

PERMITTIVITY (DIELECTRIC CONSTANT) OF WATER AT VARIOUS FREQUENCIES

The permittivity of liquid water in the radiofrequency and microwave regions can be represented by the Debye equation (References 1 and 2):

$$\epsilon' = \epsilon_{\infty} + \frac{\epsilon_s - \epsilon_{\infty}}{1 + \omega^2 \tau^2}$$

$$\epsilon'' = \frac{(\epsilon_s - \epsilon_{\infty}) \omega \tau}{1 + \omega^2 \tau^2}$$

where $\epsilon = \epsilon' + i \epsilon''$ is the (complex) relative permittivity (i.e., the absolute permittivity divided by the permittivity of free space $\epsilon_0 = 8.854 \cdot 10^{-12} \text{ F m}^{-1}$). Here ϵ_s is the static permittivity (see Reference 3 and the table "Properties of Water in the Range 0–100°C" in this Section); ϵ_{∞} is a parameter describing the permittivity in the high frequency limit; τ is the relaxation time for molecular orientation; and $\omega = 2\pi f$ is the angular frequency. The values in this table have been calculated from parameters given in Reference 2:

| | 0°C | 25°C | 50°C |
|---------------------|-------|------|------|
| ϵ_{∞} | 5.7 | 5.2 | 4.0 |
| τ/ps | 17.67 | 8.27 | 4.75 |

Other useful quantities that can be calculated from the values in the table are the loss tangent:

$$\tan \delta = \epsilon'' / \epsilon'$$

and the absorption coefficient α which describes the power attenuation per unit length ($P = P_0 e^{-\alpha l}$):

$$\alpha = \frac{\pi f \epsilon''}{c \sqrt{\epsilon'}}$$

and c is the speed of light. The last equation is valid when $\epsilon''/\epsilon' \ll 1$.

References

1. Fernendez, D. P., Mulev, Y., Goodwin, A. R. H., and Levett Sengers, J. M. H., *J. Phys. Chem. Ref. Data*, 24, 33, 1995.
2. Kaatze, U., *J. Chem. Eng. Data*, 34, 371, 1989.
3. Archer, D. G., and Wang, P., *J. Phys. Chem. Ref. Data*, 12, 817, 1983.

| Frequency | 0°C | | 25°C | | 50°C | |
|-----------|-------------|--------------|-------------|--------------|-------------|--------------|
| | ϵ' | ϵ'' | ϵ' | ϵ'' | ϵ' | ϵ'' |
| 0 | 87.90 | 0.00 | 78.36 | 0.00 | 69.88 | 0.00 |
| 1 kHz | 87.90 | 0.00 | 78.36 | 0.00 | 69.88 | 0.00 |
| 1 MHz | 87.90 | 0.01 | 78.36 | 0.00 | 69.88 | 0.00 |
| 10 MHz | 87.90 | 0.09 | 78.36 | 0.04 | 69.88 | 0.02 |
| 100 MHz | 87.89 | 0.91 | 78.36 | 0.38 | 69.88 | 0.20 |
| 200 MHz | 87.86 | 1.82 | 78.35 | 0.76 | 69.88 | 0.39 |
| 500 MHz | 87.65 | 4.55 | 78.31 | 1.90 | 69.87 | 0.98 |
| 1 GHz | 86.90 | 9.01 | 78.16 | 3.79 | 69.82 | 1.96 |
| 2 GHz | 84.04 | 17.39 | 77.58 | 7.52 | 69.65 | 3.92 |
| 3 GHz | 79.69 | 24.64 | 76.62 | 11.13 | 69.36 | 5.85 |
| 4 GHz | 74.36 | 30.49 | 75.33 | 14.58 | 68.95 | 7.75 |
| 5 GHz | 68.54 | 34.88 | 73.73 | 17.81 | 68.45 | 9.62 |
| 10 GHz | 42.52 | 40.88 | 62.81 | 29.93 | 64.49 | 18.05 |
| 20 GHz | 19.56 | 30.78 | 40.37 | 36.55 | 52.57 | 28.99 |
| 30 GHz | 12.50 | 22.64 | 26.53 | 33.25 | 40.57 | 32.74 |
| 40 GHz | 9.67 | 17.62 | 18.95 | 28.58 | 31.17 | 32.43 |
| 50 GHz | 8.28 | 14.34 | 14.64 | 24.53 | 24.42 | 30.47 |