

Chapter 6 Additional and Advanced Exercises

Volume and Length

1. A solid is generated by revolving about the x -axis the region bounded by the graph of the positive continuous function $y = f(x)$, the x -axis, and the fixed line $x = a$ and the variable line $x = b$, $b > a$. Its volume, for all b , is $b^2 - ab$. Find $f(x)$.
2. A solid is generated by revolving about the x -axis the region bounded by the graph of the positive continuous function $y = f(x)$, the x -axis, and the lines $x = 0$ and $x = a$. Its volume, for all $a > 0$, is $a^2 + a$. Find $f(x)$.
3. Suppose that the increasing function $f(x)$ is smooth for $x \geq 0$ and that $f(0) = a$. Let $s(x)$ denote the length of the graph of f from $(0, a)$ to $(x, f(x))$, $x > 0$. Find $f(x)$ if $s(x) = Cx$ for some constant C . What are the allowable values for C ?
4. a. Show that for $0 < \alpha \leq \pi/2$,

$$\int_0^\alpha \sqrt{1 + \cos^2 \theta} \, d\theta > \sqrt{\alpha^2 + \sin^2 \alpha}.$$

- b. Generalize the result in part (a).

Moments and Centers of Mass

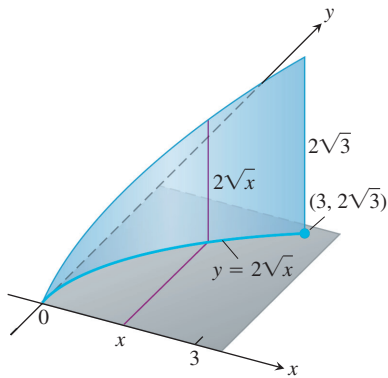
5. Find the centroid of the region bounded below by the x -axis and above by the curve $y = 1 - x^n$, n an even positive integer. What is the limiting position of the centroid as $n \rightarrow \infty$?
6. If you haul a telephone pole on a two-wheeled carriage behind a truck, you want the wheels to be 3 ft or so behind the pole's center of mass to provide an adequate "tongue" weight. NYNEX's class 1.40-ft wooden poles have a 27-in. circumference at the top and a

43.5-in. circumference at the base. About how far from the top is the center of mass?

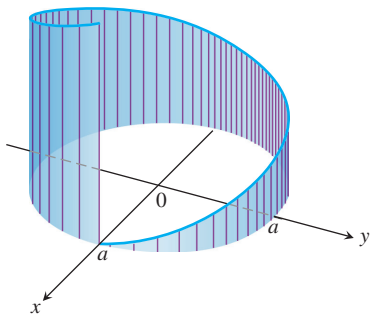
7. Suppose that a thin metal plate of area A and constant density δ occupies a region R in the xy -plane, and let M_y be the plate's moment about the y -axis. Show that the plate's moment about the line $x = b$ is
 - a. $M_y - b\delta A$ if the plate lies to the right of the line, and
 - b. $b\delta A - M_y$ if the plate lies to the left of the line.
8. Find the center of mass of a thin plate covering the region bounded by the curve $y^2 = 4ax$ and the line $x = a$, $a =$ positive constant, if the density at (x, y) is directly proportional to **(a)** x , **(b)** $|y|$.
9. a. Find the centroid of the region in the first quadrant bounded by two concentric circles and the coordinate axes, if the circles have radii a and b , $0 < a < b$, and their centers are at the origin.
 - b. Find the limits of the coordinates of the centroid as a approaches b and discuss the meaning of the result.
10. A triangular corner is cut from a square 1 ft on a side. The area of the triangle removed is 36 in.^2 . If the centroid of the remaining region is 7 in. from one side of the original square, how far is it from the remaining sides?

Surface Area

11. At points on the curve $y = 2\sqrt{x}$, line segments of length $h = y$ are drawn perpendicular to the xy -plane. (See accompanying figure.) Find the area of the surface formed by these perpendiculars from $(0, 0)$ to $(3, 2\sqrt{3})$.



12. At points on a circle of radius a , line segments are drawn perpendicular to the plane of the circle, the perpendicular at each point P being of length ks , where s is the length of the arc of the circle measured counterclockwise from $(a, 0)$ to P and k is a positive constant, as shown here. Find the area of the surface formed by the perpendiculars along the arc beginning at $(a, 0)$ and extending once around the circle.



Work

13. A particle of mass m starts from rest at time $t = 0$ and is moved along the x -axis with constant acceleration a from $x = 0$ to $x = h$ against a variable force of magnitude $F(t) = t^2$. Find the work done.
14. **Work and kinetic energy** Suppose a 1.6-oz golf ball is placed on a vertical spring with force constant $k = 2$ lb/in. The spring is compressed 6 in. and released. About how high does the ball go (measured from the spring's rest position)?

Fluid Force

15. A triangular plate ABC is submerged in water with its plane vertical. The side AB , 4 ft long, is 6 ft below the surface of the water, while the vertex C is 2 ft below the surface. Find the force exerted by the water on one side of the plate.
16. A vertical rectangular plate is submerged in a fluid with its top edge parallel to the fluid's surface. Show that the force exerted by the fluid on one side of the plate equals the average value of the pressure up and down the plate times the area of the plate.
17. The *center of pressure* on one side of a plane region submerged in a fluid is defined to be the point at which the total force exerted by the fluid can be applied without changing its total moment about any axis in the plane. Find the depth to the center of pressure (a) on a vertical rectangle of height h and width b if its upper edge is in the surface of the fluid; (b) on a vertical triangle of height h and base b if the vertex opposite b is a ft and the base b is $(a + h)$ ft below the surface of the fluid.