Chapter 8 Practice Exercises

Integration Using Substitutions

Evaluate the integrals in Exercises 1–82. To transform each integral into a recognizable basic form, it may be necessary to use one or more of the techniques of algebraic substitution, completing the square, separating fractions, long division, or trigonometric substitution.

1. $\int x\sqrt{4x^2 - 9}\, dx$
2. $\int 6x\sqrt{3x^2 + 5}\, dx$
3. $\int x(2x + 1)^{1/2}\, dx$
4. $\int x(1 - x)^{-1/2}\, dx$
5. $\int \frac{x\, dx}{\sqrt{8x^2 + 1}}$
6. $\int \frac{x\, dx}{\sqrt{9 - 4x^2}}$
7. $\int \frac{y\, dy}{25 + y^2}$
8. $\int \frac{y^3\, dy}{4 + y^4}$
9. $\int \frac{t^3\, dt}{\sqrt{9 - 4t^4}}$
10. $\int \frac{2t\, dt}{t^4 + 1}$
11. $\int z^{2/3}(z^{5/3} + 1)^{2/3}\, dz$
12. $\int z^{-1/5}(1 + z^{4/5})^{-1/2}\, dz$
13. $\int \frac{\sin 2\theta\, d\theta}{(1 - \cos 2\theta)^2}$
14. $\int \frac{\cos\theta\, d\theta}{(1 + \sin\theta)^{1/2}}$
15. $\int \frac{\sin t}{3 + 4\cos t}\, dt$
16. $\int \frac{\cos 2t}{1 + \sin 2t}\, dt$
17. $\int \sin 2x\, e^{\cos 2x}\, dx$
18. $\int \sec x \tan x\, e^{\sec x}\, dx$
19. $\int e^{\theta}\sin(e^{\theta})\cos^2(e^{\theta})\, d\theta$
20. $\int e^{\theta}\sec^2(e^{\theta})\, d\theta$
21. $\int 2^{x-1}\, dx$
22. $\int 5^{x\sqrt{2}}\, dx$
23. $\int \frac{dv}{v \ln v}$
24. $\int \frac{dv}{v(2 + \ln v)}$
25. $\int \frac{dx}{(x^2 + 1)(2 + \tan^{-1} x)}$
26. $\int \frac{\sin^{-1} x}{\sqrt{1 - x^2}}\, dx$
27. $\int \frac{2\, dx}{\sqrt{1 - 4x^2}}$
28. $\int \frac{dx}{\sqrt{49 - x^2}}$
29. $\int \frac{dt}{\sqrt{16 - 9t^2}}$
30. $\int \frac{dt}{\sqrt{9 - 4t^2}}$
31. $\int \frac{dt}{9 + t^2}$
32. $\int \frac{dt}{1 + 25t^2}$
33. $\int \frac{4\, dx}{5x\sqrt{25x^2 - 16}}$
34. $\int \frac{6\, dx}{x\sqrt{4x^2 - 9}}$
35. $\int \frac{dx}{\sqrt{4x - x^2}}$
36. $\int \frac{dx}{\sqrt{4x - x^2 - 3}}$
37. $\int \frac{dy}{y^2 - 4y + 8}$
38. $\int \frac{dt}{t^2 + 4t + 5}$
39. $\int \frac{dx}{(x - 1)\sqrt{x^2 - 2x}}$
40. $\int \frac{dv}{(v + 1)\sqrt{v^2 + 2v}}$
41. $\int \sin^2 x\, dx$
42. $\int \cos^2 3x\, dx$

43. $\int \sin^3 \frac{\theta}{2}\, d\theta$

44. $\int \sin^3 \theta \cos^2 \theta\, d\theta$

45. $\int \tan^3 2t\, dt$

46. $\int 6 \sec^4 t\, dt$

47. $\int \frac{dx}{2 \sin x \cos x}$

48. $\int \frac{2\, dx}{\cos^2 x - \sin^2 x}$

49. $\int_{\pi/4}^{\pi/2} \sqrt{\csc^2 y - 1}\, dy$

50. $\int_{\pi/4}^{3\pi/4} \sqrt{\cot^2 t + 1}\, dt$

51. $\int_0^{\pi} \sqrt{1 - \cos^2 2x}\, dx$

52. $\int_0^{2\pi} \sqrt{1 - \sin^2 \frac{x}{2}}\, dx$

53. $\int_{-\pi/2}^{\pi/2} \sqrt{1 - \cos 2t}\, dt$

54. $\int_{\pi}^{2\pi} \sqrt{1 + \cos 2t}\, dt$

55. $\int \frac{x^2}{x^2 + 4}\, dx$

56. $\int \frac{x^3}{9 + x^2}\, dx$

57. $\int \frac{4x^2 + 3}{2x - 1}\, dx$

58. $\int \frac{2x}{x - 4}\, dx$

59. $\int \frac{2y - 1}{y^2 + 4}\, dy$

60. $\int \frac{y + 4}{y^2 + 1}\, dy$

61. $\int \frac{t + 2}{\sqrt{4 - t^2}}\, dt$

62. $\int \frac{2t^2 + \sqrt{1 - t^2}}{t\sqrt{1 - t^2}}\, dt$

63. $\int \frac{\tan x\, dx}{\tan x + \sec x}$

64. $\int \frac{\cot x}{\cot x + \csc x}\, dx$

65. $\int \sec (5 - 3x)\, dx$

66. $\int x \csc (x^2 + 3)\, dx$

67. $\int \cot \left(\frac{x}{4}\right) dx$

68. $\int \tan (2x - 7)\, dx$

69. $\int x\sqrt{1 - x}\, dx$

70. $\int 3x\sqrt{2x + 1}\, dx$

71. $\int \sqrt{z^2 + 1}\, dz$

72. $\int (16 + z^2)^{-3/2}\, dz$

73. $\int \frac{dy}{\sqrt{25 + y^2}}$

74. $\int \frac{dy}{\sqrt{25 + 9y^2}}$

75. $\int \frac{dx}{x^2\sqrt{1 - x^2}}$

76. $\int \frac{x^3\, dx}{\sqrt{1 - x^2}}$

77. $\int \frac{x^2\, dx}{\sqrt{1 - x^2}}$

78. $\int \sqrt{4 - x^2}\, dx$

79. $\int \frac{dx}{\sqrt{x^2 - 9}}$

80. $\int \frac{12\, dx}{(x^2 - 1)^{3/2}}$

81. $\int \frac{\sqrt{w^2 - 1}}{w}\, dw$

82. $\int \frac{\sqrt{z^2 - 16}}{z}\, dz$

Integration by Parts

Evaluate the integrals in Exercises 83–90 using integration by parts.

83. $\int \ln (x + 1)\, dx$

84. $\int x^2 \ln x\, dx$

85. $\int \tan^{-1} 3x\, dx$

86. $\int \cos^{-1} \left(\frac{x}{2}\right) dx$

87. $\int (x + 1)^2 e^x\, dx$

88. $\int x^2 \sin (1 - x)\, dx$

89. $\int e^x \cos 2x\, dx$

90. $\int e^{-2x} \sin 3x\, dx$

Partial Fractions

Evaluate the integrals in Exercises 91–110. It may be necessary to use a substitution first.

91. $\int \frac{x\, dx}{x^2 - 3x + 2}$

92. $\int \frac{x\, dx}{x^2 + 4x + 3}$

93. $\int \frac{dx}{x(x + 1)^2}$

94. $\int \frac{x + 1}{x^2(x - 1)}\, dx$

95. $\int \frac{\sin \theta\, d\theta}{\cos^2 \theta + \cos \theta - 2}$

96. $\int \frac{\cos \theta\, d\theta}{\sin^2 \theta + \sin \theta - 6}$

97. $\int \frac{3x^2 + 4x + 4}{x^3 + x}\, dx$

98. $\int \frac{4x\, dx}{x^3 + 4x}$

99. $\int \frac{v + 3}{2v^3 - 8v}\, dv$

100. $\int \frac{(3v - 7)\, dv}{(v - 1)(v - 2)(v - 3)}$

101. $\int \frac{dt}{t^4 + 4t^2 + 3}$

102. $\int \frac{t\, dt}{t^4 - t^2 - 2}$

103. $\int \frac{x^3 + x^2}{x^2 + x - 2}\, dx$

104. $\int \frac{x^3 + 1}{x^3 - x}\, dx$

105. $\int \frac{x^3 + 4x^2}{x^2 + 4x + 3}\, dx$

106. $\int \frac{2x^3 + x^2 - 21x + 24}{x^2 + 2x - 8}\, dx$

107. $\int \frac{dx}{x\left(3\sqrt{x + 1}\right)}$

108. $\int \frac{dx}{x\left(1 + \sqrt[3]{x}\right)}$

109. $\int \frac{ds}{e^s - 1}$

110. $\int \frac{ds}{\sqrt{e^s + 1}}$

Trigonometric Substitutions

Evaluate the integrals in Exercises 111–114 **(a)** without using a trigonometric substitution, **(b)** using a trigonometric substitution.

111. $\int \frac{y\, dy}{\sqrt{16 - y^2}}$

112. $\int \frac{x\, dx}{\sqrt{4 + x^2}}$

113. $\int \frac{x\, dx}{4 - x^2}$

114. $\int \frac{t\, dt}{\sqrt{4t^2 - 1}}$

Quadratic Terms

Evaluate the integrals in Exercises 115–118.

115. $\int \frac{x\, dx}{9 - x^2}$

116. $\int \frac{dx}{x(9 - x^2)}$

117. $\int \frac{dx}{9 - x^2}$

118. $\int \frac{dx}{\sqrt{9 - x^2}}$

Trigonometric Integrals

Evaluate the integrals in Exercises 119–126.

119. $\int \sin^3 x \cos^4 x\, dx$

120. $\int \cos^5 x \sin^5 x\, dx$

121. $\int \tan^4 x \sec^2 x\, dx$

122. $\int \tan^3 x \sec^3 x\, dx$

123. $\int \sin 5\theta \cos 6\theta\, d\theta$

124. $\int \cos 3\theta \cos 3\theta\, d\theta$

125. $\int \sqrt{1 + \cos(t/2)}\, dt$

126. $\int e^t \sqrt{\tan^2 e^t + 1}\, dt$

Numerical Integration

127. According to the error-bound formula for Simpson's Rule, how many subintervals should you use to be sure of estimating the value of

$$\ln 3 = \int_1^3 \frac{1}{x}\, dx$$

by Simpson's Rule with an error of no more than 10^{-4} in absolute value? (Remember that for Simpson's Rule, the number of subintervals has to be even.)

128. A brief calculation shows that if $0 \le x \le 1$, then the second derivative of $f(x) = \sqrt{1 + x^4}$ lies between 0 and 8. Based on this, about how many subdivisions would you need to estimate the integral of f from 0 to 1 with an error no greater than 10^{-3} in absolute value using the Trapezoidal Rule?

129. A direct calculation shows that

$$\int_0^{\pi} 2 \sin^2 x\, dx = \pi.$$

How close do you come to this value by using the Trapezoidal Rule with $n = 6$? Simpson's Rule with $n = 6$? Try them and find out.

130. You are planning to use Simpson's Rule to estimate the value of the integral

$$\int_1^2 f(x)\, dx$$

with an error magnitude less than 10^{-5}. You have determined that $|f^{(4)}(x)| \le 3$ throughout the interval of integration. How many subintervals should you use to assure the required accuracy? (Remember that for Simpson's Rule the number has to be even.)

131. Mean temperature Compute the average value of the temperature function

$$f(x) = 37 \sin\left(\frac{2\pi}{365}(x - 101)\right) + 25$$

for a 365-day year. This is one way to estimate the annual mean air temperature in Fairbanks, Alaska. The National Weather Service's official figure, a numerical average of the daily normal mean air temperatures for the year, is 25.7°F, which is slightly higher than the average value of $f(x)$.

132. Heat capacity of a gas Heat capacity C_v is the amount of heat required to raise the temperature of a given mass of gas with constant volume by 1°C, measured in units of cal/deg-mol (calories per degree gram molecular weight). The heat capacity of oxygen depends on its temperature T and satisfies the formula

$$C_v = 8.27 + 10^{-5}(26T - 1.87T^2).$$

Find the average value of C_v for $20° \le T \le 675°$C and the temperature at which it is attained.

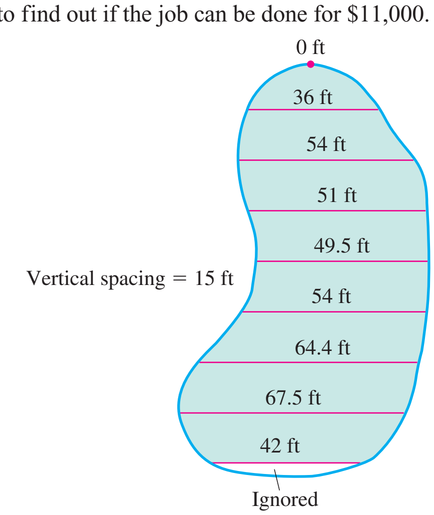
133. Fuel efficiency An automobile computer gives a digital readout of fuel consumption in gallons per hour. During a trip, a passenger recorded the fuel consumption every 5 min for a full hour of travel.

Time	Gal/h	Time	Gal/h
0	2.5	35	2.5
5	2.4	40	2.4
10	2.3	45	2.3
15	2.4	50	2.4
20	2.4	55	2.4
25	2.5	60	2.3
30	2.6		

a. Use the Trapezoidal Rule to approximate the total fuel consumption during the hour.

b. If the automobile covered 60 mi in the hour, what was its fuel efficiency (in miles per gallon) for that portion of the trip?

134. A new parking lot To meet the demand for parking, your town has allocated the area shown here. As the town engineer, you have been asked by the town council to find out if the lot can be built for \$11,000. The cost to clear the land will be \$0.10 a square foot, and the lot will cost \$2.00 a square foot to pave. Use Simpson's Rule to find out if the job can be done for \$11,000.



Improper Integrals

Evaluate the improper integrals in Exercises 135–144.

135. $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$

136. $\int_0^1 \ln x\, dx$

137. $\int_{-1}^1 \frac{dy}{y^{2/3}}$

138. $\int_{-2}^0 \frac{d\theta}{(\theta+1)^{3/5}}$

139. $\int_3^\infty \frac{2\,du}{u^2-2u}$

140. $\int_1^\infty \frac{3v-1}{4v^3-v^2}\,dv$

141. $\int_0^\infty x^2 e^{-x}\,dx$

142. $\int_{-\infty}^0 xe^{3x}\,dx$

143. $\int_{-\infty}^\infty \frac{dx}{4x^2+9}$

144. $\int_{-\infty}^\infty \frac{4dx}{x^2+16}$

Convergence or Divergence

Which of the improper integrals in Exercises 145–150 converge and which diverge?

145. $\int_6^\infty \frac{d\theta}{\sqrt{\theta^2+1}}$

146. $\int_0^\infty e^{-u}\cos u\,du$

147. $\int_1^\infty \frac{\ln z}{z}\,dz$

148. $\int_1^\infty \frac{e^{-t}}{\sqrt{t}}\,dt$

149. $\int_{-\infty}^\infty \frac{2\,dx}{e^x+e^{-x}}$

150. $\int_{-\infty}^\infty \frac{dx}{x^2(1+e^x)}$

Assorted Integrations

Evaluate the integrals in Exercises 151–218. The integrals are listed in random order.

151. $\int \frac{x\,dx}{1+\sqrt{x}}$

152. $\int \frac{x^3+2}{4-x^2}\,dx$

153. $\int \frac{dx}{x(x^2+1)^2}$

154. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

155. $\int \frac{dx}{\sqrt{-2x-x^2}}$

156. $\int \frac{(t-1)\,dt}{\sqrt{t^2-2t}}$

157. $\int \frac{du}{\sqrt{1+u^2}}$

158. $\int e^t \cos e^t\,dt$

159. $\int \frac{2-\cos x+\sin x}{\sin^2 x}\,dx$

160. $\int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$

161. $\int \frac{9\,dv}{81-v^4}$

162. $\int \frac{\cos x\,dx}{1+\sin^2 x}$

163. $\int \theta\cos(2\theta+1)\,d\theta$

164. $\int_2^\infty \frac{dx}{(x-1)^2}$

165. $\int \frac{x^3\,dx}{x^2-2x+1}$

166. $\int \frac{d\theta}{\sqrt{1+\sqrt{\theta}}}$

167. $\int \frac{2\sin\sqrt{x}\,dx}{\sqrt{x}\sec\sqrt{x}}$

168. $\int \frac{x^5\,dx}{x^4-16}$

169. $\int \frac{dy}{\sin y\cos y}$

170. $\int \frac{d\theta}{\theta^2-2\theta+4}$

171. $\int \frac{\tan x}{\cos^2 x}\,dx$

172. $\int \frac{dr}{(r+1)\sqrt{r^2+2r}}$

173. $\int \frac{(r+2)\,dr}{\sqrt{-r^2-4r}}$

174. $\int \frac{y\,dy}{4+y^4}$

175. $\int \frac{\sin 2\theta\,d\theta}{(1+\cos 2\theta)^2}$

176. $\int \frac{dx}{(x^2-1)^2}$

177. $\int_{\pi/4}^{\pi/2} \sqrt{1+\cos 4x}\,dx$

178. $\int (15)^{2x+1}\,dx$

179. $\int \frac{x\,dx}{\sqrt{2-x}}$

180. $\int \frac{\sqrt{1-v^2}}{v^2}\,dv$

181. $\int \frac{dy}{y^2-2y+2}$

182. $\int \ln\sqrt{x-1}\,dx$

183. $\int \theta^2\tan(\theta^3)\,d\theta$

184. $\int \frac{x\,dx}{\sqrt{8-2x^2-x^4}}$

185. $\int \frac{z+1}{z^2(z^2+4)}\,dz$

186. $\int x^3 e^{(x^2)}\,dx$

187. $\int \frac{t\,dt}{\sqrt{9-4t^2}}$

188. $\int_0^{\pi/10} \sqrt{1+\cos 5\theta}\,d\theta$

189. $\int \frac{\cot\theta\,d\theta}{1+\sin^2\theta}$

190. $\int \frac{\tan^{-1}x}{x^2}\,dx$

191. $\int \frac{\tan\sqrt{y}}{2\sqrt{y}}\,dy$

192. $\int \frac{e^t\,dt}{e^{2t}+3e^t+2}$

193. $\int \frac{\theta^2 d\theta}{4-\theta^2}$

194. $\int \frac{1-\cos 2x}{1+\cos 2x}\,dx$

195. $\int \frac{\cos(\sin^{-1}x)}{\sqrt{1-x^2}}\,dx$

196. $\int \frac{\cos x\,dx}{\sin^3 x-\sin x}$

197. $\int \sin\frac{x}{2}\cos\frac{x}{2}\,dx$

198. $\int \frac{x^2-x+2}{(x^2+2)^2}\,dx$

199. $\int \frac{e^t\,dt}{1+e^t}$

200. $\int \tan^3 t\,dt$

201. $\int_1^\infty \frac{\ln y}{y^3}\,dy$

202. $\int \frac{3+\sec^2 x+\sin x}{\tan x}\,dx$

203. $\int \frac{\cot v\,dv}{\ln\sin v}$

204. $\int \frac{dx}{(2x-1)\sqrt{x^2-x}}$

205. $\int e^{\ln\sqrt{x}}\,dx$

206. $\int e^\theta\sqrt{3+4e^\theta}\,d\theta$

207. $\int \frac{\sin 5t\,dt}{1+(\cos 5t)^2}$

208. $\int \frac{dv}{\sqrt{e^{2v}-1}}$

209. $\int (27)^{3\theta+1}\,d\theta$

210. $\int x^5\sin x\,dx$

211. $\int \frac{dr}{1+\sqrt{r}}$

212. $\int \frac{4x^3-20x}{x^4-10x^2+9}\,dx$

213. $\int \frac{8\,dy}{y^3(y+2)}$

214. $\int \frac{(t+1)\,dt}{(t^2+2t)^{2/3}}$

215. $\int \frac{8\,dm}{m\sqrt{49m^2-4}}$

216. $\displaystyle\int \frac{dt}{t(1 + \ln t)\sqrt{(\ln t)(2 + \ln t)}}$

217. $\displaystyle\int_0^1 3(x - 1)^2\left(\int_0^x \sqrt{1 + (t - 1)^4}\, dt\right) dx$

218. $\displaystyle\int_2^\infty \frac{4v^3 + v - 1}{v^2(v - 1)(v^2 + 1)}\, dv$

219. Suppose for a certain function f it is known that

$$f'(x) = \frac{\cos x}{x}, \quad f(\pi \;\; 2) = a, \quad \text{and} \quad f(3\pi \;\; 2) = b.$$

Use integration by parts to evaluate

$$\int_{\pi/2}^{3\pi/2} f(x)\, dx.$$

220. Find a positive number a satisfying

$$\int_0^a \frac{dx}{1 + x^2} = \int_a^\infty \frac{dx}{1 + x^2}.$$