

# PHYSICAL PROPERTIES OF THE RARE EARTH METALS

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**TABLE 1. Data for the Trivalent Ions of the Rare Earth Elements**

| Rare earth   | Symbol | Atomic no. | Atomic wt. <sup>a</sup> | Electronic configuration for R <sup>3+</sup> |     |   |      | Spectroscopic ground state symbol |
|--------------|--------|------------|-------------------------|--|-----|---|------|-----------------------------------|
|              |        |            |                         | No. 4f electrons                             | S   | L | J    |                                   |
| Scandium     | Sc     | 21         | 44.955910               | 0  | —   | — | —    | —                                 |
| Yttrium      | Y      | 39         | 88.90585                | 0  | —   | — | —    | —                                 |
| Lanthanum    | La     | 57         | 138.9055                | 0  | —   | — | —    | —                                 |
| Cerium       | Ce     | 58         | 140.115                 | 1  | 1/2 | 3 | 5/2  | <sup>2</sup> F <sub>5/2</sub>     |
| Praseodymium | Pr     | 59         | 140.90765               | 2  | 1   | 5 | 4    | <sup>3</sup> H <sub>4</sub>       |
| Neodymium    | Nd     | 60         | 144.24                  | 3  | 3/2 | 6 | 9/2  | <sup>4</sup> I <sub>9/2</sub>     |
| Promethium   | Pm     | 61         | (145)                   | 4  | 2   | 6 | 4    | <sup>5</sup> I <sub>4</sub>       |
| Samarium     | Sm     | 62         | 150.36                  | 5  | 5/2 | 5 | 5/2  | <sup>6</sup> H <sub>5/2</sub>     |
| Europium     | Eu     | 63         | 151.965                 | 6  | 3   | 3 | 0    | <sup>7</sup> F <sub>0</sub>       |
| Gadolinium   | Gd     | 64         | 157.25                  | 7  | 7/2 | 0 | 7/2  | <sup>8</sup> S <sub>7/2</sub>     |
| Terbium      | Tb     | 65         | 158.92534               | 8  | 3   | 3 | 6    | <sup>7</sup> F <sub>6</sub>       |
| Dysprosium   | Dy     | 66         | 162.50                  | 9  | 5/2 | 5 | 15/2 | <sup>6</sup> H <sub>15/2</sub>    |
| Holmium      | Ho     | 67         | 164.93032               | 10   | 2   | 6 | 8    | <sup>5</sup> I <sub>8</sub>       |
| Erbium       | Er     | 68         | 167.26                  | 11   | 3/2 | 6 | 15/2 | <sup>4</sup> I <sub>15/2</sub>    |
| Thulium      | Tm     | 69         | 168.93421               | 12   | 1   | 5 | 6    | <sup>3</sup> H <sub>6</sub>       |
| Ytterbium    | Yb     | 70         | 173.04                  | 13   | 1/2 | 3 | 7/2  | <sup>2</sup> F <sub>7/2</sub>     |
| Lutetium     | Lu     | 71         | 174.967                 | 14   | —   | — | —    | —                                 |

Note: For additional information, see Goldschmidt, Z.B., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978; DeLaeter, J.R., and Heumann, K.G., *J. Phys. Chem. Ref. Data*, 20, 1313, 1991; *Pure Appl. Chem.*, 66, 2423, 1994.

<sup>a</sup> 1993 standard atomic weights.

**TABLE 2. Crystallographic Data for the Rare Earth Metals at 24°C (297 K) or Below**

| Rare earth metal  | Crystal structure <sup>a</sup> | Lattice constants (Å) |                       |                       | Metallic radius CN = 12 (Å) | Atomic volume (cm <sup>3</sup> /mol) | Density (g/cm <sup>3</sup> ) |
|-------------------|--------------------------------|-----------------------|-----------------------|-----------------------|-----------------------------|--------------------------------------|------------------------------|
|                   |                                | <i>a</i> <sub>o</sub> | <i>b</i> <sub>o</sub> | <i>c</i> <sub>o</sub> |                             |                                      |                              |
| αSc               | hcp                            | 3.3088                | —                     | 5.2680                | 1.6406                      | 15.039                               | 2.989                        |
| αY                | hcp                            | 3.6482                | —                     | 5.7318                | 1.8012                      | 19.893                               | 4.469                        |
| αLa               | dhcp                           | 3.7740                | —                     | 12.171                | 1.8791                      | 22.602                               | 6.146                        |
| αCe <sup>b</sup>  | fcc                            | 4.85 <sup>b</sup>     | —                     | —                     | 1.72 <sup>b</sup>           | 17.2 <sup>b</sup>                    | 8.16 <sup>b</sup>            |
| βCe               | dhcp                           | 3.6810                | —                     | 11.857                | 1.8321                      | 20.947                               | 6.689                        |
| γCe <sup>c</sup>  | fcc                            | 5.1610                | —                     | —                     | 1.8247                      | 20.696                               | 6.770                        |
| αPr               | dhcp                           | 3.6721                | —                     | 11.8326               | 1.8279                      | 20.803                               | 6.773                        |
| αNd               | dhcp                           | 3.6582                | —                     | 11.7966               | 1.8214                      | 20.583                               | 7.008                        |
| αPm               | dhcp                           | 3.65                  | —                     | 11.65                 | 1.811                       | 20.24                                | 7.264                        |
| αSm               | rhomb <sup>d</sup>             | 3.6290 <sup>d</sup>   | —                     | 26.207                | 1.8041                      | 20.000                               | 7.520                        |
| Eu                | bcc                            | 4.5827                | —                     | —                     | 2.0418                      | 28.979                               | 5.244                        |
| αGd               | hcp                            | 3.6336                | —                     | 5.7810                | 1.8013                      | 19.903                               | 7.901                        |
| α'Tb <sup>e</sup> | ortho                          | 3.605 <sup>e</sup>    | 6.244 <sup>e</sup>    | 5.706 <sup>e</sup>    | 1.784 <sup>e</sup>          | 19.34 <sup>e</sup>                   | 8.219 <sup>e</sup>           |
| αTb               | hcp                            | 3.6055                | —                     | 5.6966                | 1.7833                      | 19.310                               | 8.230                        |
| α'Dy <sup>f</sup> | ortho                          | 3.595 <sup>f</sup>    | 6.184 <sup>f</sup>    | 5.678 <sup>f</sup>    | 1.774 <sup>f</sup>          | 19.00 <sup>f</sup>                   | 8.551 <sup>f</sup>           |
| αDy               | hcp                            | 3.5915                | —                     | 5.6501                | 1.7740                      | 19.004                               | 8.551                        |
| Ho                | hcp                            | 3.5778                | —                     | 5.6178                | 1.7661                      | 18.752                               | 8.795                        |
| Er                | hcp                            | 3.5592                | —                     | 5.5850                | 1.7566                      | 18.449                               | 9.066                        |
| Tm                | hcp                            | 3.5375                | —                     | 5.5540                | 1.7462                      | 18.124                               | 9.321                        |
| αYb <sup>g</sup>  | hcp                            | 3.8799 <sup>g</sup>   | —                     | 6.3859 <sup>g</sup>   | 1.9451 <sup>g</sup>         | 25.067 <sup>g</sup>                  | 6.903 <sup>g</sup>           |
| βYb               | fcc                            | 5.4848                | —                     | —                     | 1.9392                      | 24.841                               | 6.966                        |
| Lu                | hcp                            | 3.5052                | —                     | 5.5494                | 1.7349                      | 17.779                               | 9.841                        |

Note: For additional information, see Gschneidner, K.A., Jr. and Calderwood, F.W., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 8, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1986; Gschneidner, K.A., Jr., Pecharsky, V.K., Cho, Jaephil and Martin, S.W., *Scripta Mater.*, 1996, to be published.

<sup>a</sup> hcp = hexagonal close-packed; P6<sub>3</sub>/mmc, hP2, A3, Mg-type; dhcp = double-c hexagonal close-packed; P6<sub>3</sub>/mmc, hP4, A3', αLa-type; fcc = face-centered cubic; Fm<sup>3</sup>m, cF4, A1, Cu-type; rhomb = rhombohedral; R<sup>3</sup>m, hR3, αSm-type; bcc = body-centered cubic; Im<sup>3</sup>m, cl2, A2, W-type; ortho = orthorhombic; Cmcm, oC4, α'Dy-type.

<sup>b</sup> At 77 K (-196°C).

<sup>c</sup> Equilibrium room temperature (standard state) phase.

<sup>d</sup> Rhombohedral is the primitive cell. Lattice parameters given are for the nonprimitive hexagonal cell.

<sup>e</sup> At 220 K (-53°C).

<sup>f</sup> At 86 K (-187°C).

<sup>g</sup> At 23°C.

TABLE 3. Crystallographic Data for Rare Earth Metals at High Temperature

| Rare earth metal  | Structure | Lattice parameter ( $\text{\AA}$ ) | Temp. (°C)       | Metallic radius<br>CN = 8 ( $\text{\AA}$ ) | Metallic radius<br>CN = 12 ( $\text{\AA}$ ) | Atomic volume<br>( $\text{cm}^3/\text{mol}$ ) | Density (g/ $\text{cm}^3$ ) |
|-------------------|-----------|------------------------------------|------------------|--|---|---|-----------------------------|
| $\beta\text{Sc}$  | bcc       | 3.73 (est.)                        | 1337             | 1.62                                       | 1.66  | 15.6  | 2.88                        |
| $\beta\text{Y}$   | bcc       | 4.10 <sup>a</sup>                  | 1478             | 1.78                                       | 1.83  | 20.8  | 4.28                        |
| $\beta\text{La}$  | fcc       | 5.303                              | 325              | —  | 1.875                                       | 22.45   | 6.187                       |
| $\gamma\text{La}$ | bcc       | 4.26                               | 887              | 1.84                                       | 1.90  | 23.3  | 5.97                        |
| $\delta\text{Ce}$ | bcc       | 4.12                               | 757              | 1.78                                       | 1.84  | 21.1  | 6.65                        |
| $\beta\text{Pr}$  | bcc       | 4.13                               | 821              | 1.79                                       | 1.84  | 21.2  | 6.64                        |
| $\beta\text{Nd}$  | bcc       | 4.13                               | 883              | 1.79                                       | 1.84  | 21.2  | 6.80                        |
| $\beta\text{Pm}$  | bcc       | 4.10 (est.)                        | 890              | 1.78                                       | 1.83  | 20.8  | 6.99                        |
| $\beta\text{Sm}$  | hcp       | $a = 3.6630$<br>$c = 5.8448$       | 450 <sup>b</sup> | —  | 1.8176                                      | 20.450  | 7.353                       |
| $\gamma\text{Sm}$ | bcc       | 4.10 (est.)                        | 922              | 1.77                                       | 1.82  | 20.8  | 7.25                        |
| $\beta\text{Gd}$  | bcc       | 4.06                               | 1265             | 1.76                                       | 1.81  | 20.2  | 7.80                        |
| $\beta\text{Tb}$  | bcc       | 4.07 <sup>a</sup>                  | 1289             | 1.76                                       | 1.81  | 20.3  | 7.82                        |
| $\beta\text{Dy}$  | bcc       | 4.03 <sup>a</sup>                  | 1381             | 1.75                                       | 1.80  | 19.7  | 8.23                        |
| $\gamma\text{Yb}$ | bcc       | 4.44                               | 763 <sup>c</sup> | 1.92                                       | 1.98  | 26.4  | 6.57                        |

Note: The rare earths Eu, Ho, Er, Tm, and Lu are monomorphic. For additional information, see Gschneidner, K.A., Jr. and Calderwood, F.W., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 8, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1986, 1.

<sup>a</sup> Determined by extrapolation to 0% solute of a vs. composition data for R-Mg alloys at 24°C and corrected for thermal expansion to temperature given.

<sup>b</sup> The hcp phase was stabilized by impurities and the temperature of measurement was below the equilibrium transition temperature (see Table 4).

<sup>c</sup> The bcc phase was stabilized by impurities and the temperature of measurement was below the equilibrium transition temperature (see Table 4).

TABLE 4. High Temperature Transition Temperatures and Melting Point of Rare Earth Metals

| Rare earth metal  | Transition I ( $\alpha - \beta$ ) <sup>a</sup> |  | Transition II ( $\beta - \gamma$ ) <sup>a</sup> |  | Melting point (°C) |
|-------------------|--|--|---|--|--------------------|
|                   | Temp. (°C)                                     | Phases   | Temp. (°C)                                      | Phases   |                    |
| Sc                | 1337   | hcp $\rightleftharpoons$ bcc                       | —   | —  | 1541               |
| Y                 | 1478   | hcp $\rightleftharpoons$ bcc                       | —   | —  | 1522               |
| La <sup>b</sup>   | 310  | dhcp $\rightleftharpoons$ fcc                      | 865   | fcc $\rightleftharpoons$ bcc                       | 918                |
| Ce <sup>c,d</sup> | 139  | dhcp $\rightleftharpoons$ fcc ( $\beta - \gamma$ ) | 726   | fcc $\rightleftharpoons$ bcc ( $\gamma - \delta$ ) | 798                |
| Pr                | 795  | dhcp $\rightleftharpoons$ bcc                      | —   | —  | 931                |
| Nd                | 863  | dhcp $\rightleftharpoons$ bcc                      | —   | —  | 1021               |
| Pm                | 890  | dhcp $\rightleftharpoons$ bcc                      | —   | —  | 1042               |
| Sm <sup>e</sup>   | 734  | rhom $\rightleftharpoons$ hcp                      | 922   | hcp $\rightleftharpoons$ bcc                       | 1074               |
| Eu                | —  | —  | —   | —  | 822                |
| Gd                | 1235   | hcp $\rightleftharpoons$ bcc                       | —   | —  | 1313               |
| Tb                | 1289   | hcp $\rightleftharpoons$ bcc                       | —   | —  | 1356               |
| Dy                | 1381   | hcp $\rightleftharpoons$ bcc                       | —   | —  | 1412               |
| Ho                | —  | —  | —   | —  | 1474               |
| Er                | —  | —  | —   | —  | 1529               |
| Tm                | —  | —  | —   | —  | 1545               |
| Yb                | 795  | fcc $\rightleftharpoons$ bcc ( $\beta - \gamma$ )  | —   | —  | 819                |
| Lu                | —  | —  | —   | —  | 1663               |

Note: For additional information, see Gschneidner, K.A., Jr. and Calderwood, F.W., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 8, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1986; Gschneidner, K.A., Jr., Pecharsky, V.K., Cho, Jaephil and Martin, S.W., *Scripta Mater.*, 34, 1717, 1996.

<sup>a</sup> For all the transformations listed, unless otherwise noted.

<sup>b</sup> On cooling, fcc  $\rightarrow$  dhcp ( $\beta \rightarrow \alpha$ ), 260°C.

<sup>c</sup> The  $\beta \rightleftharpoons \gamma$  equilibrium transition temperature is  $10 \pm 5^\circ\text{C}$ .

<sup>d</sup> On cooling, fcc  $\rightarrow$  dhcp ( $\gamma \rightarrow \beta$ ), -16°C.

<sup>e</sup> On cooling, hcp  $\rightarrow$  rhomb ( $\beta \rightarrow \alpha$ ), 727°C.

TABLE 5. Low Temperature Transition Temperatures of the Rare Earth Metals

| Rare earth metal | Cooling                      |      | Rare earth metal | Heating        |                                     |      |
|------------------|------------------------------|------|------------------|----------------|-------------------------------------|------|
|                  | Transformation               | °C   |                  | Transformation | °C                                  | K    |
| Ce               | $\gamma \rightarrow \beta^a$ | -16  | 257              | Ce             | $\alpha \rightarrow \beta$          | -148 |
|                  | $\gamma \rightarrow \alpha$  | -172 | 101              |                | $\alpha \rightarrow \beta + \gamma$ | -104 |
|                  | $\beta \rightarrow \alpha$   | -228 | 45               |                | $\beta \rightarrow \gamma^a$        | 139  |
| Tb               | $\alpha \rightarrow \alpha'$ | -53  | 220              | Yb             | $\alpha \rightarrow \beta$          | 7    |
| Dy               | $\alpha \rightarrow \alpha'$ | -187 | 86               |                |                                     | 280  |
| Yb               | $\beta \rightarrow \alpha$   | -13  | 260              |                |                                     |      |

Note: For additional information, see Beaudry, B.J. and Gschneidner, K.A., Jr., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 173; Koskenmaki, D.C. and Gschneidner, K.A., Jr., 1978, in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 337; Gschneidner, K.A., Jr., Pecharsky, V.K., Cho, Jaephil and Martin, S.W., *Scripta Mater.*, 34, 1717, 1996.

<sup>a</sup> The  $\beta \rightleftharpoons \gamma$  equilibrium transition temperature is  $10 \pm 5^\circ\text{C}$  ( $283 \pm 5^\circ\text{K}$ ).

TABLE 6. Heat Capacity, Standard Entropy, Heats of Transformation, and Fusion of the Rare Earth Metals

| Rare earth metal | Heat capacity at 298 K (J/mol K) | Standard entropy $S^{\circ}_{298}$ (J/mol K) | Heat of transformation (kJ/mol)   |                   |                                    |                   | Heat of fusion (kJ/mol) |
|------------------|----------------------------------|--|-----------------------------------|-------------------|------------------------------------|-------------------|-------------------------|
|                  |                                  |  | trans. 1                          | $\Delta H_{tr}^1$ | trans. 2                           | $\Delta H_{tr}^2$ |                         |
| Sc               | 25.5                             | 34.6   | $\alpha \rightleftharpoons \beta$ | 4.00              | —                                  | —                 | 14.1                    |
| Y                | 26.5                             | 44.4   | $\alpha \rightleftharpoons \beta$ | 4.99              | —                                  | —                 | 11.4                    |
| La               | 27.1                             | 56.9   | $\alpha \rightleftharpoons \beta$ | 0.36              | $\beta \rightleftharpoons \gamma$  | 3.12              | 6.20                    |
| Ce               | 26.9                             | 72.0   | $\beta \rightleftharpoons \gamma$ | 0.05              | $\gamma \rightleftharpoons \delta$ | 2.99              | 5.46                    |
| Pr               | 27.2                             | 73.2   | $\alpha \rightleftharpoons \beta$ | 3.17              | —                                  | —                 | 6.89                    |
| Nd               | 27.5                             | 71.5   | $\alpha \rightleftharpoons \beta$ | 3.03              | —                                  | —                 | 7.14                    |
| Pm               | 27.3 <sup>a</sup>                | 71.6 <sup>a</sup>                            | $\alpha \rightleftharpoons \beta$ | 3.0 <sup>a</sup>  | —                                  | —                 | 7.7 <sup>a</sup>        |
| Sm               | 29.5                             | 69.6   | $\alpha \rightleftharpoons \beta$ | 0.2 <sup>a</sup>  | $\beta \rightleftharpoons \gamma$  | 3.11              | 8.62                    |
| Eu               | 27.7                             | 77.8   | —                                 | —                 | —                                  | —                 | 9.21                    |
| Gd               | 37.0                             | 68.1   | $\alpha \rightleftharpoons \beta$ | 3.91              | —                                  | —                 | 10.0                    |
| Tb               | 28.9                             | 73.2   | $\alpha \rightleftharpoons \beta$ | 5.02              | —                                  | —                 | 10.79                   |
| Dy               | 27.7                             | 75.6   | $\alpha \rightleftharpoons \beta$ | 4.16              | —                                  | —                 | 11.06                   |
| Ho               | 27.2                             | 75.3   | —                                 | —                 | —                                  | —                 | 17.0 <sup>a</sup>       |
| Er               | 28.1                             | 73.2   | —                                 | —                 | —                                  | —                 | 19.9                    |
| Tm               | 27.0                             | 74.0   | —                                 | —                 | —                                  | —                 | 16.8                    |
| Yb               | 26.7                             | 59.9   | $\beta \rightleftharpoons \gamma$ | 1.75              | —                                  | —                 | 7.66                    |
| Lu               | 26.9                             | 51.0   | —                                 | —                 | —                                  | —                 | 22 <sup>a</sup>         |

Note: For additional information, see Hultgren, R., Desai, P.D., Hawkins, D.T., Gleiser, M., Kelley, K.K., and Wagman, D.D., *Selected Values of the Thermodynamic Properties of the Elements*, ASM International, Metals Park, Ohio, 1973; Wagman, D.D., Evans, W.H., Parker, V.B., Schumm, R.H., Halow, I., Bailey, S.M., Churney, K.L., and Nuttal, R.L., *The NBS Tables of Chemical Thermodynamic Properties*, *J. Phys. Chem. Ref. Data*, Vol. 11, Suppl 2, 1982; Amitin, E.B., Bessergenev, W.G., Kovalevskaya, Yu. A., and Paukov, I.E., *J. Chem. Thermodyn.*, 15, 181, 1983; Amitin, E.B., Bessergenev, W.G., Kovalevskaya, Yu. A., and Paukov, I.E., *J. Chem. Thermodyn.*, 15, 181, 1983.

<sup>a</sup> Estimated.

TABLE 7. Vapor Pressures, Boiling Points, and Heats of Sublimation of Rare Earth Metals

| Rare earth metal | Temperature in °C <sup>a</sup> for a vapor pressure of |                          |                         |                         | Boiling point <sup>a</sup> (°C) | Heat of sublimation at 25°C (kJ/mol) |
|------------------|--|--------------------------|-------------------------|-------------------------|---------------------------------|--------------------------------------|
|                  | $10^{-8}$ atm (0.001 Pa)                               | $10^{-6}$ atm (0.101 Pa) | $10^{-4}$ atm (10.1 Pa) | $10^{-2}$ atm (1013 Pa) |                                 |                                      |
| Sc               | 1036   | 1243                     | 1533                    | 1999                    | 2836                            | 377.8                                |
| Y                | 1222   | 1460                     | 1812                    | 2360                    | 3345                            | 424.7                                |
| La               | 1301   | 1566                     | 1938                    | 2506                    | 3464                            | 431.0                                |
| Ce               | 1290   | 1554                     | 1926                    | 2487                    | 3443                            | 422.6                                |
| Pr               | 1083   | 1333                     | 1701                    | 2305                    | 3520                            | 355.6                                |
| Nd               | 955  | 1175                     | 1500                    | 2029                    | 3074                            | 327.6                                |
| Pm               | —  | —                        | —                       | —                       | 3000 <sup>b</sup>               | 348 <sup>b</sup>                     |
| Sm               | 508  | 642                      | 835                     | 1150                    | 1794                            | 206.7                                |
| Eu               | 399  | 515                      | 685                     | 964                     | 1529                            | 175.3                                |
| Gd               | 1167   | 1408                     | 1760                    | 2306                    | 3273                            | 397.5                                |
| Tb               | 1124   | 1354                     | 1698                    | 2237                    | 3230                            | 388.7                                |
| Dy               | 804  | 988                      | 1252                    | 1685                    | 2567                            | 290.4                                |
| Ho               | 845  | 1036                     | 1313                    | 1771                    | 2700                            | 300.8                                |
| Er               | 908  | 1113                     | 1405                    | 1896                    | 2868                            | 317.1                                |
| Tm               | 599  | 748                      | 964                     | 1300                    | 1950                            | 232.2                                |
| Yb               | 301  | 400                      | 541                     | 776                     | 1196                            | 152.1                                |
| Lu               | 1241   | 1483                     | 1832                    | 2387                    | 3402                            | 427.6                                |

Note: For additional information, see Hultgren, R., Desai, P.D., Hawkins, D.T., Gleiser, M., Kelley, K.K., and Wagman, D.D., *Selected Values of the Thermodynamic Properties of the Elements*, ASM International, Metals Park, Ohio, 1973; Beaudry, B.J. and Gschneidner, K.A., Jr., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 173.

<sup>a</sup> International Temperature Scale of 1990 (ITS-90) values.

<sup>b</sup> Estimated.

TABLE 8. Magnetic Properties of the Rare Earth Metals

| Rare earth metal | $\chi_A \times 10^6$ at 298 K (emu/mol) | Effective magnetic moment |       |                       |                  | Easy axis       | Néel temp. $T_N$ (K) |             | Curie temp. $T_c$ (K) | $\theta_p$ (K) |           |                    |  |  |
|------------------|---|---------------------------|-------|-----------------------|------------------|-----------------|----------------------|-------------|-----------------------|----------------|-----------|--------------------|--|--|
|                  |   | Paramagnetic at ~298 K    |       | Ferromagnetic at ~0 K |                  |                 | Hex sites            | Cubic sites |                       | $\parallel c$  | $\perp c$ | Polycryst. or avg. |  |  |
|                  |   | Theory <sup>a</sup>       | Obs.  | Theory <sup>b</sup>   | Obs.             |                 |                      |             |                       |                |           |                    |  |  |
| $\alpha$ Sc      | 295.2                                   | —                         | —     | —                     | —                | —               | —                    | —           | —                     | —              | —         | —                  |  |  |
| $\alpha$ Y       | 187.7                                   | —                         | —     | —                     | —                | —               | —                    | —           | —                     | —              | —         | —                  |  |  |
| $\alpha$ La      | 95.9                                    | —                         | —     | —                     | —                | —               | —                    | —           | —                     | —              | —         | —                  |  |  |
| $\beta$ La       | 105                                     | —                         | —     | —                     | —                | —               | —                    | —           | —                     | —              | —         | —                  |  |  |
| $\gamma$ Ce      | 2,270                                   | 2.54                      | 2.52  | 2.14                  | —                | —               | —                    | 14.4        | —                     | —              | —         | -50                |  |  |
| $\beta$ Ce       | 2,500                                   | 2.54                      | 2.61  | 2.14                  | —                | —               | 13.7                 | 12.5        | —                     | —              | —         | -41                |  |  |
| $\alpha$ Pr      | 5,530                                   | 3.58                      | 3.56  | 3.20                  | 2.7 <sup>c</sup> | <b>a</b>        | 0.03                 | —           | —                     | —              | —         | 0                  |  |  |
| $\alpha$ Nd      | 5,930                                   | 3.62                      | 3.45  | 3.27                  | 2.2 <sup>c</sup> | <b>b</b>        | 19.9                 | 7.5         | —                     | 0              | 5         | 3.3                |  |  |
| $\alpha$ Pm      | —                                       | 2.68                      | —     | 2.40                  | —                | —               | —                    | —           | —                     | —              | —         | —                  |  |  |
| $\alpha$ Sm      | 1,278 <sup>d</sup>                      | 0.85                      | 1.74  | 0.71                  | 0.5 <sup>c</sup> | <b>a</b>        | 109                  | 14.0        | —                     | —              | —         | —                  |  |  |
| Eu               | 30,900                                  | 7.94                      | 8.48  | 7.0                   | 5.9              | <110>           | —                    | 90.4        | —                     | —              | —         | 100                |  |  |
| $\alpha$ Gd      | 185,000 <sup>e</sup>                    | 7.94                      | 7.98  | 7.0                   | 7.63             | 30° to <b>c</b> | —                    | —           | 293.4                 | 317            | 317       | 317                |  |  |
| $\alpha$ Tb      | 170,000                                 | 9.72                      | 9.77  | —                     | —                | —               | 230.0                | —           | —                     | 195            | 239       | 224                |  |  |
| $\alpha'$ Tb     | —                                       | —                         | —     | 9.0                   | 9.34             | <b>b</b>        | —                    | —           | 219.5                 | —              | —         | —                  |  |  |
| $\alpha$ Dy      | 98,000                                  | 10.64                     | 10.83 | —                     | —                | —               | 180.2                | —           | —                     | 121            | 169       | 153                |  |  |
| $\alpha'$ Dy     | —                                       | —                         | —     | 10.0                  | 10.33            | <b>a</b>        | —                    | —           | 90.5 <sup>g</sup>     | —              | —         | —                  |  |  |
| Ho               | 72,900                                  | 10.60                     | 11.2  | 10.0                  | 10.34            | <b>b</b>        | 132                  | —           | 19.5                  | 73.0           | 88.0      | 83.0               |  |  |
| Er               | 48,000                                  | 9.58                      | 9.9   | 9.0                   | 9.1              | 30° to <b>c</b> | 85                   | —           | 18.7                  | 61.7           | 32.5      | 42.2               |  |  |
| Tm               | 24,700                                  | 7.56                      | 7.61  | 7.0                   | 7.14             | <b>c</b>        | 58                   | —           | 32.0                  | 41.0           | -17.0     | 2.3                |  |  |
| $\beta$ Yb       | 67 <sup>d</sup>                         | —                         | —     | —                     | —                | —               | —                    | —           | —                     | —              | —         | —                  |  |  |
| Lu               | 182.9                                   | —                         | —     | —                     | —                | —               | —                    | —           | —                     | —              | —         | —                  |  |  |

Note: For additional information, see McEwen, K.A., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 411; Legvold, S., in *Ferromagnetic Materials*, Vol. 1, Wohlfarth, E.P., Ed., North-Holland Physics, Amsterdam, 1980, 183; Pecharsky, V.K., Gschneidner, K.A., Jr. and Fort, D., *Phys. Rev. B*, 47, 5063, 1993; Pecharsky, V.K., Gschneidner, K.A., Jr. and Fort, D., 1996, to be published; Steward, A.M. and Collocott, S.J., *J. Phys.: Condens. Matter*, 1, 677, 1988.

<sup>a</sup>  $g[J(J+1)]^{1/2}$ .

<sup>b</sup> gJ.

<sup>c</sup> At 38 T and 4.2 K.

<sup>d</sup> At 290 K.

<sup>e</sup> At 350 K.

<sup>g</sup> On cooling  $T_c = 89.6$  K and on warming  $T_c = 91.5$  K.

TABLE 9. Room Temperature Coefficient of Thermal Expansion, Thermal Conductivity, Electrical Resistance, and Hall Coefficient

| Rare earth metal | Expansion ( $\alpha_i \times 10^6$ ) ( $^{\circ}\text{C}^{-1}$ ) |                   |                        | Thermal conductivity (W/cm K) | Electrical resistance ( $\mu\Omega\cdot\text{cm}$ ) |          |                      | Hall coefficient ( $R_i \times 10^{12}$ ) (V·cm/A·Oe) |       |                    |
|------------------|--|-------------------|------------------------|-------------------------------|---|----------|----------------------|---|-------|--------------------|
|                  | $\alpha_a$   | $\alpha_c$        | $\alpha_{\text{poly}}$ |                               | $\rho_a$  | $\rho_c$ | $\rho_{\text{poly}}$ | $R_a$   | $R_c$ | $R_{\text{poly}}$  |
| $\alpha$ Sc      | 7.6  | 15.3              | 10.2                   | 0.158                         | 70.9  | 26.9     | 56.2 <sup>a</sup>    | —   | —     | -0.13              |
| $\alpha$ Y       | 6.0  | 19.7              | 10.6                   | 0.172                         | 72.5  | 35.5     | 59.6                 | -0.27   | -1.6  | —                  |
| $\alpha$ La      | 4.5  | 27.2              | 12.1                   | 0.134                         | —   | —        | 61.5                 | —   | —     | -0.35              |
| bCe              | —  | —                 | —                      | —                             | —   | —        | 82.8                 | —   | —     | —                  |
| $\gamma$ Ce      | 6.3  | —                 | 6.3                    | 0.113                         | —   | —        | 74.4                 | —   | —     | +1.81              |
| $\alpha$ Pr      | 4.5  | 11.2              | 6.7                    | 0.125                         | —   | —        | 70.0                 | —   | —     | +0.709             |
| $\alpha$ Nd      | 7.6  | 13.5              | 9.6                    | 0.165                         | —   | —        | 64.3                 | —   | —     | +0.971             |
| $\alpha$ Pm      | 9 <sup>b</sup>   | 16 <sup>b</sup>   | 11 <sup>b</sup>        | 0.15 <sup>b</sup>             | —   | —        | 75 <sup>b</sup>      | —   | —     | —                  |
| $\alpha$ Sm      | 9.6  | 19.0              | 12.7                   | 0.133                         | —   | —        | 94.0                 | —   | —     | -0.21              |
| Eu               | 35.0   | —                 | 35.0                   | 0.139 <sup>b</sup>            | —   | —        | 90.0                 | —   | —     | +24.4              |
| $\alpha$ Gd      | 9.1 <sup>c</sup>   | 10.0 <sup>c</sup> | 9.4 <sup>c</sup>       | 0.105                         | 135.1   | 121.7    | 131.0                | -10   | -54   | -4.48 <sup>d</sup> |
| $\alpha$ Tb      | 9.3  | 12.4              | 10.3                   | 0.111                         | 123.5   | 101.5    | 115.0                | -1.0  | -3.7  | —                  |
| $\alpha$ Dy      | 7.1  | 15.6              | 9.9                    | 0.107                         | 111.0   | 76.6     | 92.6                 | -0.3  | -3.7  | —                  |
| Ho               | 7.0  | 19.5              | 11.2                   | 0.162                         | 101.5   | 60.5     | 81.4                 | +0.2  | -3.2  | —                  |
| Er               | 7.9  | 20.9              | 12.2                   | 0.145                         | 94.5  | 60.3     | 86.0                 | +0.3  | -3.6  | —                  |
| Tm               | 8.8  | 22.2              | 13.3                   | 0.169                         | 88.0  | 47.2     | 67.6                 | —   | —     | -1.8               |
| $\beta$ Yb       | 26.3   | —                 | 26.3                   | 0.385                         | —   | —        | 25.0                 | —   | —     | +3.77              |
| Lu               | 4.8  | 20.0              | 9.9                    | 0.164                         | 76.6  | 34.7     | 58.2                 | +0.45   | -2.6  | -0.535             |

Note: For additional information, see Beaudry, B. J. and Gschneidner, K.A., Jr., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 173; McEwen, K.A., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 411.

<sup>a</sup> Calculated from single crystal values.

<sup>b</sup> Estimated.

<sup>c</sup> At 100°C.

<sup>d</sup> At 77°C.

**TABLE 10. Electronic Specific Heat Constant ( $\gamma$ ), Electron-Electron (Coulomb) Coupling Constant ( $\mu^*$ ), Electron-Phonon Coupling Constant ( $\lambda$ ), Debye Temperature at 0 K ( $\theta_D$ ), and Superconducting Transition Temperature**

| Rare earth metal | $\gamma$ (mJ/mol·K <sup>2</sup> ) | $\mu^*$ | $\lambda$         | $\theta_D$ (K) from |                   | Superconducting temperature (K) |
|------------------|-----------------------------------|---------|-------------------|---------------------|-------------------|---------------------------------|
|                  |                                   |         |                   | Heat capacity       | Elastic constants |                                 |
| $\alpha$ Sc      | 10.334                            | 0.16    | 0.30              | 345.3               | —                 | 0.050 <sup>a</sup>              |
| $\alpha$ Y       | 7.878                             | 0.15    | 0.30              | 244.4               | 258               | 1.3 <sup>b</sup>                |
| $\alpha$ La      | 9.45                              | 0.08    | 0.76              | 150                 | 154               | 5.10                            |
| $\beta$ La       | 11.5                              | —       | —                 | 140                 | —                 | 6.00                            |
| $\alpha$ Ce      | 12.8                              | —       | —                 | 179                 | —                 | 0.022 <sup>c</sup>              |
| $\alpha$ Pr      | 20                                | —       | 1.07 <sup>d</sup> | 155 <sup>e</sup>    | 153               | —                               |
| $\alpha$ Nd      | f                                 | —       | 0.86 <sup>d</sup> | 157 <sup>e</sup>    | 163               | —                               |
| $\alpha$ Pm      | —                                 | —       | —                 | 159 <sup>e</sup>    | —                 | —                               |
| $\alpha$ Sm      | 8.1 ± 1.5 <sup>g</sup>            | —       | 0.81 <sup>d</sup> | 162 <sup>e,f</sup>  | 169               | —                               |
| Eu               | f                                 | —       | —                 | f                   | 118               | —                               |
| $\alpha$ Gd      | 4.48                              | —       | 0.30              | 169                 | 182               | —                               |
| $\alpha'$ Tb     | 3.71                              | —       | 0.34 <sup>d</sup> | 169.6               | 177               | —                               |
| $\alpha'$ Dy     | 4.9                               | —       | 0.32 <sup>d</sup> | 192                 | 183               | —                               |
| Ho               | 2.1                               | —       | 0.30 <sup>d</sup> | 175 <sup>e</sup>    | 190               | —                               |
| Er               | 8.7                               | —       | 0.33 <sup>d</sup> | 176.9               | 188               | —                               |
| Tm               | f                                 | —       | 0.36 <sup>d</sup> | 179 <sup>e</sup>    | 200               | —                               |
| $\alpha$ Yb      | 3.30                              | —       | —                 | 117.6               | 118               | —                               |
| $\beta$ Yb       | 8.36                              | —       | —                 | 109                 | —                 | —                               |
| Lu               | 8.194                             | 0.14    | 0.31              | 183.2               | 185               | 0.022 <sup>h</sup>              |

Note: For additional information, see Sundström, L.J., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr., and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 379; Scott, T., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 591; Probst, C. and Wittig, J., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 749; Tsang, T.-W.E., Gschneidner, K.A., Jr., Schmidt, F.A., and Thome, D.K., *Phys. Rev. B*, 31, 235, 1985; Collocott, S.J., Hill, R.W. and Stewart, A.M., *J. Phys. F*, 18, L223, 1988; Hill, R.W. and Gschneidner, K.A., Jr., *J. Phys. F*, 18, 2545, 1988; Skriver, H.L. and Mertig, I., *Phys. Rev. B*, 41, 6553, 1990. Collocott, S.J. and Stewart, A.M., *J. Phys.: Condens. Matter*, 4, 6743, 1992; Pecharsky, V.K., Gschneidner, K.A., Jr. and Fort, D., *Phys. Rev. B*, 47, 5063, 1993.

<sup>a</sup> At 18.6 GPa.

<sup>b</sup> At 11 GPa.

<sup>c</sup> At 2.2 GPa.

<sup>d</sup> Calculated value.

<sup>e</sup> Estimated.

<sup>f</sup> Heat capacity results have been reported, but the resultant  $\gamma$  and  $\theta_D$  values are unreliable because of the presence of impurities and/or there was no reliable procedure or model to correct for the magnetic contribution to the heat capacity.

<sup>g</sup> Based on the values reported for the purer Sm sample (IV).

<sup>h</sup> At 4.5 GPa.

**TABLE 11. Room Temperature Elastic Moduli and Mechanical Properties**

| Rare earth metal | Elastic moduli (GPa)      |                 |                 |                   | Mechanical properties (MPa) |                           |                        |                       | Recryst. temp. (°C) |
|------------------|---------------------------|-----------------|-----------------|-------------------|-----------------------------|---------------------------|------------------------|-----------------------|---------------------|
|                  | Young's (elastic) modulus | Shear modulus   | Bulk modulus    | Poisson's ratio   | Yield strength 0.2% offset  | Ultimate tensile strength | Uniform elongation (%) | Reduction in area (%) |                     |
| Sc               | 74.4                      | 29.1            | 56.6            | 0.279             | 173 <sup>a</sup>            | 255 <sup>a</sup>          | 5.0 <sup>a</sup>       | 8.0 <sup>a</sup>      | 550                 |
| Y                | 63.5                      | 25.6            | 41.2            | 0.243             | 42                          | 129                       | 34.0                   | —                     | 550                 |
| $\alpha$ La      | 36.6                      | 14.3            | 27.9            | 0.280             | 126 <sup>a</sup>            | 130                       | 7.9 <sup>a</sup>       | —                     | 300                 |
| $\beta$ Ce       | —                         | —               | —               | —                 | 86                          | 138                       | —                      | 24.0                  | —                   |
| $\gamma$ Ce      | 33.6                      | 13.5            | 21.5            | 0.24              | 28                          | 117                       | 22.0                   | 30.0                  | 325                 |
| $\alpha$ Pr      | 37.3                      | 14.8            | 28.8            | 0.281             | 73                          | 147                       | 15.4                   | 67.0                  | 400                 |
| $\alpha$ Nd      | 41.4                      | 16.3            | 31.8            | 0.281             | 71                          | 164                       | 25.0                   | 72.0                  | 400                 |
| $\alpha$ Pm      | 46 <sup>b</sup>           | 18 <sup>b</sup> | 33 <sup>b</sup> | 0.28 <sup>b</sup> | —                           | —                         | —                      | —                     | 400 <sup>b</sup>    |
| $\alpha$ Sm      | 49.7                      | 19.5            | 37.8            | 0.274             | 68                          | 156                       | 17.0                   | 29.5                  | 440                 |
| Eu               | 18.2                      | 7.9             | 8.3             | 0.152             | —                           | —                         | —                      | —                     | 300                 |
| $\alpha$ Gd      | 54.8                      | 21.8            | 37.9            | 0.259             | 15                          | 118                       | 37.0                   | 56.0                  | 500                 |
| $\alpha$ Tb      | 55.7                      | 22.1            | 38.7            | 0.261             | —                           | —                         | —                      | —                     | 500                 |
| $\alpha$ Dy      | 61.4                      | 24.7            | 40.5            | 0.247             | 43                          | 139                       | 30.0                   | 30.0                  | 550                 |
| Ho               | 64.8                      | 26.3            | 40.2            | 0.231             | —                           | —                         | —                      | —                     | 520                 |
| Er               | 69.9                      | 28.3            | 44.4            | 0.237             | 60                          | 136                       | 11.5                   | 11.9                  | 520                 |
| Tm               | 74.0                      | 30.5            | 44.5            | 0.213             | —                           | —                         | —                      | —                     | 600                 |
| $\beta$ Yb       | 23.9                      | 9.9             | 30.5            | 0.207             | 7                           | 58                        | 43.0                   | 92.0                  | 300                 |
| Lu               | 68.6                      | 27.2            | 47.6            | 0.261             | —                           | —                         | —                      | —                     | 600                 |

Note: For additional information, see Scott, T., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 1, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1978, 591.

<sup>a</sup> Value is questionable.

<sup>b</sup> Estimated.

TABLE 12. Liquid Metal Properties Near the Melting Point

| Rare earth metal | Density (g/cm³)  | Surface tension (N/m) | Viscosity (centipoise) | Heat capacity (J/mol K) | Thermal conductivity (W/cm K) | Magnetic susceptibility $\chi \times 10^4$ (emu/mol) | Electrical resistivity ( $\mu\Omega \cdot \text{cm}$ ) | Spectral emittance at $\lambda = 645 \text{ nm}$ |           |
|------------------|------------------|-----------------------|------------------------|-------------------------|-------------------------------|--|--|--|-----------|
|                  |                  |                       |                        |                         |                               |  |  | $\epsilon (\%)$                                  | Temp.     |
| Sc               | 2.80             | 0.954                 | —                      | 44.2 <sup>b</sup>       | —                             | —  | —  | —  | —         |
| Y                | 4.24             | 0.871                 | —                      | 43.1                    | —                             | —  | —  | 36.8   | 1522–1647 |
| La               | 5.96             | 0.718                 | 2.65                   | 34.3                    | 0.238                         | 1.20   | 133  | 25.4   | 920–1287  |
| Ce               | 6.68             | 0.706                 | 3.20                   | 37.7                    | 0.210                         | 9.37   | 130  | +1.1   | 877–1547  |
| Pr               | 6.59             | 0.707                 | 2.85                   | 43.0                    | 0.251                         | 17.3   | 139  | -0.02  | 931–1537  |
| Nd               | 6.72             | 0.687                 | —                      | 48.8                    | 0.195                         | 18.7   | 151  | -0.9   | 39.4      |
| Pm               | 6.9 <sup>b</sup> | 0.680 <sup>b</sup>    | —                      | 50 <sup>b</sup>         | —                             | —  | 160 <sup>b</sup>                                       | —  | —         |
| Sm               | 7.16             | 0.431                 | —                      | 50.2 <sup>b</sup>       | —                             | 18.3   | 182  | -3.6   | 43.7      |
| Eu               | 4.87             | 0.264                 | —                      | 38.1                    | —                             | 97   | 242  | -4.8   | —         |
| Gd               | 7.4              | 0.664                 | —                      | 37.2                    | 0.149                         | 67   | 195  | -2.0   | 34.2      |
| Tb               | 7.65             | 0.669                 | —                      | 46.5                    | —                             | 82   | 193  | -3.1   | —         |
| Dy               | 8.2              | 0.648                 | —                      | 49.9                    | 0.187                         | 95   | 210  | -4.5   | 29.7      |
| Ho               | 8.34             | 0.650                 | —                      | 43.9                    | —                             | 88   | 221  | -7.4   | —         |
| Er               | 8.6              | 0.637                 | —                      | 38.7                    | —                             | 69   | 226  | -9.0   | 37.2      |
| Tm               | 9.0 <sup>b</sup> | —                     | —                      | 41.4                    | —                             | 41   | 235 <sup>b</sup>                                       | -6.9   | —         |
| Yb               | 6.21             | 0.320                 | 2.67                   | 36.8                    | —                             | —  | 113  | -5.1   | —         |
| Lu               | 9.3              | 0.940                 | —                      | 47.9 <sup>b</sup>       | —                             | —  | 224  | -3.6   | —         |

Note: For additional information, see Van Zytveld, J., in *Handbook on the Physics and Chemistry of Rare Earths*, Vol. 12, Gschneidner, K.A., Jr. and Eyring, L., Eds., North-Holland Physics, Amsterdam, 1989, 357. Stretz, L.A. and Bautista, R.G., in *Temperature, Its Measurement and Control in Science and Industry*, Vol. 4, part I, H.H. Plumbe, Ed., Instrument Society of America, Pittsburgh, 1972, 489. King, T.S., Baria, D.N., and Bautista, R.G., *Met. Trans. B*, 7, 411, 1976; Baria, D.N., King, T.S., and Bautista, R.G., *Met. Trans. B*, 7, 577, 1976.

<sup>a</sup> Volume change on freezing.

<sup>b</sup> Estimated.

TABLE 13. Ionization Potentials (Electronvolts)

| Rare earth | I Neutral atom | II Singly ionized | III Doubly ionized | IV Triply ionized | V Quadruply ionized |
|------------|----------------|-------------------|--------------------|-------------------|---------------------|
| Sc         | 6.56144        | 12.79967          | 24.75666           | 73.4894           | 91.65               |
| Y          | 6.217          | 12.24             | 20.52              | 60.597            | 77.0                |
| La         | 5.5770         | 11.060            | 19.1773            | 49.95             | 61.6                |
| Ce         | 5.5387         | 10.85             | 20.198             | 36.758            | 65.55               |
| Pr         | 5.464          | 10.55             | 21.624             | 38.98             | 57.53               |
| Nd         | 5.5250         | 10.73             | 22.1               | 40.41             | —                   |
| Pm         | 5.554          | 10.90             | 22.3               | 41.1              | —                   |
| Sm         | 5.6437         | 11.07             | 23.4               | 41.4              | —                   |
| Eu         | 5.6704         | 11.241            | 24.92              | 42.7              | —                   |
| Gd         | 6.1500         | 12.09             | 20.63              | 44.0              | —                   |
| Tb         | 5.8639         | 11.52             | 21.91              | 39.79             | —                   |
| Dy         | 5.9389         | 11.67             | 22.8               | 41.47             | —                   |
| Ho         | 6.0216         | 11.80             | 22.84              | 42.5              | —                   |
| Er         | 6.1078         | 11.93             | 22.74              | 42.7              | —                   |
| Tm         | 6.18431        | 12.05             | 23.68              | 42.7              | —                   |
| Yb         | 6.25416        | 12.1761           | 25.05              | 43.56             | —                   |
| Lu         | 5.42585        | 13.9              | 20.9594            | 45.25             | 66.8                |

Note: For references, see the table "Ionization Potentials of Atoms and Atomic Ions" in Section 10.

TABLE 14. Effective Ionic Radii ( $\text{\AA}$ )<sup>a</sup>

| Rare earth ion | $R^{2+}$ |        | $R^{3+}$ |        |         | $R^{4+}$ |        |
|----------------|----------|--------|----------|--------|---------|----------|--------|
|                | CN = 6   | CN = 8 | CN = 6   | CN = 8 | CN = 12 | CN = 6   | CN = 8 |
| Sc             | —        | —      | 0.745    | 0.87   | 1.116   | —        | —      |
| Y              | —        | —      | 0.900    | 1.015  | 1.220   | —        | —      |
| La             | —        | —      | 1.045    | 1.18   | 1.320   | —        | —      |
| Ce             | —        | —      | 1.010    | 1.14   | 1.290   | 0.80     | 0.97   |
| Pr             | —        | —      | 0.997    | 1.14   | 1.286   | 0.78     | 0.96   |
| Nd             | —        | —      | 0.983    | 1.12   | 1.276   | —        | —      |
| Pm             | —        | —      | 0.97     | 1.10   | 1.267   | —        | —      |
| Sm             | 1.19     | 1.27   | 0.958    | 1.09   | 1.260   | —        | —      |
| Eu             | 1.17     | 1.25   | 0.947    | 1.07   | 1.252   | —        | —      |
| Gd             | —        | —      | 0.938    | 1.06   | 1.246   | —        | —      |
| Tb             | —        | —      | 0.923    | 1.04   | 1.236   | 0.76     | 0.88   |
| Dy             | —        | —      | 0.912    | 1.03   | 1.228   | —        | —      |
| Ho             | —        | —      | 0.901    | 1.02   | 1.221   | —        | —      |
| Er             | —        | —      | 0.890    | 1.00   | 1.214   | —        | —      |
| Tm             | —        | —      | 0.880    | 0.99   | 1.207   | —        | —      |
| Yb             | 1.00     | 1.07   | 0.868    | 0.98   | 1.199   | —        | —      |
| Lu             | —        | —      | 0.861    | 0.97   | 1.194   | —        | —      |

Note: For additional information, see Shannon, R.D. and Prewitt, C.T., *Acta Cryst.*, 25, 925, 1969 and Shannon, R.D. and Prewitt, C.T., *Acta Cryst.*, 26, 1046, 1970.

<sup>a</sup> Radius of  $O^{2-}$  is 1.40 Å for a coordination number (CN) of 6.