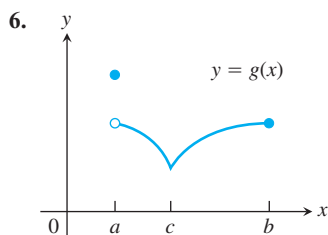
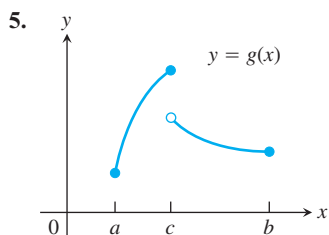
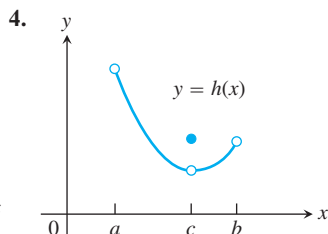
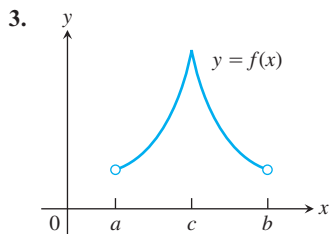
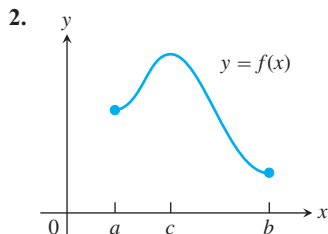
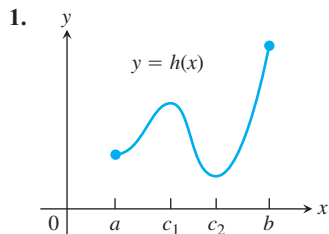


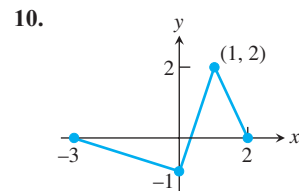
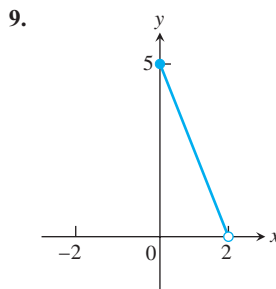
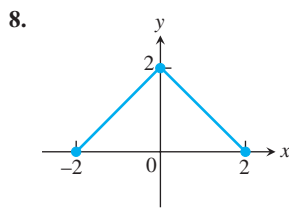
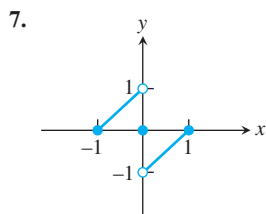
EXERCISES 4.1

Finding Extrema from Graphs

In Exercises 1–6, determine from the graph whether the function has any absolute extreme values on $[a, b]$. Then explain how your answer is consistent with Theorem 1.



In Exercises 7–10, find the extreme values and where they occur.



In Exercises 11–14, match the table with a graph.

11.

x	$f'(x)$
a	0
b	0
c	5

12.

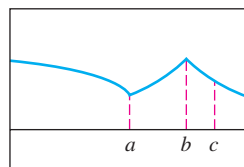
x	$f'(x)$
a	0
b	0
c	-5

13.

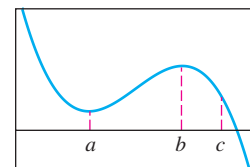
x	$f'(x)$
a	does not exist
b	0
c	-2

14.

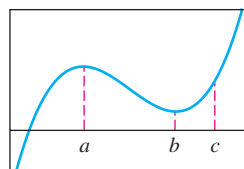
x	$f'(x)$
a	does not exist
b	does not exist
c	-1.7



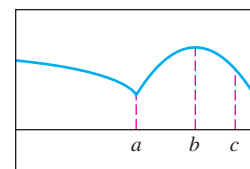
(a)



(b)



(c)



(d)

Absolute Extrema on Finite Closed Intervals

In Exercises 15–30, find the absolute maximum and minimum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates.

15. $f(x) = \frac{2}{3}x - 5, \quad -2 \leq x \leq 3$

16. $f(x) = -x - 4, \quad -4 \leq x \leq 1$

17. $f(x) = x^2 - 1, \quad -1 \leq x \leq 2$

18. $f(x) = 4 - x^2, \quad -3 \leq x \leq 1$

19. $F(x) = -\frac{1}{x^2}, \quad 0.5 \leq x \leq 2$

20. $F(x) = -\frac{1}{x}, \quad -2 \leq x \leq -1$

21. $h(x) = \sqrt[3]{x}, \quad -1 \leq x \leq 8$

22. $h(x) = -3x^{2/3}, \quad -1 \leq x \leq 1$

23. $g(x) = \sqrt{4 - x^2}, \quad -2 \leq x \leq 1$

24. $g(x) = -\sqrt{5 - x^2}, \quad -\sqrt{5} \leq x \leq 0$

25. $f(\theta) = \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{5\pi}{6}$

26. $f(\theta) = \tan \theta, \quad -\frac{\pi}{3} \leq \theta \leq \frac{\pi}{4}$

27. $g(x) = \csc x, \quad \frac{\pi}{3} \leq x \leq \frac{2\pi}{3}$

28. $g(x) = \sec x, \quad -\frac{\pi}{3} \leq x \leq \frac{\pi}{6}$

29. $f(t) = 2 - |t|, \quad -1 \leq t \leq 3$

30. $f(t) = |t - 5|, \quad 4 \leq t \leq 7$

In Exercises 31–34, find the function's absolute maximum and minimum values and say where they are assumed.

31. $f(x) = x^{4/3}, \quad -1 \leq x \leq 8$

32. $f(x) = x^{5/3}, \quad -1 \leq x \leq 8$

33. $g(\theta) = \theta^{3/5}, \quad -32 \leq \theta \leq 1$

34. $h(\theta) = 3\theta^{2/3}, \quad -27 \leq \theta \leq 8$

Finding Extreme Values

In Exercises 35–44, find the extreme values of the function and where they occur.

35. $y = 2x^2 - 8x + 9$

37. $y = x^3 + x^2 - 8x + 5$

39. $y = \sqrt{x^2 - 1}$

41. $y = \frac{1}{\sqrt[3]{1 - x^2}}$

43. $y = \frac{x}{x^2 + 1}$

36. $y = x^3 - 2x + 4$

38. $y = x^3 - 3x^2 + 3x - 2$

40. $y = \frac{1}{\sqrt{1 - x^2}}$

42. $y = \sqrt{3 + 2x - x^2}$

44. $y = \frac{x + 1}{x^2 + 2x + 2}$

Local Extrema and Critical Points

In Exercises 45–52, find the derivative at each critical point and determine the local extreme values.

45. $y = x^{2/3}(x + 2)$

46. $y = x^{2/3}(x^2 - 4)$

47. $y = x\sqrt{4 - x^2}$

48. $y = x^2\sqrt{3 - x}$

49. $y = \begin{cases} 4 - 2x, & x \leq 1 \\ x + 1, & x > 1 \end{cases}$

50. $y = \begin{cases} 3 - x, & x < 0 \\ 3 + 2x - x^2, & x \geq 0 \end{cases}$

51. $y = \begin{cases} -x^2 - 2x + 4, & x \leq 1 \\ -x^2 + 6x - 4, & x > 1 \end{cases}$

52. $y = \begin{cases} -\frac{1}{4}x^2 - \frac{1}{2}x + \frac{15}{4}, & x \leq 1 \\ x^3 - 6x^2 + 8x, & x > 1 \end{cases}$

In Exercises 53 and 54, give reasons for your answers.

53. Let $f(x) = (x - 2)^{2/3}$.

a. Does $f'(2)$ exist?b. Show that the only local extreme value of f occurs at $x = 2$.

c. Does the result in part (b) contradict the Extreme Value Theorem?

d. Repeat parts (a) and (b) for $f(x) = (x - a)^{2/3}$, replacing 2 by a .

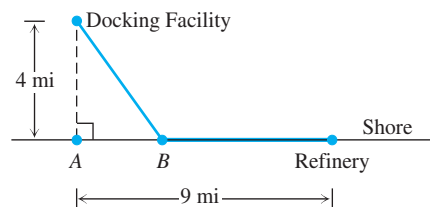
54. Let $f(x) = |x^3 - 9x|$.

a. Does $f'(0)$ exist?b. Does $f'(3)$ exist?c. Does $f'(-3)$ exist?d. Determine all extrema of f .

Optimization Applications

Whenever you are maximizing or minimizing a function of a single variable, we urge you to graph the function over the domain that is appropriate to the problem you are solving. The graph will provide insight before you begin to calculate and will furnish a visual context for understanding your answer.

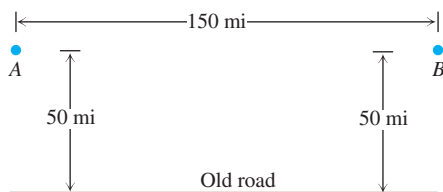
55. Constructing a pipeline Supertankers off-load oil at a docking facility 4 mi offshore. The nearest refinery is 9 mi east of the shore point nearest the docking facility. A pipeline must be constructed connecting the docking facility with the refinery. The pipeline costs \$300,000 per mile if constructed underwater and \$200,000 per mile if overland.



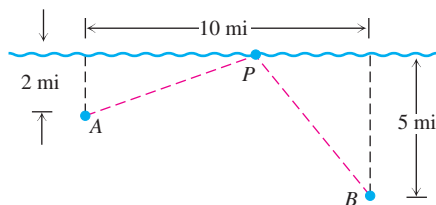
a. Locate Point B to minimize the cost of the construction.

- b. The cost of underwater construction is expected to increase, whereas the cost of overland construction is expected to stay constant. At what cost does it become optimal to construct the pipeline directly to Point A ?

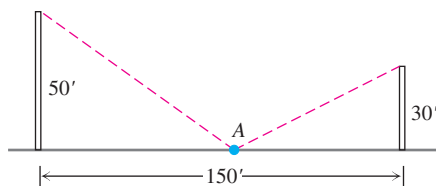
56. **Upgrading a highway** A highway must be constructed to connect Village A with Village B . There is a rudimentary roadway that can be upgraded 50 mi south of the line connecting the two villages. The cost of upgrading the existing roadway is \$300,000 per mile, whereas the cost of constructing a new highway is \$500,000 per mile. Find the combination of upgrading and new construction that minimizes the cost of connecting the two villages. Clearly define the location of the proposed highway.



57. **Locating a pumping station** Two towns lie on the south side of a river. A pumping station is to be located to serve the two towns. A pipeline will be constructed from the pumping station to each of the towns along the line connecting the town and the pumping station. Locate the pumping station to minimize the amount of pipeline that must be constructed.



58. **Length of a guy wire** One tower is 50 ft high and another tower is 30 ft high. The towers are 150 ft apart. A guy wire is to run from Point A to the top of each tower.



- Locate Point A so that the total length of guy wire is minimal.
 - Show in general that regardless of the height of the towers, the length of guy wire is minimized if the angles at A are equal.
59. The function

$$V(x) = x(10 - 2x)(16 - 2x), \quad 0 < x < 5,$$

models the volume of a box.

- Find the extreme values of V .

- Interpret any values found in part (a) in terms of volume of the box.

60. The function

$$P(x) = 2x + \frac{200}{x}, \quad 0 < x < \infty,$$

models the perimeter of a rectangle of dimensions x by $100/x$.

- Find any extreme values of P .
 - Give an interpretation in terms of perimeter of the rectangle for any values found in part (a).
61. **Area of a right triangle** What is the largest possible area for a right triangle whose hypotenuse is 5 cm long?
62. **Area of an athletic field** An athletic field is to be built in the shape of a rectangle x units long capped by semicircular regions of radius r at the two ends. The field is to be bounded by a 400-m racetrack.
- Express the area of the rectangular portion of the field as a function of x alone or r alone (your choice).
 - What values of x and r give the rectangular portion the largest possible area?
63. **Maximum height of a vertically moving body** The height of a body moving vertically is given by

$$s = -\frac{1}{2}gt^2 + v_0t + s_0, \quad g > 0,$$

with s in meters and t in seconds. Find the body's maximum height.

64. **Peak alternating current** Suppose that at any given time t (in seconds) the current i (in amperes) in an alternating current circuit is $i = 2 \cos t + 2 \sin t$. What is the peak current for this circuit (largest magnitude)?

Theory and Examples

65. **A minimum with no derivative** The function $f(x) = |x|$ has an absolute minimum value at $x = 0$ even though f is not differentiable at $x = 0$. Is this consistent with Theorem 2? Give reasons for your answer.
66. **Even functions** If an even function $f(x)$ has a local maximum value at $x = c$, can anything be said about the value of f at $x = -c$? Give reasons for your answer.
67. **Odd functions** If an odd function $g(x)$ has a local minimum value at $x = c$, can anything be said about the value of g at $x = -c$? Give reasons for your answer.
68. We know how to find the extreme values of a continuous function $f(x)$ by investigating its values at critical points and endpoints. But what if there *are* no critical points or endpoints? What happens then? Do such functions really exist? Give reasons for your answers.
69. **Cubic functions** Consider the cubic function

$$f(x) = ax^3 + bx^2 + cx + d.$$

- Show that f can have 0, 1, or 2 critical points. Give examples and graphs to support your argument.
- How many local extreme values can f have?

T 70. Functions with no extreme values at endpoints

- a. Graph the function

$$f(x) = \begin{cases} \sin \frac{1}{x}, & x > 0 \\ 0, & x = 0. \end{cases}$$

Explain why $f(0) = 0$ is not a local extreme value of f .

- b. Construct a function of your own that fails to have an extreme value at a domain endpoint.

T Graph the functions in Exercises 71–74. Then find the extreme values of the function on the interval and say where they occur.

71. $f(x) = |x - 2| + |x + 3|$, $-5 \leq x \leq 5$

72. $g(x) = |x - 1| - |x - 5|$, $-2 \leq x \leq 7$

73. $h(x) = |x + 2| - |x - 3|$, $-\infty < x < \infty$

74. $k(x) = |x + 1| + |x - 3|$, $-\infty < x < \infty$

COMPUTER EXPLORATIONS

In Exercises 75–80, you will use a CAS to help find the absolute extrema of the given function over the specified closed interval. Perform the following steps.

- Plot the function over the interval to see its general behavior there.
- Find the interior points where $f' = 0$. (In some exercises, you may have to use the numerical equation solver to approximate a solution.) You may want to plot f' as well.
- Find the interior points where f' does not exist.
- Evaluate the function at all points found in parts (b) and (c) and at the endpoints of the interval.
- Find the function's absolute extreme values on the interval and identify where they occur.

75. $f(x) = x^4 - 8x^2 + 4x + 2$, $[-20/25, 64/25]$

76. $f(x) = -x^4 + 4x^3 - 4x + 1$, $[-3/4, 3]$

77. $f(x) = x^{2/3}(3 - x)$, $[-2, 2]$

78. $f(x) = 2 + 2x - 3x^{2/3}$, $[-1, 10/3]$

79. $f(x) = \sqrt{x} + \cos x$, $[0, 2\pi]$

80. $f(x) = x^{3/4} - \sin x + \frac{1}{2}$, $[0, 2\pi]$