2(2-x) = x^3$ at $(1, 1)$.



61. **The devil's curve (Gabriel Cramer [the Cramer of Cramer's rule], 1750)** Find the slopes of the devil's curve $y^4 - 4y^2 = x^4 - 9x^2$ at the four indicated points.



62. **The folium of Descartes** (See Figure 3.38)
- Find the slope of the folium of Descartes, $x^3 + y^3 - 9xy = 0$ at the points $(4, 2)$ and $(2, 4)$.
 - At what point other than the origin does the folium have a horizontal tangent?
 - Find the coordinates of the point A in Figure 3.38, where the folium has a vertical tangent.

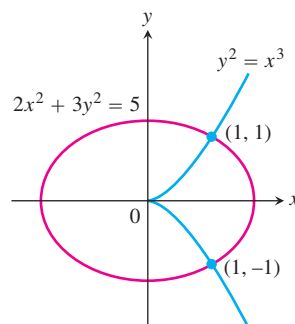
Implicitly Defined Parametrizations

Assuming that the equations in Exercises 63–66 define x and y implicitly as differentiable functions $x = f(t)$, $y = g(t)$, find the slope of the curve $x = f(t)$, $y = g(t)$ at the given value of t .

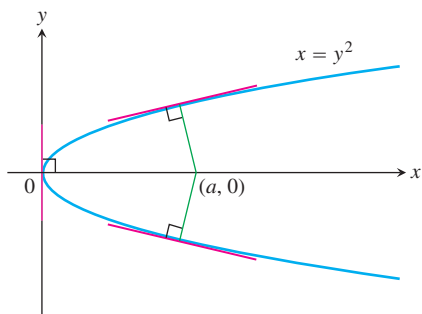
63. $x^2 - 2tx + 2t^2 = 4$, $2y^3 - 3t^2 = 4$, $t = 2$
64. $x = \sqrt{5 - \sqrt{t}}$, $y(t-1) = \sqrt{t}$, $t = 4$
65. $x + 2x^{3/2} = t^2 + t$, $y\sqrt{t+1} + 2t\sqrt{y} = 4$, $t = 0$
66. $x \sin t + 2x = t$, $t \sin t - 2t = y$, $t = \pi$

Theory and Examples

67. Which of the following could be true if $f''(x) = x^{-1/3}$?
- $f(x) = \frac{3}{2}x^{2/3} - 3$
 - $f(x) = \frac{9}{10}x^{5/3} - 7$
 - $f'''(x) = -\frac{1}{3}x^{-4/3}$
 - $f'(x) = \frac{3}{2}x^{2/3} + 6$
68. Is there anything special about the tangents to the curves $y^2 = x^3$ and $2x^2 + 3y^2 = 5$ at the points $(1, \pm 1)$? Give reasons for your answer.



- 69. Intersecting normal** The line that is normal to the curve $x^2 + 2xy - 3y^2 = 0$ at $(1, 1)$ intersects the curve at what other point?
- 70. Normals parallel to a line** Find the normals to the curve $xy + 2x - y = 0$ that are parallel to the line $2x + y = 0$.
- 71. Normals to a parabola** Show that if it is possible to draw three normals from the point $(a, 0)$ to the parabola $x = y^2$ shown here, then a must be greater than $1/2$. One of the normals is the x -axis. For what value of a are the other two normals perpendicular?



- 72.** What is the geometry behind the restrictions on the domains of the derivatives in Example 6(b) and Example 7(a)?

T In Exercises 73 and 74, find both dy/dx (treating y as a differentiable function of x) and dx/dy (treating x as a differentiable function of y). How do dy/dx and dx/dy seem to be related? Explain the relationship geometrically in terms of the graphs.

73. $xy^3 + x^2y = 6$ **74.** $x^3 + y^2 = \sin^2 y$

COMPUTER EXPLORATIONS

- 75. a.** Given that $x^4 + 4y^2 = 1$, find dy/dx two ways: (1) by solving for y and differentiating the resulting functions in the usual way and (2) by implicit differentiation. Do you get the same result each way?
- b.** Solve the equation $x^4 + 4y^2 = 1$ for y and graph the resulting functions together to produce a complete graph of the equation $x^4 + 4y^2 = 1$. Then add the graphs of the first derivatives of these functions to your display. Could you have

predicted the general behavior of the derivative graphs from looking at the graph of $x^4 + 4y^2 = 1$? Could you have predicted the general behavior of the graph of $x^4 + 4y^2 = 1$ by looking at the derivative graphs? Give reasons for your answers.

- 76. a.** Given that $(x - 2)^2 + y^2 = 4$ find dy/dx two ways: (1) by solving for y and differentiating the resulting functions with respect to x and (2) by implicit differentiation. Do you get the same result each way?
- b.** Solve the equation $(x - 2)^2 + y^2 = 4$ for y and graph the resulting functions together to produce a complete graph of the equation $(x - 2)^2 + y^2 = 4$. Then add the graphs of the functions' first derivatives to your picture. Could you have predicted the general behavior of the derivative graphs from looking at the graph of $(x - 2)^2 + y^2 = 4$? Could you have predicted the general behavior of the graph of $(x - 2)^2 + y^2 = 4$ by looking at the derivative graphs? Give reasons for your answers.

Use a CAS to perform the following steps in Exercises 77–84.

- a.** Plot the equation with the implicit plotter of a CAS. Check to see that the given point P satisfies the equation.
- b.** Using implicit differentiation, find a formula for the derivative dy/dx and evaluate it at the given point P .
- c.** Use the slope found in part (b) to find an equation for the tangent line to the curve at P . Then plot the implicit curve and tangent line together on a single graph.
- 77.** $x^3 - xy + y^3 = 7$, $P(2, 1)$
- 78.** $x^5 + y^3x + yx^2 + y^4 = 4$, $P(1, 1)$
- 79.** $y^2 + y = \frac{2+x}{1-x}$, $P(0, 1)$
- 80.** $y^3 + \cos xy = x^2$, $P(1, 0)$
- 81.** $x + \tan\left(\frac{y}{x}\right) = 2$, $P\left(1, \frac{\pi}{4}\right)$
- 82.** $xy^3 + \tan(x + y) = 1$, $P\left(\frac{\pi}{4}, 0\right)$
- 83.** $2y^2 + (xy)^{1/3} = x^2 + 2$, $P(1, 1)$
- 84.** $x\sqrt{1 + 2y} + y = x^2$, $P(1, 0)$