EXERCISES 5.5

Evaluating Integrals

Evaluate the indefinite integrals in Exercises 1–12 by using the given substitutions to reduce the integrals to standard form.

1. $\int \sin 3x \, dx, \quad u = 3x$

2. $\int x \sin(2x^2) \, dx, \quad u = 2x^2$

3. $\int \sec 2t \tan 2t \, dt, \quad u = 2t$

4. $\int \left(1 - \cos \frac{t}{2}\right)^2 \sin \frac{t}{2} \, dt, \quad u = 1 - \cos \frac{t}{2}$

5. $\int 28(7x - 2)^{-5}\,dx, \quad u = 7x - 2$

6. $\int x^3(x^4 - 1)^2\,dx, \quad u = x^4 - 1$

7. $\int \frac{9r^2\,dr}{\sqrt{1 - r^3}}, \quad u = 1 - r^3$

8. $\int 12(y^4 + 4y^2 + 1)^2(y^3 + 2y)\,dy, \quad u = y^4 + 4y^2 + 1$

9. $\int \sqrt{x}\,\sin^2(x^{3/2} - 1)\,dx, \quad u = x^{3/2} - 1$

10. $\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx, \quad u = -\frac{1}{x}$

11. $\int \csc^2 2\theta \cot 2\theta\,d\theta$

a. Using $u = \cot 2\theta$ **b.** Using $u = \csc 2\theta$

12. $\int \frac{dx}{\sqrt{5x + 8}}$

a. Using $u = 5x + 8$ **b.** Using $u = \sqrt{5x + 8}$

Evaluate the integrals in Exercises 13–48.

13. $\int \sqrt{3 - 2s}\,ds$

14. $\int (2x + 1)^3\,dx$

15. $\int \frac{1}{\sqrt{5s + 4}}\,ds$

16. $\int \frac{3\,dx}{(2 - x)^2}$

17. $\int \theta\sqrt[4]{1 - \theta^2}\,d\theta$

18. $\int 8\theta\sqrt[3]{\theta^2 - 1}\,d\theta$

19. $\int 3y\sqrt{7 - 3y^2}\,dy$

20. $\int \frac{4y\,dy}{\sqrt{2y^2 + 1}}$

21. $\int \frac{1}{\sqrt{x}\,(1 + \sqrt{x})^2}\,dx$

22. $\int \frac{(1 + \sqrt{x})^3}{\sqrt{x}}\,dx$

23. $\int \cos(3z + 4)\,dz$

24. $\int \sin(8z - 5)\,dz$

25. $\int \sec^2(3x + 2)\,dx$

26. $\int \tan^2 x \sec^2 x\,dx$

27. $\int \sin^5\frac{x}{3}\cos\frac{x}{3}\,dx$

28. $\int \tan^7\frac{x}{2}\sec^2\frac{x}{2}\,dx$

29. $\int r^2\left(\frac{r^3}{18} - 1\right)^5 dr$

30. $\int r^4\left(7 - \frac{r^5}{10}\right)^3 dr$

31. $\int x^{1/2}\sin(x^{3/2} + 1)\,dx$

32. $\int x^{1/3}\sin(x^{4/3} - 8)\,dx$

33. $\int \sec\left(v + \frac{\pi}{2}\right)\tan\left(v + \frac{\pi}{2}\right)dv$

34. $\int \csc\left(\frac{v - \pi}{2}\right)\cot\left(\frac{v - \pi}{2}\right)dv$

35. $\int \frac{\sin(2t + 1)}{\cos^2(2t + 1)}\,dt$

36. $\int \frac{6\cos t}{(2 + \sin t)^3}\,dt$

37. $\int \sqrt{\cot y}\,\csc^2 y\,dy$

38. $\int \frac{\sec z \tan z}{\sqrt{\sec z}}\,dz$

39. $\int \frac{1}{t^2}\cos\left(\frac{1}{t} - 1\right)dt$

40. $\int \frac{1}{\sqrt{t}}\cos(\sqrt{t} + 3)\,dt$

41. $\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\cos\frac{1}{\theta}\,d\theta$

42. $\int \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\sin^2\sqrt{\theta}}\,d\theta$

43. $\int (s^3 + 2s^2 - 5s + 5)(3s^2 + 4s - 5)\,ds$

44. $\int (\theta^4 - 2\theta^2 + 8\theta - 2)(\theta^3 - \theta + 2)\,d\theta$

45. $\int t^3(1 + t^4)^3\,dt$

46. $\int \sqrt{\frac{x - 1}{x^5}}\,dx$

47. $\int x^3\sqrt{x^2 + 1}\,dx$

48. $\int 3x^5\sqrt{x^3 + 1}\,dx$

Simplifying Integrals Step by Step

If you do not know what substitution to make, try reducing the integral step by step, using a trial substitution to simplify the integral a bit and then another to simplify it some more. You will see what we mean if you try the sequences of substitutions in Exercises 49 and 50.

49. $\int \frac{18\tan^2 x \sec^2 x}{(2 + \tan^3 x)^2}\,dx$

a. $u = \tan x$, followed by $v = u^3$, then by $w = 2 + v$

b. $u = \tan^3 x$, followed by $v = 2 + u$

c. $u = 2 + \tan^3 x$

50. $\int \sqrt{1 + \sin^2(x - 1)}\,\sin(x - 1)\cos(x - 1)\,dx$

a. $u = x - 1$, followed by $v = \sin u$, then by $w = 1 + v^2$

b. $u = \sin(x - 1)$, followed by $v = 1 + u^2$

c. $u = 1 + \sin^2(x - 1)$

Evaluate the integrals in Exercises 51 and 52.

51. $\int \frac{(2r - 1)\cos\sqrt{3(2r - 1)^2 + 6}}{\sqrt{3(2r - 1)^2 + 6}}\,dr$

52. $\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta\cos^3\sqrt{\theta}}}\,d\theta$

Initial Value Problems

Solve the initial value problems in Exercises 53–58.

53. $\frac{ds}{dt} = 12t(3t^2 - 1)^3, \quad s(1) = 3$

54. $\frac{dy}{dx} = 4x(x^2 + 8)^{-1/3}, \quad y(0) = 0$

55. $\frac{ds}{dt} = 8\sin^2\left(t + \frac{\pi}{12}\right), \quad s(0) = 8$

56. $\frac{dr}{d\theta} = 3\cos^2\left(\frac{\pi}{4} - \theta\right), \quad r(0) = \frac{\pi}{8}$

57. $\frac{d^2s}{dt^2} = -4\sin\left(2t - \frac{\pi}{2}\right), \quad s'(0) = 100, \quad s(0) = 0$

58. $\frac{d^2y}{dx^2} = 4\sec^2 2x \tan 2x, \quad y'(0) = 4, \quad y(0) = -1$

59. The velocity of a particle moving back and forth on a line is $v = ds/dt = 6\sin 2t$ m/sec for all t. If $s = 0$ when $t = 0$, find the value of s when $t = \pi/2$ sec.

60. The acceleration of a particle moving back and forth on a line is $a = d^2s/dt^2 = \pi^2 \cos \pi t$ m/sec^2 for all t. If $s = 0$ and $v = 8$ m/sec when $t = 0$, find s when $t = 1$ sec.

Theory and Examples

61. It looks as if we can integrate $2\sin x \cos x$ with respect to x in three different ways:

a. $$\int 2\sin x\cos x\,dx = \int 2u\,du \qquad u = \sin x,$$
$$= u^2 + C_1 = \sin^2 x + C_1$$

b. $$\int 2\sin x\cos x\,dx = \int -2u\,du \qquad u = \cos x,$$
$$= -u^2 + C_2 = -\cos^2 x + C_2$$

c. $$\int 2\sin x\cos x\,dx = \int \sin 2x\,dx \qquad 2\sin x\cos x = \sin 2x$$
$$= -\frac{\cos 2x}{2} + C_3.$$

Can all three integrations be correct? Give reasons for your answer.

62. The substitution $u = \tan x$ gives

$$\int \sec^2 x \tan x\,dx = \int u\,du = \frac{u^2}{2} + C = \frac{\tan^2 x}{2} + C.$$

The substitution $u = \sec x$ gives

$$\int \sec^2 x \tan x\,dx = \int u\,du = \frac{u^2}{2} + C = \frac{\sec^2 x}{2} + C.$$

Can both integrations be correct? Give reasons for your answer.

63. (*Continuation of Example 9.*)

a. Show by evaluating the integral in the expression

$$\frac{1}{(1/60) - 0}\int_0^{1/60} V_{max} \sin 120\,\pi t\,dt$$

that the average value of $V = V_{max} \sin 120\,\pi t$ over a full cycle is zero.

b. The circuit that runs your electric stove is rated 240 volts rms. What is the peak value of the allowable voltage?

c. Show that

$$\int_0^{1/60} (V_{max})^2 \sin^2 120\,\pi t\,dt = \frac{(V_{max})^2}{120}.$$