EXERCISES 7.2

Using the Properties of Logarithms

1. Express the following logarithms in terms of ln 2 and ln 3.
   a. $\ln 0.75$ b. $\ln (4/9)$ c. $\ln (1/2)$
   d. $\ln \sqrt[3]{9}$ e. $\ln 3\sqrt{2}$ f. $\ln \sqrt{13.5}$
2. Express the following logarithms in terms of ln 5 and ln 7.
   a. $\ln (1/125)$ b. $\ln 9.8$ c. $\ln 7\sqrt{7}$
   d. $\ln 1225$ e. $\ln 0.056$
   f. $(\ln 35 + \ln (1/7))/(\ln 25)$

Use the properties of logarithms to simplify the expressions in Exercises 3 and 4.

3. a. $\ln \sin \theta - \ln \left(\dfrac{\sin \theta}{5}\right)$ b. $\ln (3x^2 - 9x) + \ln \left(\dfrac{1}{3x}\right)$
   c. $\dfrac{1}{2} \ln (4t^4) - \ln 2$
4. a. $\ln \sec \theta + \ln \cos \theta$ b. $\ln (8x + 4) - 2 \ln 2$
   c. $3 \ln \sqrt[3]{t^2 - 1} - \ln (t + 1)$

Derivatives of Logarithms

In Exercises 5–36, find the derivative of y with respect to x, t, or θ, as appropriate.

5. $y = \ln 3x$
6. $y = \ln kx$, k constant
7. $y = \ln (t^2)$
8. $y = \ln (t^{3/2})$
9. $y = \ln \dfrac{3}{x}$
10. $y = \ln \dfrac{10}{x}$
11. $y = \ln (\theta + 1)$
12. $y = \ln (2\theta + 2)$
13. $y = \ln x^3$
14. $y = (\ln x)^3$
15. $y = t(\ln t)^2$
16. $y = t\sqrt{\ln t}$
17. $y = \dfrac{x^4}{4} \ln x - \dfrac{x^4}{16}$
18. $y = \dfrac{x^3}{3} \ln x - \dfrac{x^3}{9}$
19. $y = \dfrac{\ln t}{t}$
20. $y = \dfrac{1 + \ln t}{t}$
21. $y = \dfrac{\ln x}{1 + \ln x}$
22. $y = \dfrac{x \ln x}{1 + \ln x}$
23. $y = \ln (\ln x)$
24. $y = \ln (\ln (\ln x))$
25. $y = \theta(\sin (\ln \theta) + \cos (\ln \theta))$
26. $y = \ln (\sec \theta + \tan \theta)$
27. $y = \ln \dfrac{1}{x\sqrt{x + 1}}$
28. $y = \dfrac{1}{2} \ln \dfrac{1 + x}{1 - x}$
29. $y = \dfrac{1 + \ln t}{1 - \ln t}$
30. $y = \sqrt{\ln \sqrt{t}}$
31. $y = \ln (\sec (\ln \theta))$
32. $y = \ln \left(\dfrac{\sqrt{\sin \theta \cos \theta}}{1 + 2 \ln \theta}\right)$
33. $y = \ln \left(\dfrac{(x^2 + 1)^5}{\sqrt{1 - x}}\right)$
34. $y = \ln \sqrt{\dfrac{(x + 1)^5}{(x + 2)^{20}}}$
35. $y = \displaystyle\int_{x^2/2}^{x^2} \ln \sqrt{t}\, dt$
36. $y = \displaystyle\int_{\sqrt{x}}^{\sqrt[3]{x}} \ln t\, dt$

Integration

Evaluate the integrals in Exercises 37–54.

37. $\displaystyle\int_{-3}^{-2} \frac{dx}{x}$
38. $\displaystyle\int_{-1}^{0} \frac{3\, dx}{3x - 2}$
39. $\displaystyle\int \frac{2y\, dy}{y^2 - 25}$
40. $\displaystyle\int \frac{8r\, dr}{4r^2 - 5}$
41. $\displaystyle\int_{0}^{\pi} \frac{\sin t}{2 - \cos t}\, dt$
42. $\displaystyle\int_{0}^{\pi/3} \frac{4 \sin \theta}{1 - 4 \cos \theta}\, d\theta$
43. $\displaystyle\int_{1}^{2} \frac{2 \ln x}{x}\, dx$
44. $\displaystyle\int_{2}^{4} \frac{dx}{x \ln x}$
45. $\displaystyle\int_{2}^{4} \frac{dx}{x(\ln x)^2}$
46. $\displaystyle\int_{2}^{16} \frac{dx}{2x\sqrt{\ln x}}$
47. $\displaystyle\int \frac{3 \sec^2 t}{6 + 3 \tan t}\, dt$
48. $\displaystyle\int \frac{\sec y \tan y}{2 + \sec y}\, dy$
49. $\displaystyle\int_{0}^{\pi/2} \tan \frac{x}{2}\, dx$
50. $\displaystyle\int_{\pi/4}^{\pi/2} \cot t\, dt$
51. $\displaystyle\int_{\pi/2}^{\pi} 2 \cot \frac{\theta}{3}\, d\theta$
52. $\displaystyle\int_{0}^{\pi/12} 6 \tan 3x\, dx$
53. $\displaystyle\int \frac{dx}{2\sqrt{x} + 2x}$
54. $\displaystyle\int \frac{\sec x\, dx}{\sqrt{\ln (\sec x + \tan x)}}$

Logarithmic Differentiation

In Exercises 55–68, use logarithmic differentiation to find the derivative of y with respect to the given independent variable.

55. $y = \sqrt{x(x+1)}$

56. $y = \sqrt{(x^2+1)(x-1)^2}$

57. $y = \sqrt{\dfrac{t}{t+1}}$

58. $y = \sqrt{\dfrac{1}{t(t+1)}}$

59. $y = \sqrt{\theta + 3}\sin\theta$

60. $y = (\tan\theta)\sqrt{2\theta + 1}$

61. $y = t(t+1)(t+2)$

62. $y = \dfrac{1}{t(t+1)(t+2)}$

63. $y = \dfrac{\theta + 5}{\theta\cos\theta}$

64. $y = \dfrac{\theta\sin\theta}{\sqrt{\sec\theta}}$

65. $y = \dfrac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$

66. $y = \sqrt{\dfrac{(x+1)^{10}}{(2x+1)^5}}$

67. $y = \sqrt[3]{\dfrac{x(x-2)}{x^2+1}}$

68. $y = \sqrt[3]{\dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$

Theory and Applications

69. Locate and identify the absolute extreme values of

a. $\ln(\cos x)$ on $[-\pi/4, \pi/3]$,

b. $\cos(\ln x)$ on $[1/2, 2]$.

70. **a.** Prove that $f(x) = x - \ln x$ is increasing for $x > 1$.

b. Using part (a), show that $\ln x < x$ if $x > 1$.

71. Find the area between the curves $y = \ln x$ and $y = \ln 2x$ from $x = 1$ to $x = 5$.

72. Find the area between the curve $y = \tan x$ and the x-axis from $x = -\pi/4$ to $x = \pi/3$.

73. The region in the first quadrant bounded by the coordinate axes, the line $y = 3$, and the curve $x = 2/\sqrt{y+1}$ is revolved about the y-axis to generate a solid. Find the volume of the solid.

74. The region between the curve $y = \sqrt{\cot x}$ and the x-axis from $x = \pi/6$ to $x = \pi/2$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

75. The region between the curve $y = 1/x^2$ and the x-axis from $x = 1/2$ to $x = 2$ is revolved about the y-axis to generate a solid. Find the volume of the solid.

76. In Section 6.2, Exercise 6, we revolved about the y-axis the region between the curve $y = 9x/\sqrt{x^3+9}$ and the x-axis from $x = 0$ to $x = 3$ to generate a solid of volume 36π. What volume do you get if you revolve the region about the x-axis instead? (See Section 6.2, Exercise 6, for a graph.)

77. Find the lengths of the following curves.

a. $y = (x^2/8) - \ln x, \quad 4 \le x \le 8$

b. $x = (y/4)^2 - 2\ln(y/4), \quad 4 \le y \le 12$

78. Find a curve through the point $(1, 0)$ whose length from $x = 1$ to $x = 2$ is

$$L = \int_1^2 \sqrt{1 + \frac{1}{x^2}}\,dx.$$

T **79.** **a.** Find the centroid of the region between the curve $y = 1/x$ and the x-axis from $x = 1$ to $x = 2$. Give the coordinates to two decimal places.

b. Sketch the region and show the centroid in your sketch.

80. **a.** Find the center of mass of a thin plate of constant density covering the region between the curve $y = 1/\sqrt{x}$ and the x-axis from $x = 1$ to $x = 16$.

b. Find the center of mass if, instead of being constant, the density function is $\delta(x) = 4/\sqrt{x}$.

Solve the initial value problems in Exercises 81 and 82.

81. $\dfrac{dy}{dx} = 1 + \dfrac{1}{x}, \quad y(1) = 3$

82. $\dfrac{d^2y}{dx^2} = \sec^2 x, \quad y(0) = 0$ and $y'(0) = 1$

T **83.** **The linearization of $\ln(1+x)$ at $x = 0$** Instead of approximating $\ln x$ near $x = 1$, we approximate $\ln(1+x)$ near $x = 0$. We get a simpler formula this way.

a. Derive the linearization $\ln(1+x) \approx x$ at $x = 0$.

b. Estimate to five decimal places the error involved in replacing $\ln(1+x)$ by x on the interval $[0, 0.1]$.

c. Graph $\ln(1+x)$ and x together for $0 \le x \le 0.5$. Use different colors, if available. At what points does the approximation of $\ln(1+x)$ seem best? Least good? By reading coordinates from the graphs, find as good an upper bound for the error as your grapher will allow.

84. Use the same-derivative argument, as was done to prove Rules 1 and 4 of Theorem 2, to prove the Quotient Rule property of logarithms.

Grapher Explorations

85. Graph $\ln x$, $\ln 2x$, $\ln 4x$, $\ln 8x$, and $\ln 16x$ (as many as you can) together for $0 < x \le 10$. What is going on? Explain.

86. Graph $y = \ln|\sin x|$ in the window $0 \le x \le 22$, $-2 \le y \le 0$. Explain what you see. How could you change the formula to turn the arches upside down?

87. **a.** Graph $y = \sin x$ and the curves $y = \ln(a + \sin x)$ for $a = 2$, 4, 8, 20, and 50 together for $0 \le x \le 23$.

b. Why do the curves flatten as a increases? (*Hint:* Find an a-dependent upper bound for $|y'|$.)

88. Does the graph of $y = \sqrt{x} - \ln x$, $x > 0$, have an inflection point? Try to answer the question **(a)** by graphing, **(b)** by using calculus.