# TRANSFORMATION OF INTEGRALS

If

1. $s$ is the distance along a curve “$C$” in space and is measured from some fixed point.
2. $S$ is a surface area
3. $V$ is a volume contained by a specified surface
4. $\hat{\mathbf{t}}$ = the unit tangent to $C$ at the point
5. $P$ $\hat{\mathbf{n}}$ = the unit outward pointing normal
6. $F$ is some vector function
7. $ds$ is the vector element of curve ($= \hat{\mathbf{t}}\, ds$ )
8. $dS$ is the vector element of surface ($= \hat{\mathbf{n}}\, dS$ )

then

$$\int_{(c)} \mathbf{F} \cdot \hat{\mathbf{t}}\, ds = \int_{(c)} \mathbf{F}$$

and when $\mathbf{F} = \nabla\phi$

$$\int_{(c)} (\nabla\phi) \cdot \hat{\mathbf{t}}\, ds = \int_{(c)} d\phi$$

*Gauss' Theorem*

When $S$ defines a closed region having a volume $V$:

$$\iiint_{(v)} (\nabla \cdot \mathbf{F})\, dV = \iint_{(s)} (\mathbf{F} \cdot \hat{\mathbf{n}})\, dS = \iint_{(s)} \mathbf{F} \cdot dS$$

also
$$\iiint_{(v)} (\nabla\phi)\, dV = \iint_{(s)} \phi\, \hat{\mathbf{n}}\, dS$$

and
$$\iiint_{(v)} (\nabla \times \mathbf{F})\, dV = \iint_{(s)} (\hat{\mathbf{n}} \times \mathbf{F})\, dS$$

*Stokes' Theorem*

When $C$ is closed and bounds the open surface $S$:

$$\iint_{(s)} \hat{\mathbf{n}} \cdot (\nabla \times \mathbf{F})\, dS = \int_{(c)} \mathbf{F} \cdot d\mathbf{s}$$

also
$$\iint_{(s)} (\hat{\mathbf{n}} \times \nabla\phi)\, dS = \int_{(c)} \phi\, d\mathbf{s}$$

*Green's Theorem*

$$\iint_{(s)} (\nabla\phi \cdot \nabla\theta)\, dS = \iint_{(s)} \phi\, \hat{\mathbf{n}} \cdot (\nabla\theta)\, dS = \iiint_{(v)} \phi(\nabla^2\theta)\, dV$$
$$= \iint_{(s)} \theta \cdot \hat{\mathbf{n}}(\nabla\phi) dS = \iiint_{(v)} \phi(\nabla^2\theta)\, dV$$