EXERCISES 9.1

Verifying Solutions

In Exercises 1 and 2, show that each function $y = f(x)$ is a solution of the accompanying differential equation.

1. $2y' + 3y = e^{-x}$

a. $y = e^{-x}$ **b.** $y = e^{-x} + e^{-(3/2)x}$

c. $y = e^{-x} + Ce^{-(3/2)x}$

2. $y' = y^2$

a. $y = -\dfrac{1}{x}$ **b.** $y = -\dfrac{1}{x+3}$ **c.** $y = -\dfrac{1}{x+C}$

In Exercises 3 and 4, show that the function $y = f(x)$ is a solution of the given differential equation.

3. $y = \dfrac{1}{x}\displaystyle\int_1^x \frac{e^t}{t}\,dt, \quad x^2y' + xy = e^x$

4. $y = \dfrac{1}{\sqrt{1+x^4}}\displaystyle\int_1^x \sqrt{1+t^4}\,dt, \quad y' + \frac{2x^3}{1+x^4}y = 1$

In Exercises 5–8, show that each function is a solution of the given initial value problem.

	Differential equation	Initial condition	Solution candidate
5.	$y' + y = \dfrac{2}{1+4e^{2x}}$	$y(-\ln 2) = \dfrac{\pi}{2}$	$y = e^{-x}\tan^{-1}(2e^x)$
6.	$y' = e^{-x^2} - 2xy$	$y(2) = 0$	$y = (x-2)e^{-x^2}$
7.	$xy' + y = -\sin x,$ $x > 0$	$y\left(\dfrac{\pi}{2}\right) = 0$	$y = \dfrac{\cos x}{x}$
8.	$x^2y' = xy - y^2,$ $x > 1$	$y(e) = e$	$y = \dfrac{x}{\ln x}$

Separable Equations

Solve the differential equation in Exercises 9–18.

9. $2\sqrt{xy}\,\dfrac{dy}{dx} = 1, \quad x, y > 0$

10. $\dfrac{dy}{dx} = x^2\sqrt{y}, \quad y > 0$

11. $\dfrac{dy}{dx} = e^{x-y}$

12. $\dfrac{dy}{dx} = 3x^2e^{-y}$

13. $\dfrac{dy}{dx} = \sqrt{y}\cos^2\sqrt{y}$

14. $\sqrt{2xy}\,\dfrac{dy}{dx} = 1$

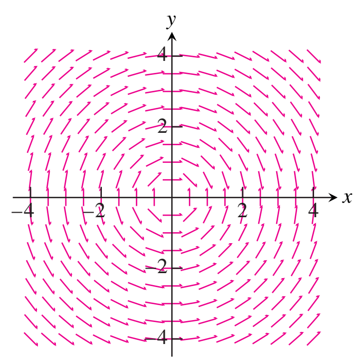
15. $\sqrt{x}\,\dfrac{dy}{dx} = e^{y+\sqrt{x}}, \quad x > 0$

16. $(\sec x)\dfrac{dy}{dx} = e^{y+\sin x}$

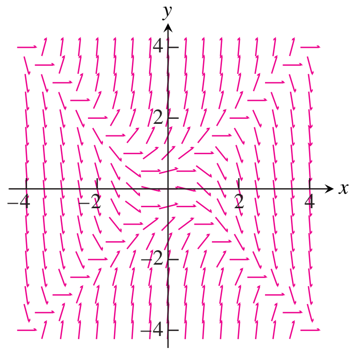
17. $\dfrac{dy}{dx} = 2x\sqrt{1-y^2}, \quad -1 < y < 1$

18. $\dfrac{dy}{dx} = \dfrac{e^{2x-y}}{e^{x+y}}$

In Exercises 19–22, match the differential equations with their slope fields, graphed here.



(a)



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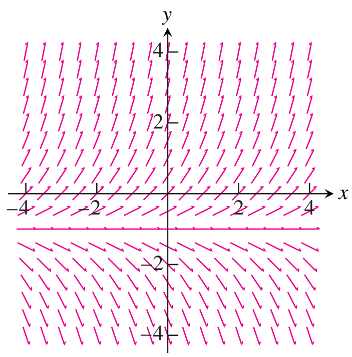

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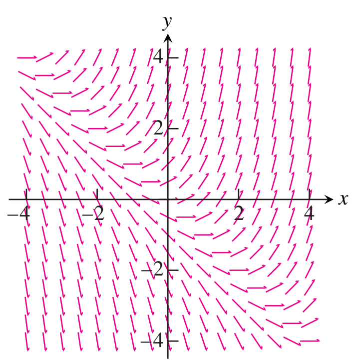


(d)

19. $y' = x + y$

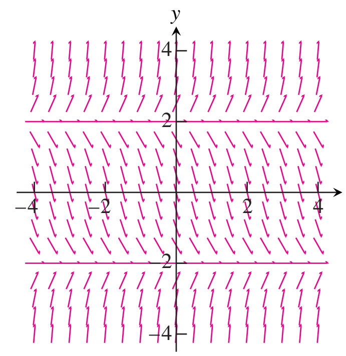
20. $y' = y + 1$

21. $y' = -\dfrac{x}{y}$

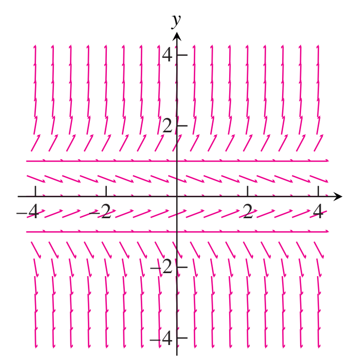
22. $y' = y^2 - x^2$

In Exercises 23 and 24, copy the slope fields and sketch in some of the solution curves.

23. $y' = (y + 2)(y - 2)$



24. $y' = y(y + 1)(y - 1)$



COMPUTER EXPLORATIONS

Slope Fields and Solution Curves

In Exercises 25–30, obtain a slope field and add to it graphs of the solution curves passing through the given points.

25. $y' = y$ with

a. $(0, 1)$ **b.** $(0, 2)$ **c.** $(0, -1)$

26. $y' = 2(y - 4)$ with

a. $(0, 1)$ **b.** $(0, 4)$ **c.** $(0, 5)$

27. $y' = y(x + y)$ with

a. $(0, 1)$ **b.** $(0, -2)$ **c.** $(0, 1/4)$ **d.** $(-1, -1)$

28. $y' = y^2$ with

a. $(0, 1)$ **b.** $(0, 2)$ **c.** $(0, -1)$ **d.** $(0, 0)$

29. $y' = (y - 1)(x + 2)$ with

a. $(0, -1)$ **b.** $(0, 1)$ **c.** $(0, 3)$ **d.** $(1, -1)$

30. $y' = \dfrac{xy}{x^2 + 4}$ with

a. $(0, 2)$ **b.** $(0, -6)$ **c.** $\left(-2\sqrt{3}, -4\right)$

In Exercises 31 and 32, obtain a slope field and graph the particular solution over the specified interval. Use your CAS DE solver to find the general solution of the differential equation.

31. **A logistic equation** $y' = y(2 - y), \quad y(0) = 1/2;$
$0 \le x \le 4, \quad 0 \le y \le 3$

32. $y' = (\sin x)(\sin y), \quad y(0) = 2; \quad -6 \le x \le 6, \quad -6 \le y \le 6$

Exercises 33 and 34 have no explicit solution in terms of elementary functions. Use a CAS to explore graphically each of the differential equations.

33. $y' = \cos(2x - y), \quad y(0) = 2; \quad 0 \le x \le 5, \quad 0 \le y \le 5;$
$y(2)$

34. **A Gompertz equation** $y' = y(1/2 - \ln y), \quad y(0) = 1/3;$
$0 \le x \le 4, \quad 0 \le y \le 3; \quad y(3)$

35. Use a CAS to find the solutions of $y' + y = f(x)$ subject to the initial condition $y(0) = 0$, if $f(x)$ is

a. $2x$ **b.** $\sin 2x$ **c.** $3e^{x/2}$ **d.** $2e^{-x/2}\cos 2x$.

Graph all four solutions over the interval $-2 \le x \le 6$ to compare the results.

36. **a.** Use a CAS to plot the slope field of the differential equation

$$y' = \frac{3x^2 + 4x + 2}{2(y - 1)}$$

over the region $-3 \le x \le 3$ and $-3 \le y \le 3$.

b. Separate the variables and use a CAS integrator to find the general solution in implicit form.

c. Using a CAS implicit function grapher, plot solution curves for the arbitrary constant values $C = -6, -4, -2, 0, 2, 4, 6$.

d. Find and graph the solution that satisfies the initial condition $y(0) = -1$.