EXERCISES 9.5

1. Coasting bicycle A 66-kg cyclist on a 7-kg bicycle starts coasting on level ground at 9 m/sec. The k in Equation (1) is about 3.9 kg/sec.

a. About how far will the cyclist coast before reaching a complete stop?

b. How long will it take the cyclist's speed to drop to 1 m/sec?

2. Coasting battleship Suppose that an Iowa class battleship has mass around 51,000 metric tons (51,000,000 kg) and a k value in Equation (1) of about 59,000 kg/sec. Assume that the ship loses power when it is moving at a speed of 9 m/sec.

a. About how far will the ship coast before it is dead in the water?

b. About how long will it take the ship's speed to drop to 1 m/sec?

3. The data in Table 9.7 were collected with a motion detector and a CBL™ by Valerie Sharritts, a mathematics teacher at St. Francis DeSales High School in Columbus, Ohio. The table shows the distance s (meters) coasted on in-line skates in t sec by her daughter Ashley when she was 10 years old. Find a model for Ashley's position given by the data in Table 9.7 in the form of Equation (2). Her initial velocity was $v_0 = 2.75$ m/sec, her mass $m = 39.92$ kg (she weighed 88 lb), and her total coasting distance was 4.91 m.

4. Coasting to a stop Table 9.8 shows the distance s (meters) coasted on in-line skates in terms of time t (seconds) by Kelly Schmitzer. Find a model for her position in the form of Equation (2). Her initial velocity was $v_0 = 0.80$ m/sec, her mass $m = 49.90$ kg (110 lb), and her total coasting distance was 1.32 m.

5. Guppy population A 2000-gal tank can support no more than 150 guppies. Six guppies are introduced into the tank. Assume that the rate of growth of the population is

$$\frac{dP}{dt} = 0.0015(150 - P)P,$$

where time t is in weeks.

TABLE 9.7 Ashley Sharritts skating data

t (sec)	s (m)	t (sec)	s (m)	t (sec)	s (m)
0	0	2.24	3.05	4.48	4.77
0.16	0.31	2.40	3.22	4.64	4.82
0.32	0.57	2.56	3.38	4.80	4.84
0.48	0.80	2.72	3.52	4.96	4.86
0.64	1.05	2.88	3.67	5.12	4.88
0.80	1.28	3.04	3.82	5.28	4.89
0.96	1.50	3.20	3.96	5.44	4.90
1.12	1.72	3.36	4.08	5.60	4.90
1.28	1.93	3.52	4.18	5.76	4.91
1.44	2.09	3.68	4.31	5.92	4.90
1.60	2.30	3.84	4.41	6.08	4.91
1.76	2.53	4.00	4.52	6.24	4.90
1.92	2.73	4.16	4.63	6.40	4.91
2.08	2.89	4.32	4.69	6.56	4.91

a. Find a formula for the guppy population in terms of t.

b. How long will it take for the guppy population to be 100? 125?

6. Gorilla population A certain wild animal preserve can support no more than 250 lowland gorillas. Twenty-eight gorillas were known to be in the preserve in 1970. Assume that the rate of growth of the population is

$$\frac{dP}{dt} = 0.0004(250 - P)P,$$

where time t is in years.

TABLE 9.8 Kelly Schmitzer skating data

t (sec)	s (m)	t (sec)	s (m)	t (sec)	s (m)
0	0	1.5	0.89	3.1	1.30
0.1	0.07	1.7	0.97	3.3	1.31
0.3	0.22	1.9	1.05	3.5	1.32
0.5	0.36	2.1	1.11	3.7	1.32
0.7	0.49	2.3	1.17	3.9	1.32
0.9	0.60	2.5	1.22	4.1	1.32
1.1	0.71	2.7	1.25	4.3	1.32
1.3	0.81	2.9	1.28	4.5	1.32

a. Find a formula for the gorilla population in terms of t.

b. How long will it take for the gorilla population to reach the carrying capacity of the preserve?

7. Pacific halibut fishery The Pacific halibut fishery has been modeled by the logistic equation

$$\frac{dy}{dt} = r(M - y)y$$

where $y(t)$ is the total weight of the halibut population in kilograms at time t (measured in years), the carrying capacity is estimated to be $M = 8 \times 10^7$ kg, and $r = 0.08875 \times 10^{-7}$ per year.

a. If $y(0) = 1.6 \times 10^7$ kg, what is the total weight of the halibut population after 1 year?

b. When will the total weight in the halibut fishery reach 4×10^7 kg?

8. Modified logistic model Suppose that the logistic differential equation in Example 2 is modified to

$$\frac{dP}{dt} = 0.001(100 - P)P - c$$

for some constant c.

a. Explain the meaning of the constant c. What values for c might be realistic for the grizzly bear population?

T **b.** Draw a direction field for the differential equation when $c = 1$. What are the equilibrium solutions (Section 9.4)?

c. Sketch several solution curves in your direction field from part (a). Describe what happens to the grizzly bear population for various initial populations.

9. Exact solutions Find the exact solutions to the following initial value problems.

a. $y' = 1 + y, \quad y(0) = 1$

b. $y' = 0.5(400 - y)y, \quad y(0) = 2$

10. Logistic differential equation Show that the solution of the differential equation

$$\frac{dP}{dt} = r(M - P)P$$

is

$$P = \frac{M}{1 + Ae^{-rMt}},$$

where A is an arbitrary constant.

11. Catastrophic solution Let k and P_0 be positive constants.

a. Solve the initial value problem?

$$\frac{dP}{dt} = kP^2, \quad P(0) = P_0$$

T **b.** Show that the graph of the solution in part (a) has a vertical asymptote at a positive value of t. What is that value of t?

12. Extinct populations Consider the population model

$$\frac{dP}{dt} = r(M - P)(P - m),$$

where $r > 0$, M is the maximum sustainable population, and m is the minimum population below which the species becomes extinct.

a. Let $m = 100$, and $M = 1200$, and assume that $m < P < M$. Show that the differential equation can be rewritten in the form

$$\left[\frac{1}{1200 - P} + \frac{1}{P - 100}\right]\frac{dP}{dt} = 1100r$$

and solve this separable equation.

b. Find the solution to part (a) that satisfies $P(0) = 300$.

c. Solve the differential equation with the restriction $m < P < M$.

Orthogonal Trajectories

In Exercises 13–18, find the orthogonal trajectories of the family of curves. Sketch several members of each family.

13. $y = mx$

14. $y = cx^2$

15. $kx^2 + y^2 = 1$

16. $2x^2 + y^2 = c^2$

17. $y = ce^{-x}$

18. $y = e^{kx}$

19. Show that the curves $2x^2 + 3y^2 = 5$ and $y^2 = x^3$ are orthogonal.

20. Find the family of solutions of the given differential equation and the family of orthogonal trajectories. Sketch both families.

a. $x\,dx + y\,dy = 0$ **b.** $x\,dy - 2y\,dx = 0$

21. Suppose a and b are positive numbers. Sketch the parabolas

$$y^2 = 4a^2 - 4ax \quad \text{and} \quad y^2 = 4b^2 + 4bx$$

in the same diagram. Show that they intersect at $\left(a - b, \pm 2\sqrt{ab}\right)$, and that each "$a$-parabola" is orthogonal to every "b-parabola."