EXERCISES 4.3

Analyzing f Given f'

Answer the following questions about the functions whose derivatives are given in Exercises 1–8:

a. What are the critical points of f?

b. On what intervals is f increasing or decreasing?

c. At what points, if any, does f assume local maximum and minimum values?

1. $f'(x) = x(x - 1)$

2. $f'(x) = (x - 1)(x + 2)$

3. $f'(x) = (x - 1)^2(x + 2)$

4. $f'(x) = (x - 1)^2(x + 2)^2$

5. $f'(x) = (x - 1)(x + 2)(x - 3)$

6. $f'(x) = (x - 7)(x + 1)(x + 5)$

7. $f'(x) = x^{-1/3}(x + 2)$

8. $f'(x) = x^{-1/2}(x - 3)$

Extremes of Given Functions

In Exercises 9–28:

a. Find the intervals on which the function is increasing and decreasing.

b. Then identify the function's local extreme values, if any, saying where they are taken on.

c. Which, if any, of the extreme values are absolute?

T **d.** Support your findings with a graphing calculator or computer grapher.

9. $g(t) = -t^2 - 3t + 3$

10. $g(t) = -3t^2 + 9t + 5$

11. $h(x) = -x^3 + 2x^2$

12. $h(x) = 2x^3 - 18x$

13. $f(\theta) = 3\theta^2 - 4\theta^3$

14. $f(\theta) = 6\theta - \theta^3$

15. $f(r) = 3r^3 + 16r$

16. $h(r) = (r + 7)^3$

17. $f(x) = x^4 - 8x^2 + 16$

18. $g(x) = x^4 - 4x^3 + 4x^2$

19. $H(t) = \frac{3}{2}t^4 - t^6$

20. $K(t) = 15t^3 - t^5$

21. $g(x) = x\sqrt{8 - x^2}$

22. $g(x) = x^2\sqrt{5 - x}$

23. $f(x) = \frac{x^2 - 3}{x - 2}, \quad x \neq 2$

24. $f(x) = \frac{x^3}{3x^2 + 1}$

25. $f(x) = x^{1/3}(x + 8)$

26. $g(x) = x^{2/3}(x + 5)$

27. $h(x) = x^{1/3}(x^2 - 4)$

28. $k(x) = x^{2/3}(x^2 - 4)$

Extreme Values on Half-Open Intervals

In Exercises 29–36:

a. Identify the function's local extreme values in the given domain, and say where they are assumed.

b. Which of the extreme values, if any, are absolute?

T **c.** Support your findings with a graphing calculator or computer grapher.

29. $f(x) = 2x - x^2, \quad -\infty < x \leq 2$

30. $f(x) = (x + 1)^2, \quad -\infty < x \leq 0$

31. $g(x) = x^2 - 4x + 4, \quad 1 \leq x < \infty$

32. $g(x) = -x^2 - 6x - 9, \quad -4 \leq x < \infty$

33. $f(t) = 12t - t^3, \quad -3 \leq t < \infty$

34. $f(t) = t^3 - 3t^2, \quad -\infty < t \leq 3$

35. $h(x) = \frac{x^3}{3} - 2x^2 + 4x, \quad 0 \leq x < \infty$

36. $k(x) = x^3 + 3x^2 + 3x + 1, \quad -\infty < x \leq 0$

Graphing Calculator or Computer Grapher

In Exercises 37–40:

a. Find the local extrema of each function on the given interval, and say where they are assumed.

T **b.** Graph the function and its derivative together. Comment on the behavior of f in relation to the signs and values of f'.

37. $f(x) = \dfrac{x}{2} - 2\sin\dfrac{x}{2}, \quad 0 \le x \le 2\pi$

38. $f(x) = -2\cos x - \cos^2 x, \quad -\pi \le x \le \pi$

39. $f(x) = \csc^2 x - 2\cot x, \quad 0 < x < \pi$

40. $f(x) = \sec^2 x - 2\tan x, \quad \dfrac{-\pi}{2} < x < \dfrac{\pi}{2}$

Theory and Examples

Show that the functions in Exercises 41 and 42 have local extreme values at the given values of θ, and say which kind of local extreme the function has.

41. $h(\theta) = 3\cos\dfrac{\theta}{2}, \quad 0 \le \theta \le 2\pi, \quad$ at $\theta = 0$ and $\theta = 2\pi$

42. $h(\theta) = 5\sin\dfrac{\theta}{2}, \quad 0 \le \theta \le \pi, \quad$ at $\theta = 0$ and $\theta = \pi$

43. Sketch the graph of a differentiable function $y = f(x)$ through the point $(1, 1)$ if $f'(1) = 0$ and

a. $f'(x) > 0$ for $x < 1$ and $f'(x) < 0$ for $x > 1$;

b. $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$;

c. $f'(x) > 0$ for $x \ne 1$;

d. $f'(x) < 0$ for $x \ne 1$.

44. Sketch the graph of a differentiable function $y = f(x)$ that has

a. a local minimum at (1, 1) and a local maximum at (3, 3);

b. a local maximum at (1, 1) and a local minimum at (3, 3);

c. local maxima at (1, 1) and (3, 3);

d. local minima at (1, 1) and (3, 3).

45. Sketch the graph of a continuous function $y = g(x)$ such that

a. $g(2) = 2, \quad 0 < g' < 1$ for $x < 2, \quad g'(x) \to 1^-$ as $x \to 2^-, \quad -1 < g' < 0$ for $x > 2$, and $g'(x) \to -1^+$ as $x \to 2^+$;

b. $g(2) = 2, \quad g' < 0$ for $x < 2, \quad g'(x) \to -\infty$ as $x \to 2^-$, $g' > 0$ for $x > 2$, and $g'(x) \to \infty$ as $x \to 2^+$.

46. Sketch the graph of a continuous function $y = h(x)$ such that

a. $h(0) = 0$, $-2 \le h(x) \le 2$ for all x, $h'(x) \to \infty$ as $x \to 0^-$, and $h'(x) \to \infty$ as $x \to 0^+$;

b. $h(0) = 0$, $-2 \le h(x) \le 0$ for all x, $h'(x) \to \infty$ as $x \to 0^-$, and $h'(x) \to -\infty$ as $x \to 0^+$.

47. As x moves from left to right through the point $c = 2$, is the graph of $f(x) = x^3 - 3x + 2$ rising, or is it falling? Give reasons for your answer.

48. Find the intervals on which the function $f(x) = ax^2 + bx + c$, $a \ne 0$, is increasing and decreasing. Describe the reasoning behind your answer.