# EXERCISES 6.6

## Springs

1. **Spring constant** It took 1800 J of work to stretch a spring from its natural length of 2 m to a length of 5 m. Find the spring's force constant.

2. **Stretching a spring** A spring has a natural length of 10 in. An 800-lb force stretches the spring to 14 in.
   - **a.** Find the force constant.
   - **b.** How much work is done in stretching the spring from 10 in. to 12 in.?
   - **c.** How far beyond its natural length will a 1600-lb force stretch the spring?

3. **Stretching a rubber band** A force of 2 N will stretch a rubber band 2 cm (0.02 m). Assuming that Hooke's Law applies, how far will a 4-N force stretch the rubber band? How much work does it take to stretch the rubber band this far?

4. **Stretching a spring** If a force of 90 N stretches a spring 1 m beyond its natural length, how much work does it take to stretch the spring 5 m beyond its natural length?

5. **Subway car springs** It takes a force of 21,714 lb to compress a coil spring assembly on a New York City Transit Authority subway car from its free height of 8 in. to its fully compressed height of 5 in.
   - **a.** What is the assembly's force constant?
   - **b.** How much work does it take to compress the assembly the first half inch? the second half inch? Answer to the nearest in.-lb.

   (Data courtesy of Bombardier, Inc., Mass Transit Division, for spring assemblies in subway cars delivered to the New York City Transit Authority from 1985 to 1987.)

6. **Bathroom scale** A bathroom scale is compressed $1/16$ in. when a 150-lb person stands on it. Assuming that the scale behaves like a spring that obeys Hooke's Law, how much does someone who compresses the scale $1/8$ in. weigh? How much work is done compressing the scale $1/8$ in.?

## Work Done By a Variable Force

7. **Lifting a rope** A mountain climber is about to haul up a 50 m length of hanging rope. How much work will it take if the rope weighs 0.624 N/m?

8. **Leaky sandbag** A bag of sand originally weighing 144 lb was lifted at a constant rate. As it rose, sand also leaked out at a constant rate. The sand was half gone by the time the bag had been

lifted to 18 ft. How much work was done lifting the sand this far? (Neglect the weight of the bag and lifting equipment.)

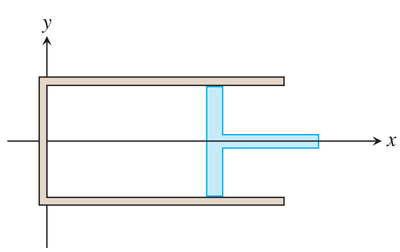
9. **Lifting an elevator cable** An electric elevator with a motor at the top has a multistrand cable weighing 4.5 lb/ft. When the car is at the first floor, 180 ft of cable are paid out, and effectively 0 ft are out when the car is at the top floor. How much work does the motor do just lifting the cable when it takes the car from the first floor to the top?

10. **Force of attraction** When a particle of mass $m$ is at $(x, 0)$, it is attracted toward the origin with a force whose magnitude is $k/x^2$. If the particle starts from rest at $x = b$ and is acted on by no other forces, find the work done on it by the time it reaches $x = a$, $0 < a < b$.

11. **Compressing gas** Suppose that the gas in a circular cylinder of cross-sectional area $A$ is being compressed by a piston. If $p$ is the pressure of the gas in pounds per square inch and $V$ is the volume in cubic inches, show that the work done in compressing the gas from state $(p_1, V_1)$ to state $(p_2, V_2)$ is given by the equation

$$\text{Work} = \int_{(p_1, V_1)}^{(p_2, V_2)} p\, dV.$$

(*Hint:* In the coordinates suggested in the figure here, $dV = A\, dx$. The force against the piston is $pA$.)




12. (*Continuation of Exercise 11.*) Use the integral in Exercise 11 to find the work done in compressing the gas from $V_1 = 243$ in.$^3$ to $V_2 = 32$ in.$^3$ if $p_1 = 50$ lb/in.$^3$ and $p$ and $V$ obey the gas law $pV^{1.4} = \text{constant}$ (for adiabatic processes).

13. **Leaky bucket** Assume the bucket in Example 4 is leaking. It starts with 2 gal of water (16 lb) and leaks at a constant rate. It finishes draining just as it reaches the top. How much work was spent lifting the water alone? (*Hint:* Do not include the rope and bucket, and find the proportion of water left at elevation $x$ ft.)

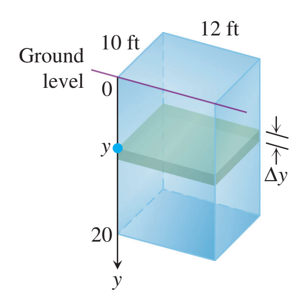
14. (*Continuation of Exercise 13.*) The workers in Example 4 and Exercise 13 changed to a larger bucket that held 5 gal (40 lb) of water, but the new bucket had an even larger leak so that it, too, was empty by the time it reached the top. Assuming that the water leaked out at a steady rate, how much work was done lifting the water alone? (Do not include the rope and bucket.)

## Pumping Liquids from Containers

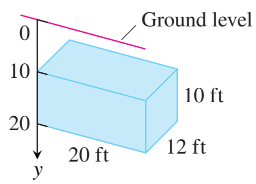
### The Weight of Water

Because of Earth's rotation and variations in its gravitational field, the weight of a cubic foot of water at sea level can vary from about 62.26 lb at the equator to as much as 62.59 lb near the poles, a variation of about 0.5%. A cubic foot that weighs about 62.4 lb in Melbourne and New York City will weigh 62.5 lb in Juneau and Stockholm. Although 62.4 is a typical figure and common textbook value, there is considerable variation.

15. **Pumping water** The rectangular tank shown here, with its top at ground level, is used to catch runoff water. Assume that the water weighs 62.4 lb/ft$^3$.
   a. How much work does it take to empty the tank by pumping the water back to ground level once the tank is full?
   b. If the water is pumped to ground level with a (5/11)-horsepower (hp) motor (work output 250 ft-lb/sec), how long will it take to empty the full tank (to the nearest minute)?
   c. Show that the pump in part (b) will lower the water level 10 ft (halfway) during the first 25 min of pumping.
   d. **The weight of water** What are the answers to parts (a) and (b) in a location where water weighs 62.26 lb/ft$^3$? 62.59 lb/ft$^3$?




16. **Emptying a cistern** The rectangular cistern (storage tank for rainwater) shown below has its top 10 ft below ground level. The cistern, currently full, is to be emptied for inspection by pumping its contents to ground level.
   a. How much work will it take to empty the cistern?
   b. How long will it take a 1/2 hp pump, rated at 275 ft-lb/sec, to pump the tank dry?
   c. How long will it take the pump in part (b) to empty the tank halfway? (It will be less than half the time required to empty the tank completely.)
   d. **The weight of water** What are the answers to parts (a) through (c) in a location where water weighs 62.26 lb/ft$^3$? 62.59 lb/ft$^3$?

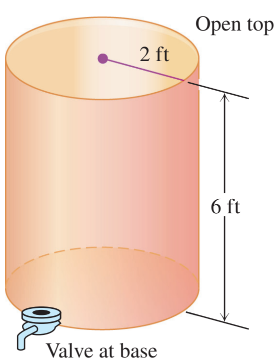



**17. Pumping oil** How much work would it take to pump oil from the tank in Example 5 to the level of the top of the tank if the tank were completely full?

**18. Pumping a half-full tank** Suppose that, instead of being full, the tank in Example 5 is only half full. How much work does it take to pump the remaining oil to a level 4 ft above the top of the tank?

**19. Emptying a tank** A vertical right circular cylindrical tank measures 30 ft high and 20 ft in diameter. It is full of kerosene weighing 51.2 lb/ft$^3$. How much work does it take to pump the kerosene to the level of the top of the tank?

**20.** The cylindrical tank shown here can be filled by pumping water from a lake 15 ft below the bottom of the tank. There are two ways to go about it. One is to pump the water through a hose attached to a valve in the bottom of the tank. The other is to attach the hose to the rim of the tank and let the water pour in. Which way will be faster? Give reasons for your answer.

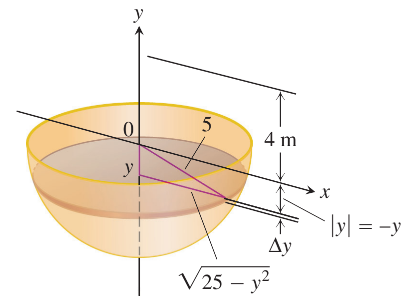



**21. a. Pumping milk** Suppose that the conical container in Example 5 contains milk (weighing 64.5 lb/ft$^3$) instead of olive oil. How much work will it take to pump the contents to the rim?

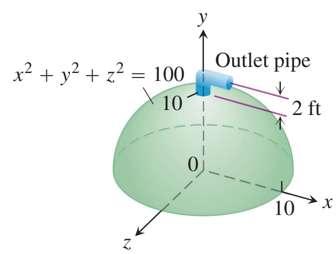
**b. Pumping oil** How much work will it take to pump the oil in Example 5 to a level 3 ft above the cone's rim?

**22. Pumping seawater** To design the interior surface of a huge stainless-steel tank, you revolve the curve $y = x^2, 0 \le x \le 4$, about the $y$-axis. The container, with dimensions in meters, is to be filled with seawater, which weighs 10,000 N/m$^3$. How much work will it take to empty the tank by pumping the water to the tank's top?

**23. Emptying a water reservoir** We model pumping from spherical containers the way we do from other containers, with the axis of integration along the vertical axis of the sphere. Use the figure here to find how much work it takes to empty a full hemispherical water reservoir of radius 5 m by pumping the water to a height of 4 m above the top of the reservoir. Water weighs 9800 N/m$^3$.




**24.** You are in charge of the evacuation and repair of the storage tank shown here. The tank is a hemisphere of radius 10 ft and is full of benzene weighing 56 lb/ft$^3$. A firm you contacted says it can empty the tank for $1/2$¢ per foot-pound of work. Find the work required to empty the tank by pumping the benzene to an outlet 2 ft above the top of the tank. If you have \$5000 budgeted for the job, can you afford to hire the firm?




## Work and Kinetic Energy

**25. Kinetic energy** If a variable force of magnitude $F(x)$ moves a body of mass $m$ along the $x$-axis from $x_1$ to $x_2$, the body's velocity $v$ can be written as $dx/dt$ (where $t$ represents time). Use Newton's second law of motion $F = m(dv/dt)$ and the Chain Rule

$$\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

to show that the net work done by the force in moving the body from $x_1$ to $x_2$ is

$$W = \int_{x_1}^{x_2} F(x)\,dx = \frac{1}{2}mv_2^{\,2} - \frac{1}{2}mv_1^{\,2},$$

where $v_1$ and $v_2$ are the body's velocities at $x_1$ and $x_2$. In physics, the expression $(1/2)mv^2$ is called the *kinetic energy* of a body of mass $m$ moving with velocity $v$. Therefore, *the work done by the force equals the change in the body's kinetic energy*, and we can find the work by calculating this change.

In Exercises 26–32, use the result of Exercise 25.

**26. Tennis** A 2-oz tennis ball was served at 160 ft/sec (about 109 mph). How much work was done on the ball to make it go this fast? (To find the ball's mass from its weight, express the weight in pounds and divide by 32 ft/sec$^2$, the acceleration of gravity.)

**27. Baseball** How many foot-pounds of work does it take to throw a baseball 90 mph? A baseball weighs 5 oz, or 0.3125 lb.

**28. Golf** A 1.6-oz golf ball is driven off the tee at a speed of 280 ft/sec (about 191 mph). How many foot-pounds of work are done on the ball getting it into the air?

**29. Tennis** During the match in which Pete Sampras won the 1990 U.S. Open men's tennis championship, Sampras hit a serve that was clocked at a phenomenal 124 mph. How much work did Sampras have to do on the 2-oz ball to get it to that speed?

**30. Football** A quarterback threw a 14.5-oz football 88 ft/sec (60 mph). How many foot-pounds of work were done on the ball to get it to this speed?

**31. Softball** How much work has to be performed on a 6.5-oz softball to pitch it 132 ft/sec (90 mph)?

**32. A ball bearing** A 2-oz steel ball bearing is placed on a vertical spring whose force constant is $k = 18$ lb/ft. The spring is compressed 2 in. and released. About how high does the ball bearing go?

**33. Pumping the funnel of the glory hole** (*Continuation of Example 6.*)

**a.** Find the radius of the cross-section (funnel portion) of the glory hole in Example 6 as a function of the height $y$ above the floor of the dam (from $y = 325$ to $y = 375$).

**b.** Find $\Delta V$ for the funnel section of the glory hole (from $y = 325$ to $y = 375$).

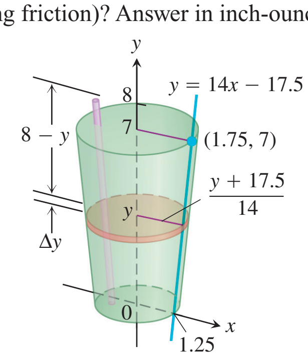
**c.** Find the work necessary to pump out the funnel section by formulating and evaluating the appropriate definite integral.

**34. Pumping water from a glory hole** (*Continuation of Exercise 33.*)

**a.** Find the total work necessary to pump out the glory hole, by adding the work necessary to pump both the throat and funnel sections.

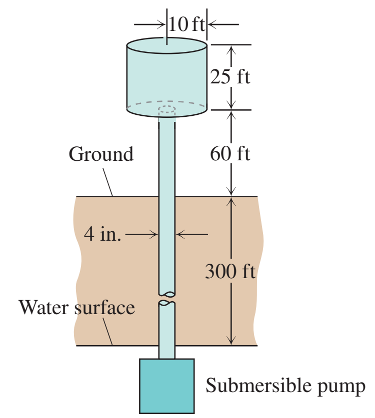
**b.** Your answer to part (a) is in foot-pounds. A more useful form is horsepower-hours, since motors are rated in horsepower. To convert from foot-pounds to horsepower-hours, divide by $1.98 \times 10^6$. How many hours would it take a 1000-horsepower motor to pump out the glory hole, assuming that the motor was fully efficient?

**35. Drinking a milkshake** The truncated conical container shown here is full of strawberry milkshake that weighs 4/9 oz/in.$^3$ As you can see, the container is 7 in. deep, 2.5 in. across at the base, and 3.5 in. across at the top (a standard size at Brigham's in Boston). The straw sticks up an inch above the top. About how much work does it take to suck up the milkshake through the straw (neglecting friction)? Answer in inch-ounces.



Dimensions in inches

**36. Water tower** Your town has decided to drill a well to increase its water supply. As the town engineer, you have determined that a water tower will be necessary to provide the pressure needed for distribution, and you have designed the system shown here. The water is to be pumped from a 300 ft well through a vertical 4 in. pipe into the base of a cylindrical tank 20 ft in diameter and 25 ft high. The base of the tank will be 60 ft aboveground. The pump is a 3 hp pump, rated at 1650 ft · lb/sec. To the nearest hour, how long will it take to fill the tank the first time? (Include the time it takes to fill the pipe.) Assume that water weighs 62.4 lb/ft$^3$.



NOT TO SCALE

**37. Putting a satellite in orbit** The strength of Earth's gravitational field varies with the distance $r$ from Earth's center, and the magnitude of the gravitational force experienced by a satellite of mass $m$ during and after launch is

$$F(r) = \frac{mMG}{r^2}.$$

Here, $M = 5.975 \times 10^{24}$ kg is Earth's mass, $G = 6.6720 \times 10^{-11}$ N · m$^2$ kg$^{-2}$ is the universal gravitational constant, and $r$ is measured in meters. The work it takes to lift a 1000-kg satellite from Earth's surface to a circular orbit 35,780 km above Earth's center is therefore given by the integral

$$\text{Work} = \int_{6{,}370{,}000}^{35{,}780{,}000} \frac{1000MG}{r^2}\,dr \text{ joules}.$$

Evaluate the integral. The lower limit of integration is Earth's radius in meters at the launch site. (This calculation does not take into account energy spent lifting the launch vehicle or energy spent bringing the satellite to orbit velocity.)

**38. Forcing electrons together** Two electrons $r$ meters apart repel each other with a force of

$$F = \frac{23 \times 10^{-29}}{r^2} \text{ newtons}.$$

**a.** Suppose one electron is held fixed at the point (1, 0) on the $x$-axis (units in meters). How much work does it take to move a second electron along the $x$-axis from the point $(-1, 0)$ to the origin?

**b.** Suppose an electron is held fixed at each of the points $(-1, 0)$ and (1, 0). How much work does it take to move a third electron along the $x$-axis from (5, 0) to (3, 0)?