EXERCISES 7.3

Algebraic Calculations with the Exponential and Logarithm

Find simpler expressions for the quantities in Exercises 1–4.

1. **a.** $e^{\ln 7.2}$ **b.** $e^{-\ln x^2}$ **c.** $e^{\ln x - \ln y}$

2. **a.** $e^{\ln(x^2+y^2)}$ **b.** $e^{-\ln 0.3}$ **c.** $e^{\ln \pi x - \ln 2}$

3. **a.** $2\ln\sqrt{e}$ **b.** $\ln(\ln e^e)$ **c.** $\ln(e^{-x^2-y^2})$

4. **a.** $\ln(e^{\sec\theta})$ **b.** $\ln(e^{(e^x)})$ **c.** $\ln(e^{2\ln x})$

Solving Equations with Logarithmic or Exponential Terms

In Exercises 5–10, solve for y in terms of t or x, as appropriate.

5. $\ln y = 2t + 4$

6. $\ln y = -t + 5$

7. $\ln(y - 40) = 5t$

8. $\ln(1 - 2y) = t$

9. $\ln(y - 1) - \ln 2 = x + \ln x$

10. $\ln(y^2 - 1) - \ln(y + 1) = \ln(\sin x)$

In Exercises 11 and 12, solve for k.

11. **a.** $e^{2k} = 4$ **b.** $100e^{10k} = 200$ **c.** $e^{k/1000} = a$

12. **a.** $e^{5k} = \frac{1}{4}$ **b.** $80e^k = 1$ **c.** $e^{(\ln 0.8)k} = 0.8$

In Exercises 13–16, solve for t.

13. **a.** $e^{-0.3t} = 27$ **b.** $e^{kt} = \frac{1}{2}$ **c.** $e^{(\ln 0.2)t} = 0.4$

14. **a.** $e^{-0.01t} = 1000$ **b.** $e^{kt} = \frac{1}{10}$ **c.** $e^{(\ln 2)t} = \frac{1}{2}$

15. $e^{\sqrt{t}} = x^2$

16. $e^{(x^2)}e^{(2x+1)} = e^t$

Derivatives

In Exercises 17–36, find the derivative of y with respect to x, t, or θ, as appropriate.

17. $y = e^{-5x}$

18. $y = e^{2x/3}$

19. $y = e^{5-7x}$

20. $y = e^{(4\sqrt{x}+x^2)}$

21. $y = xe^x - e^x$

22. $y = (1 + 2x)e^{-2x}$

23. $y = (x^2 - 2x + 2)e^x$

24. $y = (9x^2 - 6x + 2)e^{3x}$

25. $y = e^\theta(\sin\theta + \cos\theta)$

26. $y = \ln(3\theta e^{-\theta})$

27. $y = \cos(e^{-\theta^2})$

28. $y = \theta^3 e^{-2\theta}\cos 5\theta$

29. $y = \ln(3te^{-t})$

30. $y = \ln(2e^{-t}\sin t)$

31. $y = \ln\left(\frac{e^\theta}{1 + e^\theta}\right)$

32. $y = \ln\left(\frac{\sqrt{\theta}}{1 + \sqrt{\theta}}\right)$

33. $y = e^{(\cos t + \ln t)}$

34. $y = e^{\sin t}(\ln t^2 + 1)$

35. $y = \int_0^{\ln x} \sin e^t \, dt$

36. $y = \int_{e^{4\sqrt{x}}}^{e^{2x}} \ln t \, dt$

In Exercises 37–40, find dy/dx.

37. $\ln y = e^y \sin x$

38. $\ln xy = e^{x+y}$

39. $e^{2x} = \sin(x + 3y)$

40. $\tan y = e^x + \ln x$

Integrals

Evaluate the integrals in Exercises 41–62.

41. $\int (e^{3x} + 5e^{-x})\, dx$

42. $\int (2e^x - 3e^{-2x})\, dx$

43. $\int_{\ln 2}^{\ln 3} e^x \, dx$

44. $\int_{-\ln 2}^{0} e^{-x}\, dx$

45. $\int 8e^{(x+1)}\, dx$

46. $\int 2e^{(2x-1)}\, dx$

47. $\int_{\ln 4}^{\ln 9} e^{x/2}\, dx$

48. $\int_0^{\ln 16} e^{x/4}\, dx$

49. $\int \frac{e^{\sqrt{r}}}{\sqrt{r}}\, dr$

50. $\int \frac{e^{-\sqrt{r}}}{\sqrt{r}}\, dr$

51. $\int 2t\, e^{-t^2}\, dt$

52. $\int t^3 e^{(t^4)}\, dt$

53. $\int \frac{e^{1/x}}{x^2}\, dx$

54. $\int \frac{e^{-1/x^2}}{x^3}\, dx$

55. $\int_0^{\pi/4} (1 + e^{\tan\theta})\sec^2\theta\, d\theta$

56. $\int_{\pi/4}^{\pi/2} (1 + e^{\cot\theta})\csc^2\theta\, d\theta$

57. $\int e^{\sec \pi t}\sec \pi t \tan \pi t\, dt$

58. $\int e^{\csc(\pi + t)}\csc(\pi + t)\cot(\pi + t)\, dt$

59. $\displaystyle\int_{\ln(\pi/6)}^{\ln(\pi/2)} 2e^{v}\cos e^{v}\,dv$

60. $\displaystyle\int_{0}^{\sqrt{\ln \pi}} 2x\,e^{x^2}\cos\left(e^{x^2}\right)dx$

61. $\displaystyle\int \frac{e^{r}}{1+e^{r}}\,dr$

62. $\displaystyle\int \frac{dx}{1+e^{x}}$

Initial Value Problems

Solve the initial value problems in Exercises 63–66.

63. $\dfrac{dy}{dt} = e^{t}\sin\left(e^{t}-2\right), \quad y(\ln 2) = 0$

64. $\dfrac{dy}{dt} = e^{-t}\sec^{2}\left(\pi e^{-t}\right), \quad y(\ln 4) = 2/\pi$

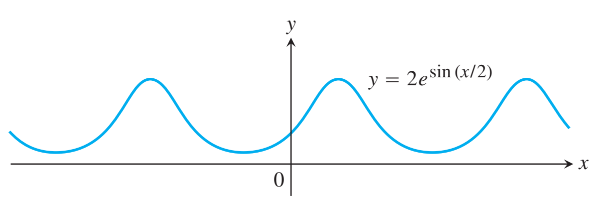
65. $\dfrac{d^2y}{dx^2} = 2e^{-x}, \quad y(0) = 1 \quad \text{and} \quad y'(0) = 0$

66. $\dfrac{d^2y}{dt^2} = 1 - e^{2t}, \quad y(1) = -1 \quad \text{and} \quad y'(1) = 0$

Theory and Applications

67. Find the absolute maximum and minimum values of $f(x) = e^{x} - 2x$ on $[0, 1]$.

68. Where does the periodic function $f(x) = 2e^{\sin(x/2)}$ take on its extreme values and what are these values?



69. Find the absolute maximum value of $f(x) = x^2 \ln(1/x)$ and say where it is assumed.

70. Graph $f(x) = (x-3)^2 e^{x}$ and its first derivative together. Comment on the behavior of f in relation to the signs and values of f'. Identify significant points on the graphs with calculus, as necessary.

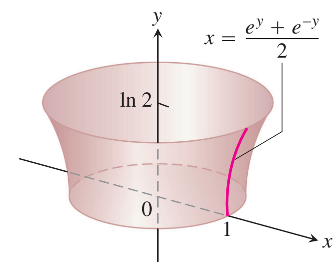
71. Find the area of the "triangular" region in the first quadrant that is bounded above by the curve $y = e^{2x}$, below by the curve $y = e^{x}$, and on the right by the line $x = \ln 3$.

72. Find the area of the "triangular" region in the first quadrant that is bounded above by the curve $y = e^{x/2}$, below by the curve $y = e^{-x/2}$, and on the right by the line $x = 2\ln 2$.

73. Find a curve through the origin in the xy-plane whose length from $x = 0$ to $x = 1$ is

$$L = \int_0^1 \sqrt{1 + \frac{1}{4}e^{x}}\,dx.$$

74. Find the area of the surface generated by revolving the curve $x = (e^{y} + e^{-y})/2, 0 \le y \le \ln 2$, about the y-axis.



75. **a.** Show that $\int \ln x\,dx = x\ln x - x + C$.

b. Find the average value of $\ln x$ over $[1, e]$.

76. Find the average value of $f(x) = 1/x$ on $[1, 2]$.

77. **The linearization of e^{x} at $x = 0$**

a. Derive the linear approximation $e^{x} \approx 1 + x$ at $x = 0$.

b. Estimate to five decimal places the magnitude of the error involved in replacing e^{x} by $1 + x$ on the interval $[0, 0.2]$.

c. Graph e^{x} and $1 + x$ together for $-2 \le x \le 2$. Use different colors, if available. On what intervals does the approximation appear to overestimate e^{x}? Underestimate e^{x}?

78. **Laws of Exponents**

a. Starting with the equation $e^{x_1}e^{x_2} = e^{x_1+x_2}$, derived in the text, show that $e^{-x} = 1/e^{x}$ for any real number x. Then show that $e^{x_1}/e^{x_2} = e^{x_1-x_2}$ for any numbers x_1 and x_2.

b. Show that $(e^{x_1})^{x_2} = e^{x_1x_2} = (e^{x_2})^{x_1}$ for any numbers x_1 and x_2.

79. **A decimal representation of e** Find e to as many decimal places as your calculator allows by solving the equation $\ln x = 1$.

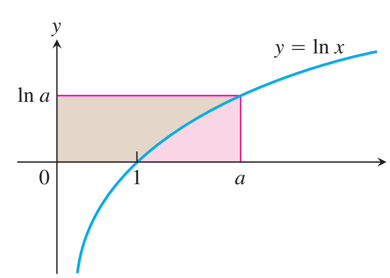
80. **The inverse relation between e^{x} and $\ln x$** Find out how good your calculator is at evaluating the composites

$$e^{\ln x} \quad \text{and} \quad \ln(e^{x}).$$

81. Show that for any number $a > 1$

$$\int_1^a \ln x\,dx + \int_0^{\ln a} e^{y}\,dy = a\ln a.$$

(See accompanying figure.)



82. **The geometric, logarithmic, and arithmetic mean inequality**

a. Show that the graph of e^{x} is concave up over every interval of x-values.

b. Show, by reference to the accompanying figure, that if $0 < a < b$ then

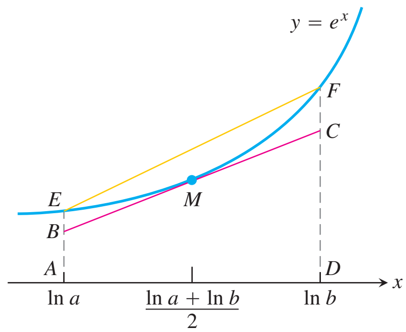
$$e^{(\ln a + \ln b)/2} \cdot (\ln b - \ln a) < \int_{\ln a}^{\ln b} e^x \, dx < \frac{e^{\ln a} + e^{\ln b}}{2} \cdot (\ln b - \ln a).$$

c. Use the inequality in part (b) to conclude that

$$\sqrt{ab} < \frac{b - a}{\ln b - \ln a} < \frac{a + b}{2}.$$

This inequality says that the geometric mean of two positive numbers is less than their logarithmic mean, which in turn is less than their arithmetic mean.

(For more about this inequality, see "The Geometric, Logarithmic, and Arithmetic Mean Inequality" by Frank Burk, *American Mathematical Monthly*, Vol. 94, No. 6, June–July 1987, pp. 527–528.)



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