

## RELATION OF ANGULAR FUNCTIONS IN TERMS OF ONE ANOTHER

### Trigonometric Functions

Function	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$	$\sec \alpha$	$\csc \alpha$
$\sin \alpha$	$\sin \alpha$	$\pm\sqrt{1-\cos^2 \alpha}$	$\frac{\tan \alpha}{\pm\sqrt{1+\tan^2 \alpha}}$	$\frac{1}{\pm\sqrt{1+\cot^2 \alpha}}$	$\frac{\pm\sqrt{\sec^2 \alpha-1}}{\sec \alpha}$	$\frac{1}{\csc \alpha}$
$\cos \alpha$	$\pm\sqrt{1-\sin^2 \alpha}$	$\cos \alpha$	$\frac{1}{\pm\sqrt{1+\tan^2 \alpha}}$	$\frac{\cot \alpha}{\pm\sqrt{1+\cot^2 \alpha}}$	$\frac{1}{\sec \alpha}$	$\frac{\pm\sqrt{\csc^2 \alpha-1}}{\csc \alpha}$
$\tan \alpha$	$\frac{\sin \alpha}{\pm\sqrt{1-\sin^2 \alpha}}$	$\frac{\pm\sqrt{1-\cos^2 \alpha}}{\cos \alpha}$	$\tan \alpha$	$\frac{1}{\cot \alpha}$	$\pm\sqrt{\sec^2 \alpha-1}$	$\frac{1}{\pm\sqrt{\csc^2 \alpha-1}}$
$\cot \alpha$	$\frac{\pm\sqrt{1-\sin^2 \alpha}}{\sin \alpha}$	$\frac{\cos \alpha}{\pm\sqrt{1-\cos^2 \alpha}}$	$\frac{1}{\tan \alpha}$	$\cot \alpha$	$\frac{1}{\pm\sqrt{\sec^2 \alpha-1}}$	$\pm\sqrt{\csc^2 \alpha-1}$
$\sec \alpha$	$\frac{1}{\pm\sqrt{1-\sin^2 \alpha}}$	$\frac{1}{\cos \alpha}$	$\pm\sqrt{1+\tan^2 \alpha}$	$\frac{\pm\sqrt{1+\cot^2 \alpha}}{\cot \alpha}$	$\sec \alpha$	$\frac{\csc \alpha}{\pm\sqrt{\csc^2 \alpha-1}}$
$\csc \alpha$	$\frac{1}{\sin \alpha}$	$\frac{1}{\pm\sqrt{1-\cos^2 \alpha}}$	$\frac{\pm\sqrt{1+\tan^2 \alpha}}{\tan \alpha}$	$\pm\sqrt{1+\cot^2 \alpha}$	$\frac{\sec \alpha}{\pm\sqrt{\sec^2 \alpha-1}}$	$\csc \alpha$

Note: The choice of sign depends upon the quadrant in which the angle terminates.

### Hyperbolic Functions

Function	$\sinh x$	$\cosh x$	$\tanh x$
$\sinh x =$	$\sinh x$	$+ \sqrt{\cosh^2 x - 1}$	$\frac{\tanh x}{\sqrt{1-\tanh^2 x}}$
$\cosh x =$	$\sqrt{1+\sinh^2 x}$	$\cosh x$	$\frac{1}{\sqrt{1-\tanh^2 x}}$
$\tanh x =$	$\frac{\sinh x}{\sqrt{1+\sinh^2 x}}$	$\pm \frac{\sqrt{\cosh^2 x-1}}{\cosh x}$	$\tanh x$
$\operatorname{sech} x =$	$\frac{1}{\sinh x}$	$\pm \frac{1}{\sqrt{\cosh^2 x-1}}$	$\frac{\sqrt{1-\tanh^2 x}}{\tanh x}$
$\operatorname{sech} x =$	$\frac{1}{\sqrt{1+\sinh^2 x}}$	$\frac{1}{\cosh x}$	$\sqrt{1-\tanh^2 x}$
$\coth x =$	$\frac{\sqrt{1+\sinh^2 x}}{\sinh x}$	$\frac{\pm \cosh x}{\sqrt{\cosh^2 x-1}}$	$\frac{1}{\tanh x}$

Function	$\operatorname{cosech} x$	$\operatorname{sech} x$	$\operatorname{coth} x$
$\sinh x =$	$\frac{1}{\operatorname{cosech} x}$	$\pm \frac{\sqrt{1-\operatorname{sech}^2 x}}{\operatorname{sech} x}$	$\frac{\pm 1}{\sqrt{\operatorname{coth}^2 x-1}}$
$\cosh x =$	$\pm \frac{\sqrt{\operatorname{cosech}^2 x+1}}{\operatorname{cosech} x}$	$\frac{1}{\operatorname{sech} x}$	$\pm \frac{\operatorname{coth} x}{\sqrt{\operatorname{coth}^2 x-1}}$
$\tanh x =$	$\frac{1}{\sqrt{\operatorname{cosech}^2 x+1}}$	$\pm \sqrt{1+\operatorname{sech}^2 x}$	$\frac{1}{\operatorname{coth} x}$
$\operatorname{cosech} x =$	$\operatorname{cosech} x$	$\pm \frac{\operatorname{sech} x}{\sqrt{1-\operatorname{sech}^2 x}}$	$\pm \frac{\sqrt{\operatorname{coth}^2 x-1}}{1}$
$\operatorname{sech} x =$	$\pm \frac{\operatorname{cosech} x}{\sqrt{\operatorname{cosech}^2 x+1}}$	$\operatorname{sech} x$	$\pm \frac{\sqrt{\operatorname{coth}^2 x-1}}{\operatorname{coth} x}$
$\coth x =$	$\sqrt{\operatorname{cosech}^2 x+1}$	$\pm \frac{1}{\sqrt{1-\operatorname{sech}^2 x}}$	$\operatorname{coth} x$

Whenever two signs are shown, choose + sign if  $x$  is positive, - sign if  $x$  is negative.