

## EXERCISES 9.3

## Calculating Euler Approximations

In Exercises 1–6, use Euler's method to calculate the first three approximations to the given initial value problem for the specified increment size. Calculate the exact solution and investigate the accuracy of your approximations. Round your results to four decimal places.

1.  $y' = 1 - \frac{y}{x}$ ,  $y(2) = -1$ ,  $dx = 0.5$

2.  $y' = x(1 - y)$ ,  $y(1) = 0$ ,  $dx = 0.2$

3.  $y' = 2xy + 2y$ ,  $y(0) = 3$ ,  $dx = 0.2$

4.  $y' = y^2(1 + 2x)$ ,  $y(-1) = 1$ ,  $dx = 0.5$

**T** 5.  $y' = 2xe^{x^2}$ ,  $y(0) = 2$ ,  $dx = 0.1$

**T** 6.  $y' = y + e^x - 2$ ,  $y(0) = 2$ ,  $dx = 0.5$

7. Use the Euler method with  $dx = 0.2$  to estimate  $y(1)$  if  $y' = y$  and  $y(0) = 1$ . What is the exact value of  $y(1)$ ?

8. Use the Euler method with  $dx = 0.2$  to estimate  $y(2)$  if  $y' = y/x$  and  $y(1) = 2$ . What is the exact value of  $y(2)$ ?

**T** 9. Use the Euler method with  $dx = 0.5$  to estimate  $y(5)$  if  $y' = y^2/\sqrt{x}$  and  $y(1) = -1$ . What is the exact value of  $y(5)$ ?

**T** 10. Use the Euler method with  $dx = 1/3$  to estimate  $y(2)$  if  $y' = y - e^{2x}$  and  $y(0) = 1$ . What is the exact value of  $y(2)$ ?

## Improved Euler's Method

In Exercises 11 and 12, use the improved Euler's method to calculate the first three approximations to the given initial value problem. Compare the approximations with the values of the exact solution.

11.  $y' = 2y(x + 1)$ ,  $y(0) = 3$ ,  $dx = 0.2$

(See Exercise 3 for the exact solution.)

12.  $y' = x(1 - y)$ ,  $y(1) = 0$ ,  $dx = 0.2$

(See Exercise 2 for the exact solution.)

## COMPUTER EXPLORATIONS

## Euler's Method

In Exercises 13–16, use Euler's method with the specified step size to estimate the value of the solution at the given point  $x^*$ . Find the value of the exact solution at  $x^*$ .

13.  $y' = 2xe^{x^2}$ ,  $y(0) = 2$ ,  $dx = 0.1$ ,  $x^* = 1$

14.  $y' = y + e^x - 2$ ,  $y(0) = 2$ ,  $dx = 0.5$ ,  $x^* = 2$

15.  $y' = \sqrt{x}/y$ ,  $y > 0$ ,  $y(0) = 1$ ,  $dx = 0.1$ ,  $x^* = 1$

16.  $y' = 1 + y^2$ ,  $y(0) = 0$ ,  $dx = 0.1$ ,  $x^* = 1$

In Exercises 17 and 18, (a) find the exact solution of the initial value problem. Then compare the accuracy of the approximation with  $y(x^*)$  using Euler's method starting at  $x_0$  with step size (b) 0.2, (c) 0.1, and (d) 0.05.

17.  $y' = 2y^2(x - 1)$ ,  $y(2) = -1/2$ ,  $x_0 = 2$ ,  $x^* = 3$

18.  $y' = y - 1$ ,  $y(0) = 3$ ,  $x_0 = 0$ ,  $x^* = 1$

## Improved Euler's Method

In Exercises 19 and 20, compare the accuracy of the approximation with  $y(x^*)$  using the improved Euler's method starting at  $x_0$  with step size

a. 0.2      b. 0.1      c. 0.05

d. Describe what happens to the error as the step size decreases.

19.  $y' = 2y^2(x - 1)$ ,  $y(2) = -1/2$ ,  $x_0 = 2$ ,  $x^* = 3$

(See Exercise 17 for the exact solution.)

20.  $y' = y - 1$ ,  $y(0) = 3$ ,  $x_0 = 0$ ,  $x^* = 1$

(See Exercise 18 for the exact solution.)

### Exploring Differential Equations Graphically

Use a CAS to explore graphically each of the differential equations in Exercises 21–24. Perform the following steps to help with your explorations.

- a. Plot a slope field for the differential equation in the given  $xy$ -window.
  - b. Find the general solution of the differential equation using your CAS DE solver.
  - c. Graph the solutions for the values of the arbitrary constant  $C = -2, -1, 0, 1, 2$  superimposed on your slope field plot.
  - d. Find and graph the solution that satisfies the specified initial condition over the interval  $[0, b]$ .
  - e. Find the Euler numerical approximation to the solution of the initial value problem with 4 subintervals of the  $x$ -interval and plot the Euler approximation superimposed on the graph produced in part (d).
  - f. Repeat part (e) for 8, 16, and 32 subintervals. Plot these three Euler approximations superimposed on the graph from part (e).
  - g. Find the error  $(y(\text{exact}) - y(\text{Euler}))$  at the specified point  $x = b$  for each of your four Euler approximations. Discuss the improvement in the percentage error.
21.  $y' = x + y, \quad y(0) = -7/10; \quad -4 \leq x \leq 4, \quad -4 \leq y \leq 4; \quad b = 1$
  22.  $y' = -x/y, \quad y(0) = 2; \quad -3 \leq x \leq 3, \quad -3 \leq y \leq 3; \quad b = 2$
  23. **A logistic equation**  $y' = y(2 - y), \quad y(0) = 1/2; \quad 0 \leq x \leq 4, \quad 0 \leq y \leq 3; \quad b = 3$
  24.  $y' = (\sin x)(\sin y), \quad y(0) = 2; \quad -6 \leq x \leq 6, \quad -6 \leq y \leq 6; \quad b = 3\pi/2$