Chapter 3 Questions to Guide Your Review

1. What is the derivative of a function f? How is its domain related to the domain of f? Give examples.
2. What role does the derivative play in defining slopes, tangents, and rates of change?
3. How can you sometimes graph the derivative of a function when all you have is a table of the function's values?
4. What does it mean for a function to be differentiable on an open interval? On a closed interval?
5. How are derivatives and one-sided derivatives related?
6. Describe geometrically when a function typically does *not* have a derivative at a point.
7. How is a function's differentiability at a point related to its continuity there, if at all?
8. Could the unit step function

$$U(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

possibly be the derivative of some other function on $[-1, 1]$? Explain.

9. What rules do you know for calculating derivatives? Give some examples.
10. Explain how the three formulas

a. $\dfrac{d}{dx}(x^n) = nx^{n-1}$

b. $\dfrac{d}{dx}(cu) = c\dfrac{du}{dx}$

c. $\dfrac{d}{dx}(u_1 + u_2 + \cdots + u_n) = \dfrac{du_1}{dx} + \dfrac{du_2}{dx} + \cdots + \dfrac{du_n}{dx}$

enable us to differentiate any polynomial.

11. What formula do we need, in addition to the three listed in Question 10, to differentiate rational functions?
12. What is a second derivative? A third derivative? How many derivatives do the functions you know have? Give examples.
13. What is the relationship between a function's average and instantaneous rates of change? Give an example.
14. How do derivatives arise in the study of motion? What can you learn about a body's motion along a line by examining the derivatives of the body's position function? Give examples.
15. How can derivatives arise in economics?
16. Give examples of still other applications of derivatives.
17. What do the limits $\lim_{h\to 0}((\sin h)/h)$ and $\lim_{h\to 0}((\cos h - 1)/h)$ have to do with the derivatives of the sine and cosine functions? What *are* the derivatives of these functions?
18. Once you know the derivatives of $\sin x$ and $\cos x$, how can you find the derivatives of $\tan x$, $\cot x$, $\sec x$, and $\csc x$? What *are* the derivatives of these functions?
19. At what points are the six basic trigonometric functions continuous? How do you know?
20. What is the rule for calculating the derivative of a composite of two differentiable functions? How is such a derivative evaluated? Give examples.
21. What is the formula for the slope dy/dx of a parametrized curve $x = f(t), y = g(t)$? When does the formula apply? When can you expect to be able to find d^2y/dx^2 as well? Give examples.
22. If u is a differentiable function of x, how do you find $(d/dx)(u^n)$ if n is an integer? If n is a rational number? Give examples.
23. What is implicit differentiation? When do you need it? Give examples.
24. How do related rates problems arise? Give examples.
25. Outline a strategy for solving related rates problems. Illustrate with an example.
26. What is the linearization $L(x)$ of a function $f(x)$ at a point $x = a$? What is required of f at a for the linearization to exist? How are linearizations used? Give examples.
27. If x moves from a to a nearby value $a + dx$, how do you estimate the corresponding change in the value of a differentiable function $f(x)$? How do you estimate the relative change? The percentage change? Give an example.