Chapter 7 Practice Exercises

Differentiation

In Exercises 1–24, find the derivative of y with respect to the appropriate variable.

1. $y = 10e^{-x/5}$

2. $y = \sqrt{2}e^{\sqrt{2}x}$

3. $y = \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$

4. $y = x^2e^{-2/x}$

5. $y = \ln(\sin^2\theta)$

6. $y = \ln(\sec^2\theta)$

7. $y = \log_2(x^2/2)$

8. $y = \log_5(3x - 7)$

9. $y = 8^{-t}$

10. $y = 9^{2t}$

11. $y = 5x^{3.6}$

12. $y = \sqrt{2}x^{-\sqrt{2}}$

13. $y = (x + 2)^{x+2}$

14. $y = 2(\ln x)^{x/2}$

15. $y = \sin^{-1}\sqrt{1 - u^2}, \quad 0 < u < 1$

16. $y = \sin^{-1}\left(\frac{1}{\sqrt{v}}\right), \quad v > 1$

17. $y = \ln\cos^{-1}x$

18. $y = z\cos^{-1}z - \sqrt{1 - z^2}$

19. $y = t\tan^{-1}t - \frac{1}{2}\ln t$

20. $y = (1 + t^2)\cot^{-1}2t$

21. $y = z\sec^{-1}z - \sqrt{z^2 - 1}, \quad z > 1$

22. $y = 2\sqrt{x - 1}\,\sec^{-1}\sqrt{x}$

23. $y = \csc^{-1}(\sec\theta), \quad 0 < \theta < \pi/2$

24. $y = (1 + x^2)e^{\tan^{-1}x}$

Logarithmic Differentiation

In Exercises 25–30, use logarithmic differentiation to find the derivative of y with respect to the appropriate variable.

25. $y = \dfrac{2(x^2 + 1)}{\sqrt{\cos 2x}}$

26. $y = \sqrt[10]{\dfrac{3x + 4}{2x - 4}}$

27. $y = \left(\dfrac{(t + 1)(t - 1)}{(t - 2)(t + 3)}\right)^5, \quad t > 2$

28. $y = \dfrac{2u2^u}{\sqrt{u^2 + 1}}$

29. $y = (\sin\theta)^{\sqrt{\theta}}$

30. $y = (\ln x)^{1/(\ln x)}$

Integration

Evaluate the integrals in Exercises 31–78.

31. $\displaystyle\int e^x \sin(e^x)\,dx$

32. $\displaystyle\int e^t \cos(3e^t - 2)\,dt$

33. $\int e^x \sec^2(e^x - 7)\,dx$

34. $\int e^y \csc(e^y + 1)\cot(e^y + 1)\,dy$

35. $\int \sec^2(x)e^{\tan x}\,dx$

36. $\int \csc^2 x\, e^{\cot x}\,dx$

37. $\int_{-1}^{1} \frac{dx}{3x - 4}$

38. $\int_{1}^{e} \frac{\sqrt{\ln x}}{x}\,dx$

39. $\int_{0}^{\pi} \tan\frac{x}{3}\,dx$

40. $\int_{1/6}^{1/4} 2\cot \pi x\,dx$

41. $\int_{0}^{4} \frac{2t}{t^2 - 25}\,dt$

42. $\int_{-\pi/2}^{\pi/6} \frac{\cos t}{1 - \sin t}\,dt$

43. $\int \frac{\tan(\ln v)}{v}\,dv$

44. $\int \frac{dv}{v \ln v}$

45. $\int \frac{(\ln x)^{-3}}{x}\,dx$

46. $\int \frac{\ln(x - 5)}{x - 5}\,dx$

47. $\int \frac{1}{r}\csc^2(1 + \ln r)\,dr$

48. $\int \frac{\cos(1 - \ln v)}{v}\,dv$

49. $\int x3^{x^2}\,dx$

50. $\int 2^{\tan x}\sec^2 x\,dx$

51. $\int_{1}^{7} \frac{3}{x}\,dx$

52. $\int_{1}^{32} \frac{1}{5x}\,dx$

53. $\int_{1}^{4} \left(\frac{x}{8} + \frac{1}{2x}\right)dx$

54. $\int_{1}^{8} \left(\frac{2}{3x} - \frac{8}{x^2}\right)dx$

55. $\int_{-2}^{-1} e^{-(x+1)}\,dx$

56. $\int_{-\ln 2}^{0} e^{2w}\,dw$

57. $\int_{0}^{\ln 5} e^r(3e^r + 1)^{-3/2}\,dr$

58. $\int_{0}^{\ln 9} e^{\theta}(e^{\theta} - 1)^{1/2}\,d\theta$

59. $\int_{1}^{e} \frac{1}{x}(1 + 7\ln x)^{-1/3}\,dx$

60. $\int_{e}^{e^2} \frac{1}{x\sqrt{\ln x}}\,dx$

61. $\int_{1}^{3} \frac{(\ln(v + 1))^2}{v + 1}\,dv$

62. $\int_{2}^{4} (1 + \ln t)t\ln t\,dt$

63. $\int_{1}^{8} \frac{\log_4 \theta}{\theta}\,d\theta$

64. $\int_{1}^{e} \frac{8\ln 3 \log_3 \theta}{\theta}\,d\theta$

65. $\int_{-3/4}^{3/4} \frac{6\,dx}{\sqrt{9 - 4x^2}}$

66. $\int_{-1/5}^{1/5} \frac{6\,dx}{\sqrt{4 - 25x^2}}$

67. $\int_{-2}^{2} \frac{3\,dt}{4 + 3t^2}$

68. $\int_{\sqrt{3}}^{3} \frac{dt}{3 + t^2}$

69. $\int \frac{dy}{y\sqrt{4y^2 - 1}}$

70. $\int \frac{24\,dy}{y\sqrt{y^2 - 16}}$

71. $\int_{\sqrt{2}/3}^{2/3} \frac{dy}{|y|\sqrt{9y^2 - 1}}$

72. $\int_{-2/\sqrt{5}}^{-\sqrt{6}/\sqrt{5}} \frac{dy}{|y|\sqrt{5y^2 - 3}}$

73. $\int \frac{dx}{\sqrt{-2x - x^2}}$

74. $\int \frac{dx}{\sqrt{-x^2 + 4x - 1}}$

75. $\int_{-2}^{-1} \frac{2\,dv}{v^2 + 4v + 5}$

76. $\int_{-1}^{1} \frac{3\,dv}{4v^2 + 4v + 4}$

77. $\int \frac{dt}{(t + 1)\sqrt{t^2 + 2t - 8}}$

78. $\int \frac{dt}{(3t + 1)\sqrt{9t^2 + 6t}}$

Solving Equations with Logarithmic or Exponential Terms

In Exercises 79–84, solve for y.

79. $3^y = 2^{y+1}$

80. $4^{-y} = 3^{y+2}$

81. $9e^{2y} = x^2$

82. $3^y = 3\ln x$

83. $\ln(y - 1) = x + \ln y$

84. $\ln(10\ln y) = \ln 5x$

Evaluating Limits

Find the limits in Exercises 85–96.

85. $\lim_{x\to 0} \frac{10^x - 1}{x}$

86. $\lim_{\theta\to 0} \frac{3^{\theta} - 1}{\theta}$

87. $\lim_{x\to 0} \frac{2^{\sin x} - 1}{e^x - 1}$

88. $\lim_{x\to 0} \frac{2^{-\sin x} - 1}{e^x - 1}$

89. $\lim_{x\to 0} \frac{5 - 5\cos x}{e^x - x - 1}$

90. $\lim_{x\to 0} \frac{4 - 4e^x}{xe^x}$

91. $\lim_{t\to 0^+} \frac{t - \ln(1 + 2t)}{t^2}$

92. $\lim_{x\to 4} \frac{\sin^2(\pi x)}{e^{x-4} + 3 - x}$

93. $\lim_{t\to 0^+} \left(\frac{e^t}{t} - \frac{1}{t}\right)$

94. $\lim_{y\to 0^+} e^{-1/y}\ln y$

95. $\lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^x$

96. $\lim_{x\to 0^+} \left(1 + \frac{3}{x}\right)^x$

Comparing Growth Rates of Functions

97. Does f grow faster, slower, or at the same rate as g as $x \to \infty$? Give reasons for your answers.

  a. $f(x) = \log_2 x$, $g(x) = \log_3 x$

  b. $f(x) = x$, $g(x) = x + \frac{1}{x}$

  c. $f(x) = x/100$, $g(x) = xe^{-x}$

  d. $f(x) = x$, $g(x) = \tan^{-1} x$

  e. $f(x) = \csc^{-1} x$, $g(x) = 1/x$

  f. $f(x) = \sinh x$, $g(x) = e^x$

98. Does f grow faster, slower, or at the same rate as g as $x \to \infty$? Give reasons for your answers.

  a. $f(x) = 3^{-x}$, $g(x) = 2^{-x}$

  b. $f(x) = \ln 2x$, $g(x) = \ln x^2$

  c. $f(x) = 10x^3 + 2x^2$, $g(x) = e^x$

  d. $f(x) = \tan^{-1}(1/x)$, $g(x) = 1/x$

  e. $f(x) = \sin^{-1}(1/x)$, $g(x) = 1/x^2$

  f. $f(x) = \operatorname{sech} x$, $g(x) = e^{-x}$

99. True, or false? Give reasons for your answers.

a. $\frac{1}{x^2} + \frac{1}{x^4} = O\left(\frac{1}{x^2}\right)$ **b.** $\frac{1}{x^2} + \frac{1}{x^4} = O\left(\frac{1}{x^4}\right)$

c. $x = o(x + \ln x)$ **d.** $\ln(\ln x) = o(\ln x)$

e. $\tan^{-1} x = O(1)$ **f.** $\cosh x = O(e^x)$

100. True, or false? Give reasons for your answers.

a. $\frac{1}{x^4} = O\left(\frac{1}{x^2} + \frac{1}{x^4}\right)$ **b.** $\frac{1}{x^4} = o\left(\frac{1}{x^2} + \frac{1}{x^4}\right)$

c. $\ln x = o(x + 1)$ **d.** $\ln 2x = O(\ln x)$

e. $\sec^{-1} x = O(1)$ **f.** $\sinh x = O(e^x)$

Theory and Applications

101. The function $f(x) = e^x + x$, being differentiable and one-to-one, has a differentiable inverse $f^{-1}(x)$. Find the value of df^{-1}/dx at the point $f(\ln 2)$.

102. Find the inverse of the function $f(x) = 1 + (1/x), x \neq 0$. Then show that $f^{-1}(f(x)) = f(f^{-1}(x)) = x$ and that

$$\left.\frac{df^{-1}}{dx}\right|_{f(x)} = \frac{1}{f'(x)}.$$

In Exercises 103 and 104, find the absolute maximum and minimum values of each function on the given interval.

103. $y = x \ln 2x - x, \quad \left[\frac{1}{2e}, \frac{e}{2}\right]$

104. $y = 10x(2 - \ln x), \quad (0, e^2]$

105. Area Find the area between the curve $y = 2(\ln x)/x$ and the x-axis from $x = 1$ to $x = e$.

106. Area

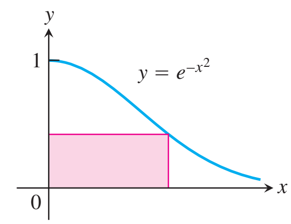
a. Show that the area between the curve $y = 1/x$ and the x-axis from $x = 10$ to $x = 20$ is the same as the area between the curve and the x-axis from $x = 1$ to $x = 2$.

b. Show that the area between the curve $y = 1/x$ and the x-axis from ka to kb is the same as the area between the curve and the x-axis from $x = a$ to $x = b$ $(0 < a < b, k > 0)$.

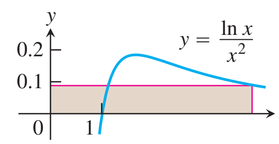
107. A particle is traveling upward and to the right along the curve $y = \ln x$. Its x-coordinate is increasing at the rate $(dx/dt) = \sqrt{x}$ m/sec. At what rate is the y-coordinate changing at the point $(e^2, 2)$?

108. A girl is sliding down a slide shaped like the curve $y = 9e^{-x/3}$. Her y-coordinate is changing at the rate $dy/dt = (-1/4)\sqrt{9 - y}$ ft/sec. At approximately what rate is her x-coordinate changing when she reaches the bottom of the slide at $x = 9$ ft? (Take e^3 to be 20 and round your answer to the nearest ft/sec.)

109. The rectangle shown here has one side on the positive y-axis, one side on the positive x-axis, and its upper right-hand vertex on the curve $y = e^{-x^2}$. What dimensions give the rectangle its largest area, and what is that area?



110. The rectangle shown here has one side on the positive y-axis, one side on the positive x-axis, and its upper right-hand vertex on the curve $y = (\ln x)/x^2$. What dimensions give the rectangle its largest area, and what is that area?



111. The functions $f(x) = \ln 5x$ and $g(x) = \ln 3x$ differ by a constant. What constant? Give reasons for your answer.

112. a. If $(\ln x)/x = (\ln 2)/2$, must $x = 2$?

b. If $(\ln x)/x = -2 \ln 2$, must $x = 1/2$?

Give reasons for your answers.

113. The quotient $(\log_4 x)/(\log_2 x)$ has a constant value. What value? Give reasons for your answer.

T **114.** $\log_x(2)$ **vs.** $\log_2(x)$ How does $f(x) = \log_x(2)$ compare with $g(x) = \log_2(x)$? Here is one way to find out.

a. Use the equation $\log_a b = (\ln b)/(\ln a)$ to express $f(x)$ and $g(x)$ in terms of natural logarithms.

b. Graph f and g together. Comment on the behavior of f in relation to the signs and values of g.

T **115.** Graph the following functions and use what you see to locate and estimate the extreme values, identify the coordinates of the inflection points, and identify the intervals on which the graphs are concave up and concave down. Then confirm your estimates by working with the functions' derivatives.

a. $y = (\ln x)/\sqrt{x}$ **b.** $y = e^{-x^2}$ **c.** $y = (1 + x)e^{-x}$

T **116.** Graph $f(x) = x \ln x$. Does the function appear to have an absolute minimum value? Confirm your answer with calculus.

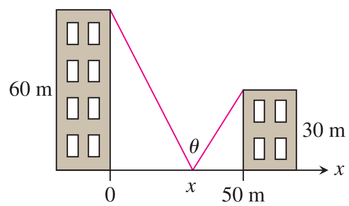
117. What is the age of a sample of charcoal in which 90% of the carbon-14 originally present has decayed?

118. Cooling a pie A deep-dish apple pie, whose internal temperature was 220°F when removed from the oven, was set out on a breezy 40°F porch to cool. Fifteen minutes later, the pie's internal temperature was 180°F. How long did it take the pie to cool from there to 70°F?

119. Locating a solar station You are under contract to build a solar station at ground level on the east–west line between the two buildings shown here. How far from the taller building should you place the station to maximize the number of hours it will be in the sun on a day when the sun passes directly overhead? Begin by observing that

$$\theta = \pi - \cot^{-1}\frac{x}{60} - \cot^{-1}\frac{50 - x}{30}.$$

Then find the value of x that maximizes θ.



120. A round underwater transmission cable consists of a core of copper wires surrounded by nonconducting insulation. If x denotes the ratio of the radius of the core to the thickness of the insulation, it is known that the speed of the transmission signal is given by the equation $v = x^2 \ln(1/x)$. If the radius of the core is 1 cm, what insulation thickness h will allow the greatest transmission speed?