EXERCISES 8.4

Products of Powers of Sines and Cosines

Evaluate the integrals in Exercises 1–14.

1. $\displaystyle\int_0^{\pi/2} \sin^5 x\, dx$

2. $\displaystyle\int_0^{\pi} \sin^5 \frac{x}{2}\, dx$

3. $\displaystyle\int_{-\pi/2}^{\pi/2} \cos^3 x\, dx$

4. $\displaystyle\int_0^{\pi/6} 3\cos^5 3x\, dx$

5. $\displaystyle\int_0^{\pi/2} \sin^7 y\, dy$

6. $\displaystyle\int_0^{\pi/2} 7\cos^7 t\, dt$

Exercises

7. $\int_0^{\pi} 8 \sin^4 x \, dx$

8. $\int_0^{1} 8 \cos^4 2\pi x \, dx$

9. $\int_{-\pi/4}^{\pi/4} 16 \sin^2 x \cos^2 x \, dx$

10. $\int_0^{\pi} 8 \sin^4 y \cos^2 y \, dy$

11. $\int_0^{\pi/2} 35 \sin^4 x \cos^3 x \, dx$

12. $\int_0^{\pi} \sin 2x \cos^2 2x \, dx$

13. $\int_0^{\pi/4} 8 \cos^3 2\theta \sin 2\theta \, d\theta$

14. $\int_0^{\pi/2} \sin^2 2\theta \cos^3 2\theta \, d\theta$

Integrals with Square Roots

Exercises

Evaluate the integrals in Exercises 15–22.

15. $\int_0^{2\pi} \sqrt{\frac{1 - \cos x}{2}} \, dx$

16. $\int_0^{\pi} \sqrt{1 - \cos 2x} \, dx$

17. $\int_0^{\pi} \sqrt{1 - \sin^2 t} \, dt$

18. $\int_0^{\pi} \sqrt{1 - \cos^2 \theta} \, d\theta$

19. $\int_{-\pi/4}^{\pi/4} \sqrt{1 + \tan^2 x} \, dx$

20. $\int_{-\pi/4}^{\pi/4} \sqrt{\sec^2 x - 1} \, dx$

21. $\int_0^{\pi/2} \theta \sqrt{1 - \cos 2\theta} \, d\theta$

22. $\int_{-\pi}^{\pi} (1 - \cos^2 t)^{3/2} \, dt$

Powers of Tan *x* and Sec *x*

Exercises

Evaluate the integrals in Exercises 23–32.

23. $\int_{-\pi/3}^{0} 2 \sec^3 x \, dx$

24. $\int e^x \sec^3 e^x \, dx$

25. $\int_0^{\pi/4} \sec^4 \theta \, d\theta$

26. $\int_0^{\pi/12} 3 \sec^4 3x \, dx$

27. $\int_{\pi/4}^{\pi/2} \csc^4 \theta \, d\theta$

28. $\int_{\pi/2}^{\pi} 3 \csc^4 \frac{\theta}{2} \, d\theta$

29. $\int_0^{\pi/4} 4 \tan^3 x \, dx$

30. $\int_{-\pi/4}^{\pi/4} 6 \tan^4 x \, dx$

31. $\int_{\pi/6}^{\pi/3} \cot^3 x \, dx$

32. $\int_{\pi/4}^{\pi/2} 8 \cot^4 t \, dt$

Products of Sines and Cosines

Exercises

Evaluate the integrals in Exercises 33–38.

33. $\int_{-\pi}^{0} \sin 3x \cos 2x \, dx$

34. $\int_0^{\pi/2} \sin 2x \cos 3x \, dx$

35. $\int_{-\pi}^{\pi} \sin 3x \sin 3x \, dx$

36. $\int_0^{\pi/2} \sin x \cos x \, dx$

37. $\int_0^{\pi} \cos 3x \cos 4x \, dx$

38. $\int_{-\pi/2}^{\pi/2} \cos x \cos 7x \, dx$

Exercises

Theory and Examples

39. Surface area Find the area of the surface generated by revolving the arc

$$x = t^{2/3}, \quad y = t^2/2, \quad 0 \le t \le 2,$$

about the x-axis.

40. Arc length Find the length of the curve

$$y = \ln(\cos x), \quad 0 \le x \le \pi/3.$$

41. Arc length Find the length of the curve

$$y = \ln(\sec x), \quad 0 \le x \le \pi/4.$$

42. Center of gravity Find the center of gravity of the region bounded by the x-axis, the curve $y = \sec x$, and the lines $x = -\pi/4$, $x = \pi/4$.

43. Volume Find the volume generated by revolving one arch of the curve $y = \sin x$ about the x-axis.

44. Area Find the area between the x-axis and the curve $y = \sqrt{1 + \cos 4x}$, $0 \le x \le \pi$.

45. Orthogonal functions Two functions f and g are said to be **orthogonal** on an interval $a \le x \le b$ if $\int_a^b f(x)g(x) \, dx = 0$.

a. Prove that $\sin mx$ and $\sin nx$ are orthogonal on any interval of length 2π provided m and n are integers such that $m^2 \ne n^2$.

b. Prove the same for $\cos mx$ and $\cos nx$.

c. Prove the same for $\sin mx$ and $\cos nx$ even if $m = n$.

46. Fourier series A finite Fourier series is given by the sum

$$f(x) = \sum_{n=1}^{N} a_n \sin nx$$
$$= a_1 \sin x + a_2 \sin 2x + \cdots + a_N \sin Nx$$

Show that the mth coefficient a_m is given by the formula

$$a_m = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin mx \, dx.$$