Chapter 9 Practice Exercises

In Exercises 1–20 solve the differential equation.

1. $\frac{dy}{dx} = \sqrt{y}\cos^2\sqrt{y}$
2. $y' = \frac{3y(x+1)^2}{y-1}$
3. $yy' = \sec y^2 \sec^2 x$
4. $y\cos^2 x\, dy + \sin x\, dx = 0$
5. $y' = xe^y\sqrt{x-2}$
6. $y' = xye^{x^2}$
7. $\sec x\, dy + x\cos^2 y\, dx = 0$
8. $2x^2\, dx - 3\sqrt{y}\csc x\, dy = 0$
9. $y' = \frac{e^y}{xy}$
10. $y' = xe^{x-y}\csc y$
11. $x(x-1)\, dy - y\, dx = 0$
12. $y' = (y^2 - 1)x^{-1}$
13. $2y' - y = xe^{x/2}$
14. $\frac{y'}{2} + y = e^{-x}\sin x$
15. $xy' + 2y = 1 - x^{-1}$
16. $xy' - y = 2x\ln x$
17. $(1 + e^x)\, dy + (ye^x + e^{-x})\, dx = 0$
18. $e^{-x}\, dy + (e^{-x}y - 4x)\, dx = 0$
19. $(x + 3y^2)\, dy + y\, dx = 0$ (*Hint:* $d(xy) = y\, dx + x\, dy$)
20. $x\, dy + (3y - x^{-2}\cos x)\, dx = 0, \quad x > 0$

Initial Value Problems

In Exercises 21–30 solve the initial value problem.

21. $\frac{dy}{dx} = e^{-x-y-2}, \quad y(0) = -2$
22. $\frac{dy}{dx} = \frac{y\ln y}{1 + x^2}, \quad y(0) = e^2$
23. $(x+1)\frac{dy}{dx} + 2y = x, \quad x > -1, \quad y(0) = 1$
24. $x\frac{dy}{dx} + 2y = x^2 + 1, \quad x > 0, \quad y(1) = 1$
25. $\frac{dy}{dx} + 3x^2 y = x^2, \quad y(0) = -1$
26. $x\, dy + (y - \cos x)\, dx = 0, \quad y\left(\frac{\pi}{2}\right) = 0$
27. $x\, dy - \left(y + \sqrt{y}\right) dx = 0, \quad y(1) = 1$
28. $y^{-2}\frac{dx}{dy} = \frac{e^x}{e^{2x} + 1}, \quad y(0) = 1$
29. $xy' + (x-2)y = 3x^3e^{-x}, \quad y(1) = 0$
30. $y\, dx + (3x - xy + 2)\, dy = 0, \quad y(2) = -1, \quad y < 0$

Euler's Method

In Exercises 31 and 32, use the stated method to solve the initial value problem on the given interval starting at x_0 with $dx = 0.1$.

T 31. **Euler:** $y' = y + \cos x, \quad y(0) = 0; \quad 0 \le x \le 2; \quad x_0 = 0$

T 32. **Improved Euler:** $y' = (2 - y)(2x + 3), \quad y(-3) = 1; \quad -3 \le x \le -1; \quad x_0 = -3$

In Exercises 33 and 34, use the stated method with $dx = 0.05$ to estimate $y(c)$ where y is the solution to the given initial value problem.

T 33. **Improved Euler:**

$$c = 3; \quad \frac{dy}{dx} = \frac{x - 2y}{x + 1}, \quad y(0) = 1$$

T 34. **Euler:**

$$c = 4; \quad \frac{dy}{dx} = \frac{x^2 - 2y + 1}{x}, \quad y(1) = 1$$

In Exercises 35 and 36, use the stated method to solve the initial value problem graphically, starting at $x_0 = 0$ with

a. $dx = 0.1$. **b.** $dx = -0.1$.

T 35. **Euler:**

$$\frac{dy}{dx} = \frac{1}{e^{x+y+2}}, \quad y(0) = -2$$

T 36. **Improved Euler:**

$$\frac{dy}{dx} = -\frac{x^2 + y}{e^y + x}, \quad y(0) = 0$$

Slope Fields

In Exercises 37–40, sketch part of the equation's slope field. Then add to your sketch the solution curve that passes through the point $P(1, -1)$. Use Euler's method with $x_0 = 1$ and $dx = 0.2$ to estimate $y(2)$. Round your answers to four decimal places. Find the exact value of $y(2)$ for comparison.

37. $y' = x$ **38.** $y' = 1/x$

39. $y' = xy$ **40.** $y' = 1/y$

Autonomous Differential Equations and Phase Lines

In Exercises 41 and 42.

a. Identify the equilibrium values. Which are stable and which are unstable?

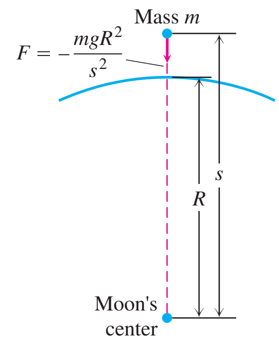
b. Construct a phase line. Identify the signs of y' and y''.

c. Sketch a representative selection of solution curves.

41. $\frac{dy}{dx} = y^2 - 1$ **42.** $\frac{dy}{dx} = y - y^2$

Applications

43. Escape velocity The gravitational attraction F exerted by an airless moon on a body of mass m at a distance s from the moon's center is given by the equation $F = -mg\,R^2 s^{-2}$, where g is the acceleration of gravity at the moon's surface and R is the moon's radius (see accompanying figure). The force F is negative because it acts in the direction of decreasing s.



a. If the body is projected vertically upward from the moon's surface with an initial velocity v_0 at time $t = 0$, use Newton's second law, $F = ma$, to show that the body's velocity at position s is given by the equation

$$v^2 = \frac{2gR^2}{s} + {v_0}^2 - 2gR.$$

Thus, the velocity remains positive as long as $v_0 \geq \sqrt{2gR}$. The velocity $v_0 = \sqrt{2gR}$ is the moon's **escape velocity**. A body projected upward with this velocity or a greater one will escape from the moon's gravitational pull.

b. Show that if $v_0 = \sqrt{2gR}$, then

$$s = R\left(1 + \frac{3v_0}{2R}t\right)^{2/3}.$$

44. Coasting to a stop Table 9.9 shows the distance s (meters) coasted on in-line skates in t sec by Johnathon Krueger. Find a model for his position in the form of Equation (2) of Section 9.5. His initial velocity was $v_0 = 0.86$ m/sec, his mass $m = 30.84$ kg (he weighed 68 lb), and his total coasting distance 0.97 m.

TABLE 9.9 Johnathon Krueger skating data

t (sec)	*s* (m)	*t* (sec)	*s* (m)	*t* (sec)	*s* (m)
0	0	0.93	0.61	1.86	0.93
0.13	0.08	1.06	0.68	2.00	0.94
0.27	0.19	1.20	0.74	2.13	0.95
0.40	0.28	1.33	0.79	2.26	0.96
0.53	0.36	1.46	0.83	2.39	0.96
0.67	0.45	1.60	0.87	2.53	0.97
0.80	0.53	1.73	0.90	2.66	0.97