# EXERCISES 7.4

## Algebraic Calculations With $a^x$ and $\log_a x$

Simplify the expressions in Exercises 1–4.

**1. a.** $5^{\log_5 7}$ **b.** $8^{\log_8 \sqrt{2}}$ **c.** $1.3^{\log_{1.3} 75}$

**d.** $\log_4 16$ **e.** $\log_3 \sqrt{3}$ **f.** $\log_4 \left(\frac{1}{4}\right)$

**2. a.** $2^{\log_2 3}$ **b.** $10^{\log_{10}(1/2)}$ **c.** $\pi^{\log_\pi 7}$

**d.** $\log_{11} 121$ **e.** $\log_{121} 11$ **f.** $\log_3 \left(\frac{1}{9}\right)$

**3. a.** $2^{\log_4 x}$ **b.** $9^{\log_3 x}$ **c.** $\log_2\left(e^{(\ln 2)(\sin x)}\right)$

**4. a.** $25^{\log_5(3x^2)}$ **b.** $\log_e(e^x)$ **c.** $\log_4\left(2^{e^x \sin x}\right)$

Express the ratios in Exercises 5 and 6 as ratios of natural logarithms and simplify.

**5. a.** $\dfrac{\log_2 x}{\log_3 x}$ **b.** $\dfrac{\log_2 x}{\log_8 x}$ **c.** $\dfrac{\log_x a}{\log_{x^2} a}$

**6. a.** $\dfrac{\log_9 x}{\log_3 x}$ **b.** $\dfrac{\log_{\sqrt{10}} x}{\log_{\sqrt{2}} x}$ **c.** $\dfrac{\log_a b}{\log_b a}$

Solve the equations in Exercises 7–10 for $x$.

**7.** $3^{\log_3(7)} + 2^{\log_2(5)} = 5^{\log_5(x)}$

**8.** $8^{\log_8(3)} - e^{\ln 5} = x^2 - 7^{\log_7(3x)}$

**9.** $3^{\log_3(x^2)} = 5e^{\ln x} - 3 \cdot 10^{\log_{10}(2)}$

**10.** $\ln e + 4^{-2\log_4(x)} = \frac{1}{x}\log_{10}(100)$

## Derivatives

In Exercises 11–38, find the derivative of $y$ with respect to the given independent variable.

**11.** $y = 2^x$ **12.** $y = 3^{-x}$

**13.** $y = 5^{\sqrt{s}}$ **14.** $y = 2^{(s^2)}$

**15.** $y = x^\pi$ **16.** $y = t^{1-e}$

**17.** $y = (\cos\theta)^{\sqrt{2}}$ **18.** $y = (\ln\theta)^\pi$

**19.** $y = 7^{\sec\theta}\ln 7$ **20.** $y = 3^{\tan\theta}\ln 3$

**21.** $y = 2^{\sin 3t}$ **22.** $y = 5^{-\cos 2t}$

**23.** $y = \log_2 5\theta$ **24.** $y = \log_3(1 + \theta\ln 3)$

**25.** $y = \log_4 x + \log_4 x^2$ **26.** $y = \log_{25} e^x - \log_5 \sqrt{x}$

**27.** $y = \log_2 r \cdot \log_4 r$ **28.** $y = \log_3 r \cdot \log_9 r$

**29.** $y = \log_3\left(\left(\dfrac{x+1}{x-1}\right)^{\ln 3}\right)$ **30.** $y = \log_5\sqrt{\left(\dfrac{7x}{3x+2}\right)^{\ln 5}}$

**31.** $y = \theta\sin(\log_7\theta)$ **32.** $y = \log_7\left(\dfrac{\sin\theta\cos\theta}{e^\theta 2^\theta}\right)$

**33.** $y = \log_5 e^x$ **34.** $y = \log_2\left(\dfrac{x^2e^2}{2\sqrt{x+1}}\right)$

**35.** $y = 3^{\log_2 t}$ **36.** $y = 3\log_8(\log_2 t)$

**37.** $y = \log_2(8t^{\ln 2})$ **38.** $y = t\log_3\left(e^{(\sin t)(\ln 3)}\right)$

## Logarithmic Differentiation

In Exercises 39–46, use logarithmic differentiation to find the derivative of $y$ with respect to the given independent variable.

**39.** $y = (x+1)^x$ **40.** $y = x^{(x+1)}$

**41.** $y = (\sqrt{t})^t$ **42.** $y = t^{\sqrt{t}}$

**43.** $y = (\sin x)^x$ **44.** $y = x^{\sin x}$

**45.** $y = x^{\ln x}$ **46.** $y = (\ln x)^{\ln x}$

## Integration

Evaluate the integrals in Exercises 47–56.

**47.** $\displaystyle\int 5^x\,dx$ **48.** $\displaystyle\int (1.3)^x\,dx$

**49.** $\int_0^1 2^{-\theta}\, d\theta$

**50.** $\int_{-2}^0 5^{-\theta}\, d\theta$

**51.** $\int_1^{\sqrt{2}} x2^{(x^2)}\, dx$

**52.** $\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}\, dx$

**53.** $\int_0^{\pi/2} 7^{\cos t} \sin t\, dt$

**54.** $\int_0^{\pi/4} \left(\frac{1}{3}\right)^{\tan t} \sec^2 t\, dt$

**55.** $\int_2^4 x^{2x}(1 + \ln x)\, dx$

**56.** $\int_1^2 \frac{2^{\ln x}}{x}\, dx$

Evaluate the integrals in Exercises 57–60.

**57.** $\int 3x^{\sqrt{3}}\, dx$

**58.** $\int x^{\sqrt{2}-1}\, dx$

**59.** $\int_0^3 (\sqrt{2} + 1)x^{\sqrt{2}}\, dx$

**60.** $\int_1^e x^{(\ln 2)-1}\, dx$

Evaluate the integrals in Exercises 61–70.

**61.** $\int \frac{\log_{10} x}{x}\, dx$

**62.** $\int_1^4 \frac{\log_2 x}{x}\, dx$

**63.** $\int_1^4 \frac{\ln 2 \log_2 x}{x}\, dx$

**64.** $\int_1^e \frac{2 \ln 10 \log_{10} x}{x}\, dx$

**65.** $\int_0^2 \frac{\log_2 (x + 2)}{x + 2}\, dx$

**66.** $\int_{1/10}^{10} \frac{\log_{10} (10x)}{x}\, dx$

**67.** $\int_0^9 \frac{2 \log_{10} (x + 1)}{x + 1}\, dx$

**68.** $\int_2^3 \frac{2 \log_2 (x - 1)}{x - 1}\, dx$

**69.** $\int \frac{dx}{x \log_{10} x}$

**70.** $\int \frac{dx}{x(\log_8 x)^2}$

Evaluate the integrals in Exercises 71–74.

**71.** $\int_1^{\ln x} \frac{1}{t}\, dt, \quad x > 1$

**72.** $\int_1^{e^x} \frac{1}{t}\, dt$

**73.** $\int_1^{1/x} \frac{1}{t}\, dt, \quad x > 0$

**74.** $\frac{1}{\ln a} \int_1^x \frac{1}{t}\, dt, \quad x > 0$

## Theory and Applications

**75.** Find the area of the region between the curve $y = 2x/(1 + x^2)$ and the interval $-2 \le x \le 2$ of the $x$-axis.

**76.** Find the area of the region between the curve $y = 2^{1-x}$ and the interval $-1 \le x \le 1$ of the $x$-axis.

**77. Blood pH** The pH of human blood normally falls between 7.37 and 7.44. Find the corresponding bounds for $[H_3O^+]$.

**78. Brain fluid pH** The cerebrospinal fluid in the brain has a hydronium ion concentration of about $[H_3O^+] = 4.8 \times 10^{-8}$ moles per liter. What is the pH?

**79. Audio amplifiers** By what factor $k$ do you have to multiply the intensity of $I$ of the sound from your audio amplifier to add 10 dB to the sound level?

**80. Audio amplifiers** You multiplied the intensity of the sound of your audio system by a factor of 10. By how many decibels did this increase the sound level?

**81.** In any solution, the product of the hydronium ion concentration $[H_3O^+]$ (moles/L) and the hydroxyl ion concentration $[OH^-]$ (moles/L) is about $10^{-14}$.

**a.** What value of $[H_3O^+]$ minimizes the sum of the concentrations, $S = [H_3O^+] + [OH^-]$? (*Hint:* Change notation. Let $x = [H_3O^+]$.)

**b.** What is the pH of a solution in which $S$ has this minimum value?

**c.** What ratio of $[H_3O^+]$ to $[OH^-]$ minimizes $S$?

**82.** Could $\log_a b$ possibly equal $1/\log_b a$? Give reasons for your answer.

T **83.** The equation $x^2 = 2^x$ has three solutions: $x = 2$, $x = 4$, and one other. Estimate the third solution as accurately as you can by graphing.

T **84.** Could $x^{\ln 2}$ possibly be the same as $2^{\ln x}$ for $x > 0$? Graph the two functions and explain what you see.

**85. The linearization of $2^x$**

**a.** Find the linearization of $f(x) = 2^x$ at $x = 0$. Then round its coefficients to two decimal places.

T **b.** Graph the linearization and function together for $-3 \le x \le 3$ and $-1 \le x \le 1$.

**86. The linearization of $\log_3 x$**

**a.** Find the linearization of $f(x) = \log_3 x$ at $x = 3$. Then round its coefficients to two decimal places.

T **b.** Graph the linearization and function together in the window $0 \le x \le 8$ and $2 \le x \le 4$.

## Calculations with Other Bases

T **87.** Most scientific calculators have keys for $\log_{10} x$ and $\ln x$. To find logarithms to other bases, we use the Equation (5), $\log_a x = (\ln x)/(\ln a)$.

Find the following logarithms to five decimal places.

**a.** $\log_3 8$

**b.** $\log_7 0.5$

**c.** $\log_{20} 17$

**d.** $\log_{0.5} 7$

**e.** $\ln x$, given that $\log_{10} x = 2.3$

**f.** $\ln x$, given that $\log_2 x = 1.4$

**g.** $\ln x$, given that $\log_2 x = -1.5$

**h.** $\ln x$, given that $\log_{10} x = -0.7$

**88. Conversion factors**

**a.** Show that the equation for converting base 10 logarithms to base 2 logarithms is

$$\log_2 x = \frac{\ln 10}{\ln 2} \log_{10} x.$$

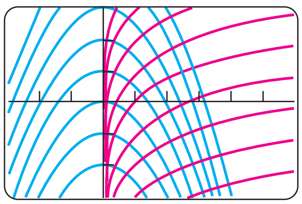
**b.** Show that the equation for converting base $a$ logarithms to base $b$ logarithms is

$$\log_b x = \frac{\ln a}{\ln b} \log_a x.$$

**89. Orthogonal families of curves** Prove that all curves in the family

$$y = -\frac{1}{2}x^2 + k$$

($k$ any constant) are perpendicular to all curves in the family $y = \ln x + c$ ($c$ any constant) at their points of intersection. (See the accompanying figure.)



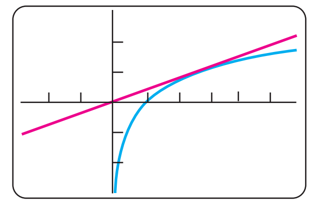
T **90. The inverse relation between $e^x$ and $\ln x$** Find out how good your calculator is at evaluating the composites

$$e^{\ln x} \quad \text{and} \quad \ln(e^x).$$

T **91. A decimal representation of $e$** Find $e$ to as many decimal places as your calculator allows by solving the equation $\ln x = 1$.

T **92. Which is bigger, $\pi^e$ or $e^\pi$?** Calculators have taken some of the mystery out of this once-challenging question. (Go ahead and check; you will see that it is a surprisingly close call.) You can answer the question without a calculator, though.

**a.** Find an equation for the line through the origin tangent to the graph of $y = \ln x$.



[–3, 6] by [–3, 3]

**b.** Give an argument based on the graphs of $y = \ln x$ and the tangent line to explain why $\ln x < x/e$ for all positive $x \neq e$.

**c.** Show that $\ln(x^e) < x$ for all positive $x \neq e$.

**d.** Conclude that $x^e < e^x$ for all positive $x \neq e$.

**e.** So which is bigger, $\pi^e$ or $e^\pi$?